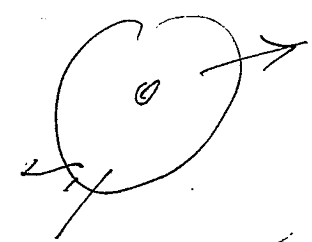
Lecture # 29

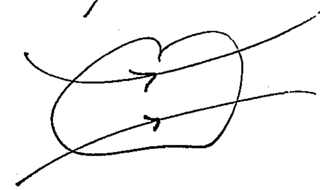
Maxwell's Equations

GAUSS' LAW, GAUSS' LAW FOR MAGNETISM, FARADAY'S LAW, AND MAXWELL-AMPERE LAW

$$\Phi_E \equiv \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_\perp \, dA = \frac{Q_{\text{inside}}}{\epsilon_0}$$

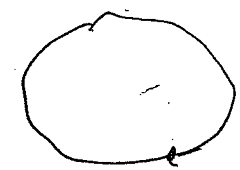


$$\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = \oint B_\perp \, dA = 0$$

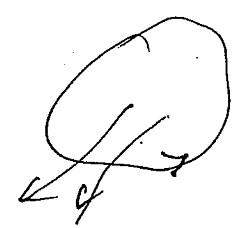


Fara

$$\mathcal{E}_{m} = \oint \mathbf{E} \cdot d\mathbf{s} = \oint E_\parallel \, ds = -\frac{d\Phi_B}{dt}$$



$$\oint \mathbf{B} \cdot d\mathbf{s} = \oint B_\parallel \, ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

