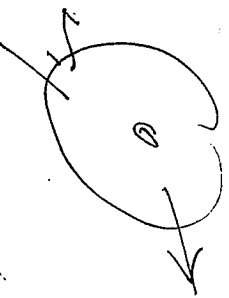


Lecture # 29

Maxwell's Equations

GAUSS' LAW, GAUSS' LAW FOR MAGNETISM, FARADAY'S LAW, AND MAXWELL-AMPERE LAW

$$\Phi_E \equiv \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_{\perp} dA = \frac{Q_{\text{inside}}}{\epsilon_0}$$



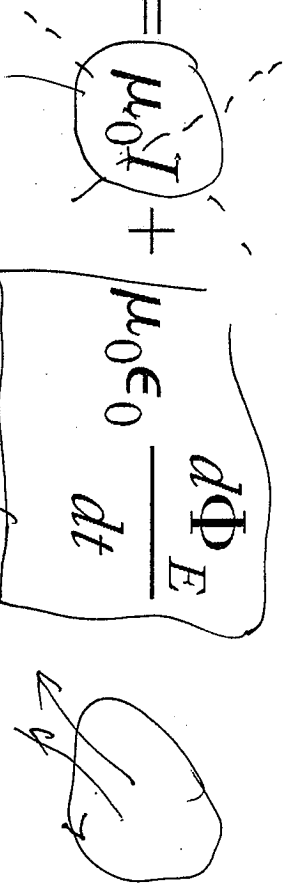
$$\Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = \oint B_{\perp} dA = 0$$



$$\text{Far-} \quad \mathcal{E} \equiv \oint_{\text{path}} \mathbf{E} \cdot d\mathbf{s} = \oint E_{\parallel} ds = - \frac{d\Phi_B}{dt}$$



$$\oint \mathbf{B} \cdot d\mathbf{s} = \oint B_{\parallel} ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



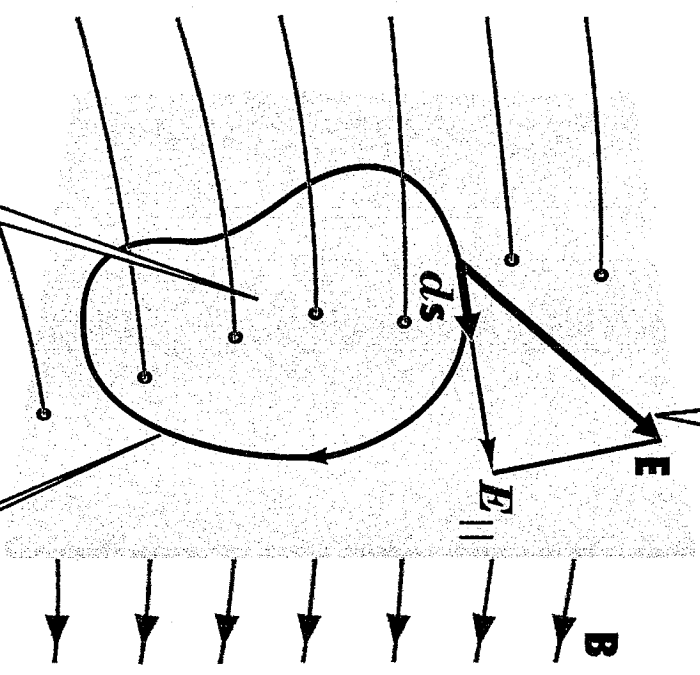
Ampere's added by Maxwell

DISPLACEMENT CURRENT



$$I_{\text{displacement}} = \epsilon_0 \frac{d\Phi_E}{dt}$$

Changing magnetic flux induces an electric field.



Changing magnetic field gives rise to a changing magnetic flux through this area.

Faraday's Law tells us the sum of $E_{||} \times$ [distance] contributions around a closed path.

