Chaos and Structures in the Magnetosphere

W. Horton

Institute for Fusion Studies, The University of Texas at Austin

Austin, Texas 78712 USA

Abstract

The nonlinear plasma transport mechanisms that control the collisionless heating in the Earth's magnetosphere and the onset of geomagnetic substorms are reviewed. In the high pressure plasma trapped in the reversed magnetic field loops on the nightside of the magnetosphere, the key issue of the role of the ion orbital chaos as the mechanism for the plasma sheet energization is examined. The energization rate is governed by a collisionless conductance and the solar wind driven dawn-to-dusk electric field. The low-frequency response function is derived and the fluctuation dissipation theorem is given for the system. Returning to the global picture the collisionless energization rate from the transport physics is the basis for a low-dimensional energy-momentum conserving dynamical model of magnetospheric substorms.

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1 Introduction

The Earth's magnetotail is the most thoroughly measured and most extensively modelled high pressure, collisionless plasma current sheet known to physicists. The Earth's central plasma sheet trapped by the reversed magnetic field serves as an important model of the reversed magnetic field configuration—the so-called FRC—for both laboratory and astrophysical plasma science. The most comparable laboratory plasma was produced in the FRC plasma experiment by the Siemon group at Los Alamos National Laboratory. The principal difference between the laboratory reversed field configuration and the magnetotail is in the fixed metallic walls versus the free boundary for the magnetospheric plasma.

In that 2-meter theta pinch the record shot number 12357 of 1991 produced an ion temperature of 1.9 keV in a dense $n \simeq 2{\times}10^{21}\,\mathrm{m}^{-3}$ plasma confined for large number of Alfvén times (Rej *et al.*, 1992). In comparison to this and other laboratory FRC experiments, much more detailed particle and field diagnostics are available for the geomagnetic tail plasma. This space physics type of plasma confinement is of intrinsic interest to magnetic fusion because of the high ratio of plasma energy density to magnetic energy, its natural divertor and its engineering simplicity.

The central plasma sheet is a self-consistent plasma pinch with a current of order $5 \times 10^7\,\mathrm{A}$ flowing from dawn-to-dusk in the nightside magnetic equator. The magnetosphere is parameterized by geocentric magnetospheric (GSM) coordinates X, Y, Z with X positive along the Earth to Sun direction, and Y perpendicular to the X-Z plane containing the Earth's magnetic dipole axis. The $J_y(X,Z)$-current sheet extends from approximately $X = -10R_E$ to beyond $-100R_E$ for a direction of length $L_y \simeq 40\,R_E$ through the cross-sectional area of height $L_z \simeq R_E$ and length of order $L_x = 100\,R_E$. At the dawn and dusk boundaries of the magnetopause the current splits with one-half closing over the northern lobe and one-half closing under the southern lobe closing the flow in the magnetopause at a nominal height of $z = \pm H \simeq \pm 20\,R_E$ above and below the center $z = 0$ of the current sheet. The inductance of these two closed current loops is $\mathcal{L} = \mu_0 H L_y / L_x \simeq 40\,\mathrm{H}$ and the capacitance C of the system arises from the polarization of the central plasma sheet $C = L_x \sum_s n_s m_s/(B_z B_x' L_y)$ where $B_x' = dB_x/dz\Big|_{z=0} = \mu_0 j_y(0)$. Thus the $\mathcal{L}C$-frequency of the magnetotail cavity is the global Alfvén frequency $\omega_A = (\mathcal{L}C)^{-1/2} = (B_z B_x'/\mu_0 \rho_m H)^{1/2}$. The value of C ranges from 10^3 to $10^4\,\mathrm{F}$ giving the global Alfvén eigenmode period of 20–60 min.

The resistance of the system is determined by the collisionless energy transfer from the E_y field to the particles through $\int j_y E_y\, d^3x = \overline{\sigma}\left\langle E_y^2 \right\rangle \Omega_{\rm cps}$ through the nonlinear resonances in the particle Hamiltonian $H(\mathbf{p}, \mathbf{x})$. Here $\Omega_{\rm cps} = L_x\, L_y\, L_z$ is the volume of the plasma sheet. The Horton-Tajima chaotic conductivity σ arises from the large ion gyro-orbits $\epsilon = \rho_i/L_z$ and is of the form of a Hall conductivity since the electrons are strongly magnetized by the small B_z in contrast to the large orbits of the ions. Horton-Tajima (1990, 1991a, 1994) have given $\overline{\sigma} = C_1(n_0 e/B_z)(\rho_i/L_z)^{1/2}$ with the constant $C_1 \sim 0.1$ determined by test particle simulations. The resulting RC-decay rate for the decay of the $\mathbf{E}\times\mathbf{B}$ kinetic energy is then $\gamma_{RC} = 1/RC = (eB_{x0}/m_i)(\rho_i/L_z)^{1/2}$. Here the net conductance $\sum = L_x L_z \overline{\sigma}/L_y$ gives the dissipative current $I_\Sigma = \Sigma E_y L_y \ll I_{ps}$ that is small compared with the pressure gradient current $I_{\rm ps}$ for $E_y/B_z \ll v_{Ti}$. The collisionless damping γ_{RC} is clearly a large ion orbit effect and vanishes in the MHD limit $\rho_i/L_z \to 0$. The collisionless damping may also be viewed as phase mixing where the stretching and folding of phase space volumes by the Hamiltonian occurs due to the chaos.

In the equilibrium state the pressure balance in the pinch determines the maximum plasma pressure $p(z=0)$ in terms of the current I through $p(0) = B_{x0}^2/2\mu_0 = \frac{1}{2\mu_0}\,(\mu_0 I/L_x)^2$. The magnetosphere based on the empirical satellite based model of Tsyganenko (1989) is shown in Fig. 1. F1

Since the Earth's magnetic tail and the FRC plasma are homologous there have been parallel developments in their analysis. The expressions for the chaotic conductivity for the magneto-tail have their parallel in the quadratic energy functional employed to examine the stability of FRC's in which the azimuthal θ-current ($z \to r,\ y \to r\theta,\ x \to z$) is carried by energetic large-orbit ions with regular or chaotic orbits and whose dynamics is governed by the Vlasov equation [Lovelace (1975, 1976), Sudan and Rosenbluth (1976, 1979), Finn and Sudan (1978, 1979), Symon *et al.* (1982), Barnes *et al.* (1986)]. This energy-functional δW is obtained by integrating the work done by the perturbed current $\delta\mathbf{J}$ against the perturbed electric field $\delta\mathbf{E}$ but the final expression for δW is expressed in terms of the plasma displacement $\boldsymbol{\xi}$ i.e. $\delta W(\boldsymbol{\xi}, \boldsymbol{\xi})$ instead of $\delta\mathbf{E}$, as in the chaotic conductivity kernel. An auto-correlation function defines the chaotic orbits of ions in this functional [Finn (1979), Finn and Sudan (1982), Krall *et al.* (1991)].

1.1 Energies and power flow in the magnetosphere

Both the laboratory and space FRC plasmas are driven by an induction electric field. In the geotail the electric field E_y arises from the solar wind $-\mathbf{v}_{\rm sw}\times\mathbf{B}_m$ acting on the Earth's magnetic field while the $E_\theta(r, z, t)$ field in the FRC experiment is induced by the discharge of a capacitor bank through a single turn coil encircling the plasma.

The principal energy components of the magnetotail are the lobe magnetic energy $W_B = \int (B^2/2\mu_0) d^3x = \frac{1}{2}\mathcal{L}I^2 \approx 3 \times 10^{15}$ J to 10^{18} J, the plasma energy $U = \int \frac{3}{2} p d^3x = \frac{3}{2}\bar{p}\Omega_{\rm cps} \simeq 10^{13}$ J to 10^{14} J. In the presence of the dawn-dusk electric field there is the $\mathbf{E} \times \mathbf{B}$ flow kinetic energy $K_E = \int \frac{1}{2}\rho v_E^2 d^3x = \frac{1}{2} C V^2 \approx 10^{12}$ J and the streaming kinetic energy $K_\parallel = \int \frac{1}{2}\rho v_\parallel^2 d^3x \approx 10^{13}$ J. For reference, Lyons and Williams (1984) estimate the total geotail energy as 3×10^{15} to 3×10^{18} J, and the solar wind input to the magnetosphere to be on average 10^{13} W. The dynamics of W_B, U, K_E and $K_\parallel$ is analyzed in Sec. 4.1.

The traditional definition of the magnetospheric substorm is based on the increase in the strength of the auroral electrojet currents as measured by changes in the horizontal component of the Earth's magnetic field by magnetometer stations at about 70° latitude. The early review article of Ferraro (1957) is a useful introduction to the aurora and ground-based magnetic disturbances.

The height integrated ionospheric current is dominated by the Hall current occurring in the altitude range 90-130 km where the ions are collisional. Thus, the current flows in a large dipolar vortex parallel to the polar cap equipotential contours in the opposite direction to the $\mathbf{E} \times \mathbf{B}$ drift. Thus, the current flows tailward over the polar cap and divides into the eastward and westward electrojets. During disturbed times the current system known as S_D shows the intensification of the westward electrojet giving the strong increase of the lower envelope of the auroral latitude ground-based magnetometers. The lower envelope signal is called the AL index and is the primary output signal used in the substorm correlation studies of Baker *et al.* (1983) and Bargatze *et al.* (1985).

A classical isolated substorm will show (i) a growth phase of about $20-25$ min in which the AL may decrease from a few nT to -50 nT, (ii) an expansion phase of about 30 min in which AL decreases to -1000 nT and higher, and (iii) a recovery stage over which AL returns to the ambient level over the period of order one hour. There are extensive statistical studies of the geomagnetic activity. The activity is well correlated with the solar wind and the direction of the interplanetary magnetic field (IMF). Substorm onsets correlate with rotation of the IMF from northward to an east-west direction as shown, for example, by the Farrugia *et al.* (1993) analysis of a controlled period during which the IMF rotates from northward to southward over a 29 hour period. This important database will be described further in Sec. 4.2. Various linear and nonlinear filter methods have been able to produce good short-time models for the solar wind-geomagnetic activity correlations. Here we are concerned with the dynamics of the particle-field interactions during the substorm activity.

1.2 Global MHD simulations

Global MHD simulations reveal the critical role of the IMF field in the triggering of the substorms and plasmoid formation. Three-dimensional simulations with a southward IMF are given by Brecht *et al.* (1982), Fedder and Lyon (1987, 1995), and with both northward and southward IMF by Walker and Ogino (1989) and Usadi *et al.* (1993). Recently, Fedder *et al.* (1995) show the change in the orientation of the plasma sheet and magnetotail as a function of the clock angle of IMF with very long ($300\,R_E$) magnetotail 3D simulations. Spicer *et al.* (1996) have developed an adaptive tetrahedra finite element MHD code that shows the details of the growth of a plasmoid with helical field lines driven by 5nT southward IMF. Here we discuss in some detail the reconnection dynamics reported by Usadi *et al.* (1993).

The Usadi *et al.* (1993) work presents a large scale supercomputer simulation of the MHD equations over a large volume with a nonuniform mesh covering one quadrant of the magnetosphere. The time advance is by fourth order Runge-Kutta-Gill and the space derivatives are second order centered difference. The effective magnetic Reynolds number is 10^4 arising from the discretization of the partial differential equations.

The global simulations show that for northward IMF the magnetic flux is peeled off the lobes resulting in a thicker and weaker geotail current sheet and a lower magnetospheric cavity plasma pressure. The solar wind parameters used on this study are $V_{\rm sw} = 300\,{\rm km/s}$, $\rho = 5\,m_p/{\rm cm}^3$ and $T = 20\,{\rm ev}$ with the $B_z^{\rm IMF} = \pm 5\,{\rm nT}$. For the southward IMF run the authors describe the fast, strong dynamics as "explosive." Reconnection begins in the near-Earth region about 20 min after the reconnection starts at the day-side magnetopause which is argued to agree with the time delay time reported by Baker *et al.* (1983). A magnetic bifurcation occurs in the quasineutral sheet with the birth of an $\times$-O pair. The O-point starts at about $-28\,R_E$ and rapidly moves tailward while the $\times$-point moves Earthward slowly. The separatrix defining the trapped plasma called the plasmoid expands rapidly and after an initial acceleration develops a tailward velocity of about $0.8\,R_E/$ min taking 1.5 hrs for the plasmoid to leave the simulation region. Figure 2 shows the dynamics over the 75 min period after the start of the dayside reconnection. In frame (b), about 25 min after the birth of the $\times$-0 pair, the separatrix has expanded to $12\,R_E$ in length and $4\,R_E$ in height. The $\times$-point (near Earth neutral line) has moved in to $X = -18\,R_E$. In frame (c) at 50 min after the plasmoid has reached a velocity of well over $100\,{\rm km/s}$. F2

During the growth phase in the time interval between the onset of reconnection on the dayside and before the bifurcation in the plasma sheet the current sheet continually thins and the current density grows 4 to 5 times stronger than in the quiet time geotail. After the formation of the plasmoid the current density remains high along the separatrix, and thus an amount up to 10% of the plasma sheet current is ejected from the tail. The corresponding fractional decrease of the stored magnetic energy is approximately twice the fractional loss of plasma current. The result is the release in a period of 20 min of up to 10^{15} J of energy: part transported to the inner magnetospheric and ionosphere and part ejected out the magnetotail.

To understand the dynamics of the collisionless current sheet before and during the substorm it clearly is necessary to know the particle orbits in response to changes in the electromagnetic fields in the current sheet. Thus, we now consider the particle orbits in some detail.

2 Particle Dynamics in the Geomagnetic Tail

A standard model for the particle-field interactions in the geomagnetic tail follows from the local dawn-dusk symmetry for $|y| < L_y \sim 20\,R_E$, and the slow variation along the Earth-sunline $\partial_x \sim 1/L_x \ll \partial_z \sim 1/L_z$ compared to the thickness L_z of the CPS and the height H of the geotail cavity. In this model the electromagnetic fields are given by the vector potentials $A_y(x,z,t)$ and $A_x = B_y z$.

The Lorentz force equations of motion for the charged particles are derived from the $N = 2\,\frac{1}{2}$ degrees-of-freedom (DOF) Hamiltonian

$$H = \frac{1}{2m}\,P_z^2 + \frac{1}{2m}\,(P_x - qB_y z)^2 + \frac{1}{2m}\left(P_y - qA_y(x,z,t)\right)^2. \tag{1}$$

Within the current sheet $|z| < L_z$ the vector potential has the polynomial expansion $A_y = -\frac{1}{2}\,B_x' z^2 + B_z x$ where $B_x' = (dB_x/dz)_0 \equiv B_{x0}/L_z$.