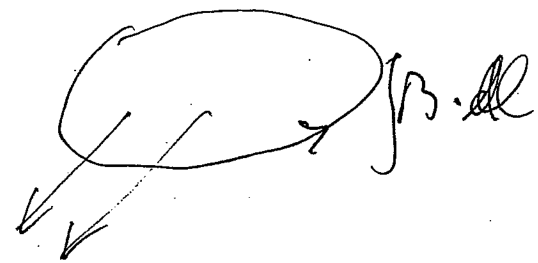


Ampere's

added by Maxwell

Equation 33-4 derivatives Physics for Engineers and Scientists 3/e

DISPLACEMENT CURRENT



$$I_{\text{displacement}} = \epsilon_0 \frac{d\Phi_E}{dt}$$

Equation 33-4 derivatives b Physics for Engineers and Scientists 3/e

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_m$$

$$= -\frac{d}{dt} \oint \vec{B} \cdot d\vec{A}$$

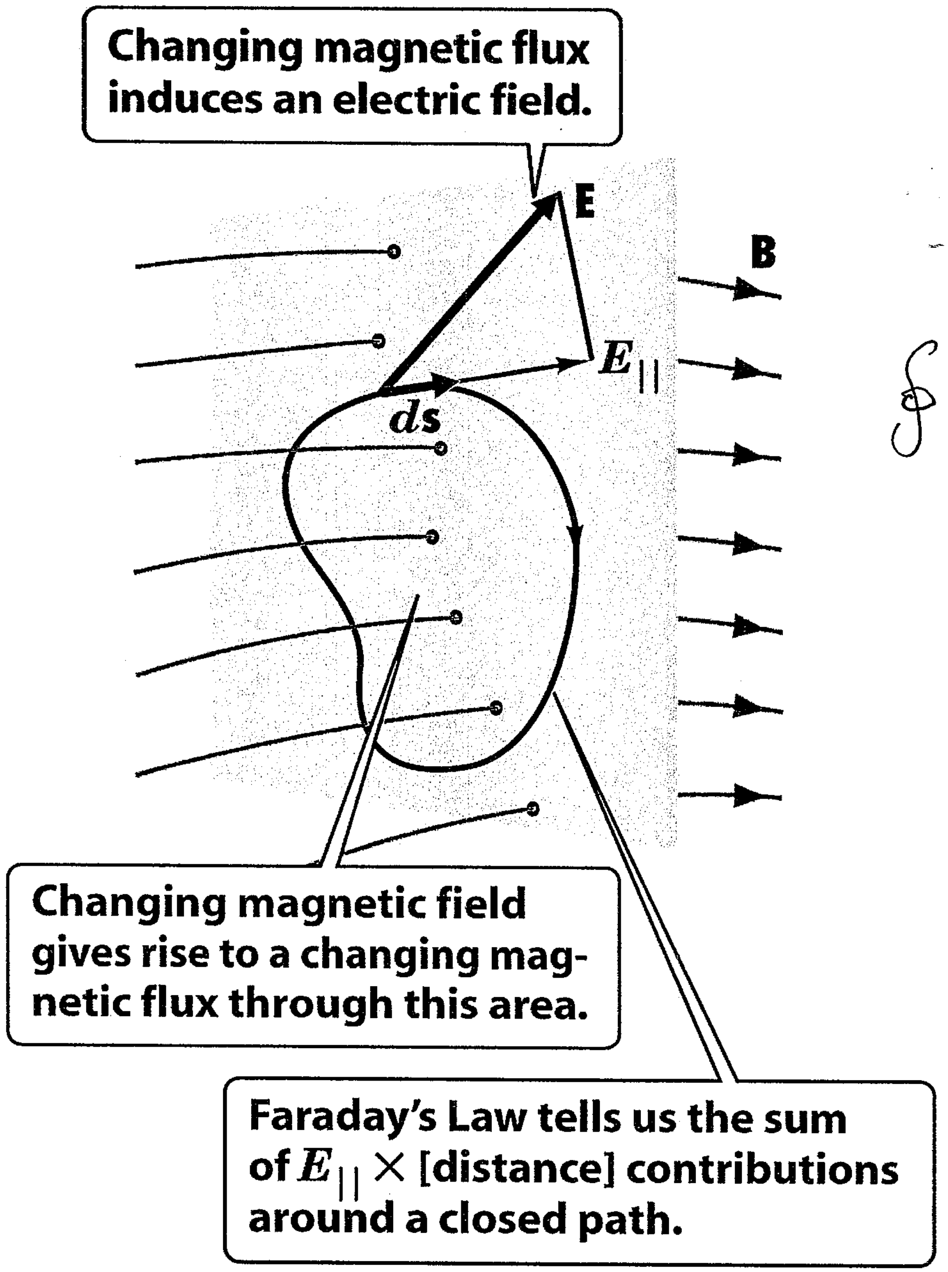


Figure 33-1 Physics for Engineers and Scientists 3/e

(a) True

(b) False

Magnetic Energy can be stored in a capacitor

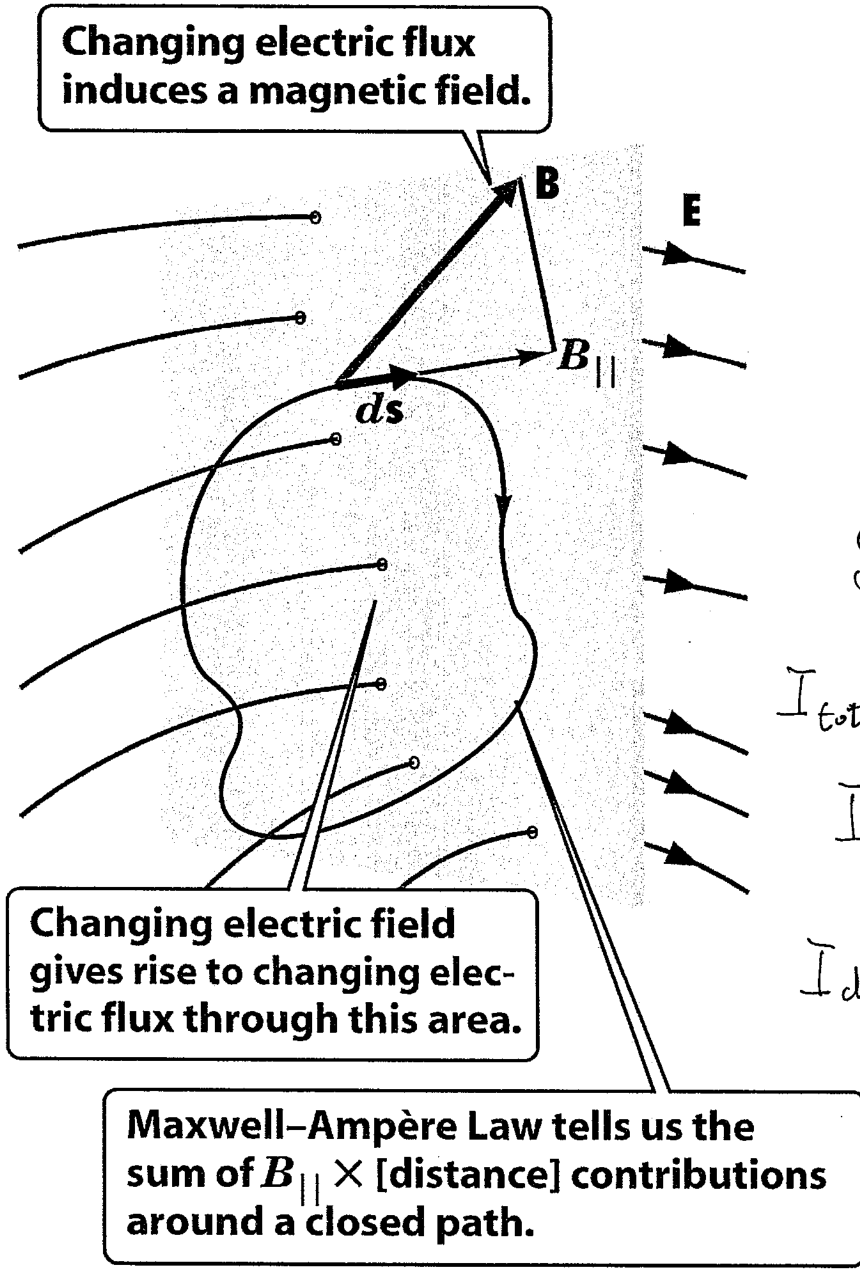


Figure 33-2 Physics for Engineers and Scientists 3/e

<u>Maxwell- Ampere Law</u>

$$\int \vec{B}\cdot d\vec{l} = \mu_0 I_{total}$$