Faraday's Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \Phi_M$$

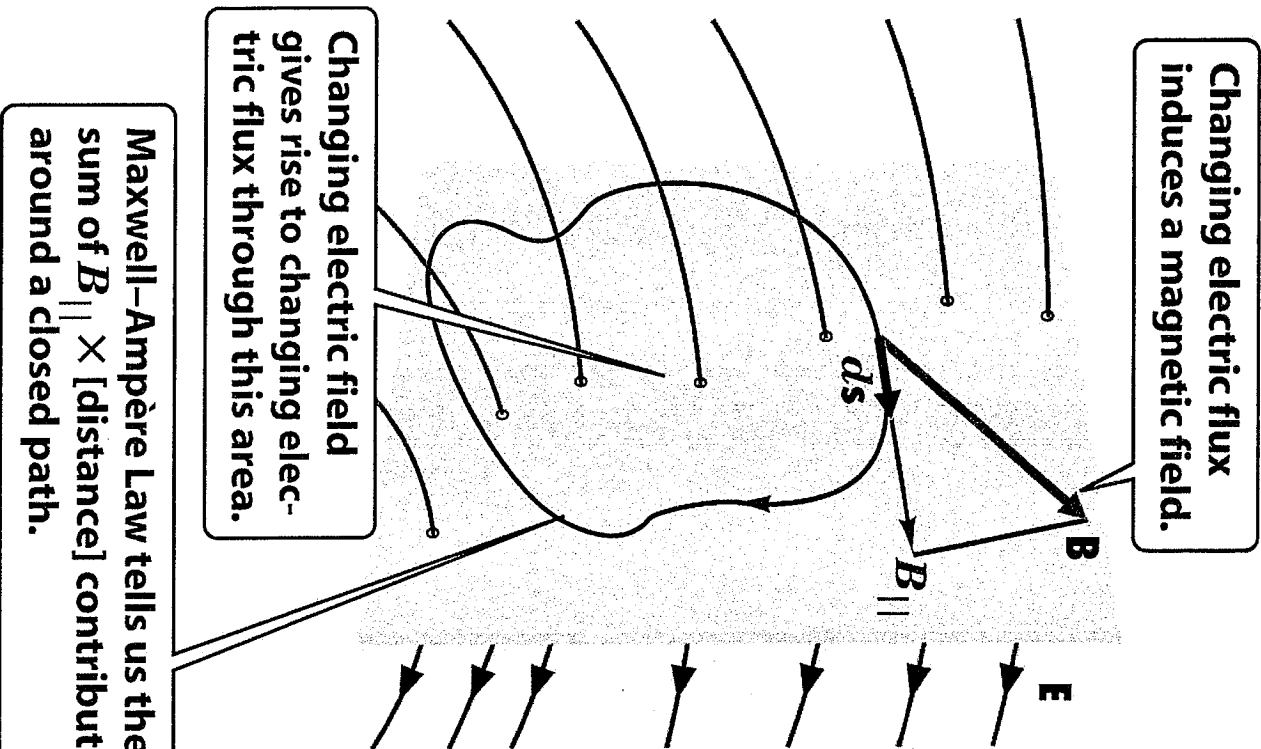
$$= -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

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(a) True

(b) False

Magnetic Energy can be stored in a capacitor



Changing electric flux induces a magnetic field.

Changing electric field gives rise to changing electric flux through this area.

Maxwell-Ampère Law tells us the sum of $B_{||} \times$ [distance] contributions around a closed path.

Maxwell-Ampere Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{total}$$

$$I_{total} = I_{conductor} + I_{displacement}$$

$$I_{conductor} = \frac{dq}{dt}$$

$$I_{displacement} = \epsilon_0 \frac{d\Phi_E}{dt}$$

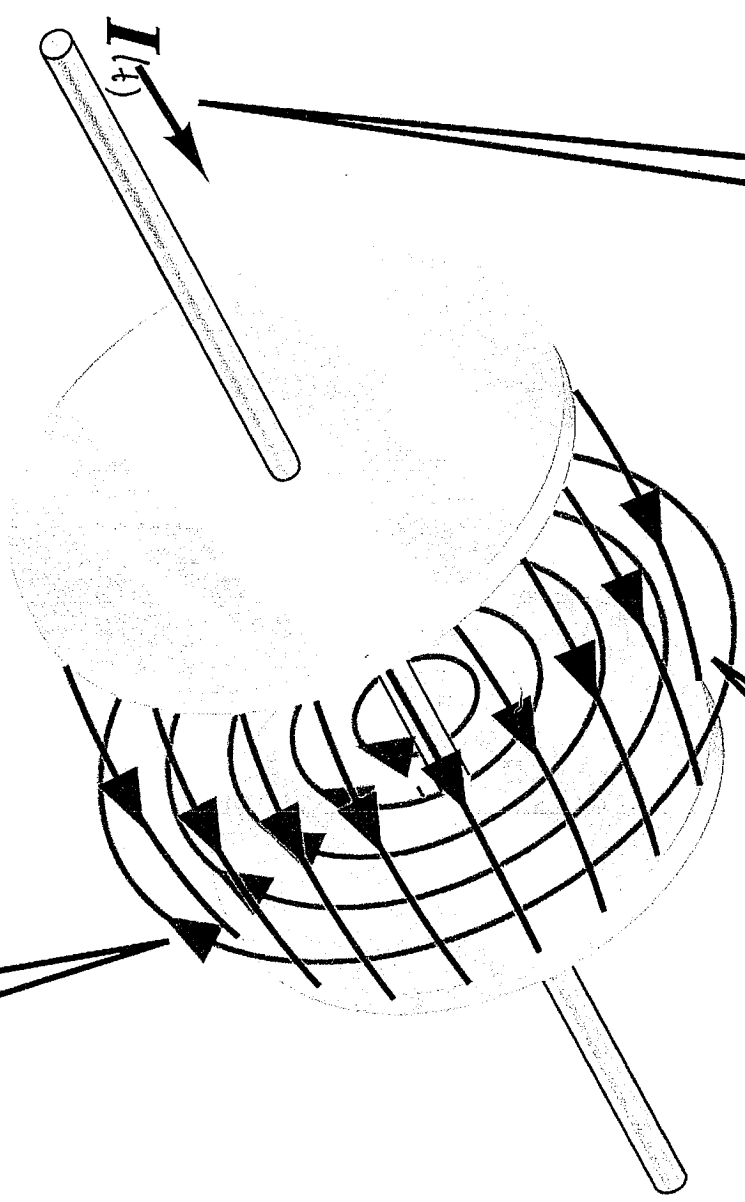
$$= \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