For $B_y = E_y = 0$ the system reduces to $N = 2$ DOF and can be we written as

$$H = \frac{p_z^2}{2m} + \frac{p_x^2}{2m} + \frac{q^2\,B_z^2}{2mc}\left(x - \frac{z^2}{2R_c}\right)^2 = H_z(p_x,x) + H_x(p_x,x) + V\,z^2 x \tag{2}$$

with the two integrable degrees of freedom

$$H_z(p_z,z) = \frac{p_z^2}{2m} + \frac{m}{8}\,\frac{\Omega^2}{R_c^2}\,z^4 \tag{3}$$

$$H_x(p_x,x) = \frac{p_x^2}{2m} + \frac{m}{2}\,\Omega^2\,x^2 \tag{4}$$

coupled by the interaction $V(z,x)$ with

$$V(z,x) = -\frac{m\Omega^2}{2R_c}\, z^2 x \tag{5}$$

where $\Omega = q\, B_z/mc$ is the cyclotron frequency in the "reconnection" or normal field component B_z and $R_c = B_z/B'$ is the radius of curvature of the field line at the reversal layer.

The action-angle variables for the current sheet Hamiltonian $H_z(p_z, z)$ are given in the Appendix. Figures 3(a) and (b) shows the bifurcation of the effective potential $V_{\rm eff}(z) = (2m)^{-1}(p_y + qB'_x z^2/2)^2$ with the change in sign of qp_y. In frame (a) there are crossing orbits and in frame (b) noncrossing cyclotron orbits for $E < p_y^2/2m$ and crossing orbits for $E > p_y^2/m$. Frame (c) shows the phase space for frame (b) where the separatrix SX provides the seed for the chaotic orbits in the full Hamiltonian in Eq. (2). F3ab

From the form of H in Eq. (2) we see that x is the bifurcation parameter with regard to nonlinear z oscillations. For $x < 0$ there is a single stable elliptic fixed point at $z = 0$; while for $x > 0$ the $z = 0$ point bifurcates into an unstable fixed point with stable fixed points at $z = \pm(2R_c x)^{1/2}$. The $x(t)$ motion through the separatrix in this configuration generates chaos in the system.

2.1 Two degrees of freedom and degeneracy

For a northward IMF the standard model is $E_y = 0$ with only the diamagnetic current forming the geotail.

While the $N = 2$ DOF system is a difficult, nonanalytic problem, the KAM theory applies under the following three conditions:

(i) non-degenerate lowest order system $H = H_0(\{I\}) + \varepsilon V(\{I, \theta\})$

$$\det\left(\frac{\partial^2 H_0}{\partial I_i\, \partial I_j}\right) \neq 0 \tag{6}$$

(ii) weak $\varepsilon \ll 1$ perturbation

$$\varepsilon V(\{I, \theta\}) \to 0 \tag{7}$$

generally equivalent to sufficiently low oscillator energies

(iii) irrational rotational transform or winding number

$$n\Omega_z(H) + m\Omega_x(H) \neq 0$$

$$q(H) = -\frac{\Omega_x(H)}{\Omega_z(H)} \neq \frac{m}{n} \tag{8}$$

The Hamiltonian (2) does not satisfy these conditions so we expect and find a rich dynamical behavior beyond the simple opening up of island structures. In particular condition (i) is not satisfied since $H_x(p_x, x)$ in Eq. (4) is a *linear* oscillator. The occurrence of degeneracy allows a global chaos to occur.

The degeneracy of the linear $H_x(p_x, x)$ oscillator with $\partial H_0/\partial I_x = \Omega = eB/mc = \text{const}$ can be removed by including a x-gradient of the B_z field. Usadi *et al.* (1995) consider the analytic model of $P(A_y) = k^2 A_y^2/2$ which gives $B_z(x, 0) = B_z e^{x/L_x}$ and report a new type of transient-stochastic orbit boundary. In this review we stay with the $L_z/L_x \to 0$ limit of the studied modified Harris sheet (Harris, 1962). In the case of the magnetotail current sheet, where this problem was originally studied, the spatial gradient of B_z is sufficiently weak that $\Omega = \text{const}$ is a good approximation.

The oscillation frequency of the Hamiltonian $H_z(p_z, z)$ in Eq. (3) is

$$\Omega_z = \left(\frac{\Omega(2H_z/m)^{1/2}}{R_c}\right)^{1/2} \left(\frac{\pi}{2K(1/2)}\right) = 0.847 \left(\frac{v}{\Omega_x R_c}\right)^{1/2} \tag{9}$$

(from the elliptic function orbits in the Appendix), and that for $H_x(p_x, x)$ is

$$\Omega_x = q\frac{B_z}{mc} = \Omega.$$

The frequency Ω_z is easily understood from the estimate that $\Omega_z = v_z/z_t$ where $v_z \simeq (2H_z/m)^{1/2}$ and z_t is the turning point where $H_z(p_z = 0, z_t) = H_z$ is the kinetic energy in the north-south z-degree of freedom.

2.2 Resonance conditions and the surface of section

A simple way to see the connection between the chaos in the two-degree of freedom autonomous system ($E_y = 0$) and the chaos in the 1-$\frac{1}{2}$-D system with a periodic $E_y(t)$ is to consider the problem of the transfer of energy between the two degrees of freedom. In particular, we may consider the case where the initial data puts most of the energy into the $H_x(p_x, x)$ oscillator which is linear and has the solution

$$p_x(t) = (2mH_x)^{1/2} \sin(\Omega t) \qquad x(t) = \frac{(2H_x/m)^{1/2}}{\Omega} \cos(\Omega t) \tag{10}$$

describing cyclotron orbits in the small region $|z| \ll R_c$. With the $x(t)$ motion substituted into the Hamiltonian equation, we see that the form of $H(p_z, z, x(t))$ in Eq. (2) becomes identical to that where the perturbation is that from a sinusoidal transverse electric field $A_y(\omega t)$ in Eq. (1) with $B_y = B_z = 0$. The periodic time torus $\phi = \omega t$ that occurs in the applied electric field problem is replaced with the abstract toroidal surface described by the two-angle variables θ, ϕ associated with the $H_z(p_z, z)$ and $H_x(p_x, x)$ components of the motion.

The winding number for the uncoupled oscillators is given by $1/q = \Omega_z/\Omega_x = 0.847(v/\Omega_x\, R_c)^{1/2}$. Thus, the winding number is controlled by the maximum of the finite Larmor radius parameter given by

$$\varepsilon = \frac{\rho}{R_c} = \frac{mcv}{q\, B_{\min}\, R_c} \tag{11}$$

with $1/q \simeq \varepsilon^{1/2} < 1$ giving the winding number less than one. In the case $\varepsilon \ll 1$ the magnetic moment $\mu = v_\perp^2/2B$ is a good adiabatic invariant. The resonance condition (8) is only satisfied for high m values with negligible (exponentially small) zones of instability.

For regimes with $\varepsilon \gtrsim 1/2$ the resonance conditions are satisfied for low m, n values, and the phase space is filled up with overlapping resonances. As discussed above, the conditions for the KAM theorem, Eqs. (6)–(8), are not satisfied so we must examine the surfaces of section to understand the nature of the chaotic motion.

For the study of the surface of section and the Lyapunov exponents of the chaotic orbits it is convenient to introduce dimensionless variables appropriate to the current sheet. For a given value of $H = \frac{1}{2}\, mv^2$ we define the rescaled $(\mathbf{z}, t)$ variables by

$$\mathbf{z} \rightarrow \left(\frac{mcv}{qB'}\right)^{1/2} \mathbf{z} \qquad \text{and} \qquad t \rightarrow \left(\frac{mc}{vqB'}\right)^{1/2} t \tag{12}$$

so that

$$\frac{d\mathbf{z}}{dt} \rightarrow v\, \frac{d\mathbf{z}}{dt}. \tag{13}$$

The dimensionless Hamiltonian is then $H = mv^2 h$ with

$$h = \frac{p_z^2}{2} + \frac{p_x^2}{2} + \frac{1}{2}\left(\kappa x - \frac{z^2}{2}\right)^2 = \frac{1}{2} \tag{14}$$

with κ (the Büchner and Zelenyĭ (1986) parameter) being the only parameter that controls the competition between the integrability and chaos. The κ parameter is

$$\kappa = \frac{B_z}{B_0(\rho/L_B)^{1/2}} = \left(\frac{R_c}{\rho_{\max}}\right)^{1/2} \tag{15}$$

where we have introduced $L_B = B_0/B'_x$ and $\Omega_0 = qB_0/mc$. The single κ parameter controls the solutions of the current sheet Hamiltonian including the winding number $\nu = 0.847(v/\Omega_x R_c)^{1/2}$ determining the resonant energies in Eq. (8). The $(m = 5, n = -1)$ resonance, for example, has $\nu = 1/5$ and $\kappa = 0.847/5 = 0.169$.

A closely related two-parameter (b, ε) form of the dimensionless Hamiltonian is introduced by Chen and Palmadesso (1986) for the global magnetic field model

$$B_x(z) = B_0 \tanh\left(\frac{z}{L}\right) \quad \text{and} \quad B_z = \text{const.}\ .$$

In this global model there are two parameters $b \equiv B_z/B_0$ and the FLR parameter $\varepsilon = \rho/L$. In the Chen and Palmadesso works the space variables are normalized by $R_c = B_z L/B_0$ and the time by $t \to t/\Omega$ with $\Omega = qB_z/mc$ so that the value of the kinetic energy $\frac{1}{2}\,mv^2$, measured relative to $mL^2\,\Omega_0^2(B_z/B_0)^4$, becomes the stochasticity parameter. Defining

$$\widehat{H}_{CP} = \frac{mv^2}{2(mR_c^2\,\Omega^2)} \tag{16}$$

with $R_c = LB_z/B_0$ we have the relationship with the Büchner-Zelenyĭ kappa parameter $\kappa = 1/(2\widehat{H}_{CP})^{1/4}$. A widely used reference parameter value is

$$\kappa_{BZ} = 0.18 \quad \text{or} \quad \widehat{H}_{CP} = 500 \quad \text{or} \quad \frac{\rho_{\max}}{R_c} = 30.9. \tag{17}$$

For motions confined well inside the current sheet $|z| \ll L$ the Chen-Palmadesso problem reduces to the Hamiltonian (14). For motion outside the current sheet $|z| > L$ the Chen-Palmadesso problem has $\mathbf{B} = B_z\,\mathbf{e}_z \pm B_0\,\mathbf{e}_x \simeq$ constant. In contrast the current sheet Hamiltonian has $|B|$ increasing as $|Z| \to \infty$. Thus the surface of sections and the regular versus chaotic phase space regions are different in the two systems.

The surface of section is constructed by finding the crossings of the $z = 0$ plane by testing each time step for the condition $z(t_n)z(t_{n-1}) < 0$ and interpolating to find the values of (x, p_x) at t_* where $z(t_*) = 0$.

The results for the CP system with the parameter values $\widehat{H}_{\rm cp} = 500$ and $b_z = 0.05$ are shown in Fig. 4 for $B_y/B_z = 0$, Fig. 5 for $B_y/B_z = 1$ and Fig. 6 $B_y/B_z = 5$. The surface of sections show the mixture of integrable orbits, chaotic orbits and the white areas for which (Speiser) orbits transit through the sheet. F4 F5 F6

The structure of the phase space is revealed by the surface of section in Fig. 4 where three types of orbits can be identified by the regions labeled A, B, and C. The orbit types are as follows.

2.2.1 Integrable (ring) orbits with $H_z \gg H_z$

When most of the energy is in the $p_z - z$ oscillations there are invariant curves in the phase space as shown in Region A. The rotation rate near the stable fixed point is approximately κ. The spatial configuration of an integrable orbit is that of oscillations on a cylindrical surface with its axis along $B_z \hat{\mathbf{e}}_z$ and radius $r = mcv/qB_z$ as shown in Fig. 7(a). F7a

2.2.2 Stochastic orbits with $H_z \sim H_x$

In the region marked B the energy in the H_z and H_x oscillations are comparable and the motions are unstable in that two neighboring trajectories diverge exponentially in time. A stochastic orbit is shown in Fig. 7(b). F7b

In this regime the effective potential for the p_z, z-oscillator is rapidly changing from the stable to the unstable configurations shown in Fig. 3—due to the periodic oscillations of the effective p_y for the p_z, z-oscillator for which

$$p_y \rightarrow p_y^{\text{eff}}(t) = b\, x(t). \tag{18}$$

At the reversal layer where $v_y(z = 0) = bx$ the effective potential has an unstable fixed point for $bx > 0$ and a stable fixed point for $bx < 0$. The energy in $H_x(p_x, x)$ oscillator forces the system to make repeated separatrix crossings in the p_z, z-phase space giving rise to the chaotic scattering of the orbit as it passes close to the $\times$-point of the unperturbed Hamiltonian.

2.2.3 Transient unbounded orbits with $H_z \ll H_x$

In this regime marked C the orbits have small pitch angles in the exterior region so that they pass through the reversal layer and make long excursions into the strong left ($z < 0$) and right ($z > 0$) exterior magnetic field regions. A transient orbit is shown in Fig. 7(c). F7c

While transiting through the reversal layer the particle is rotated in the y-x-plane by the B_z magnetic field at the angular frequency $\Omega = qB_z/mc$. Chen and Palmadesso (1986) show that when there are an integrable number n of (z, p_z) oscillations in the half period rotation time π/Ω that the particles enter and exit the white regions labeled C_1, $C_2, \ldots, C_n$ without entering into the stochastic domain B. The condition for n-oscillations is given by $\Omega_z \Delta t = (n + 1/2)\pi$ with $\Delta t = \pi/\Omega$. This resonance condition defines a sequence of resonant energies E_n since $\Omega_z \cong (v\Omega_0/L)^{1/2}$ by Eq. (8). In terms of the Chen-Palmadesso $\widehat{H}$ the resonance condition is

$$\widehat{H}^{1/4} = n + 1/2. \tag{19}$$

For $\widehat{H} = 500$ used in Fig. 4 the system is close to the $n = 5$ resonance which explains the origin of the five white regions labeled by C_1, $C_2, \ldots, C_5$. Thus, in this degenerate Hamiltonian system the $m = -1$, $n = 5$ resonance shows a different kind of phase space structure from the standard chain of resonant islands along the rational surface.

The transient orbits in region C were discovered by Speiser (1965, 1967) and are known as Speiser orbits. The Speiser orbits play an important role in the transport processes and the heating rate of the plasma trapped in the geomagnetic tail on the night side of the earth (Lyons and Speiser, 1982; Horton and Tajima, 1990, 1991a,b).

