Lecture # 28

Drift Waves

Electro - static Bernstein Waves

For simplicity $F = \frac{n_0}{(2\pi V_{thj}^2)^{3/2}} \exp\left[-\frac{V^2}{2V_{thj}^2}\right]$

$$m V_{thj}^2 = T_j$$

For electron response (ion mass $\to \infty$)

$$D(\omega, k) = k^2 + \omega_p^2 \int \frac{d^3v}{n_0} \frac{\partial F}{\partial V^2/2} \left[1 - \sum_p \frac{J_p^2\left(\frac{k_\perp v_\perp}{\omega_c}\right)\omega}{\omega - k_z v_z - n\omega_c}\right] = 0$$

$$= k^2 + \frac{\omega_p^2}{V_{the}^2}\left[1 - \omega \sum_p \frac{I_p(b)e^{-b}}{(2\pi)^{1/2} V_{the}} \int \frac{dv_z \exp(-v_z^2/2V_{th}^2)}{\omega - k_z v_z - p\omega_c}\right]$$

$$\xrightarrow{k_z \to 0} \quad k^2 + \frac{\omega_{pe}^2}{V_{the}^2}\left[1 - \omega \sum_p \frac{I_p(b)e^{-b}}{\omega - p\omega_c}\right] = 0$$

$$b = \frac{k_\perp^2 V_{the}^2}{\omega_{ce}^2} = k_\perp^2 \rho_e^2 \qquad (\rho_e = V_{the}/\omega_{ce})$$

What happened to Landau damping?

In the limit $k_\perp^2 \rho_e^2 \gg 1$, $\frac{\omega}{\omega_c} \gg 1$
one can show

$$D(\omega, \underset{\sim}{k}) = D_R(\omega, \underset{\sim}{k}; \omega_{ce}=0) - \cot\left(\frac{\pi\omega}{\omega_{ce}}\right) D_I(\omega, k, \omega_{ce})$$

$$\cot\left(\frac{\pi\omega}{\omega_{ce}}\right) = \frac{i e^{i\pi\omega/\omega_{ce}} + e^{-i\pi\omega/\omega_{ce}}}{e^{i\pi\omega/\omega_{ce}} - e^{-i\pi\omega/\omega_{ce}}}$$

$$\xrightarrow[\mathrm{Im}\left(\frac{\pi\omega}{\omega_{ce}}\right) \gg 1]{} -i$$

Here

$D(\omega, k, \omega_c = 0)$

$= D_R(\omega, k, \omega_{ce} = 0) + i\, D_I(\omega, k, \omega_{ce} = 0)$

$$= k^2 + \frac{4\pi e^2}{m} \int d^3v \frac{\underset{\sim}{k} \cdot \frac{\partial F}{\partial \underset{\sim}{v}}}{\omega - \underset{\sim}{k} \cdot \underset{\sim}{v}}$$

For Maxwellian, $\underset{\sim}{k} = k_z \hat{\underset{\sim}{z}}$

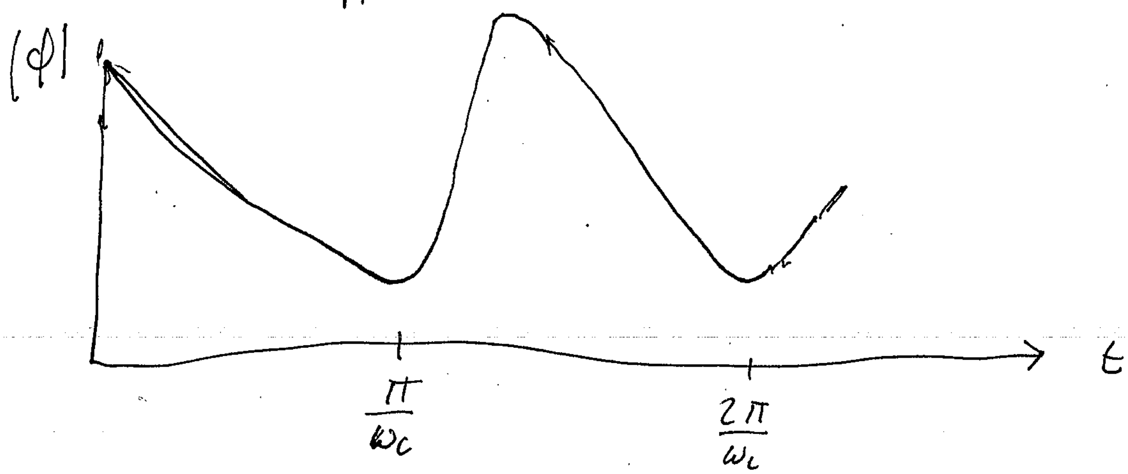
$$= k^2 + \frac{\omega_{pe}^2}{v_{the}^2} - \frac{\omega_{pe}^2}{v_{the}^2}\, \omega \int \frac{dv_z\, e^{-v_z^2/2v_{th}^2}}{\omega - k_z v_z}$$