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Find magnetic energy density in this capacitor

As charge flows onto plates,...

...E field between plates increases.



Field lines of induced B field are circles.

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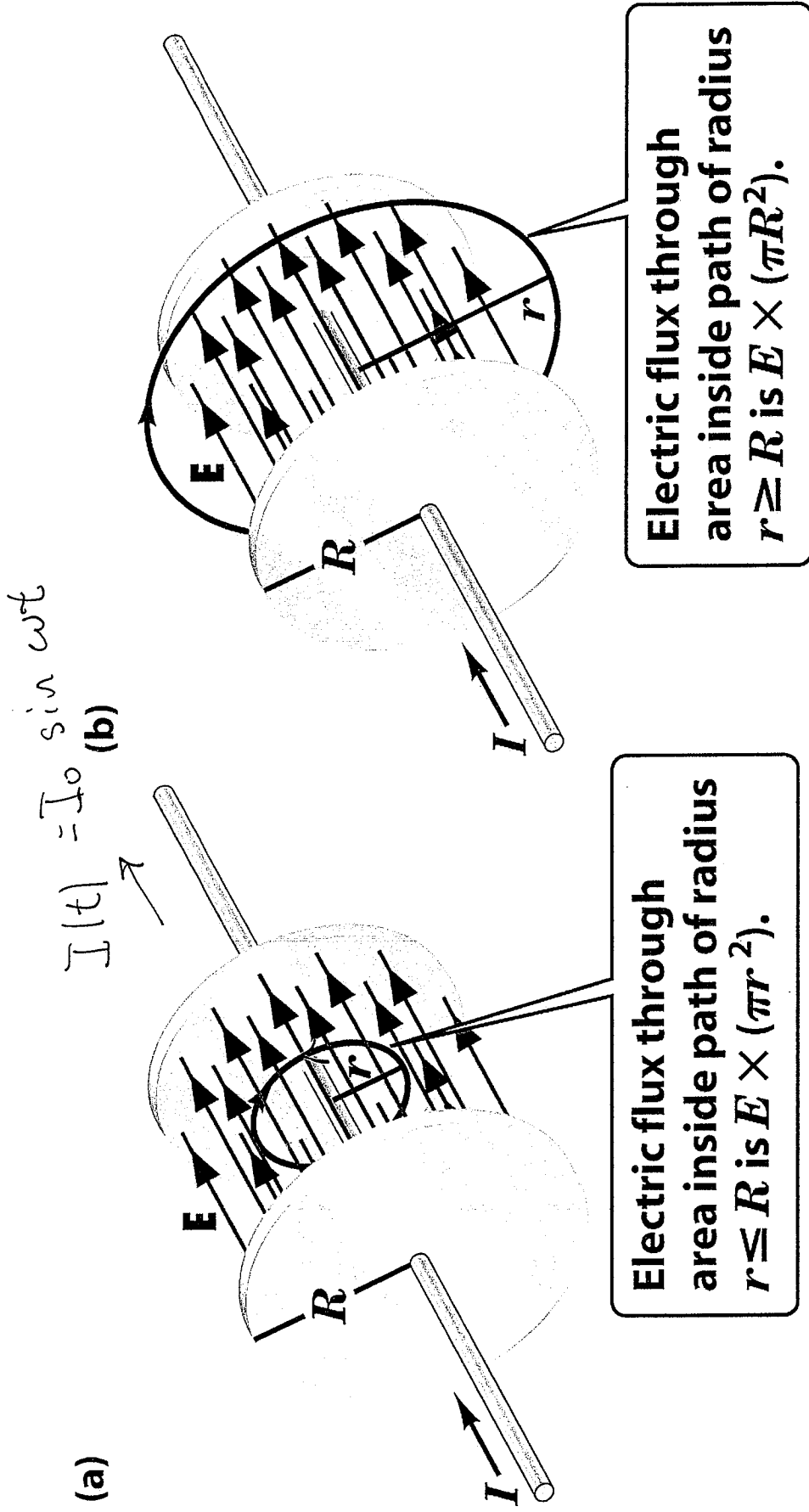