$$I_{total} = I_{chgconv} + I_{displacement}$$

$$I_{chgconv} = \frac{dq}{dt}$$

$$I_{displacement} = \epsilon_0 \frac{d\Phi_E}{dt}$$

$$= \epsilon_0 \frac{d}{dt} \int \vec{E}\cdot d\vec{A}$$

Find magnetic energy density in this capacitor

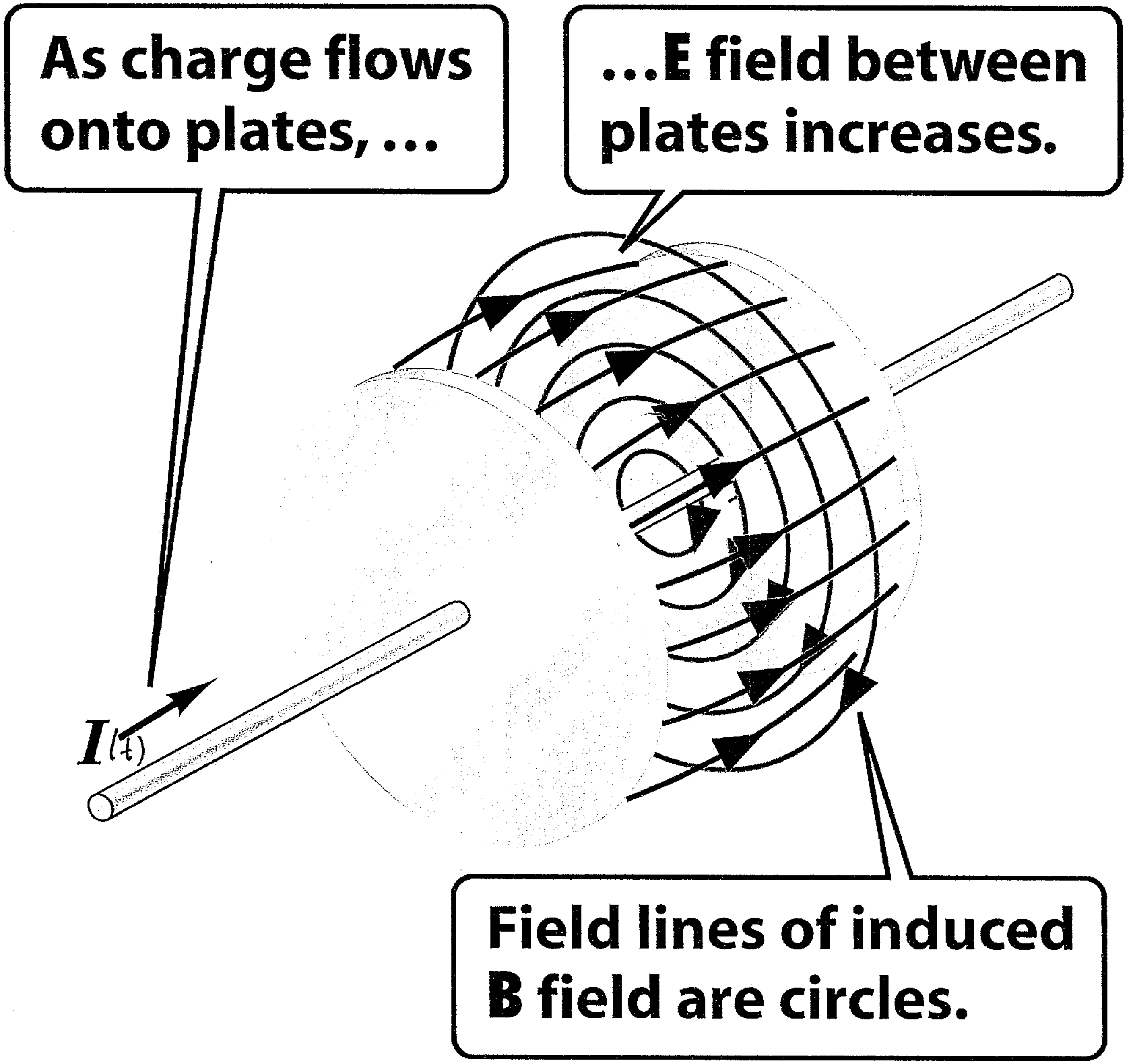


Figure 33-3 Physics for Engineers and Scientists 3/e

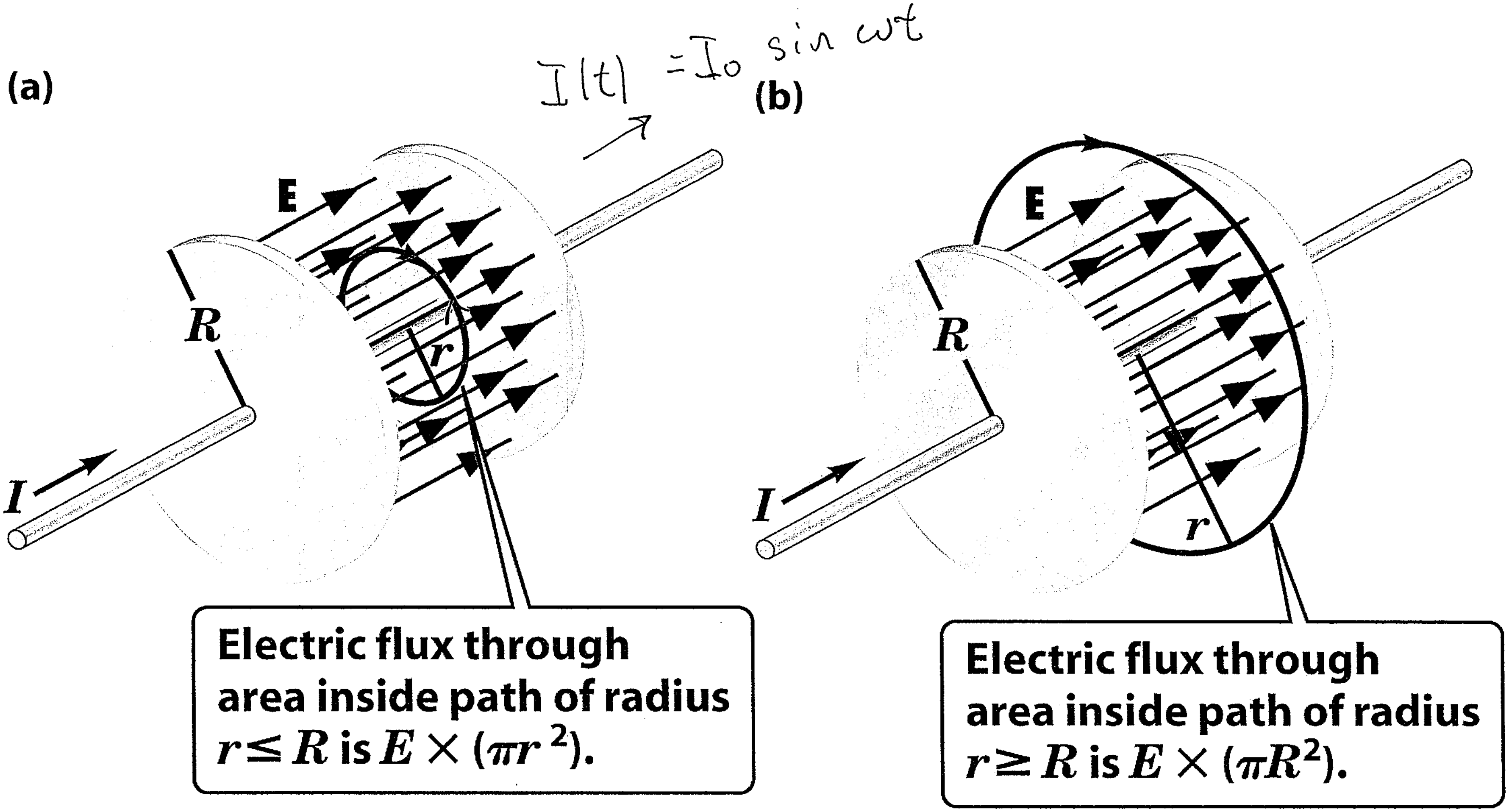


Figure 33-4 Physics for Engineers and Scientists 3/e

$$\oint B\,dl = B(r)\,2\pi r = \frac{d}{dt}\left(\frac{Q}{\epsilon_0 A(R)}\right) A(r) \overset{\mu_0\epsilon}{=} ①$$

$$=$$

$$\mu_0\epsilon \int dA \frac{\partial E}{\partial t} = \pi r^2 \frac{\partial E}{\partial t}$$

$$A = \pi R^2$$

$$A(r)\ \pi r^2$$

$$E = \frac{Q}{\epsilon_0}$$

(4)