Chen and Palmadesso (1986) have emphasized that the particle distribution function f_α for the three regions $\alpha =$ A, B, C do not mix in a collisionless, fluctuation free, plasma and thus can have different values. They use this property to predict that the value of f at the resonant energies E_n is different than the value of f off the resonance when the northern and southern lobe plasmas are different. Thus $f(E)$ should show peaks and valleys spaced at E_n defined by Eq. (19).

Subsequently, study of the energetic ion distribution function obtained from the ISEE-3 satellite in the geomagnetic tail confirmed the bumpy structures of $F(E)$ as shown in Chen-Burkhart-Huang (1990). The spacing of the resonances in energy E_n is such that $E_{n+1}^{1/4} - E_n^{1/4} =$ const proportional to $L^{1/2}$. The measured resonances confirmed well to this prediction and suggest a method of determining the current sheet thickness from the resonances of $f(E)$ and the measured values of B_z and B_x.

Chen-Rexford-Lee (1990) have investigated the boundary between region B and region C orbits. By repeatedly blowing up smaller and smaller regions of the B-C boundary they show that the boundary is fractal in nature. This boundary curve is the large ε analog of the loss cone boundary in small ε (gyroradius-to-scale length) theory. The boundary is determined numerically following the definition of a loss-cone boundary in a μ-conserving theory. Sufficiently far from the current sheet $|z| \gtrsim 5L$ where $B = B_{\max} = (B_z^2 + B_0^2)^{1/2}$ the particles are launched toward the current sheet with a nearly vanishing parallel velocity; i.e. almost 90° pitch angle. Those with smaller pitch angles pierce the $z = 0$ plane inside region C. While those corresponding to punctures just outside region C will have insufficient H_z energy to reach the large $|z|$ region. Thus, they are reflected back into the reversal layer and become the stochastic B orbits.

A thorough review of the orbit types and the nature of the boundaries between the regions and entry-exit regions in the surface of sections is to be found in Chen (1992). A resonance in the structure of the ion velocity distribution in its dependence on the distance x along the geotail arising from the $B_z(x,0)$ variations of the κ parameters is shown by Ashour *et al.* (1993)) with large scale test particle simulations in the Tsyganenko-based magnetic field model.

2.3 Collisionless conductivity in magnetic field reversed configurations (FRCs)

The rate of conversion of electromagnetic energy into mass flow and thermal energy is given by $\int d^3x\, \mathbf{J}\cdot\mathbf{E}$, which is the total power transferred by the fields to the sources in a finite volume.

Energy conservation for a system of sources and fields in a finite volume Ω with boundary $\partial\Omega$ is given by Poynting's theorem,

$$\int_\Omega d^3x\, \frac{\partial u}{\partial t} + \oint_{\partial\Omega} d\mathbf{a}\cdot\mathbf{S} = -\int_\Omega d^3x\, \mathbf{J}\cdot\mathbf{E}, \tag{20}$$

where u is the energy density of the electromagnetic field, $u = \epsilon_0 E^2/2 + B^2/2\mu_0$, and $\mathbf{S}$, the Poynting vector, represents the energy flow, $\mathbf{S} = \mathbf{E}\times\mathbf{B}/\mu_0$. Expression (20) states that the total work done by the fields on the charged particle is balanced with the rate of change of electromagnetic energy in the volume and the transport through the boundary.

For fields varying harmonically in time, $e^{-i\omega t}$, energy conservation can be written as

$$\frac{1}{2}\int d^3x\, \mathbf{J}^*_\omega\cdot\mathbf{E}_\omega + 2i\omega\int d^3x\,(u_e - u_m) + \oint d\mathbf{a}\cdot\mathbf{S}_\omega = 0, \tag{21}$$

where $u_e = \epsilon_0|E_\omega|^2/4$, $u_m = |B_\omega|^2/4\mu_0$, and $\mathbf{S}_\omega = \mathbf{E}_\omega\times\mathbf{B}^*_\omega/2\mu_0$.

The complex Poynting theorem (21) is useful because it can be used to determine the complex input impedance, $Z = R - iX$, of a general electromagnetic system. In particular, the conversion rate of electromagnetic energy into particle energy is

$$Q_\omega = \frac{1}{2}\,\mathrm{Re}\int d^3x\, \mathbf{J}^*_\omega\cdot\mathbf{E}_\omega \tag{22}$$

and the reactive or stored energy and its alternating flow is given by the imaginary part of (21),

$$\frac{1}{2}\,\mathrm{Im}\int d^3x\, \mathbf{J}^*_\omega\cdot\mathbf{E}_\omega + 2\omega\int d^3x\,(u_e - u_m) = 0. \tag{23}$$

There are two contributions to the reactive power flow: (1) the polarization current proportional to $\partial_t\mathbf{E}$ and (2) the adiabatic deformation of the current sheet with $\mathbf{J}^{\mathrm{ad}}_\omega$ proportional to $\delta\mathbf{B}_\omega$.

In order to determine the dissipation (22) necessary for magnetic reconnection in tail-like magnetic field reversals, we consider the perturbed current density $\delta\mathbf{J}$ produced as a response of the medium to tearing-like perturbations of the form

$$\delta\mathbf{A}(x,z,t) = \delta\mathbf{A}(z)e^{i(kx-\omega t)} + \text{c.c.}, \tag{24}$$

$$\delta\phi(x,z,t) = \delta\phi(z)e^{i(kx-\omega t)} + \text{c.c.}. \tag{25}$$

In the linear approximation, the perturbed distribution function f_j for particle species j can be written as

$$f_j = f_{0j} + \delta f_j, \tag{26}$$

where the equilibrium distribution satisfies

$$\mathbf{v}\cdot\frac{\partial f_{0j}}{\partial\mathbf{r}} + \frac{q_j}{m_j}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_{0j}}{\partial\mathbf{v}} = 0, \tag{27}$$

and where the perturbed part of the distribution function satisfies the linearized Vlasov equation

$$\frac{\partial\delta f_j}{\partial t} + \mathbf{v}\cdot\frac{\partial\delta f_j}{\partial\mathbf{r}} + \frac{q_j}{m_j}(\mathbf{v}\times\mathbf{B})\cdot\frac{\partial\delta f_j}{\mathbf{v}} = -\frac{q_j}{m_j}\left[\delta\mathbf{E} + \mathbf{v}\times\delta\mathbf{B}\right]\cdot\frac{\partial f_{0j}}{\partial\mathbf{v}}. \tag{28}$$

The formal solution to (28) is

$$\delta f_j(x,z,\mathbf{v},t) = -\frac{q_j}{m_j}\int_{-\infty}^{t} dt'[\delta\mathbf{E}' + \mathbf{v}'\times\delta\mathbf{B}']\cdot\frac{\partial f'_{0j}}{\partial\mathbf{v}'}, \tag{29}$$

where the integration is performed along the unperturbed trajectories $(\mathbf{x}',\mathbf{v}')$, which satisfy the initial conditions $\mathbf{x}'(t'=t) = \mathbf{x}$ and $\mathbf{v}'(t'=t) = \mathbf{v}$. The primes in the integrand of (29) mean that all the quantities are evaluated at time t'. Note also that the integration domain $(-\infty,t)$ is such as to satisfy the causality principle, which states that the correction δf cannot precede the perturbation $(\delta\mathbf{E},\delta\mathbf{B})$.

The perturbed part of the current density, $\mathbf{J} = \mathbf{J}_0 + \delta\mathbf{J}$, is given by becomes

$$\delta\mathbf{J}(x,z,t) = -\sum_j \frac{q_j^2}{m_j}\int d^3v\mathbf{v}\int_{-\infty}^{t} dt'[\delta\mathbf{E}' + \mathbf{v}'\times\mathbf{B}']\cdot\frac{\partial f'_{0j}}{\partial\mathbf{v}'}. \tag{30}$$

The equilibrium distribution is a function of the constants of the motion $f_0 = f_0(H,P_y)$, with

$$H = \frac{1}{2}mv^2 + q\phi(x,z) \quad \text{and} \quad P_y = mv_y + qA_y(x,z). \tag{31}$$

Thus we have

$$\frac{\partial f_0}{\partial \mathbf{v}} = m\,\frac{\partial f_0}{\partial P_y}\,\widehat{\mathbf{e}}_y + m\,\frac{\partial f_0}{\partial H}\,\mathbf{v}, \tag{32}$$

which for the Harris distribution,

$$f_0 = n_0\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left(-\frac{1}{T}\left[H - U_y P_y\right]\right), \tag{33}$$

becomes

$$\frac{\partial f_0}{\partial \mathbf{v}} = -\frac{m}{T}\,(\mathbf{v} - U_y\widehat{\mathbf{e}}_y) f_0. \tag{34}$$

Substituting (34) into (30) we get

$$\delta\mathbf{J} = \sum_j \frac{q_j^2}{T_j}\int d^3v\,\mathbf{v} f_{0j}\int_{-\infty}^{t} dt'[\delta\mathbf{E}' + \mathbf{v}'\times\delta\mathbf{B}']\cdot(\mathbf{v}' - U_{yj}\widehat{\mathbf{e}}_y), \tag{35}$$

where the constancy of f_{0j} along the unperturbed trajectories,

$$\left(\frac{df_{0j}}{dt}\right)_{\text{unpert.}} = 0, \tag{36}$$

has been used to take it out of the temporal integration.

From (35) we have that $\delta\mathbf{J}$ can be split into two parts,

$$\delta\mathbf{J} = \delta\mathbf{J}^{ad} + \delta\mathbf{J}^{d}, \tag{37}$$

where the adiabatic correction to the current, $\delta\mathbf{J}^{ad}$, is given by

$$\delta\mathbf{J}^{ad} = -\sum_j \frac{q_j^2 U_{yj}}{T_j}\int d^3v\mathbf{v} f_{0j}\int_{-\infty}^{t} dt'[\delta\mathbf{E}' + \mathbf{v}'\times\delta\mathbf{B}']\cdot\widehat{\mathbf{e}}_y \tag{38}$$

and the dissipative part of the perturbed current is

$$\delta\mathbf{J}^{d} = \sum_j \frac{q_j^2}{T_j}\int d^3v\mathbf{v} f_{0j}\int_{-\infty}^{t} dt'\,\mathbf{v}'\cdot\delta\mathbf{E}'. \tag{39}$$

First consider $\delta\mathbf{J}^{ad}$. Expanding the term enclosed by square brackets in Eq. (38) in terms of the perturbed potentials of Eq. (25), we get

$$[\delta\mathbf{E} + \mathbf{v}\times\delta\mathbf{B}]\cdot\widehat{\mathbf{e}}_y = -\frac{d\delta A_y}{dt}. \tag{40}$$

Substituting Eq. (40) into Eq. (38), we obtain that $\delta\mathbf{J}^{ad}$ is given by

$$\delta\mathbf{J}^{ad} = \left(\sum_j \frac{q_j^2 U_{yj}}{T_j}\int d^3v\mathbf{v} f_{0j}\right)\delta A_y. \tag{41}$$

On the other hand,

$$\frac{\partial J_{0y}}{\partial A_y} = \sum_j q_j \int d^3v v_y \frac{\partial f_{0j}}{\partial A_{0y}} = \sum_j \frac{q_j^2 U_{yj}}{T_j} \int d^3v v_y f_{0j}. \tag{42}$$

Substituting (42) into (41) we get

$$\delta\mathbf{J}^{ad}(x,z,t) = \frac{\partial J_{0y}}{\partial A_y}\,\delta A_y \widehat{\mathbf{e}}_y = -\frac{1}{\mu_0 B_x(z)}\,\frac{d^2 B_x(z)}{dz^2}\,\delta A_y \widehat{\mathbf{e}}_y. \tag{43}$$

The power associated with $\delta\mathbf{J}^{ad}$ is given by

$$Q^{ad}(k,\omega) = \frac{1}{2}\int d^3x \delta\mathbf{J}^{ad}_{k,\omega}\cdot\delta\mathbf{E}^*_{k,\omega} = i\frac{\omega}{2}\int d^3x\,\frac{\partial J_{0y}}{\partial A_y}\,|\delta A_y(k,\omega,z)|^2, \tag{44}$$

where we have used Eq. (43). The quantity Q^{ad} is purely imaginary, that is, Q^{ad} corresponds to the reactive part of energy transfer and is reversible. Q^{ad} is related to the adiabatic change of current filament interaction energy.

The energy dissipated per unit time is given by

$$Q(k,\omega) = \frac{1}{2}\,\mathrm{Re}\int d^3x \delta\mathbf{J}_{k,\omega}\cdot\delta\mathbf{E}^*_{k,\omega} = \frac{1}{2}\,\mathrm{Re}\int d^3x\delta\mathbf{J}^{d}_{k,\omega}\cdot\delta\mathbf{E}^*_{k,\omega}. \tag{45}$$

Hence, the time-averaged dissipated power is

$$\begin{aligned}\overline{Q} &= \frac{1}{2}\,\mathrm{Re}\left(\lim_{T\to\infty}\int_{-T/2}^{T/2}\frac{dt}{T}\int d^3x\delta\mathbf{J}^d\cdot\delta\mathbf{E}^*\right),\\ &:= \frac{1}{2}\,\sigma^H_{\alpha\beta}(k,\omega)\lim_{T\to\infty}\int_{-T/2}^{T/2}\frac{dt}{T}\int d^3x\delta E^*_{\alpha\,k,\omega}\,\delta E_{\beta\,k,\omega}\end{aligned} \tag{46}$$

where the time average removes the reversible part of the power transfer in $\delta\mathbf{J}\cdot\delta\mathbf{E}$ and the height integrated effective conductivity $\boldsymbol{\sigma}_{\alpha\beta}$ is defined in the second line. Note that the standard notation for the time period T for the average in Eqs. (46) and (47) is not to be confused with the use of T for temperature in Eqs. (33) and (34).

From Eq. (39) we have that

$$\begin{aligned}\lim_{T\to\infty}\int_{-T/2}^{T/2}\frac{dt}{T}\int d^3x\delta\mathbf{J}^d\cdot\delta\mathbf{E}^* &= \sum_j\frac{q_j^2}{T_j}\int d^6X_0 f_{0j}\lim_{T\to\infty}\int_{-T/2}^{T/2}\frac{dt}{T}\\ &\times\int_{-\infty}^{t} dt'\delta\mathbf{E}^*(z)\cdot\mathbf{v}\mathbf{v}'\cdot\delta\mathbf{E}(z')\times e^{i[k(x'-x)-\omega(t'-t)]},\end{aligned} \tag{47}$$

where the phase space integration is performed over the initial conditions $\mathbf{X}_0 \equiv (\mathbf{x}_0, \mathbf{v}_0)$, and where $\mathbf{x}\equiv\mathbf{x}(t;\mathbf{X}_0)$ and $\mathbf{v}\equiv\mathbf{v}(t;\mathbf{X}_0)$ are the position and velocity at time t of a charge moving in the unperturbed fields with initial conditions $\mathbf{X}_0$.

