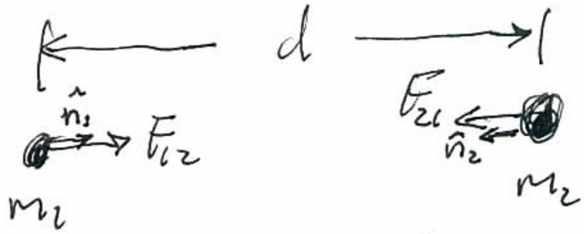
Law of gravitation

F



$$\vec{F}_{12} = -\vec{F}_{21} = \frac{G m_1 m_2}{d^2} \hat{n}$$

Gravitational potential energy

$$U = -\frac{m_1 m_2 G}{d}$$

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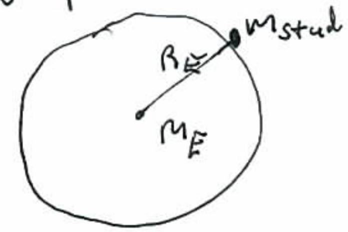
g on Earth

Newton first guessed it, but took years to prove relationship.



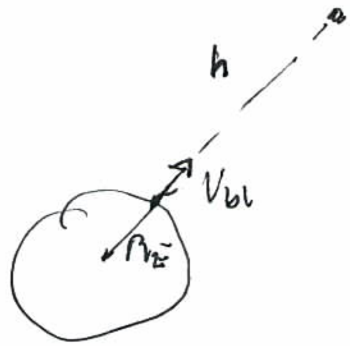
$$F = \frac{G m_{stud} M_E}{R_E^2} = g\, m_{stud}$$

$$g_E = \frac{G M_E}{R_E^2} = 9.81 \text{ m/s}^2$$

$$g_M = \frac{G M_{moon}}{R_M^2} = 1.6 \text{ m/s}^2$$

Maximum height &

Escape velocity



$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2} m v_{bi}^2 - \frac{GmM_E}{R_E} = -\frac{GmM_E}{(R_E+d)}$$

$$\boxed{\cancel{GM_e\left(\frac{1}{R_E^2} - \frac{1}{(R_E+d)^2}\right) = \frac{v_{bi}^2}{2}}}$$

$$\frac{1}{(R_E+d)} = \frac{1}{R_E} - \frac{v_{bi}^2}{2GM_E}$$

$$\frac{1}{1+d/R_E} = 1 - \frac{v_{bi}^2 R_E}{2GM_E}$$

$$1 + d/R_E = \frac{1}{1 - \frac{v_{bi}^2 R_E}{2GM_E}}$$

$$\boxed{\frac{d}{R_E} = \frac{v_{bi}^2 R_E/2GM_E}{1 - \frac{v_{bi}^2 R_E}{2GM_E}}} \qquad (2)$$

$$d = \frac{\frac{V_{bi}^2}{2}/g}{1 - \frac{V_b^2}{2 g R_E}}$$

escape velocity

$$V_{esc}^2 = 2 g R_E$$

$$\simeq 2 \times 10 \times 10^6 \times 6$$

$$V_{esc} \simeq 1 \times 10^4 \text{ m/s} \simeq 10 \text{ km/s}$$

~~If V~~

Compare escape velocity with satellite going around earth

$$m_s \frac{V_{st}^2}{R_E} = \frac{G m_s M_E}{R_E^2}$$

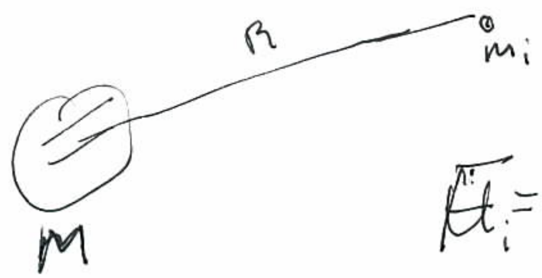
$$V_{st}^2 = \frac{G M_s}{R_E}$$

$$V_{esc}^2 = \frac{2 G M_s}{R_E}$$

$$\boxed{V_{esc} = \sqrt{2}\, V_{st}}$$

Gravitational Field:

Gravitational ~~potent~~ Force



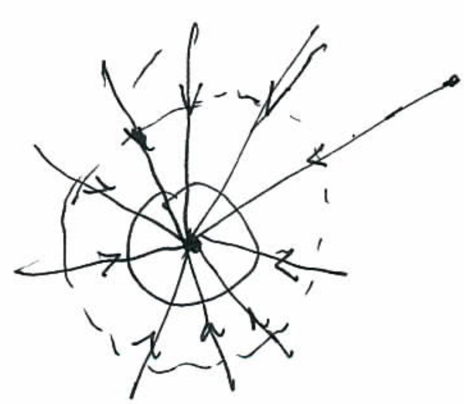
$$F_i = \frac{G M m_i}{R^2}$$

$$F_j = \frac{G M m_j}{R^2}$$

What is then in the absence of 'test' masses m_i

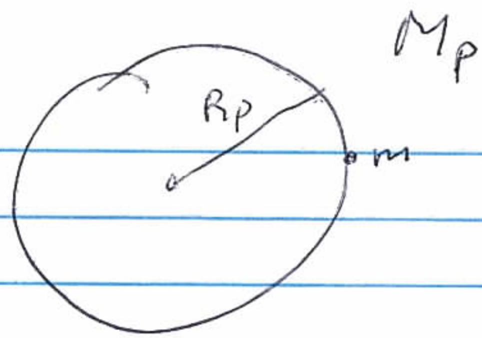
The gravitational field

$$\vec{g} = -\frac{GM}{R^2}\hat{r}$$



field lines point towards mass source

1.



Given a planet of mass M_p and radius R_p, the force on a body of mass m, on the planet's surface is,

(a) $\frac{G M_p m}{R_p}$ (b) $\frac{G M_p m}{R_p^2}$ (c) $\frac{G M_p m}{R_p^3}$

(2) The acceleration of gravity, g_p, on the planet's surface is $g_p =$

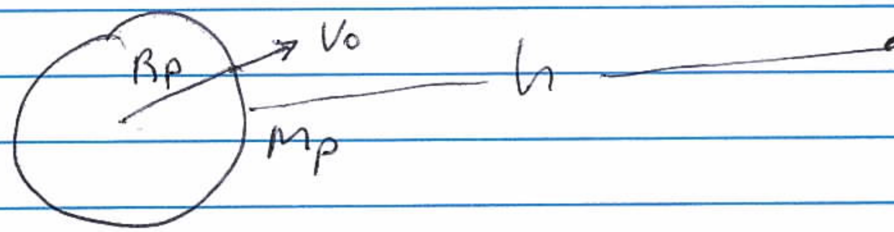
(a) $\frac{G M_p}{R_p}$ (b) $\frac{G M_p}{R_p^2}$, (c) $\frac{G M_p}{R_p^3}$

(3) The accelation of gravity at $R = 3R_p$ in terms of g_p (at planet's surface) is

(a) g_p (b) $\frac{g_p}{4}$ (c) $\frac{g_p}{9}$ (d) $\frac{g_p}{16}$ (1/

How High do we go?

How fast do we leave?



$$ME = KE + PE = \text{const}$$

$$KE_i + PE_i = KE_f + PE_f$$

(1) Do we leave?

If we leave $PE \to 0$ as $r \to \infty$

$$ME = \frac{1}{2} m V_f^2 > 0$$

If we leave

$$\frac{1}{2} m V_0^2 - \frac{G M_p m}{R_p} > 0$$

$$V_0^2 > \frac{2 G M_p}{R_p} \equiv V_{esc}^2$$

If we leave then ~~escape sp~~ final exiting speed is $V = V_\infty$ $\left(\begin{array}{c}\text{when} \\ r \to \infty\end{array}\right)$ (2)

$$ME_i = ME_f$$

$$\frac{1}{2} m V_0^2 - \frac{G m M_p}{R_p} = \frac{1}{2} m V_\infty^2 = 0$$

(if $R_p \to \infty$)

$$\therefore \quad V_\infty^2 = V_0^2 - \frac{2 G M_p}{R_p}$$

$$V_\infty^2 = V_0^2 - 2 g_p R_p$$

How High $\quad$ If $V^2 < V_{esc}^2 = 2 g_p R_p = \frac{2 M_p G}{R_p}$

$$\frac{1}{2} m V_0^2 - \frac{G m M_p}{R_p} = \underset{0}{\underset{\parallel}{KE_f}} - \frac{G m M_p}{R_p + h}$$

solve to obtain:

$$\frac{h}{R_p} = \frac{+V_0^2/2g_pR_p}{1 - V_0^2/2g_pR_p} = \frac{+V_0^2/V_{esc}^2}{1 - V_0^2/V_{esc}^2}$$