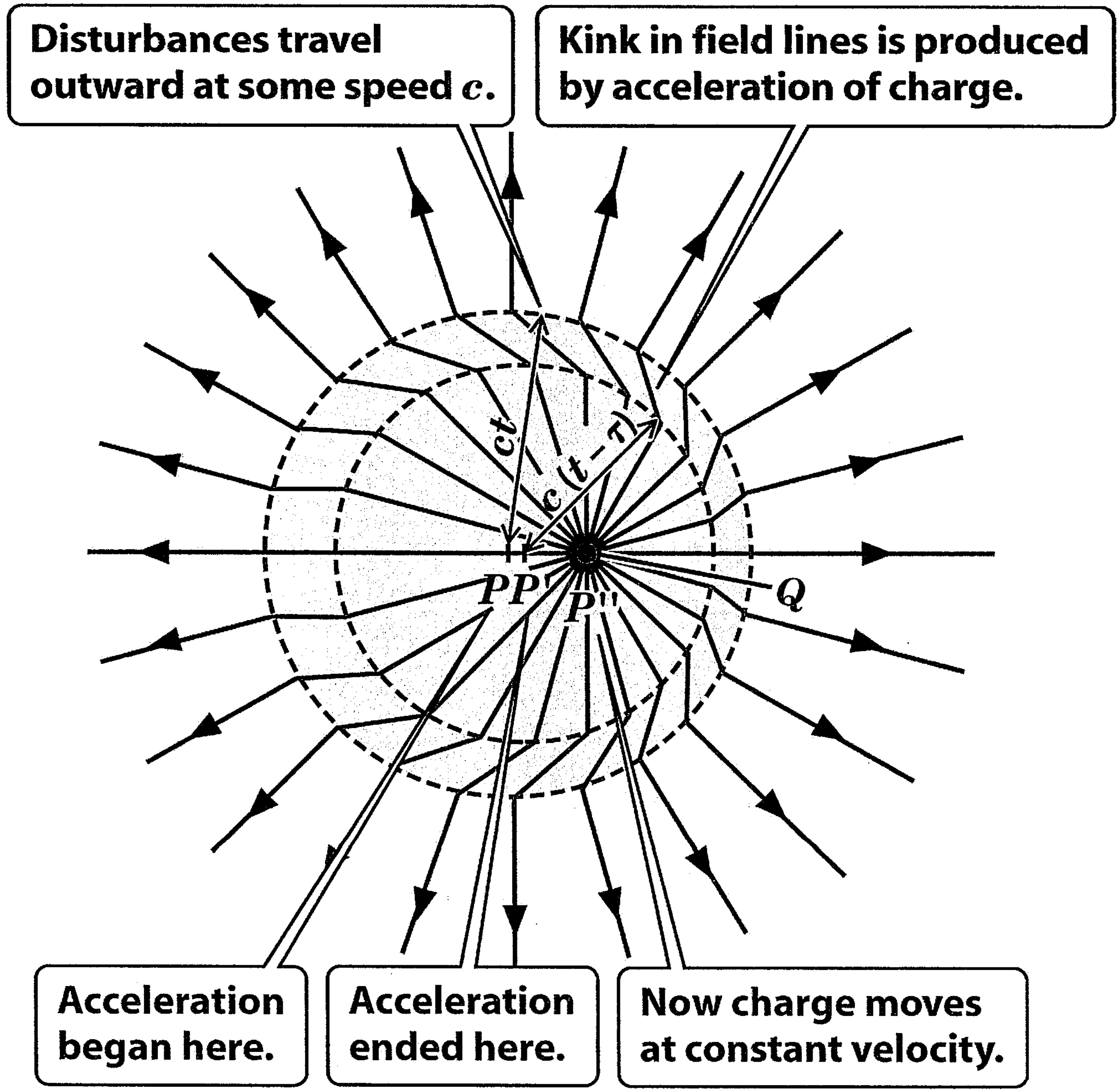


Figure 33-5 Physics for Engineers and Scientists 3/e

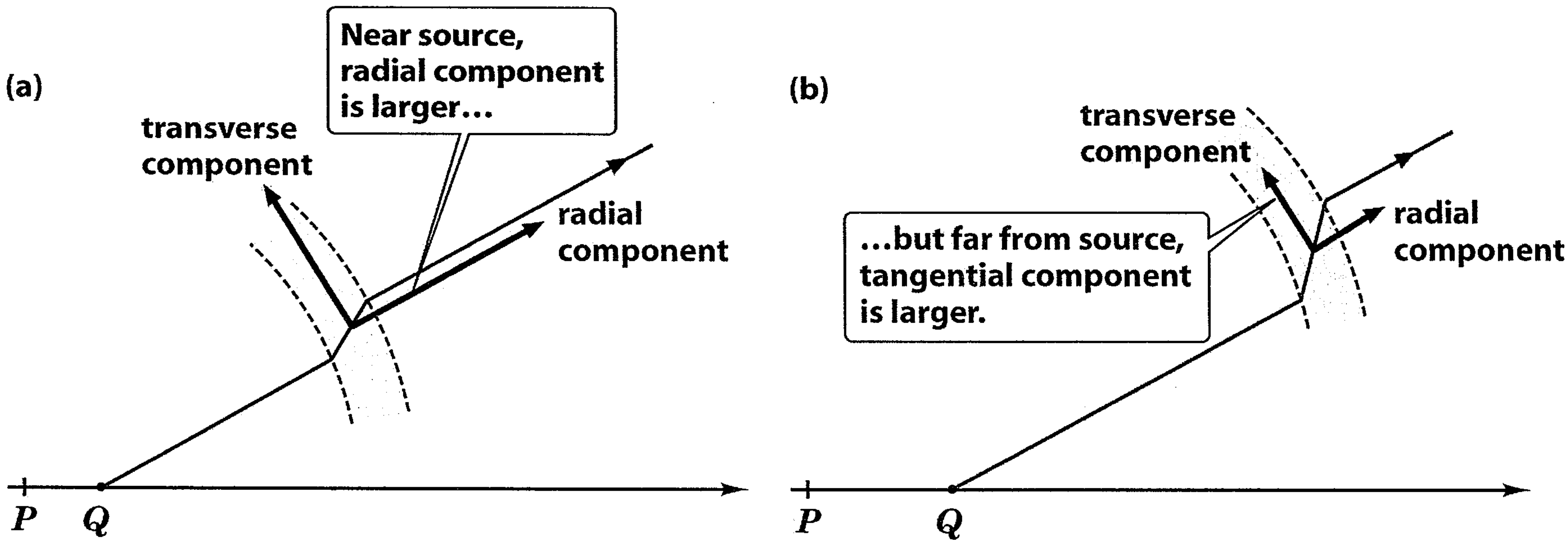


Figure 33-6 Physics for Engineers and Scientists 3/e

Electric field is perpendicular to the direction of propagation

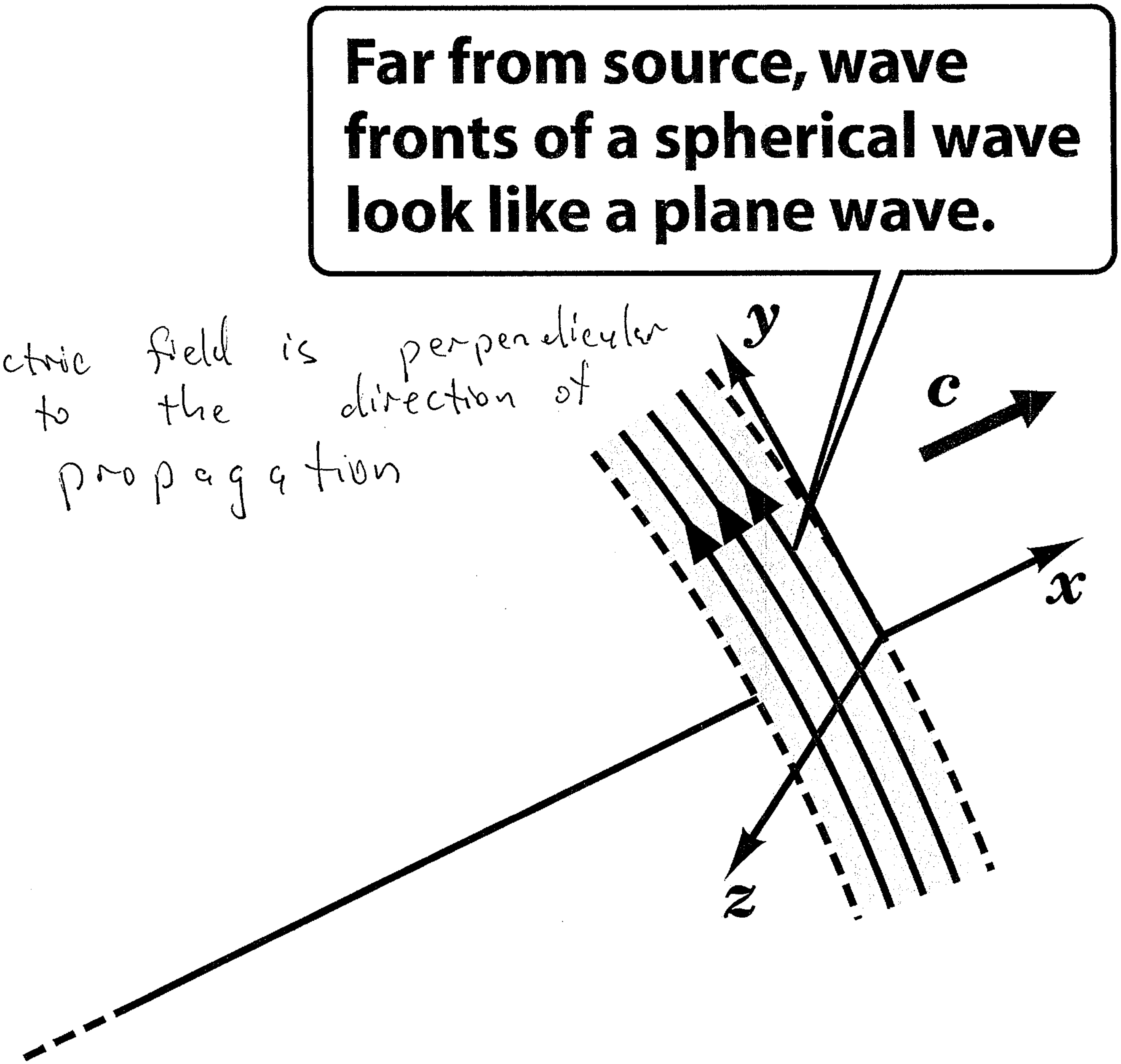


Figure 33-7 Physics for Engineers and Scientists 3/e

Changing Electric Field generates Magnetic Field in an E&M wave

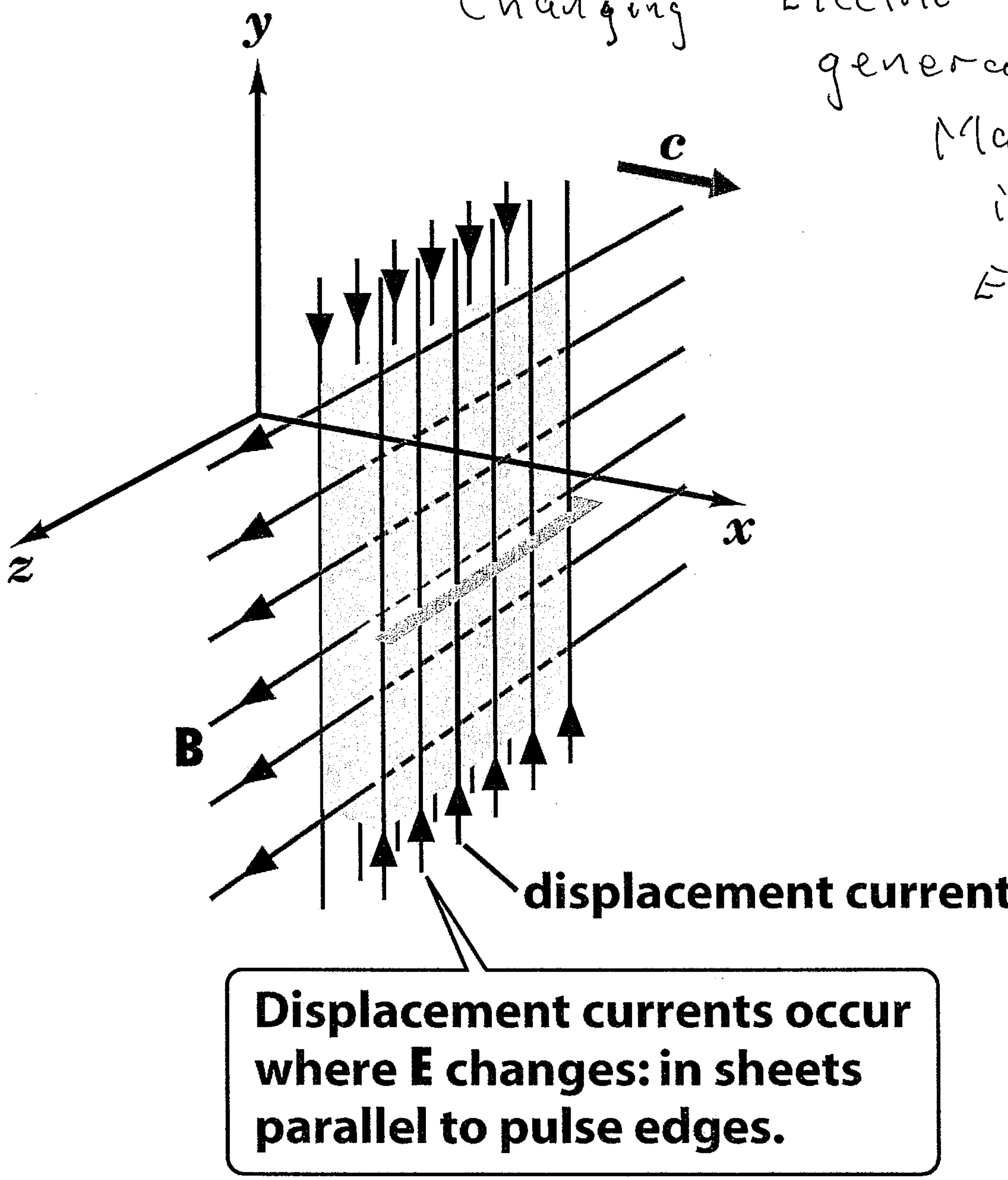


Figure 33-9 Physics for Engineers and Scientists 3/e

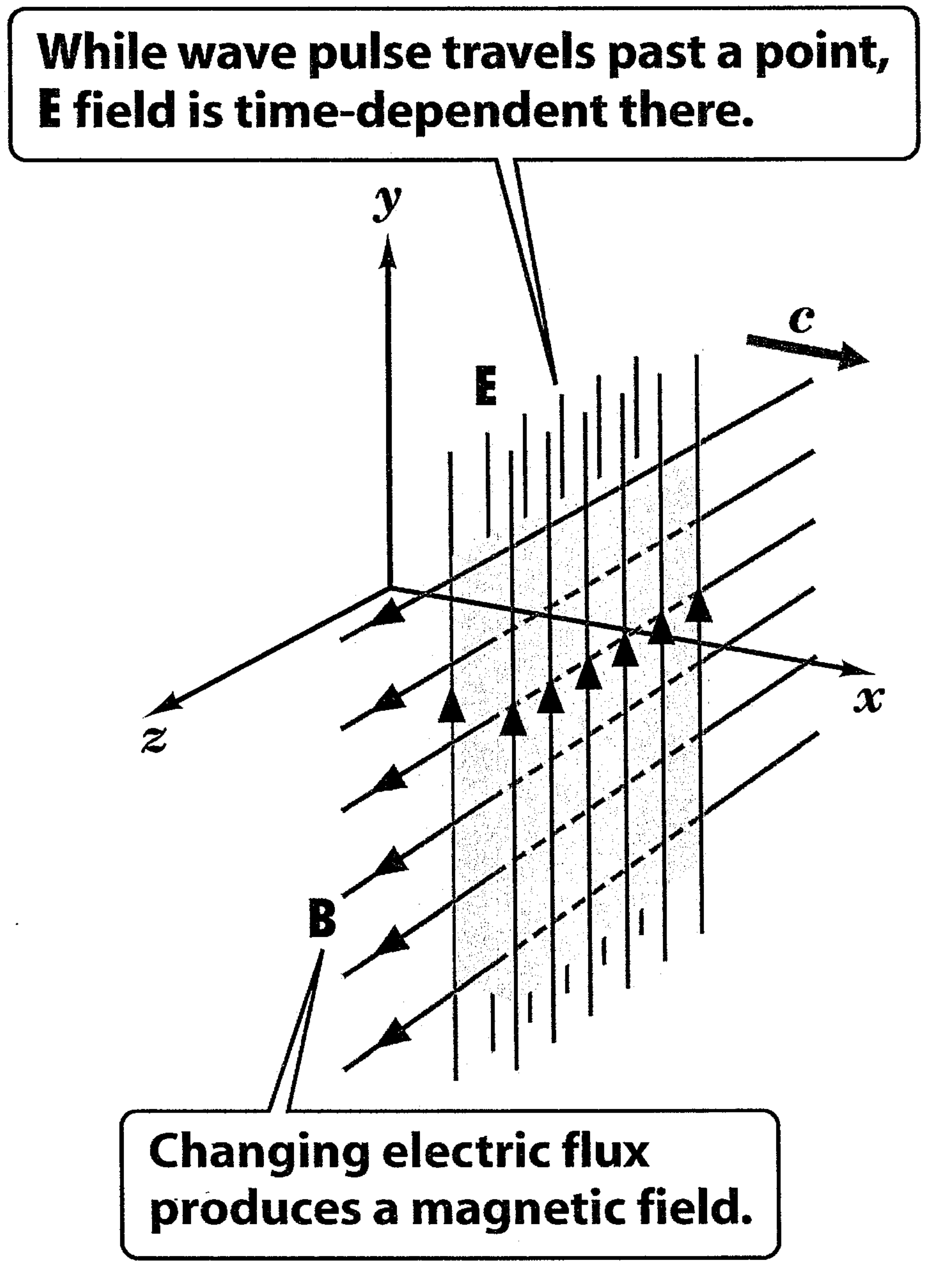


Figure 33-8 Physics for Engineers and Scientists 3/e

Changing Electric Flux Generates Magnetic Field

Without charges, the displacement current due to time varying electric flux, generates a magnetic field, while the changing magnetic flux generates an electric field, Both $E(x,t)$ and $B(x,t)$ satisfy a wave equation

$$\mu_0 \epsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2} - \frac{\partial^2 E_y(x,t)}{\partial x^2} = 0 \qquad \mu_0 \epsilon_0 = \frac{1}{c^2} = \frac{1}{(3\times10^8 \, m/s)^2}$$

$$\mu_0 \epsilon_0 \frac{\partial^2 B_z(x,t)}{\partial t^2} - \frac{\partial^2 B_z(x,t)}{\partial x^2} = 0 \qquad c \equiv \text{speed of light}$$

further $B_z = E_x/c$, $\underset{\sim}{E} \perp \underset{\sim}{B} \perp$ direction of wave propagation