Dissipation for low frequency phenomena ($\omega \ll \omega_{cx0}$), such as tearing modes, occurs only in the narrow layer where the effect of the lobe magnetic field can be neglected. The thickness of the resistive layer can be estimated by determining the critical value of z, Δ, which satisfies the condition $\rho_c = \Delta$, where ρ_c is the Larmor radius evaluated at $z = \Delta$. Approximating the magnetic field in the current sheet layer by $B_x(z) = B_{x0}z/L_z$, we get $\Delta = (\rho_0 L_z)^{1/2}$, where $\rho_0 = v_{th}/\omega_{cx0}$.

2.4 Vlasov conductivity formula

From the above analysis we have that for low frequency phenomena, dissipation occurs in the resistive layer with the characteristic half-thickness. Since the main contribution to (47) comes from the resistive layer with $|z| \lesssim \Delta$, the fields can be approximated by their values at $z = 0$ and taken out from the temporal integration in (47). Hence, the low frequency conductivity is given by

$$\sigma_{\alpha\beta}(k,\omega) = \sum_j \frac{q_j^2}{T_j} \int d^6X_0 f_{0j} \int_0^\infty d\tau e^{i\omega\tau}$$

$$\times \lim_{T\to\infty} \int_{-T/2}^{T/2} \frac{dt}{T}\, v_\alpha(t;\mathbf{X}_0) v_\beta(t-\tau;\mathbf{X}_0) \times e^{ik[x(t-\tau;\mathbf{X}_0)-x(t;\mathbf{X}_0)]}, \tag{48}$$

where the change of variables from t' to $\tau \equiv t - t'$ has been made.

At this point it is convenient to introduce some notation. From now on, the product $\mathbf{v}(t;\mathbf{X}_0)e^{-ikx(t;\mathbf{X}_0)}$ will be denoted by $\mathbf{v}(k,t;\mathbf{X}_0)$;

$$\mathbf{v}(k,t;\mathbf{X}_0) := \mathbf{v}(t;\mathbf{X}_0)e^{-ikx(t;\mathbf{X}_0)}. \tag{49}$$

Ensemble averages over the initial conditions will be denoted by triangular brackets; $\int d^6X_0 f_{0j} \ldots := n_0\langle \ldots \rangle_j$. Finally, we define single-particle, two-time velocity correlations by

$$C_{\alpha\beta}(k,\tau;\mathbf{X}_0) := \lim_{T\to\infty} \int_{-T/2}^{T/2} \frac{dt}{T}\, v_\alpha(k,t;\mathbf{X}_0) v_\beta(-k,t-\tau;\mathbf{X}_0) \tag{50}$$

and we denote the one-sided Fourier transform of $C_{\alpha\beta}(k,\tau;\mathbf{X}_0)$ by $C_{\alpha\beta}(k,\omega;\mathbf{X}_0)$, that is,

$$C_{\alpha\beta}(k,\omega;\mathbf{X}_0) := \int_0^\infty d\tau\, e^{i\omega\tau} C_{\alpha\beta}(k,\tau;\mathbf{X}_0). \tag{51}$$

Note that $\mathbf{X}_0$ has been included in the definition of Eq. (50) to stress the fact that the single particle velocity correlation tensor depends on the initial conditions. For ergodic systems the time integration in Eq. (50) can be replaced by averages over the ensemble of particles and the reference to the initial conditions $\mathbf{X}_0$ in Eq. (50) can be suppressed.

Substituting Eqs. (49)–(51) into Eq. (48), we find that the low frequency conductivity formula (48) can be written as

$$\sigma_{\alpha\beta}(k,\omega) = \sum_j \frac{n_j q_j^2}{m_j} \langle C_{\alpha\beta}(k,\omega;\mathbf{X}_0)\rangle_j. \tag{52}$$

The dissipative part of the conductivity corresponds to its Hermitian part, which according to Eq. (52) is determined by the Hermitian part of the correlation tensor,

$$C^H_{\alpha\beta}(k,\omega) := \frac{1}{2}\left[C_{\alpha\beta}(k,\omega) + C^*_{\beta\alpha}(k,\omega)\right]. \tag{53}$$

From Eq. (50) we have that $C_{\alpha\beta}(k,\tau)$ has the following properties:

$$C^*_{\alpha\beta}(k,\tau) = C_{\alpha\beta}(-k,\tau) \tag{54}$$

and

$$C_{\alpha\beta}(k,-\tau) = C_{\beta\alpha}(-k,\tau). \tag{55}$$

From these properties it is easily verified that the Hermitian part of $C_{\alpha\beta}(k,\omega)$ is

$$C^H_{\alpha\beta}(k,\omega) = \widetilde{C}_{\alpha\beta}(k,\omega), \tag{56}$$

where $\widetilde{C}_{\alpha\beta}(k,\omega)$ is the Fourier transform of $C_{\alpha\beta}(k,\tau)$,

$$\widetilde{C}_{\alpha\beta}(k,\omega) = \int_{-\infty}^{\infty} d\tau\, e^{i\omega\tau} C_{\alpha\beta}(k,\tau). \tag{57}$$

In this work we will refer to $\widetilde{C}_{\alpha\beta}(k,\omega)$ as velocity power spectrum or spectral velocity correlations.

From (52) and (56) it follows that the dissipative part of the conductivity is given by

$$\sigma^H_{\alpha\beta}(k,\omega) = \sum_j \frac{n_j q_j^2}{m_j} \tau_{cj}, \tag{58}$$

where the correlation time τ_c (the effective "collisional time") is defined as

$$\tau_{cj} = \frac{1}{2v^2_{th\,j}} \left\langle \widetilde{C}_{\alpha\beta}(k,\omega;\mathbf{X}_0)\right\rangle_j. \tag{59}$$

In tail-like magnetic field reversals the conductivity depends on the finite Larmor radius parameter $\epsilon := \rho/L_z$, and on the magnetic field components $b_y := B_y/B_{x0}$ and $b_z := B_z/B_{x0}$;

$$\sigma_{\alpha\beta}(k,\omega) = \sigma_{\alpha\beta}(k,\omega;b_y,b_z,\epsilon). \tag{60}$$

Equations (52), (58), and (59) are the main results for the conductivity.

2.5 Frequency sum rule

Consider the integral over all the frequencies of the dissipative part of the conductivity. From (58), (59), and (57) we get

$$\int_{-\infty}^{\infty} d\omega \sigma_{\alpha\beta}^{H}(k,\omega) = \pi \sum_{j} \frac{n_j q_j^2}{m_j v_{th\,j}^2} \langle C_{\alpha\beta}(k,\tau=0)\rangle_j. \tag{61}$$

Noting that

$$\langle C_{\alpha\beta}(k,\tau=0)\rangle = v_{th}^2 \delta_{\alpha\beta} + U_y^2 \delta_{\alpha y}\delta_{\beta y}, \tag{62}$$

we obtain the frequency-sum rule

$$\int_{-\infty}^{\infty} d\omega\, \sigma_{\alpha\beta}^{H}(k,\omega) = \pi \sum_{j} \frac{n_j q_j^2}{m_j} \left[\delta_{\alpha\beta} + \left(\frac{U_y}{v_{th}}\right)_j^2 \delta_{\alpha y}\delta_{\beta y}\right], \tag{63}$$

which has been verified in our numerical experiments.

The frequency-sum rule (63) means that regardless of the details of the absorption spectrum the total amount of dissipation is constant. In the lobe region ($|z/L_z| \gg 1$), where the particles execute cyclotron motion around the magnetic field lines, the main contribution to the total dissipation is due to the high frequencies centered around the cyclotron frequency ω_{cx0} for the asymptotic field B_{x0}, as shown in Fig. 3. On the other hand, in the dissipative layer ($|z| \leq (\rho L_z)^{1/2}$), chaotic particle motion spreads the dissipation to low frequencies and the frequency-sum rule (63) has contributions from a broad band of absorption frequencies. In general, for inhomogeneous systems, dissipation is spread to low frequencies ($\omega \ll \omega_{cx0}$) even if the motion is integrable (Horton *et al.*, 1994).

In this section we have developed the spectral velocity correlations formalism for the calculation of the space-time-averaged conductivity. Numerically, the formalism consists of the following steps:

(i) Launch an ensemble of N particles distributed in phase space according to the equilibrium distribution function f_0.

(ii) For each particle integrate the equations of motion in the unperturbed fields and compute the corresponding power spectra $\widetilde{C}_{\alpha\beta}^{(j)}(k,\omega;\mathbf{X}_{0j})$, $j = 1, 2, \ldots, N$.

(iii) Average over all the particles,

$$\sigma_{\alpha\beta}^{H}(k,\omega) = \frac{q^2}{m v_{th}^2} \frac{1}{N} \sum_{j=1}^{N} \widetilde{C}_{\alpha\beta}^{(j)}(k,\omega;\mathbf{X}_{0j}). \tag{64}$$

(iv) As a consistency test, check the frequency-sum rule (63).

The single particle ensemble procedure just described is *essential* when the dynamics is nonintegrable. In Sec. 2.8 we show how the procedure reproduces the analytic $\sigma^H_{\alpha\beta}(k,\omega)$ in the limit of integrable (straight-line) orbits.

Holland and Chen (1992) and Chen (1992) have criticized the above procedure in terms of the energization rate in the zero frequency limit. They point out that there is a part of the energization that is proportional to $qv\,E_y$ rather than $\sigma\,E_y^2$. They also point out that for the transient Speiser orbits the zero frequency limit of the orbit integrals in Eqs. (50)–(51) are not well defined due to the nonstationary nature of the two-time velocity correlation function for these orbits. We agree that there is a component of the heating that is proportional to E_y, and thus is reversible over a full cycle of a slowly varying $E_y(\omega t)$. The division between the $E_y v$ and E_y^2 is seen in the work of Lyons and Speiser (1982) and was reinvestigated by Baek *et al.* (1995). Baek *et al.* consider ensembles of transient ions launched from the lobe region toward the current sheet and calculate the energy gained and residence time over a wider range of initial energies (0.1 ev to 20 keV) than in previous works. They also report the effect of finite B_y. The results show that low energy ions ($v \leq E_y/B_z$ corresponding to $H \leq 1\,\mathrm{keV}$ for $E_y = 0.25\,\mathrm{mV/m}$ and $B_z = 1\,\mathrm{nT}$) gain energy as $\sigma\,E_y^2$ whereas the higher energy ions gain according to $qv\,E_y$. Thus the energization rate of the fast, transient ions is not well characterized by the collisionless conductivity apparently due to their short residence time in the current sheet.

2.6 Fluctuation dissipation relations and the collisionless conductivity

Based on the idea that for a system close to thermodynamic equilibrium, the evolution towards equilibrium does not depend on whether the system was set out of equilibrium by an external perturbation or by an spontaneous fluctuation, a link can be established between the microscopic fluctuations and the response functions of the system. In particular, linear response theory leads to the so-called Green-Kubo formulae (eg., Kubo, 1957) which are general relations connecting transport coefficients and autocorrelation functions of fluctuating quantities. Taking Fourier transforms of the Green-Kubo formulae, general relations between spectral correlations of microscopic fluctuations and the dissipative part of the transport coefficients are obtained. These general relations are called fluctuation-dissipation relations (Sitenko, 1982; Kubo *et al.*, 1985; Klimontovich, 1991).

Consider a system in thermodynamic equilibrium. In the absence of external perturbations, the Hamiltonian is independent of time and is denoted by $H_0(\Gamma)$, where Γ is an abbreviation for the $6N$ independent variables:

$$\Gamma = \{\mathbf{X}_1, \ldots, \mathbf{X}_N\}, \quad \mathbf{X}_i := \{\mathbf{x}_i,\, \mathbf{v}_i\}, \tag{65}$$

with $\mathbf{x}_i$ and $\mathbf{v}_i$ the position and the velocity of the i-th particle, respectively. When the system is perturbed by an external vector potential, $\mathbf{A}(\mathbf{x}, t)$, the Hamiltonian takes the form

$$H(\Gamma, t) = H_0(\Gamma) + \delta H(\Gamma, t) \tag{66}$$

where

$$\delta H(\Gamma, t) = -\int d^3x\, \mathbf{J}(\mathbf{x}, t; \Gamma) \cdot \mathbf{A}(\mathbf{x}, t), \tag{67}$$

with the current density

$$\mathbf{J}(\mathbf{x}, t; \Gamma) = \sum_{i=1}^{N} q_i \mathbf{v}_i(t) \delta(\mathbf{x} - \mathbf{x}_i(t)). \tag{68}$$

The linear causal response of the medium to the perturbing potential

$$\mathbf{J}(\mathbf{x}, t) = \int_{-\infty}^{t} dt' \int d^3x'\, \mathbf{K}(\mathbf{x}, \mathbf{x}', t, t') \cdot \mathbf{A}(\mathbf{x}', t'). \tag{69}$$

Then, for the Fourier components of $\mathbf{J}$ and $\mathbf{A}$ we can write

$$\mathbf{J}(\mathbf{k}, \omega) = \mathbf{K}(\mathbf{k}, \omega) \cdot \mathbf{A}(\mathbf{k}, \omega), \tag{70}$$

where $K_{\alpha\beta}$ is the response tensor of the medium.

The fluctuation-dissipation theorem relates the spectral correlations of current density fluctuations to the dissipative properties of the medium and is given (Kubo *et al.*, 1985) by

$$\langle J_\alpha^* J_\beta \rangle_{\mathbf{k}\omega} = \frac{\hbar}{\exp(\hbar\omega/T) - 1}\, i \left\{ K_{\alpha\beta}^*(\mathbf{k}, \omega) - K_{\beta\alpha}(\mathbf{k}, \omega) \right\}, \tag{71}$$

which in the classical limit ($T \gg \hbar\omega$) becomes

$$\langle J_\alpha^* J_\beta \rangle_{\mathbf{k}\omega} = \frac{T}{\omega}\, i \left\{ K_{\alpha\beta}^*(\mathbf{k}, \omega) - K_{\beta\alpha}(\mathbf{k}, \omega) \right\}. \tag{72}$$

In writing Eqs. (71) and (72) we are assuming that the system is in thermal equilibrium at the temperature T.