What does structure of magnetic field dispersion mean?

$$\phi_{\underset{\sim}{k}}(t) = \int_C \frac{d\omega\, e^{-i\omega t}\, S(\omega, \underset{\sim}{k})}{D(\omega, k_z, k_\perp, \omega_{ce} \neq 0)}$$

As long as $\omega_c t \ll 1$, we can accurately evaluate integral with $\mathrm{Im}\, C > \omega_{ce}$, and $D(\omega, k_z, k_\perp, \omega_{ce} \neq 0) \simeq D(\omega, k_z, k_\perp, \omega_{ce} = 0)$, and time response nearly the same as in zero B-field. But when $\omega_{ce} t \sim 1$, apparent wave damping reconstitutes:

Suppose $\gamma_d >> \omega_{ce}$



This is also true when $\frac{k_\parallel v_{the}}{\omega_{ce}}$ is finite (but small) except that reconstitution is not perfect.

If many resonances $\sim \frac{\omega - p\omega_{ce}}{k_z}$ for $p >> 1$, response nearly the same as $B = 0$ case.

Let us now consider low frequency oscillations, keeping density gradient. Take Maxwellian ion and electron distribution $T_e \neq T_i$, $\frac{\omega}{\omega_{ci}} \ll 1$

$$F_j = \frac{n_0(x)}{(2\pi V_{thj})^{3/2}} \exp\left(-\frac{v^2}{2V_{thj}^2}\right) \qquad T_j = m_j V_{thj}^2$$

$$T = T(x)$$

$$\frac{\partial F_j}{\partial X} = \left[\frac{\partial n_0}{n_0 \partial X} - \frac{\partial T}{T \partial X}\left(\frac{3}{2} - \frac{V^2}{2V_{the}^2}\right)\right] F_0 \; ; \quad \boxed{\eta = \frac{\frac{\partial T}{T\partial X}}{\partial n_0/\partial x / n_0} = \frac{\partial \ln T}{\partial \ln n_0}}$$

Only $p=0$ necessary

$$0 = D(\omega,k) = k^2 + \sum_s \frac{\omega_{ps}^2}{n_0} \int d^3v \left[\sum_p \frac{\left(\frac{\partial F_s}{\partial V^2/2} + \frac{k_y}{\omega_{cj}}\frac{\partial F_s}{\partial X_g}\right) J_p^2\left(\frac{k_\perp V_\perp}{\omega_{cj}}\right)}{\omega - k_\parallel V_\parallel - p\omega_{cj}} - \frac{\partial F_s}{\partial V_\perp^2/2}\right]$$

$$\simeq k^2 + \sum_s \frac{\omega_{ps}^2}{V_{ths}^2}\left(1 - \int \frac{\left(\omega - \omega_s^*\left(1 + \eta_s\left(\frac{V^2}{2V_{ths}^2} - \frac{3}{2}\right)\right)\right) d^3v\, e^{-V^2/2V_{ths}^2} J_0^2\left(\frac{k_\perp V_\perp}{\omega_c}\right)}{(\omega - k_\parallel V_\parallel)\,(2\pi V_{ths}^2)^{3/2}}\right)$$

$$\eta_s = \frac{n\, \partial T_s/\partial x}{T_s\, \partial n_0/\partial x}$$

$$\omega_j^* = \frac{-k_y \frac{\partial n_0}{\partial X} V_{thj}^2}{n_0\, \omega_{cj}} \; ; \quad \omega_e^* = -\frac{T_e}{T_i}\omega_i^*$$

$$1 - J_0^2 \simeq \frac{k_\perp^2 v_\perp^2}{2\omega_c^2} \quad \text{if} \quad k_\perp^2 \rho_s^2 \ll 1$$

assume $\quad k_\parallel v_{thi} \ll \omega \ll k_\parallel v_{the}$

For electrons we obtain $J_0\left(\frac{k_\perp v_\perp}{\omega_c}\right) \simeq 1$

$$\frac{\omega_{pe}^2}{v_{the}^2}\left(1 - i\pi \int \frac{d^2 v_\perp \, e^{-v_\perp^2/2v_{the}^2}}{(2\pi v_{the}^2)^{3/2} |k_z|}\left(\omega - \omega_e^*\left[1 + \eta\left(\frac{v_\perp^2}{2v_{th}^2} - \frac{3}{2}\right]\right)\right)$$

Do $v_\perp^2$ integration

$$\frac{\omega_{pe}^2}{v_{the}^2}\left(1 - \frac{i\pi}{(2\pi v_{the}^2)^{1/2}|k_z|}\left[\omega - \omega_e^*(1 - \eta/2)\right]\right)$$