solution of wave equation

$$E_y(x,t) = E_{y0}(x \mp ct)$$

HARMONIC TRAVELING WAVE SOLUTIONS OF THE WAVE EQUATION

$c = 3\times10^8$ m/s

$$E_y = E_0 \cos[k(x - vt)]$$

$$B_z = B_0 \cos[k(x - vt)]$$

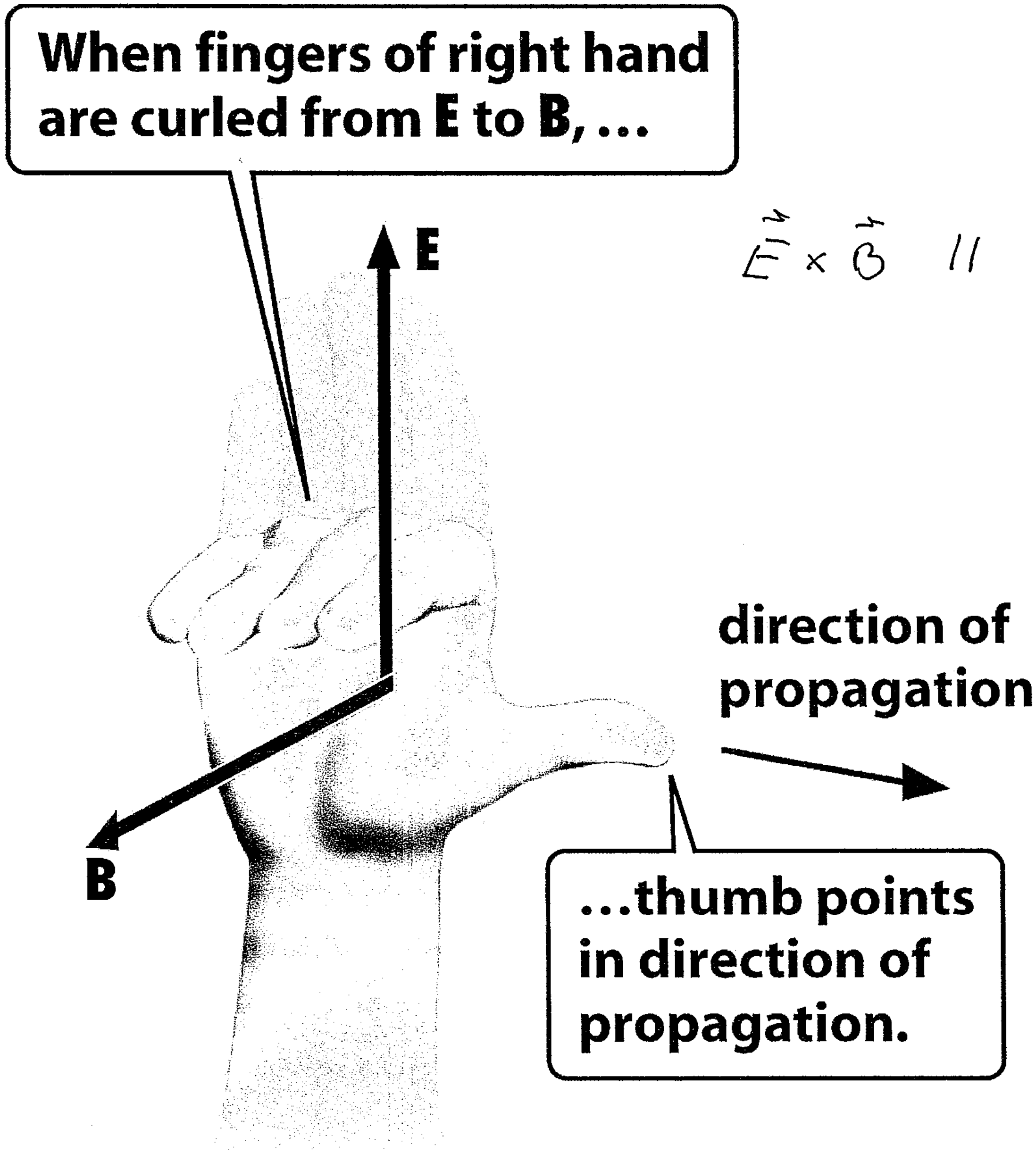
For E&M wave $v = c$

$k = \frac{2\pi}{\lambda}$, $\omega = kc$, $\omega = 2\pi f$; $c = f/\lambda$

$$E = E_0 \cos\left[2\pi\left(\frac{x}{\lambda} - ft\right)\right]$$

Waves are travelling to the right

Equation 33-34 derivatives Physics for Engineers and Scientists 3/e



$\vec{E} \times \vec{B}$ // to direction of propagation

Figure 33-10 Physics for Engineers and Scientists 3/e

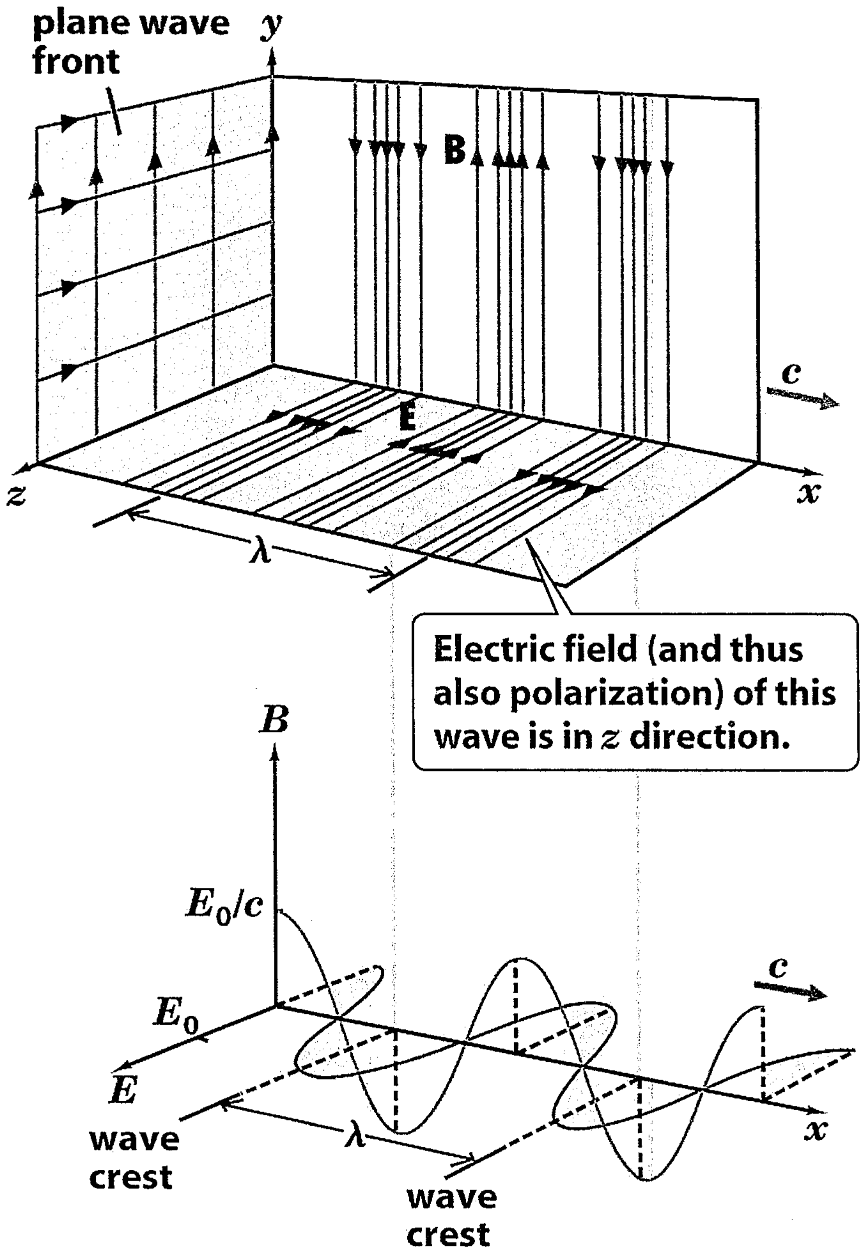


Figure 33-12 part 1 Physics for Engineers and Scientists 3/e

Another polarization

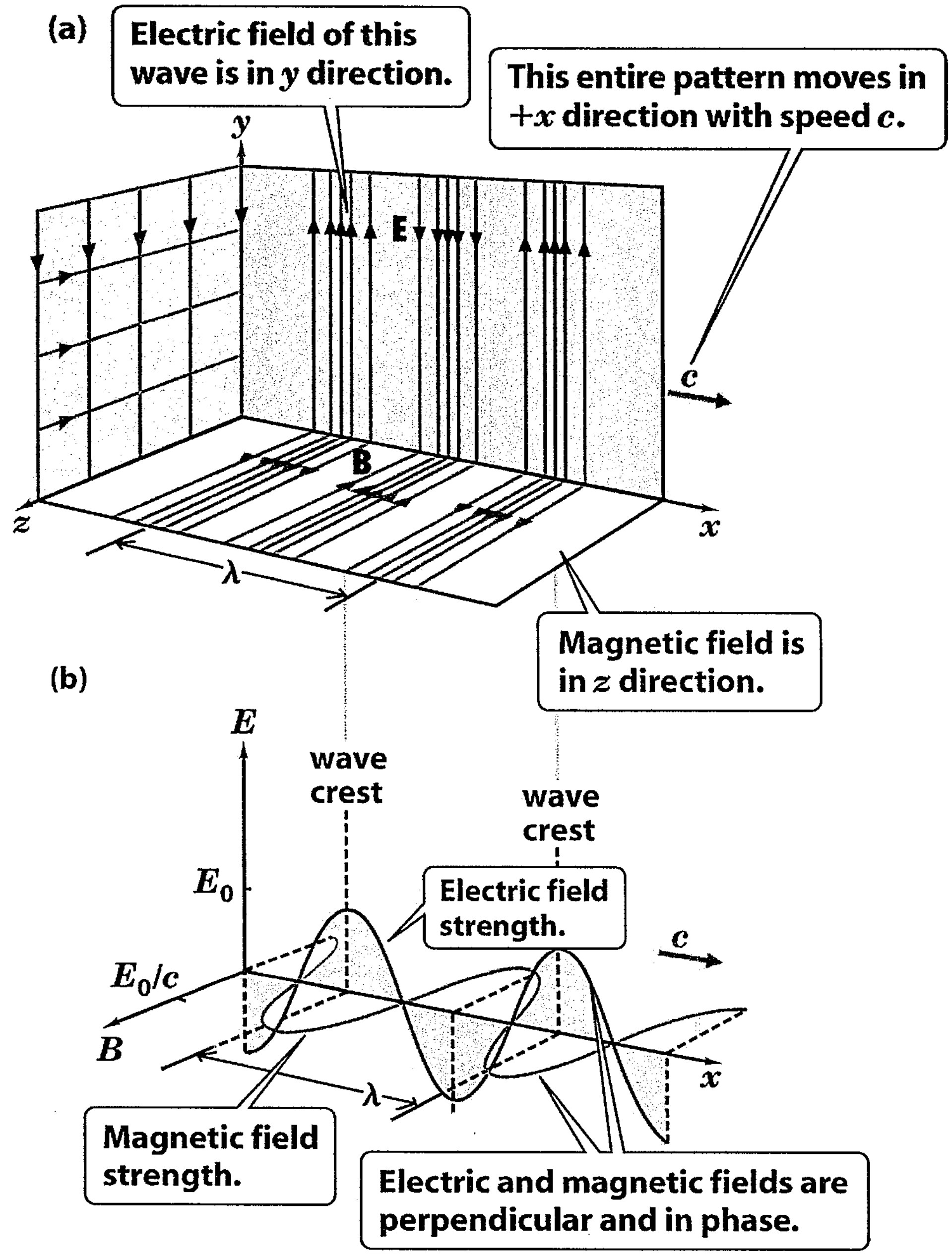


Figure 33-11 part 1 Physics for Engineers and Scientists 3/e

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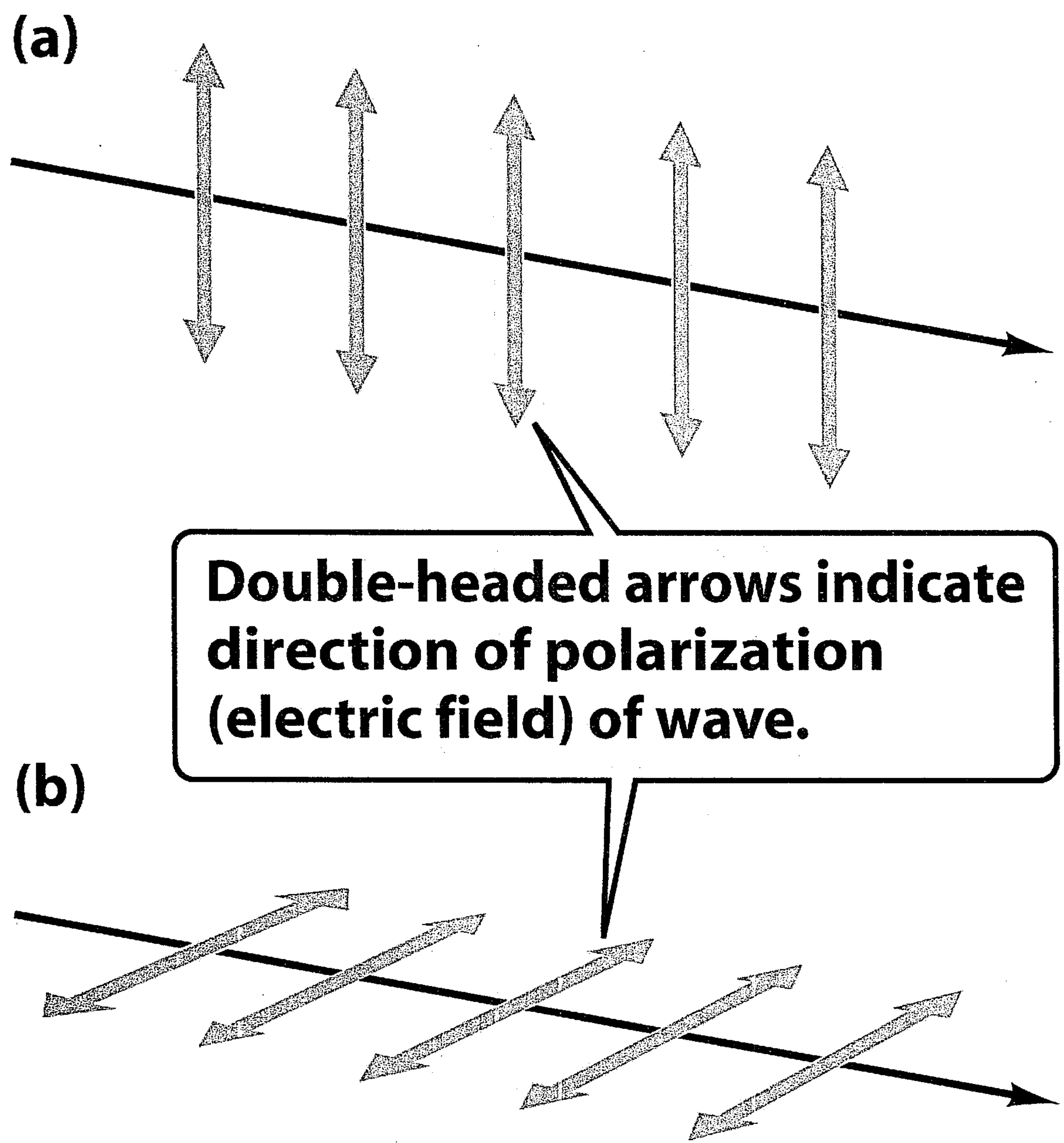