In the derivation of (71) the average energy absorbed by the medium per unit time Q is calculated using time-dependent perturbation theory, with the perturbation given by (67), and by performing two averages: one over the quantum state of the system and one over the statistical distribution of the different quantum states of the system given by the canonical distribution.

The fluctuation-dissipation theorem can be used in several ways. If the response tensor $K_{\alpha\beta}(\mathbf{k},\omega)$ is known (for example, the dielectric tensor for a uniform plasma both in the absence and in the presence of a constant magnetic field), then the spectral distribution of $\mathbf{J}$ can be found. On the other hand, if we somehow know the fluctuation spectra of $\mathbf{J}$, then we can invert (71) and determine the response properties of the medium (this is like using the absorption lines in a spectrum to determine the dielectric properties of the medium). The later approach is the one we use in this work: we want to calculate the collisionless conductivity from the numerical computation of the spectral velocity correlations.

For the case of electromagnetic perturbations, the response tensor $K_{\alpha\beta}(\mathbf{k},\omega)$ is given in terms of the electrical conductivity $\sigma_{\alpha\beta}(\mathbf{k},\omega)$ through

$$K_{\alpha\beta}(\mathbf{k},\omega) = i\omega\sigma_{\alpha\beta}(\mathbf{k},\omega), \tag{73}$$

and the fluctuation-dissipation relation which follows from substituting (73) into (72) gives

$$\sigma^{H}_{\alpha\beta}(\mathbf{k},\omega) = \sum_j \frac{1}{2m_j v_{th\,j}^2} \int_{-\infty}^{\infty} d\tau e^{i\omega\tau} \langle J_\alpha(\mathbf{k},t) J_\beta(-\mathbf{k},t-\tau)\rangle_j, \tag{74}$$

Formula (74) is called the Kubo conductivity formula (Kubo *et al.*, 1985). Expression (74) has to be compared with expression (58) for the collisionless conductivity. The space-time-averaged conductivity formula (58) is more general than the statistical equilibrium conductivity formula (74). To the extent that the orbits are ergodic so that the ensemble-averaged two-time velocity correlation function $\langle v_\alpha(\mathbf{k},t)v_\beta(-\mathbf{k},t-\tau)\rangle$ is the same as the time-averaged velocity correlation function in (50), the Kubo conductivity formula is identical to the space-time averaged conductivity (58).

2.7 Conductivity from Kubo's formula for an unmagnetized plasma

Consider a collisionless, uniform, homogeneous, and unmagnetized plasma. There are no forces acting on the system, thus the particles move along straight-line trajectories,

$$\mathbf{x}_i(t) = \mathbf{x}_i(t_0) + \mathbf{v}_i(t-t_0),$$

$$\mathbf{v}_i(t) = \mathbf{v}_i = \text{const.}, \tag{75}$$

with the microscopic distribution function

$$F(\mathbf{x},\mathbf{v},t;\Gamma) = \frac{1}{N}\sum_{i=1}^{N}\delta(\mathbf{x}-\mathbf{x}_i(t))\delta(\mathbf{v}-\mathbf{v}_i(t)). \tag{76}$$

One obtains the connection between microscopic quantities and macroscopic quantities by taking averages over ensembles of systems, which differ only in the particle states, and by considering the distribution of the systems in the different states.

The spectral correlation between fluctuations of the distribution function:

$$\langle \delta F(\mathbf{v})\delta F(\mathbf{v}')\rangle_{\mathbf{k}\omega} = 2\pi\delta(\omega-\mathbf{k}\cdot\mathbf{v})\delta(\mathbf{v}-\mathbf{v}')f_0(\mathbf{v}). \tag{77}$$

The particle density distribution $n(\mathbf{x},t;\Gamma)$ is obtained integrating Eq. (76) over $\mathbf{v}$. The result is

$$n(\mathbf{x},t;\Gamma) = \sum_{i=1}^{N}\delta(\mathbf{x}-\mathbf{x}_i(t)) = \sum_{\mathbf{k}} n_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{78}$$

where

$$n_{\mathbf{k}}(t) = \sum_{i=1}^{N} e^{-i\mathbf{k}\cdot\mathbf{x}_i(t)}. \tag{79}$$

Similarly, the current density $\mathbf{J}(\mathbf{x},t;\Gamma)$ is obtained multiplying Eq. (76) by $q\mathbf{v}$ and integrating over $\mathbf{v}$. The result is

$$\mathbf{J}(\mathbf{x},t;\Gamma) = q\sum_{i=1}^{N}\mathbf{v}_i(t)\delta(\mathbf{x}-\mathbf{x}_i(t)) = q\sum_{\mathbf{k}}\mathbf{v}_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{80}$$

where

$$\mathbf{v}_{\mathbf{k}}(t) = \sum_{i=1}^{N}\mathbf{v}_i(t)e^{-i\mathbf{k}\cdot\mathbf{x}_i(t)}. \tag{81}$$

The spectral correlations between density fluctuations are obtained integrating Eq. (77) twice over the velocities,

$$\left\langle \delta n^2\right\rangle_{\mathbf{k}\omega} = 2\pi\int d^3v\,\delta(\omega-\mathbf{k}\cdot\mathbf{v})f_0(\mathbf{v}). \tag{82}$$

Similarly, the spectral correlations between current density fluctuations are obtained multiplying Eq. (77) by $\mathbf{v}\mathbf{v}'$ and integrating twice over the velocities,

$$\langle J_\alpha J_\beta\rangle_{\mathbf{k}\omega} = 2\pi q^2\int d^3v\,v_\alpha v_\beta\delta(\omega-\mathbf{k}\cdot\mathbf{v})f_0(\mathbf{v}). \tag{83}$$

Taking the distribution $f_0(\mathbf{v})$ to be the Maxwellian distribution function $f_M(\mathbf{v})$,

$$f_M(\mathbf{v}) = n_0\,\frac{1}{(2\pi)^{3/2}\,v_{th}^3}\exp[-v^2/2v_{th}^2], \tag{84}$$

substituting (84) into (83) and substituting the result into Kubo's conductivity formula (74), we obtain the well-known Vlasov-Maxwell conductivity

$$\mathrm{Re}\,\boldsymbol{\sigma} = \sqrt{\frac{\pi}{2}}\,\frac{n_0 q^2}{m|k|v_{th}} \begin{pmatrix} \frac{\omega^2}{k^2 v_{th}^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \exp\left(-\frac{\omega^2}{2k^2 v_{th}^2}\right), \tag{85}$$

where we have taken the wave vector $\mathbf{k}$ to be along the x-axis.

2.8 Conductivity from the spectral velocity correlations (SVC) formalism

The numerical calculation of the conductivity formula from the spectral velocity correlations formalism is illustrated in Fig. 8. Figure 8 displays plots for (a) the transverse and (b) the longitudinal components of the dissipative part of the conductivity for the unmagnetized plasma. We obtained both plots by launching $N = 3000$ particles according to a Maxwellian distribution and following the spectral velocity correlations formalism. The impulse lines in the plots correspond to our numerical results for the discrete frequency components over the finite time interval, and the solid curves correspond to the analytical result from Eq. (85). We can see that the numerical results are in good agreement with the analytical formula in this limit of straight line trajectories. The frequency sum rule was verified for both the transverse and longitudinal components of the conductivity since the numerical values 3.04 and 3.028 were obtained, which are close to π, within 1/30, as desired. F8

3 Applications of the Conductivity to Tearing Modes in the Central Plasma Sheet

3.1 Collisionless conductivity for the Harris sheet

In the case of the Harris sheet the orbits, while integrable, are highly nonlinear. The only practical method of determining the conductivity is by velocity spectral correlations formalims.

Figure 9 is a plot of the dissipative part of the conductivity produced by the microscopic fluctuations from charged particles trapped in the plasma sheet. We obtained the plot by launching $N = 1000$ particles distributed according to the Harris distribution of Eq. (33) with number density $n(z) = n_0\,\mathrm{sech}^2(z/L_z)$ for $|z/L_z| < 1$, with $U_y/v_{th} = 2\epsilon = 0.08$, $\omega_{cx0}/\omega_{bz} = 5$, F9 and with $kv_{th}/\omega_{bz} = 0.02$. We find that the dc value of the conductivity is the value expected from the Galeev-Zelenyĭ conductivity (85) with the density of the effectively unmagnetized particles given by $n = n_0(\Delta/L_z) = \epsilon^{1/2} n_0 = 0.2\, n_0$.

Figure 10 displays the dissipative part of the conductivity produced by the microscopic fluctuations from charged particles located away from the plasma sheet, that is, well into the region where the magnetic field is nearly uniform, $\mathbf{B}(z) \approx B_{x0}\widehat{\mathbf{e}}_x$. Initially the particles were uniformly distributed in the range $3 \leq z/L_z \leq 4$, where the magnetic field is uniform to within 0.5%. The rest of the simulation parameters are the same as those in Fig. 9. Note that the only significant contribution to the conductivity occurs for $\omega \approx \omega_{cx0}$, which is in agreement with the assumption that the low-frequency component of the conductivity arises from particles in the current sheet. F10

The possibility of collisionless magnetic reconnection in a straight magnetic field reversal was proposed by Coppi *et al.* (1966). These authors showed that the Harris sheet is tearing-unstable for long wavelength ($kL_z \ll 1$) perturbations for which the energy released by the pinching of the current filaments exceeds the energy spent in the creation of the perturbed magnetic field. The mechanism for energy dissipation is given by the transfer of energy from the plasma to a fraction of the electrons through the resonant interaction of the waves with the particles in the thin layer of thickness $\Delta_e = (\rho_e L_z)^{1/2}$.

In the outer region we have $\delta J_y^d = 0$, and the reconnection perturbation $\delta A_y(z)$ satisfies the homogeneous equation

$$\left[\frac{d^2}{dz^2} - k^2 - \frac{B_x''(z)}{B_x(z)}\right] \delta A_y(z) = 0, \tag{86}$$

with the even-parity solution satisfying the boundary conditions

$$\lim_{|z|\to\infty} \delta A_y(z) = 0. \tag{87}$$

The even parity modes describe a pinching and then tearing: the odd modes $\delta A_y(-z) = -\delta A_y(z)$ describe a flapping or kinking of the current sheet.

The matching between the inner and outer regions is achieved through the matching parameter Δ' defined by

$$\Delta' := \left(\frac{d\ln(\delta A_y(z))}{dz}\right)_{-\Delta_e}^{+\Delta_e}, \tag{88}$$

which for the approximate solution (86) is given by

$$\Delta_k' L_z \cong \frac{2}{kL_z}\,(1 - k^2 L_z^2) \approx 2/kL_z \tag{89}$$

for $kL_z \ll 1$. The perturbation releases reversed magnetic field energy in proportion to $|\delta B|^2 \Delta'$ producing exponential growth γ. The tearing mode growth rate

$$\gamma = \frac{\Delta'_k}{2\mu_0 \sigma_{yy} \Delta}. \tag{90}$$

For the electron $\sigma_{yy} = \sigma^{G-Z} = (\pi/2)^{1/2}(n_e e^2/m_e|k_x|v_{the})$ we find that the correlation time is $\tau_{ce}^{G-Z} \approx 1\,\mathrm{s}$, the half-width of the unmagnetized layer is $(\rho_e L_z)^{1/2} = 160\,\mathrm{km}$, and the conductivity is $\sigma^{G-Z} \sim 10^{-2}\,\mathrm{mho/m}$. In contrast, For substorm time scales to be met. We would require $\tau_{ce} \sim 10^{-3}\,\mathrm{sec}$ and $\sigma_{yy} \sim 10^{-5}\,\mathrm{mho/m}$.

Using the pressure balance formula $n = B_{x0}^2/2\mu_0(T_i + T_e)$ to determine the plasma density in terms of the lobe magnetic field strength, we find that the collisionless electron tearing mode growth rate takes the form

$$\frac{\gamma}{\omega_{cx0\,e}} = \frac{2^{3/2}}{\pi^{1/2}}\left(1 + \frac{T_i}{T_e}\right)\epsilon_e^{5/2}, \tag{91}$$

and the mode is unstable for $kL_z \ll 1$. For the typical geotail parameters the growth rate of the instability is $\gamma \cong 10^{-4}$ sec^{-1}, and the growth time is $\tau \sim 3$ hr. The observed growth phase of a substorm lasts 20 min-40 min. Thus, in recent times this electron tearing mechanism in the Harris sheet is not considered as a plausible mechanisms for the substorm dynamics.

3.2 Collisionless Hall conductivity for the magnetic field reversals

In reality the small component of the magnetic field normal to the current sheet layer, $B_z \ll B_{x0}$, magnetizes the electrons, which perform gyromotion around B_z in the current sheet layer, and thus the main contribution to the low-frequency dissipative part of the conductivity is due to the unmagnetized ions. This is a type of collisionless Hall effect for the conductance. This hybrid or Hall-like conductance is also the principle of the plasma ion-diode (Golden *et al.*, 1977, 1981), where the perpendicular magnetic field in the diode is adjusted to magnetize the electrons ($\epsilon_e \ll 1$), leaving the ions, with $\epsilon_i \gtrsim 1$, to carry the current in the diode. Ion diodes are used for the generation and propagation of intense ion beams (Dreike *et al.*, 1976).

In parabolic-like magnetic field reversals with $B_z \ll B_{x0}$, ion motion becomes chaotic. The chaotic particle motion leads to a continuous broadband power spectrum and to the decay of the velocity correlations. That is, chaotic particle motion smears the resonance peaks at the orbital frequencies, thus allowing the exchange of energy between particles and waves for a continuum of frequencies.

The conductivity formula can be motivated if we follow the picture of Lyons and Speiser (1985). The main pickup of energy from $v_y E_y$ occurs when the particles are in the $\Delta_i = (\rho_i L_z)^{1/2}$ layer and when $\omega_{cz} < \omega_{bz}$. In this regime the orbits enter into the Δ_i layer, make rapid north-south oscillations, make a large section of approximately one-half of the cyclotron orbit around B_z, and then escape to one of the lobes. Particles are coherently accelerated by E_y when they are in the Δ_i layer. Taking the correlation time in the layer to be on the order of one-half of the cyclotron period, π/ω_{cz}, the low-frequency conductivity is given by the Lyons-Speiser formula

$$\sigma^{L-S} = \frac{n_0 q^2}{m} \frac{\pi}{|\omega_{cz}|}. \tag{92}$$

Horton and Tajima (1990, 1991) showed that the conductivity acquires this value only in the Δ_i layer and thus the height-averaged conductivity for the current sheet is given by

$$\sigma^{H-T} = \epsilon^{1/2} \sigma^{L-S} \ . \tag{93}$$

Formula (93) has the important property that as m/e (or the gyroradius) vanishes so that the particles are tied to the field lines, the conductivity vanishes. This is in contrast to Eq. (92) where σ is independent of m/e.