For ions we have

$$\frac{\omega_{pi}^2}{v_{thi}^2} \int \frac{d^3 v \, e^{-v^2/2v_{thi}^2}}{(v_{thi})^3 (2\pi)^{3/2}}\left(1 - \frac{J_0^2 \left[\omega - \omega_*^i\left(1 + \eta_i\left(\frac{v^2}{v_{thi}^2} - \frac{3}{2}\right)\right)\right]}{(\omega - k_z v_z)}\right)$$

Assume $\quad k_z v_{thi}/\omega \ll 1 \quad , \quad k^2 \rho_i^2 \ll 1$

$$\frac{\omega_{pi}^2}{v_{thi}^2}\left[-\int \frac{d^3 v}{v_{thi}^3} \frac{\exp\left(-\frac{v^2}{2v_{thi}^2}\right)}{(2\pi)^{3/2}}\right]\left[\frac{k_\perp^2 v_\perp^2}{2\omega_{ci}^2} + \frac{k_z^2 v_z^2 (\omega - \omega_{Ti}^*)}{\omega^2 \omega} + \frac{\omega_i^*}{\omega}\left(1 + \eta_i\left(\frac{v^2}{2v_{thi}^2} - \frac{3}{2}\right)\right)\right]$$

$$\boxed{\omega_{Ti}^* = \omega_i^*\left(1 - \eta_i\left(\frac{v^2}{v_{thi}^2} - \frac{3}{2}\right)\right)}$$

Ion term continued:

η_i cancels out of this term

$$+\frac{\omega_{pi}^2}{\omega_{c0}^2}k_\perp^2 - \frac{\omega_i^*}{\omega} - \frac{k_z^2 V_{thi}^2}{\omega^2}\left(1-\omega_i^*(1+\eta_i)\right)$$

Thus dispersion relation is

$$0 = k^2\lambda_{De}^2 + 1 + \frac{i(\pi)^{1/2}\left(\omega - \omega_e^*(1-\eta_e/2)\right)}{\sqrt{2}\,V_{the}|k_z|}$$

$$+\tau\left(k_\perp^2 a_i^2 + \frac{\omega_i^*}{\omega} - \frac{k_z^2 V_{thi}^2}{\omega^2}\left(1-\frac{\omega_i^*}{\omega}(1+\eta_i)\right)\right)$$

assume $k_\perp^2\lambda_{De}^2 \ll 1$, $\rho_s^2 = a_i^2 T_e/T_i$

$$\frac{\omega_e^*}{\omega} = 1 + k^2\lambda_{De}^2 + k_\perp^2\rho_s^2 - \frac{k_z^2 V_{thi}^2}{\omega^2}\tau\left(1+\frac{\omega_e^*}{\tau\omega}(1+\eta_i)\right)$$

$$+ i\left(\frac{\pi}{2}\right)^{1/2}\frac{\left[\omega - \omega_e^*(1-\eta_e/2)\right]}{|k_z|\,V_{the}}$$

Invert using corrections to $\frac{\omega}{\omega_e^*} - 1$ are small. $k_\parallel \ll k_\perp$,

$$\omega = \omega_e^*\left(1 - k_\perp^2(\lambda_{De}^2+\rho_s^2) + \frac{k_z^2 V_{thi}^2 \tau}{\omega_e^{*2}}\left(1+\frac{1}{\tau}(1+\eta_i)\right)\right.$$

$$\left. + i\left(\frac{\pi}{2}\right)^{1/2}\frac{\omega_e^{*2}}{|k_z|\,V_{the}}\left(k_\perp^2(\lambda_{De}^2+\rho_s^2) - \frac{k_z^2 V_{thi}^2}{\omega_e^{*2}}(\tau + (1+\eta_i)) - \eta_e/2\right)\right)$$

$$\frac{\omega}{\omega_e^*} = 1 - k_\perp^2 \rho_s^2 + \frac{k_z^2 v_{thi}^2}{\omega_e^{*2}} \left(\tau + \omega_e^*(1+\eta_i)\right)$$

$\Leftarrow$ destabilizing $\quad \Leftarrow$ stabilizing

$$+ i \left(\frac{\pi}{2}\right)^{1/2} \frac{\omega_e^*}{|k_z| v_{the}} \left\{ k_\perp^2 (\lambda_{De}^2 + \rho_s^2) - \frac{k_z^2 v_{thi}^2}{\omega_e^{*2}} \left(\tau + (1+\eta_i)\right) - \eta_e/2 \right\}$$

$\Leftarrow$ stabilizing

electron density gradient instability.

Arises from inverse Landau damping, because phase velocity of wave is less than diamagnetic drift velocity.

Distribution is "inverted" in energy by the diamagnetic drift velocity: $-\frac{v_{the}^2}{\omega_{ce} n_0} k_y \frac{\partial n_0}{\partial x}$