Figure 33-13 Physics for Engineers and Scientists 3/e

A small window in the E&M spectrum

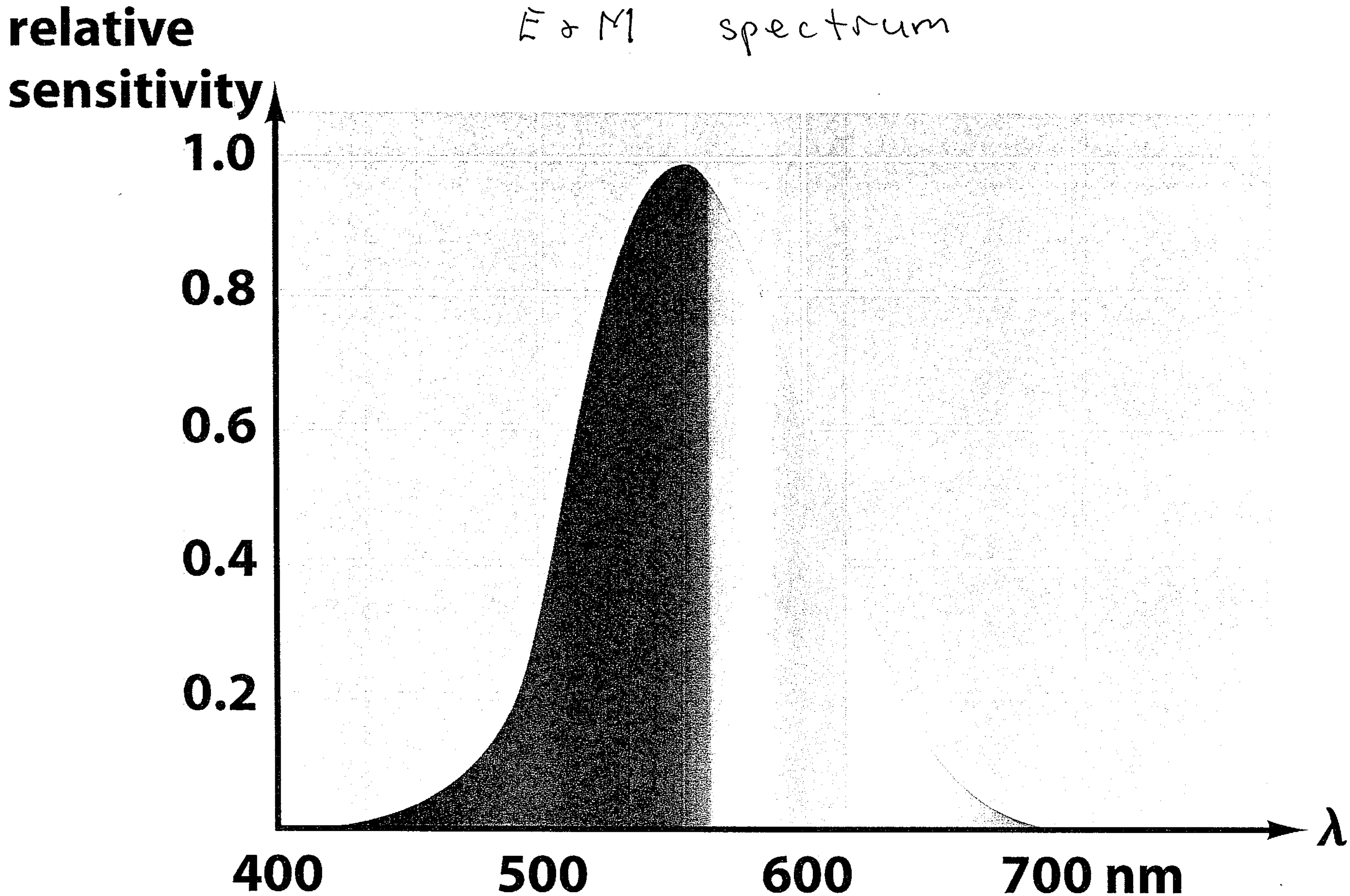

Figure 33-24 Physics for Engineers and Scientists 3/e

E & M Spectra Very Broad

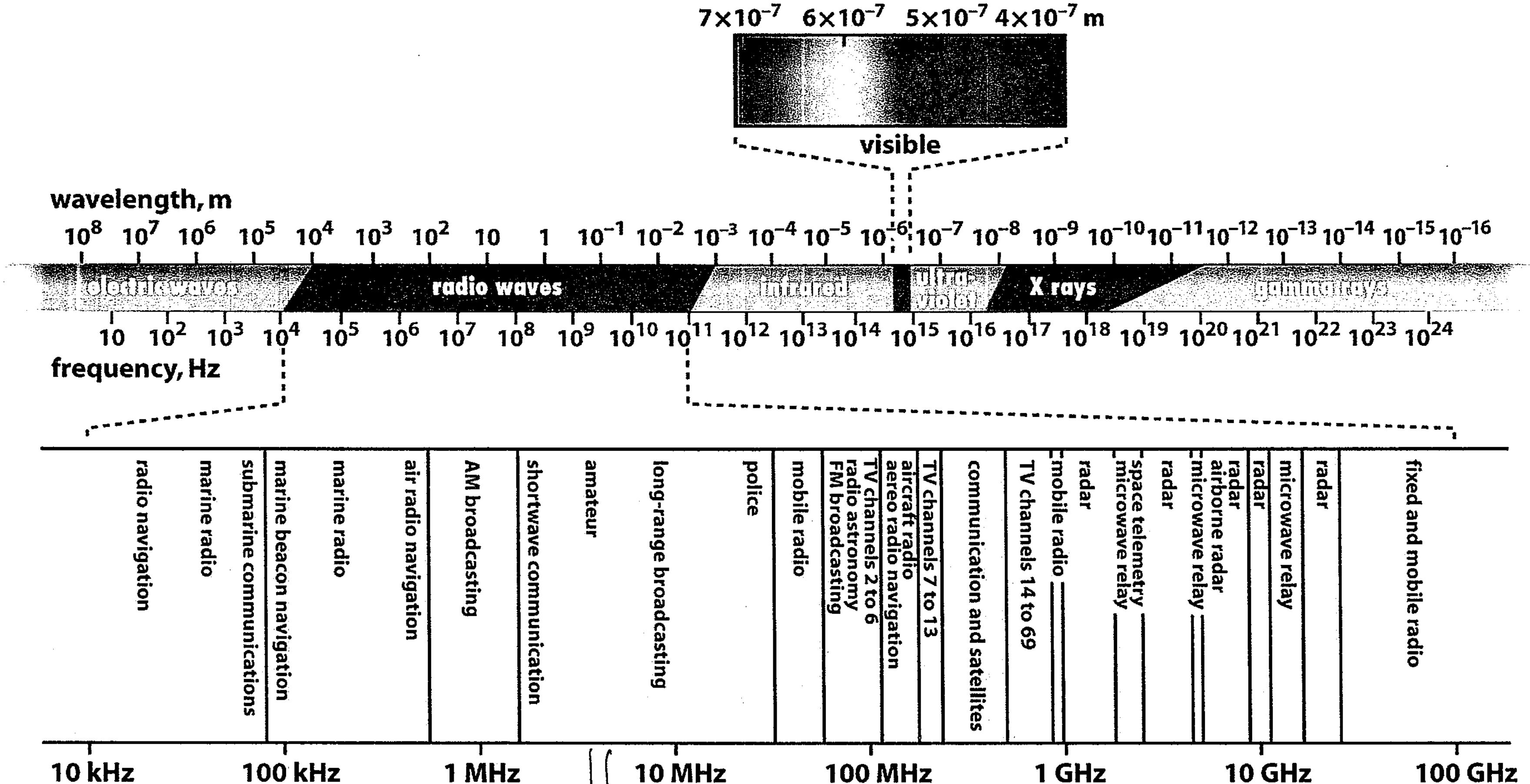

Figure 33-23 Physics for Engineers and Scientists 3/e

Spectrum of Today's Man Made Technology

ENERGY DENSITY IN ELECTROMAGNETIC WAVE

$W_E = W_B$ in E & M wave because $B = \frac{E}{c}$

$$W_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\frac{E^2}{c^2} = \frac{\epsilon\mu_0 E^2}{2\mu_0} = \frac{\epsilon E^2}{2} = W_E$$

$$w_{EM} = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 = \epsilon_0 E^2$$