When the period $\Delta t = \pi/|\omega_{cz}|$ is shorter than the streaming time $1/kv_{th}$, the cyclotron frequency at $z = 0$ determines the correlation time and the conductivity, rather than the phase mixing rate kv_{th}. Combining the characteristics given above we obtain the conductivity formula

$$\sigma^{H-T} = \frac{nq^2}{m} \frac{\epsilon^{1/2}}{c_1|\omega_{cz}| + c_2|k|v_{th}} = \frac{nq^2}{m|\omega_{cx0}|} \frac{\epsilon^{1/2}}{c_1 b_z + c_2|k|\rho}, \tag{94}$$

where c_1 and c_2 are constants which are determined by test particle simulations. Note that in the limit that $\epsilon = \rho/L_z \to 0$, the particle motion becomes strictly adiabatic, executing only $\mathbf{E} \times \mathbf{B}$ drift motion, and the low-frequency conductivity (94) and the mobility vanish. Note also that for long wavelengths, $kv_{th} \lesssim \omega_{cz}$, the second formula in (94) shows clearly the role of the κ_{BZ} parameter defined in Eq. (15) in determining the conductance.

The variation of the conductivity (94) with ϵ, b_z, and kL_z has been tested (Horton and Tajima, 1991a and Hernández *et al.* (1993)) by numerical simulations following the procedure given in Sec. 2.5. Good agreement with Eq. (94) has been obtained with $c_1 \approx 10$ and $c_2 \approx 2$. The relatively large value of the ratio c_1/c_2 implies that the partial magnetization of the orbits is more effective than the phase mixing from the Landau resonance at $v = \omega/k$ in determining the low-frequency conductivity when $kv_{th}/\omega_{cz} \ll c_1/c_2 \simeq 5$.

Outside the Δ_i layer the particles are magnetized and the dominant current is given by the polarization current $J_y = (nq^2/m\omega_{cx}^2)dE_y/dt$, and the δJ^{ad} in Eq. (41) which are completely reversible or reactive and does not contribute to the time-averaged $\langle J_y E_y \rangle$.

The reconnection growth rate γ obtained from the substitution of (94) into (90) is given by

$$\frac{\gamma}{|\omega_{cx0}|} = \epsilon^{3/2} \frac{c_1 b_z + c_2 k L_z \epsilon}{k L_z} \left(1 + \frac{T_e}{T_i}\right). \tag{95}$$

Thus the widely used collisionless ion tearing mode growth rate $\gamma^{G-Z} = |\omega_{cx0}|\epsilon^{5/2}$ of Galeev and Zelenyĭ (1976) increases to $\gamma = c_1 b_z |\omega_{cx0}|\epsilon^{3/2}$, due to the low value of the conductivity (94). For $kL_z > c_1 b_z / c_2 \epsilon$ the growth rate reduces to the Galeev-Zelenyĭ value $\gamma^{G-Z}/|\omega_{cx0}| \approx c_2 \epsilon^{5/2}$ giving $1/\gamma^{G-Z} \sim 1$ hour. For $kL_z < c_1 b_z / c_2 \epsilon$ and $b_z < \epsilon^{1/2}$ the growth rate is increased to $\gamma/|\omega_{cx0}| \approx c_1 \epsilon^{3/2}$ giving $1/\gamma \sim 1$ min.

For typical geotail parameters, and for $kL_z \cong 0.5$ the correlation time is

$$\tau_c = \frac{\epsilon^{1/2}|\omega_{cx0}|^{-1}}{c_1 b_z + c_2 k\rho} = 0.15 \text{ sec}, \tag{96}$$

the unmagnetized layer half-width is $\Delta_i = (\rho_i L_z)^{1/2} = 980\,\text{km}$, the conductivity is $\sigma^{H-T} = 1.2 \times 10^{-6}\,\text{mho/m}$, the growth rate is $\gamma = 0.9\ \text{min}^{-1}$, and the growth time is $\tau_\gamma = 1/\gamma \simeq 1$ min.

Now we analyze the global implications of the onset of the tearing mode with a growth time of order one minute. The trigger condition is that the current sheet thins sufficiently that $L_z \ll b_z L_x/\epsilon$ and $b_z < \epsilon^{1/2}$. Within a time of order 20 min the plasmoid is fully developed.

The reader should note that the ion tearing mode mechanism is actively debated after 20 years of research following its introduction by Galeev and Zelenyĭ (1976). Lui and his collaborators, Yoon and Lui (1996), present the Wiebel instability driven by the Z-gradient of the ion drift velocity as alternative mechanism. This purely electromagnetic mode can also filament the current density. Nevertheless, the 3D MHD simulations show the tearing mode mechanism for the plasmoid formation occurs with the onset of a southward IMF field as discussed in Sec. 1.2.

4 Substorm Correlations with the Interplanetary Plasma: Global Model

4.1 Global substorm model

The correlation of the magnetospheric substorm activity with solar wind and the orientation of the interplanetary magnetic field (IMF) has been extensively investigated over the past thirty years. The early stages of the studies are succinctly summarized by Cliver (1994). Quantitative correlation studies began with Perrault and Akasofu (1978), Akasofu (1980, 1994) and with the rather precisely defined time series of solar wind driving voltage $v_{sw}[B_z]_s$ as input and auroral electrojet current $AL(t)$ as output by Bargatze *et al.* (1985). In the past ten years many important, quantitative correlation studies have been repeated. In this section we give the global model of the substorm that is associated with plasma sheet conductivity and the tearing mode as the fast plasma unloading mechanism. We describe two correlation studies that we view as supporting the theoretical model developed here.

The global low-dimensional dynamical model based on the physical processes of the collisionless Hall conductivity Σ and the fast plasma unloading U_0 triggered by magnetic reconnection in the plasma sheet is given by four ode's for $I(t), V(t) = L_y E_y(z=0,t), p_0(t)$ and $K_\parallel(t)$. In the introduction estimates of the global magnetotail energies $W_B = \frac{1}{2}\mathcal{L}I^2$, $K_E = \frac{1}{2}CV^2, U = \frac{3}{2}\bar{p}\Omega_{\rm cps}$ and $K_\parallel = \int \frac{1}{2}\rho v_\parallel^2 d^3x$ associated with these four key state variables are given along with the values of the lobe inductance $\mathcal{L}$ and central plasma sheet capacitance C. The solar wind input voltage $V_{\rm sw}(t)$ is taken as the standard rectified signal

$$V_{\rm sw}(t) = \beta v_x^{\rm sw}\left[B_z(t)\right]_s L_y \tag{97}$$

where the bracket with subscript s indicates that the voltage is zero for $B_z > 0$ and given by the magnitude of the southern component for $B_z < 0$. Here $\beta \simeq 0.1$ gives the fraction of the solar wind electric field loading magnetic flux in the geotail (Hill, 1975). This input voltage is the well-accepted input signal used in the numerous solar wind-geomagnetic activity studies, e.g. Bargatze *et al.* (1985). The alternative input is the Perrault-Akasofu (1978) input power $\mathcal{E}^{\rm PA}(\theta)$ given below with a partial rectification in terms of the IMF clock angle θ ($\tan\theta = B_y^{\rm IMF}/B_z^{\rm IMF}$).

The low-dimensional dynamical model is given by

$$\mathcal{L}\,\frac{dI}{dt} = V_{\rm sw}(t) - V(t) \tag{98}$$

$$C\ \frac{dV}{dt} = I - I_{ps} - \Sigma V \tag{99}$$

$$\tfrac{3}{2}\ \frac{dp}{dt} = \frac{\Sigma V^2}{\Omega_{\rm cps}} - u_0\, K_\parallel^{1/2}\Theta(I - I_c)p \tag{100}$$

$$\frac{dK_\parallel}{dt} = I_{ps}V - \frac{K_\parallel}{\tau_\parallel} \tag{101}$$

where $I_{ps}(t)/L_x = (8p(t)/\mu_0)^{1/2}$ follows from the self-consistent pinch equilibrium. The critical current I_c for the onset of the fast unloading of plasma pressure follows from onset of the tearing mode with the thinning of the current sheet discussed in Secs. 1.2 and 3 and shown in Fig. 2. The critical current may be estimated by setting a limit on $\Delta' \geq \Delta'_{\rm crit} = L_x^{\rm NL}/L^2_{z,{\rm crit}}$ for the release of sufficient reversal field energy by δJ^{ad} in Eqs. (86)–(90). Here we take the tailward location of the near-Earth neutral line point $L_x^{\rm NL}$ to determine the relevant mode number through $k_x L_x^{\rm NL} = \pi$. This choice is guided by the 3D-MHD simulations for the onset position of the X-O bifurcation in the reversed magnetic field. The dynamical properties of the low-dimensional model defined by Eqs. (99)–(102) is that the conductivity Σ is based on the Hamiltonian dynamics with internal resonances given rise to the transfer of energy from the fields to the ions. As a consequence when the driving voltage $V_{\rm sw}$ and unloading are switched off the system is energy conserving with

$$\frac{d}{dt}\left[\tfrac{1}{2}\,\mathcal{L}\,I^2 + \tfrac{1}{2}\,CV^2 + \tfrac{3}{2}\,p(t)\Omega_{\rm cps} + K_\parallel\right] = 0 \tag{102}$$

when $V_{\rm sw} = u_0 = 0$. The global dynamical model given by Eqs. (99)–(102) is closely related to the Klimas *et al.* (1992, 1994) "dripping faucet" model as explained in Horton and Doxas (1995). In the Klimas *et al.* (1995a) model there is a critical current above which there is fast unloading with the rate of loading chosen to be proportional to dI/dt, rather than $K_\parallel^{1/2}$, at the moment that the current reaches the critical value.

In the Horton-Doxas model the plasma loss rate is taken to be given by the flux limit parameter u_0 in Eq. (100) through the mean parallel flow velocity. The detailed picture of how the flow of plasma is divided toward the inner magnetosphere and the distant tail is provided by the 3D MHD simulations. The Horton-Doxas model contains kinetic effects that are beyond the simple closure of the fluid moment equations. The parallel thermal loss $q_\parallel$ modeled by $u_0 K_\parallel^{1/2} p$ and the nonadiabatic heating from ΣV^2 are effects contained in test particle codes. Ashour-Abdalla *et al.* (1994) report large scale test particle simulations of the transport of ions in the geotail in a Tsygananko-based geomagnetic field. Usadi *et al.* (1996) investigate the global ion transport problem with simpler, analytic X-dependent geotail model fields. Both of these simulations show the importance of fast parallel flows out of dynamically active regions.

4.2 Substorm recurrence during southward IMF

The Klimas *et al.* model and the Horton-Doxas model appear capable of explaining the dominant features of the solar wind-geomagnetic activity databases. Here we restrict the discussion to three complementary correlation studies those of Bargatze *et al.* (1985), Farrugia *et al.* (1993) and Blanchard and McPherron (1994). We also comment on the Borovsky *et al.* (1993) database. The Bargatze *et al.* (1985) data is characterized by 2.5 min sampling of the IMF B_z and solar wind velocity $v_x^{\rm sw}$ and the AL index over long periods from 1 to 2 days, containing both active and quiet periods. The Farrugia *et al.* (1993) data contain a 29 hour period in which a strong IMF field rotates from northward $B_z > 0$ (so $vB_s = 0$) for the first 11 hours, to then $B_z < 0$ (so that $vB_s = |B_z| v_x^{\rm sw} \sim -4$ to $-10\,{\rm mv/m}$) during the following 18 hour period. This data justifies the choice of the rectified input voltage of $vB_s(t)$ which is switched on and off according to whether the IMF is southward or northward. A strongly rectified solar wind input signal such as the Perrault-Akasofu (1978) input power $\mathcal{E}(\theta) = \rho v_{\rm sw}^3 \ell^2 \sin^4(\theta/2)$ where θ is the clock angle of the IMF is not ruled out by the Farrugia *et al.* (1993) data. Furthermore, the southward IMF period of the Farrugia *et al.* (1993) data sets limits on substorm modeling. In particular, for the 18 hour period with $VB_s \sim -4$ to $-10\,{\rm mv/m}$ there is a sequence of 23 substorms with the mean recurrence period between onsets of 50 min within the range of time intervals between 25 min to 100 min during this period.

Further details of the Farrugia *et al.* (1993) database are that during a 30-hour period the IMF rotates through 240° from a northeasterly to a southerly orientation. The maximum northward orientation occurs at 1530(UT) with $B_z^{\rm max} \sim 24\,{\rm nT}$. At the transition between the two regimes the field is purely westward with $B_y = -31\,{\rm nT}$ and then the IMF monotonically decreases in strength over an 18 hour interval decaying from $B_z = -18\,{\rm nT}$ to $-4\,{\rm nT}$.

In contrast to both these data sets the Blanchard and McPherron database is a set of well-isolated substorms with a range of $(AL)_{\rm max} = -170$ to $-1140\,{\rm nT}$ with active period lengths ranging from 90 min to 600 minutes. Of the 124 data sets some show the classic two-time delayed responses of $\Delta t_1 \sim 20\,{\rm min}$ and $\Delta t_2 \sim 60\,{\rm min}$ very clearly and thus provide support for the bi-modal response model put forth in Bargatze *et al.* (1985) and developed further by McPherron and Blanchard (1994).

Borovsky *et al.* (1993, 1994) report two patterns of substorm recurrences: one is the train of substorms with a well-defined mean recurrence time as in the Farrugia *et al.* data, except now the mean recurrence time is 3.1 hr, and the second is a random sequence of events with an exponential distribution of recurrence times consistent with a Poisson process. The mean recurrence time here is 5 hrs.

Linear and nonlinear prediction filters have been employed by numerous groups to address the issues of the type of internal nonlinear dynamics versus the external fluctuating solar wind responsible for the substorms. The question is whether the internal magnetosphere dynamics can be accurately characterized by a low-dimensional chaotic attractor. The theoretical support for such a characterization stems in part from the observation that the extended geomagnetic tail is a highly stressed system close to the MHD stability limit. For such systems the theory of self-organized criticality (Chang (1992) states that only a few degrees of freedom are required to describe the system. Support for this description follows from the analysis of Sharma *et al.* (1993) and Vassiliadis *et al.* (1995). Using the singular value decomposition method to extract the coherent dynamics from the stochastic solar wind driven component, a rather clear convergence of the correlation dimension to $\nu \simeq 2.5$ is shown for the Bargatze AE time series.

4.3 Observations of plasma sheet heating

Huang *et al.* (1992) report detailed correlations of "density dropouts" and *nonadiabatic* plasma heating with the expansion phase of substorms. The data is based on particle detectors on ISEE satellite while in the CPS and the PSBL in the range $X = -9R_E$ to $-23\,R_E$. The energy range for the ions is $24-65\,\mathrm{keV}$ and electrons $23-75\,\mathrm{keV}$. The principal conclusion of their work is that the CPS is a major sink of substorm energy requiring powers of up to 5×10^{10} W. The conclusion is based on using the observed changes in the particle densities and energies applied uniformly to the region $\Delta x = 14\,R_E$, $\Delta z = 13\,R_E$ and $\Delta y/x \cong 15^\circ$ azimuth for the plasma volume $\Omega \sim 700\,R_E^3$ in which the inferred thermal (non-streaming) energy increase is $\Delta E \sim 1.5\times 10^{12}$ J in the time $\Delta t \sim 6\,\mathrm{min}$, giving 5×10^{10} watts. Based on the observations, Huang *et al.* (1992) argue that the plasma sheet heating is a result of and not a cause of the substorm.

The energy conserving nonlinear dynamics substorm model explains quantitatively the heating of CPS plasma since it explicitly contains a heating of the thermal plasma which increases sharply with substorm onset expansion (EXO) phase. The model given here is consistent with that of Doxas *et al.* (1994) which also correlates with the Huang *et al.* (1992) data.

These local particle observations place constraints on the time sequence of the oscillations in W_B, W_p, K_E, and $K_{\|}$. They also constrain the heating rate and the loss rate of the thermal balance equation W_p. The test particle simulations of Doxas *et al.* (1990) address these observations using a reconnection perturbation for the NENL paradigm. The test particle simulations of Ashour-Abdalla *et al.* (1994) show the fast energization of the ions and formation of parallel streaming beams that are consistent with the observed plasma distributions and the nonlinear model (98)–(101). These test particle simulations intrinsically contain a nonlinear form of the large gyroradius conductivity since they integrate the full Lorentz force dynamics in prescribed tearing modes.

To summarize, collisionless heating and reconnection follow from the Horton-Tajima (volume averaged) conductivity formula

$$\sigma^{H-T} = \frac{nq^2}{m_i} \frac{\epsilon^{1/2}}{c_1|\omega_{cz}|}, \tag{103}$$

where $c_1 \approx 10$ is a constant determined by test particle simulations, ω_{cz} is the cyclotron frequency of the ions for the component of the magnetic field normal to the current sheet (B_z) and $\epsilon = \rho/L_z$ is the finite Larmor radius parameter for the ions. Ion temperatures in the central plasma sheet are in the range $T_i = 1 - 5\,\text{keV}$. For $T_i = 1\,\text{keV}$ the conductivity is $\sigma^{H-T} = 1 \times 10^{-5}\,\text{mho/m}$ which corresponds to $R_m = 330$. The Horton-Tajima conductivity is valid for vanishing B_y. In the $B_y = 0$ case the electrons are tied to the magnetic field lines and cannot move across them in the y-direction in response to the E_y field. When $B_y \neq 0$ the electrons can move along y and start playing an important role (Hernández *et al.* 1993b) in the substorm dynamics.

On the other side of the issue, however, is the evidence from the studies of Prichard and Price (1992), Prichard *et al.* (1995) that the evidence for low-dimensional nonlinear dynamics for the magnetosphere is not convincing. Prichard *et al.* (1995) arguments are based on the Borovsky *et al.* database and the issues are thoroughly debated in the reply by Klimas *et al.* (1995b).

5 Conclusions

Returning to the global view of the solar-wind driven dynamics of the magnetosphere we see that there is an important role in the global dynamics from the microscale electrical transport properties in the plasma sheet. In this high plasma pressure-to-magnetic pressure reversed field current sheet the ion orbits are large, comparable to the radius of curvature of the magnetic field line, and thus their motion is highly complex as shown in Sec. 2 through the surface of sections in Figs. 4 to 7. A direct result of the chaotic orbits is to yield a short correlation time for the transfer of energy-momentum between the electromagnetic fields and the ions. The ion-fluctuation resonance is thus broad and a technique called the velocity spectral correlation function method is used to calculate the self-consistent field dynamics (Horton-Tajima, 1990, 1991a; Hernańdez *et al.*, 1993; Horton *et al.*, 1994). The resulting conductivity is anomalously low and is of the form of a collisionless Hall conductivity. This low conductivity gives a magnetic Reynolds number comparable to that used in resistive MHD codes (Birn and Hesse, 1990) and global MHD simulations which range from 4×10^2 to 10^4 (Usadi *et al.*, 1994). In the terminology of tearing mode theory this collisionless Hall conductivity determines the growth rate through the energy transfer rate in the dissipation layer which in the magnetosphere is clearly a collisionless Hamiltonian resonance between the particles and electromagnetic fields. Such a collisionless dissipation is related to the microscopic velocity correlation function in Secs. 2.4–2.5 given through the fluctuation-dissipation relation given in Sec. 2.6.

To complete the global tearing mode theory the energy released δW_{ext} by a pinching perturbation of the current sheet must be calculated and may be expressed in terms of $\delta B_z = ik_x \delta A_y$ through $\delta W_{\text{ext}} = (\pi L_y L_z / 2\mu_0 k_x^2)|\delta B_z|^2 \Delta'_{\text{ext}}$. The calculation of the external Δ'_{ext}, or equivalently δW_{ext}, is a difficult numerical problem with different models given rather different results according to the inclusion or exclusion of effects associated with B_z/B_{x0}, B_y/B_{x0}, L_z/H and the value of $k_x L_z$. In essence, however, the global 3D MHD simulations calculate the energy available in the evolving equilibria with mass flows. Returning to the discussion in Sec. 1.2 it appears that for a northward IMF the magnetotail is sufficiently "dipolar" that $\Delta'_{\text{ext}} \sim L_x^{\text{NL}}/L_z^2$ is not large enough to overcome stabilizing influences, particularly of B_z/B_{x0}. As the IMF rotates to $\theta = \tan^{-1}(B_y/B_z) \sim \pi/2$, and clearly for $\theta \sim \pi$, the Δ'_{ext} increases dramatically due to the thinning of the current sheet. There is then a rapid growth of the tearing reconnection perturbation develops according to collisionless ion conductivity. The electrons remain magnetized except possibly at the much smaller scale of order c/ω_{pe} where they finally break loose from the magnetic field lines. In this electron microscale the plasma is decoupled from the magnetic field and electrons heat.

Other potentially unstable modes of importance are the pressure gradient driven interchanges and the kinking or flapping of the current sheet. As in the case of the laboratory FRC confinement experiments, these modes appear to be anomalously stable in comparison with the predictions of ideal MHD theory. The current view in laboratory FRC research is that the large ion orbits provide stabilizing kinetic currents. In the work of Krall-Seyler-Sudan (1991) the method of calculating the large ion orbit current is closely related to the method of Horton and his collaborators for the tearing mode. Krall *et al.* call their method the calculation of phase-space autocorrelation functions. They conclude that the decorrelation time τ_d for the exchange of energy, expressed in terms of δW, is $\tau_d \sim 10/\omega_{ci}$. Although requiring further study, this result appears consistent with the Horton-Tajima conductivity formula.

From the theories and simulations reviewed here it is clear that for the FRC system the kinetic response of the ions exerts a strong control over the large scale dynamics of the system.

In the author's view the complex and chaotic orbits may be thought of as providing a collisionless dissipation which would be expected to provide the fast, collisionless reconnection and the enhanced stability of pressure gradient driven interchange modes.

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Appendix A: Current Sheet Hamiltonian

Charged particles trapped in the current sheet have strongly nonlinear oscillations due to the reversing of the magnetic field direction as they cross the sheet. The lowest order Hamiltonian thus has the cubic nonlinear Lorentz restoring force rather than linear. The Hamiltonian for $P_y = 0$ is

$$H = \tfrac{1}{2}\, m\dot{z}^2 + \tfrac{1}{2}\, m \left(\frac{qB'_x}{2m} \right)^2 z^4 \tag{A1}$$

where the restoring force is proportional to the square of the current density

$$\mu_0 j_y(0) = \left.\frac{dB_x}{dz}\right|_{z=0} = B'_x. \tag{A2}$$

For a given energy $H = E$ the velocity at $z = 0$ is $v = \dot{z} = (2E/m)^{1/2}$ and the tuning point is $z_t = (2E/m)^{1/4}(2m/qB'_x)^{1/2} = 2^{1/2}(vL/\Omega_{x0})^{1/2}$ where introduce the scale length $L = B_{x0}/B'_x(0)$ and define the cyclotron frequency $\Omega_{x0} = qB_{x0}/m$.

It is important to know the details of the orbits of Eq. (A1) for the perturbation theory and KAM theory of the system. To find the orbits let

$$z = z_t \, \tan\theta$$

$$\dot{z} = z_t \, \sec^2\theta\dot{\theta}$$

and

$$E = \frac{m}{2}\, z_t^2 \, \sec^4\theta\dot{\theta}^2 + \frac{m}{8}\left(\frac{qB'_x}{m}\right)^2 z_t^4 \, \tan^4\theta$$

$$\frac{mz_t^2}{2E}\,(\dot{\theta})^2 = \frac{1-\tan^4\theta}{\sec^4\theta} = \frac{1-\tan^2\theta}{\sec^2\theta} = \cos^2 - \sin^2\theta = 1 - 2\sin^2\theta. \tag{A3}$$

Now, we let $\sin\phi = \sqrt{2}\,\sin\theta$ with $\theta_{\max} = \pi/4$ corresponding to $\phi_{\max} = \pi/2$ and obtain

$$\int\limits_0^\phi \frac{d\phi}{\sqrt{1-\frac{1}{2}\,\sin^2\phi}} = \int \left(\frac{4E}{mz_t^2}\right)^{1/2} dt = u.$$

The orbit is then given by the $sn(u,m)-$ Jacobi elliptic function through

$$\sin\phi = sn(u,m)$$

with the quarter period $T/4$ given by

$$\frac{T}{4} = \left(\frac{mz_t^2}{4E}\right)^{1/2} \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \frac{1}{2}\sin^2\phi}}$$

and thus angular frequency of the motion is

$$\Omega = \frac{dH}{dI} = \frac{2\pi}{T} = \frac{\pi}{2K(\frac{1}{2})}\left(\frac{4E}{mz_t^2}\right)^{1/2}$$

$$= \left(\frac{v\Omega}{L}\right)^{1/2}\left(\frac{\pi}{2K(\frac{1}{2})}\right) = 0.847\left(\frac{v\Omega}{L}\right)^{1/2}.$$

This frequency is the geometric mean of the free transit rate v/L and the cyclotron frequency at the edge $z = L$ of the current sheet.

The Fourier series expansion of their orbit and other properties with respect to the KAM theorem are given in Chirikov (1979). The action I readily obtained by integrating $I = \int_0^H dH/\Omega(H) = I_\theta H^{3/4} \propto v^{3/2}$ giving the action-angle Hamiltonian $H = (I/I_0)^{4/3}$.

References

[1] Akasofu, S.-I., The solar-wind magnetosphere energy coupling and magnetospheric disturbances, Planet. Space Sci. **28**, 495 (1980).

[2] Akasofu, S.-I., Assessing the magnetic reconnection paradigm," EoS Transactions **75**(22), 249 (1994).

[3] Ashour-Abdalla, M., L.M. Zelenyĭ, and V. Peroomian, and R.L. Richard, Consequences of magnetotail ion dynamics, J. Geophys. Res. **99**, 14,891–14,916 (1994).

[4] Baek, S., C. Choi, and W. Horton, Dawn-dusk magnetic field effects on ions accelerated in the current sheet, J. Geophys. Res. **100**, 14,935–14,942 (1995).

[5] Baker, D.N., A.J. Klimas, T.I. Pulkkinar, and R.L. McPherron, Re-examination of the driven and unloading aspects of magnetospheric substorms, Adv. Space Res. **13**, 75 (1983).

[6] Bargatze, L.F., D.N. Baker, R.L. McPherron, and E.W. Hones, Magnetospheric impulse response for many levels of geomagnetic activity, J. Geophys. Res. **90**, 6387 (1985).

[7] Barnes, D.C., J.L. Schwarzmeier, H.R. Lewis and C.E. Seyler, Phys. Fluids **29**, 2616 (1986).

[8] Birn, J., and M. Hesse, The magnetic topology of the plasmoid flux rope in a MHD-simulation of magnetotail reconnection, in *Physics of Magnetic Flux Ropes*, Geophys. Monogr. Ser. **58**, 655, AGU, Washington, D.C. (1990).

[9] Blanchard, G.T., and R.L. McPherron, A bimodal representation of the response function relating the solar wind electric field to the AL index, Adv. Space Res. **13**, (4)71 (1993).

[10] Borovsky, J.E., R.J. Nemzek, and R.D. Belian, The occurrence rate of magnetospheric substorm onsets: random and periodic substorms, J. Geophys. Res. **98**, 3897 (1993).

[11] Borovsky, J.E., R.J. Nemzek, C.W. Smith, and R.D. Belian, Periodic substorms and random substorms: driving by the solar wind, submitted to J. Geophys. Res. (1994).

[12] Brecht, S.H., J.G. Lyon, J.A. Fedder, and K. Hain, A time dependent three-dimensional simulation of the Earth's magnetosphere: Reconnection events, J. Geophys. Res. **87**, 6098 (1982).

[13] Büchner, J., and L.M. Zelenyĭ, Deterministic chaos in the dynamics of charged particles near a magnetic field reversal, Phys. Lett. A. **92**, 13,395 (1986).

[14] Chang, T., Low-dimensional behavior and symmetry breaking of stochastic systems near criticality—Can these effects be observed in space and in the laboratory?, IEEE Transactions on Plasma Sci. **20**, 691 (1992).

[15] Chen, J., and P.J. Palmadesso, Chaos and nonlinear dynamics of single-particle orbits in a magnetotail-like magnetic field, J. Geophys. Res. **91**, 1499 (1986).

[16] Chen, J., G.R. Burkhart, and C.Y. Huang, Geophys. Res. Lett. **17**, 2237 (1990).

[17] Chen, J., J.L. Rexford, and Y.C. Lee, Fractal boundaries in magnetotail particle dynamics, Geophys. Res. Lett. **17**, 1049 (1990).

[18] Chen, J., Nonlinear dynamics of charged particles in the magnetotail, J. Geophys. Res. **97**, 15,011–15,050 (1992).