Equation 33-17 Physics for Engineers and Scientists 3/e

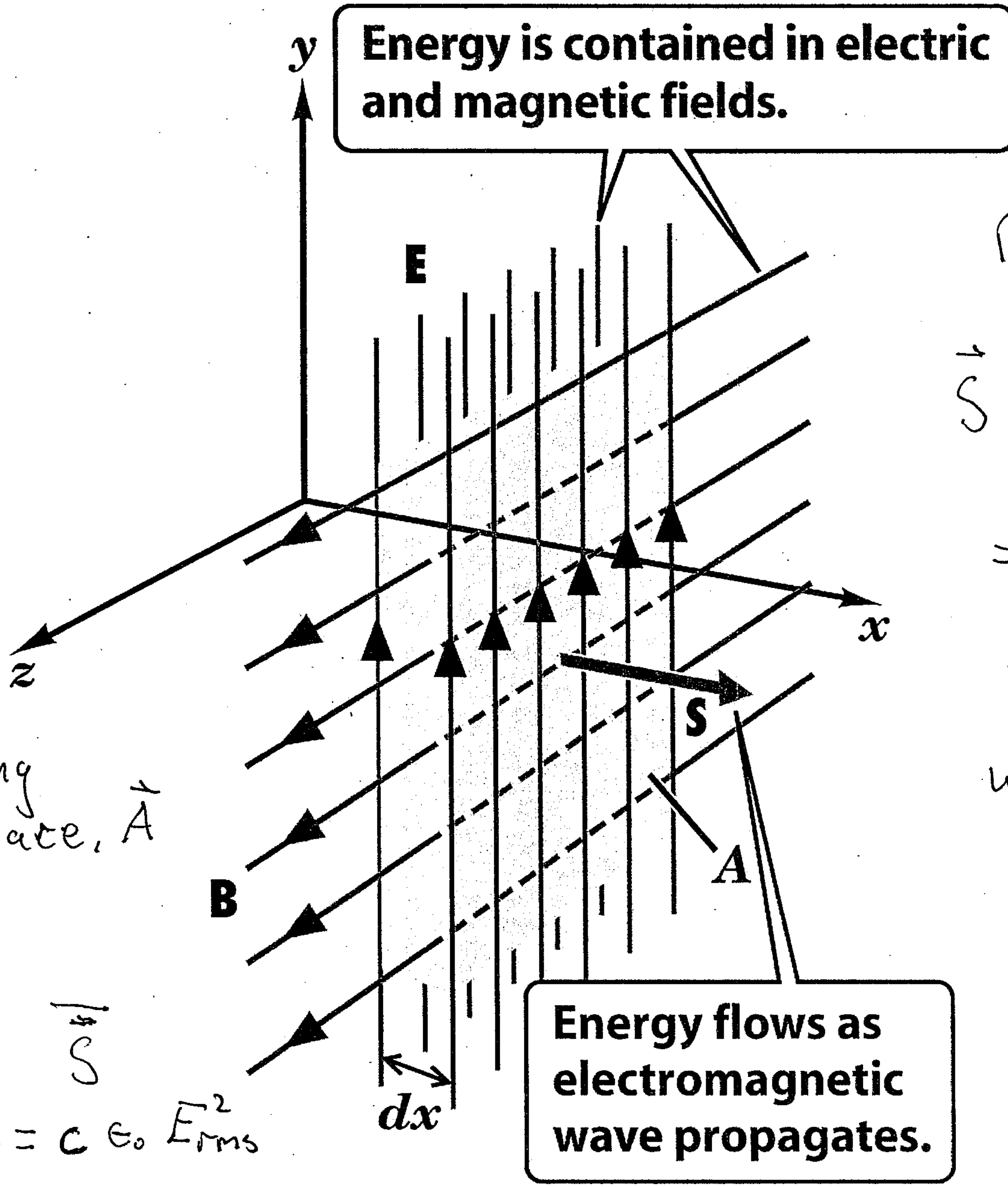

Figure 33-25 Physics for Engineers and Scientists 3/e

E+M
Power, P, impinging on a surface, $\vec{A}$

$$P = \vec{S} \cdot \vec{A}$$

Intensity $I = \overline{\vec{S}}$

$$= c\,\epsilon_0 E_{rms}^2$$

Poynting's Vector

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$= \vec{c}\; W_{EM}$$

$$= \vec{c}\left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}\right)$$

watts / m^2

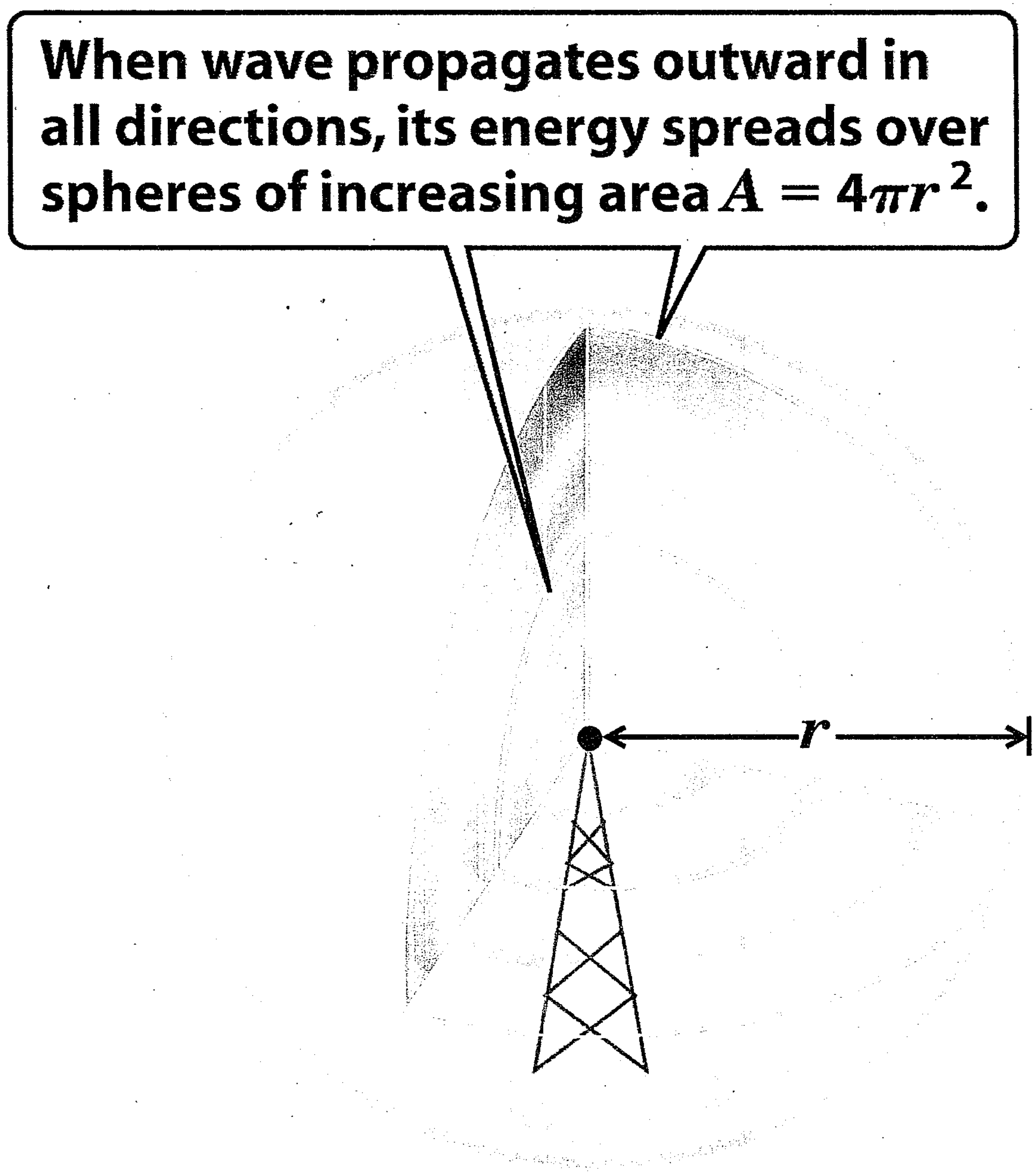

Figure 33-26 Physics for Engineers and Scientists 3/e

PRESSURE OF ELECTROMAGNETIC WAVE

$$[\text{pressure}] = \begin{cases} \dfrac{S}{c} & \text{(absorption)} \\[2ex] \dfrac{2S}{c} & \text{(reflection)} \end{cases}$$

Equation 33-27 derivatives Physics for Engineers and Scientists 3/e

(21)