[19] Chirikov, B.V., A universal instability of many-dimensional oscillator systems, Phys. Rep. **52**, 263 (1979).

[20] Cliver, E.W., Solar activity and geomagnetic storms: The first 40 years, EOS Transactions, American Geophysical Union **75**, 566 and 609 (1994).

[21] Coppi, B., G. Laval, and R. Pellat, Dynamics of the geomagnetic tail, Phys. Rev. Lett **16**, 1207 (1966).

[22] Doxas, I., W. Horton, K. Sandusky, T. Tajima, and R. Steinfolson, Numerical study of the current sheet and plasma sheet boundary layer in a magnetotail model, J. Geophys. Res. **95**, 12033 (1990).

[23] Doxas, I., T.W. Speiser, P.B. Dusenbery, and W. Horton, A proposed neutral line signature, J. Geophys. Res. **99**, 2375 (1994).

[24] Dreike, P.L., C. Eischenberger, S. Humphries, Jr., and R.N. Sudan, Production of intense proton fluxes in a magnetically insulated diode, J. Appl. Phys. **47**, 2382 (1976).

[25] Farrugia, C.J., M.P. Freeman, L.F. Burlaga, R.P. Lepping, and K. Takahashi, The earth's magnetosphere under continued forcing: Substorm activity during the passage of an interplanetary magnetic cloud, J. Geophys. Res. **98**, 7657 (1993).

[26] Fedder, J.A., and J.G. Lyon, The solar wind-magnetosphere current-voltage relationship, Geophys. Res. Lett. **14**, 880 (1987).

[27] Fedder, J.A., and J.G. Lyon, The Earth's magnetosphere is 165 R_E long: Self-consistent currents, convection, magnetospheric structure, and processes for northward interplanetary magnetic field, J. Geophys. Res. **100**, 3623–3635 (1995).

[28] Fedder, J.A., S.P. Slinker, J.G. Lyon, and R.D. Elphinstone, "Global numerical simulation of the growth phase and the expansion onset for a substorm observed by Viking," J. Geophys. Res. **100**, 19,083–19,093 (1995).

[29] Ferraro, V.C.A., "Aurorae and magnetic storms," in *The Earth and its Atmoshpere*, edited by D.R. Bates (Basic Books, Inc., Pergamon Press, 1957).

[30] Finn, J.M., and R.N. Sudan, Phys. Rev. Lett. **41**, 695 (1978).

[31] Finn, J.M., Plasma Phys. **21**, 405 (1979).

[32] Finn, J.M., and R.N. Sudan, Phys. Fluids **22**, 1148 (1979).

[33] Finn, J.M., and R.N. Sudan, Nucl. Fusion **22**, 1443 (1982).

[34] Galeev, A.A., and L.M. Zelenyĭ, Tearing instability in plasma configurations, Sov. Phys. JETP **43**, 1113 (1976).

[35] Golden, J., C.A. Kapetanakos, S.J. Marsh, and S.J. Stephanakis, Generation of 0.2-TW proton pulses, Phys. Rev. Lett. **38**, 130 (1977).

[36] Golden, J., C.A. Kapetanakos, J.A. Pasour, and R.A. Mahaffey, The generation and application of intense pulsed ion beams, Am. Sci. **69**, 130 (1981).

[37] Harris, E.G., On a plasma sheath separating regions of oppositely directed magnetic field, Nuovo Cim. **23**, 115 (1962).

[38] Hernández, J.V., W. Horton, and T. Tajima, Low-frequency mobility response functions for the central plasma sheet with applications to tearing modes, J. Geophys. Res. **98**, A4, 5893 (1993).

[39] Hernández, J., T. Tajima, and W. Horton, Neural net forecasting for geomagnetic activity, Geophys. Res. Lett. **20**, 2707 (1993b).

[40] Hill, T.W., Magnetic merging in a collisionless plasma, J. Geophys. Res. **80**, 4689 (1975).

[41] Holland, D.L., and J. Chen, On chaotic conductivity in the magnetotail, Geophys. Res. Lett. **19**, 1231–1234, (1992).

[42] Horton, W., and T. Tajima, Decay of correlations and the collisionless conductivity in the geomagnetic tail, Geophys. Res. Lett. **17**, 123 (1990).

[43] Horton, W. and T. Tajima, Collisionless conductivity and stochastic heating of the plasma sheet in the geomagnetic tail, J. Geophys. Res. **96**, 15811 (1991).

[44] Horton, W., J.V. Hernández, T. Tajima, and A. Dykhne, Fluctuation dissipation relations for plasmas in strongly inhomogeneous magnetic fields, Physica D **71**, 249 (1994).

[45] Horton, W., T. Tajima, and I. Doxas, in *Physics of Space Plasmas (1995). Number 14*, T. Chang and J.R. Jasperse, eds. (MIT Center for Theoretical Geo/Cosmo Plasma Physics, Cambridge, MA, 1996); also Horton, W., and I. Doxas, A low-dimensional energy conserving state space model for substorm dynamics, submitted to J. Geophys. Res. (1995).

[46] Huang, C.Y., L.A. Frank, G. Rostoker, J. Fennell, and D.G. Mitchell, Nonadiabatic heating of the central plasma sheet at substorm onset, J. Geophys. Res. **97**, 1481 (1992).

[47] Klimas, A.J., D.N. Baker, D.A. Roberts, D.H. Fairfield, and J. Büchner, A nonlinear analog model of geomagnetic activity, J. Geophys. Res. **97**, 12253 (1992).

[48] Klimas, A.J., D.N. Baker, D. Vassiliadis, and D.A. Roberts, Substorm recurrence during steady and variable solar wind driving: Evidence for a normal mode in the unloading dynamics of the magnetosphere," J. Geophys. Res. **99**, 14855 (1994).

[49] Klimas, A.J., D.N. Baker, D. Vassiliadis, and D.A. Roberts, J. Geophys. Res. **100**, 23,003–22,005 (1995a).

[50] Klimas, A.J., D.N. Baker, D. Vassiliadis, and D.A. Roberts, Comment on "Substorm recurrence during steady and variable solar wind driving: Evidence for a normal mode in the unloading dynamics of the magnetosphere," J. Geophys. Res. **100**, 21,995–22,001 (1995b).

[51] Klimontovich, Yu.L., *Turbulent Motion and the Structure of Chaos* (Kluwer Academic Publishers, Boston, Mass, 1991), pp. 135–183.

[52] Krall, J., C.E. Seyler, and R.N. Sudan, Kinetic stabilization of interchange modes in an axisymmetric mirror by large orbit radius thermal ions, Phys. Fluids B **3**, 1015–1025 (1991).

[53] Kubo, R., Statistical-mechanical theory of irreversible processes: 1, General theory and simple applications to magnetic conduction problems, J. Phys. Soc. Jpn. **12**, 570 (1957).

[54] Kubo, R., M. Toda, and N. Hashitsume, *Statistical Physics II* (Springer-Verlag, New York, NY, 1985), pp. 167–174.

[55] Lovelace, R.V., Phys. Rev. Lett. **35**, 162 (1975).

[56] Lovelace, R.V., Phys. Fluids **19**, 723 (1976).

[57] Lyons, L.R., and T.W. Speiser, Evidence for current sheet acceleration in the geomagnetic tail, J. Geophys. Res. **87**, 2276 (1982).

[58] Lyons, L.R., and D.J. Williams, *Quantitative Aspects of magnetospheric Physics* (D. Reidel Publishing Company, AA Dordrecht, Holland, 1984).

[59] Lyons, L.R. and T.W. Speiser, Ohm's law for a current sheet, J. Geophys. Res. **90**, 8543 (1985).

[60] McPherron, R.L., and G. Blanchard, private communication on AL/VBs databases, 1994.

[61] Perrault, P., and S.-I. Akasofu, A study of magnetic storms, Geophys. J. R. Astron. Soc. **54**, 547 (1978).

[62] Prichard, D., and C.P. Price, Spurious dimension estimates from time series of geomagnetic indices, Geophys. Res. Lett. **19**, 1623 (1992).

[63] Prichard, D., J.E. Borovsky, and C.P. Price, Comments on "Substorm recurrence during steady and variable solar wind driving: Evidence for a normal mode in the unloading dynamics of the magnetosphere," J. Geophys. Res. **100**(21), 995 (1995).

[64] Rej, D.J., D.P. Taggart, M.H. Baron, R.E. Chrien, R.J. Gribble, M. Tuszewski, W.J. Waganaar, and B.L. Wright, High power magnetic-compression heating of field-reversed configurations, Phys. Fluids B **4**(7), 1909–1919 (1992).

[65] Sharma, A.S., D.V. Vassiliadis, and K. Papadapoulous, Reconstruction of low-dimensional magnetospheric dynamics by singular spectrum analysis, Geophys. Res. Lett. **20**, 335 (1993).

[66] Sitenko, A.G., *Fluctuations and Nonlinear Wave Interaction in Plasmas* (Pergamon Press, Oxford, 1982).

[67] Speiser, T.W., Particle trajectories in model current sheets, J. Geophys. Res. **70**, 4219 (1965).

[68] Speiser, T.W., J. Geophys. Res. **72**, 3919 (1967).

[69] Spicer, D.S., S.T. Zalesak, R. Löhner, and S. Curtis, Simulation of the magnetosphere with a three-dimensional MHD code and adaptive mesh refinement: Preliminary results, preprint (1996).

[70] Sudan, R.N., and M.N. Rosenbluth, Phys. Rev. Lett. **36**, 972 (1976).

[71] Sudan, R.N., and M.N. Rosenbluth, Phys. Fluids **22**, 282 (1979).

[72] Symon, K.R., C.E. Seyler, and H.R. Lewis, J. Plasma Phys. **27**, 13 (1982).

[73] Tsyganenko, N.A., Global quantitative models of the geomagnetic field in the cislunar magnetosphere for different disturbance levels, Planet Space Sci. **35**, 1347 (1987).

[74] Tsyganenko, N.A., A magnetospheric magnetic field model with a warped tail current sheet, Planet Space Sci. **37**, 5 (1989).

[75] Usadi, A., A. Kageyama, K. Watanabe, and T. Sato, A global simulation of the magnetosphere with a long tail: Southward and northward interplanetary magnetic field, J. Geophys. Res. **98**, 7503–7517 (1993).

[76] Usadi, A., R.A. Wolf, M. Heinemann, and W. Horton, does chaos alter the ensemble averaged drift equations?, J. Geophys. Res. **101**, to appear (1996).

[77] Vassiliadis, D., A.J. Klimas, D.N. Baker, and D.A. Roberts, A description of the solar wind-magnetosphere coupling based on nonlinear filters, J. Geophys. Res. **100**, 3495-3512 (1995).

[78] Walker, R.J., and T. Ogino, Global magnetohydrodynamic simulations of the magnetosphere, IEEE Trans. Plasma Sci. **17**, 135 (1989).

[79] Yoon, P.H., and A.T.Y. Lui, Nonlocal ion-Weibel instability in the geomagnetotail, J. Geophys. Res. **101**, 4899 (1996).

Figure Captions

1. The magnetic field lines forming the Earth's magnetosphere as given by the widely used magnetic field model Tsyganenko (1987) based on satellite data. The solar wind from the left stretches the dipole field out into the extended magnetotail to the right.

2. Equatorial and meridian cross-sections showing the plasma flow lines and magnetic field lines over a 1.5-hour period for the case when a southward IMF is present in the solar wind. Because of reconnection in the plasma sheet, a plasmoid develops and is accelerated toward the tail boundary. Note that Figs. 2(a)-(d) are drawn for $-90R_E \leq x \leq 20R_E$, $0 \leq y \leq 40R_E$, and $0 \leq z \leq 40R_E$, while actual simulations were executed for $-95R_E \leq x \leq 30R_E$, $0 \leq y \leq 50R_E$, and $0 \leq z \leq 50R_E$.

3. The effective potential $V_{p_y}(z)$ controlling the motion of a charged particle in the presence of a nonuniform magnetic field. (a) the shape of the effective potential for the case of a reversed field for $qp_y > 0$. (b) the effective potential for the case of particles with $qp_y < 0$. (c) the phase space diagrams for the Hamiltonian flow for case (b) with the phase space separatrix SX separating the crossing and non-crossing orbits. The critical energy is $H_c = p_y^2/2m$.

4. Surface of section plot for the reversed field configuration with $b_z = 0.05$ and $\kappa = 0.18$ ($\widehat{H}_{\rm CP} = 500$) and $B_y/B_z = 0$.

5. Surface of section plot for the reversed field configuration with $b_z = 0.05$ and $\kappa = 0.18$ ($\widehat{H}_{\rm CP} = 500$) and $B_y/B_z = 1$.

6. Surface of section plot for the reversed field configuration with $b_z = 0.05$ and $\kappa = 0.18$ ($\widehat{H}_{\rm CP} = 500$) and $B_y/B_z = 5$.

7. Three-dimensional perspective views for examples of the three orbit classes for the current sheet Hamiltonian with $\kappa = 0.18$ and $b_z = 0.05$ and $B_y/B_z = 0$. (a) a ring orbit, (b) a quasi-trapped or stochastic orbit and (c) a transient or Speiser orbit.

8. Conductivity for unmagnetized plasma. (a) transverse and (b) longitudinal components of the dissipative part of the conductivity for perturbations propagating along the x-axis. In both plots the solid curves represent the theoretical conductivities (85) and the impulse lines represent the conductivities from microscopic fluctuations obtained numerically with the SVC formalism of Sec. 4.2. The results were generated by launching $N = 3000$ particles distributed according to a Maxwellian distribution in velocity and uniformly distributed along the x-axis from $x = 0$ up to $x = 2\pi/k$.

9. Conductivity for the Harris sheet: $|z/L_z| < 1$.(a) Transverse component of the dissipative part of the conductivity (Re σ_{yy}) produced by the microscopic fluctuations from charged particles trapped in the plasma sheet and (b) blow up of the low-frequency portion of Re σ_{yy}.

10. Conductivity for the Harris sheet: $3 \le z/L_z \le 4$. (a) transverse component of the dissipative part of the conductivity (Re σ_{yy}) produced by the microscopic fluctuations from charged particles away from the plasma sheet, that is, well into the lobe region where the magnetic field is nearly uniform, $\mathbf{B}(z) \approx B_{x0}\widehat{\mathbf{e}}_x$; (b) blow up of the low-frequency portion of Re σ_{yy}. Note the difference between the low frequency portions of the transverse conductivity for the inhomogeneous case (Fig. 9(b)) and for the homogeneous case (Fig. (10(b)). This is an example of the enhancement of the low frequency spectrum due to inhomogeneities in the plasma.