Figure 33-4 Physics for Engineers and Scientists 3/e
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$$\int \mathbf{B} \cdot d\mathbf{l} = B(\omega) 2\pi r = \frac{d}{dt} \left(\frac{Q}{\epsilon_0} A(r) \right) \cos \epsilon = 0$$

$$\int \mathbf{E} \cdot d\mathbf{A} \frac{\partial E}{\partial t} = \pi r^2 \frac{\partial E}{\partial t} = \pi R^2 \frac{\partial E}{\partial t}$$

$$E = \frac{Q}{\epsilon_0 A}$$

$$A(r) = \pi r^2$$

$$A(R) = \pi R^2$$

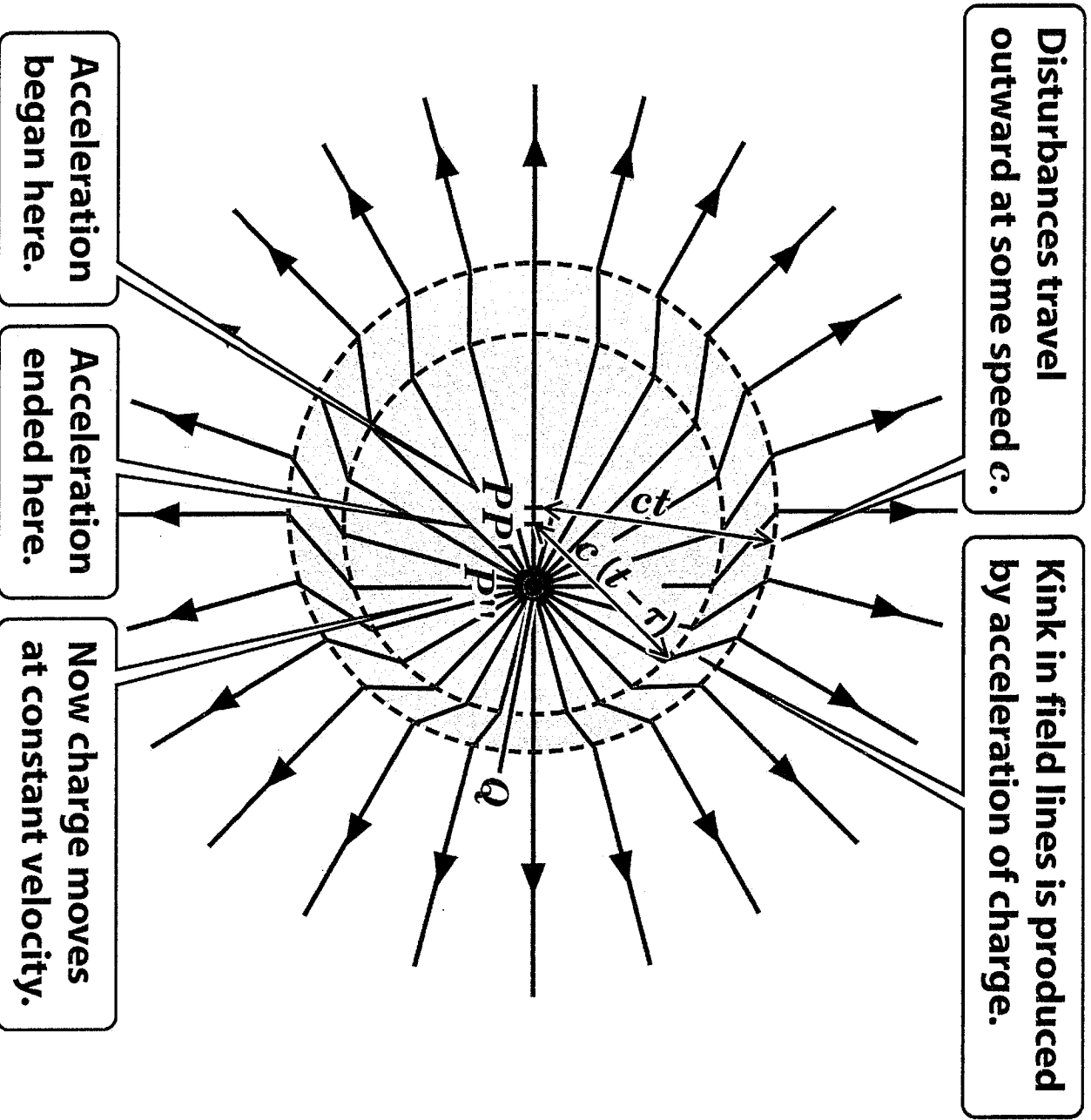


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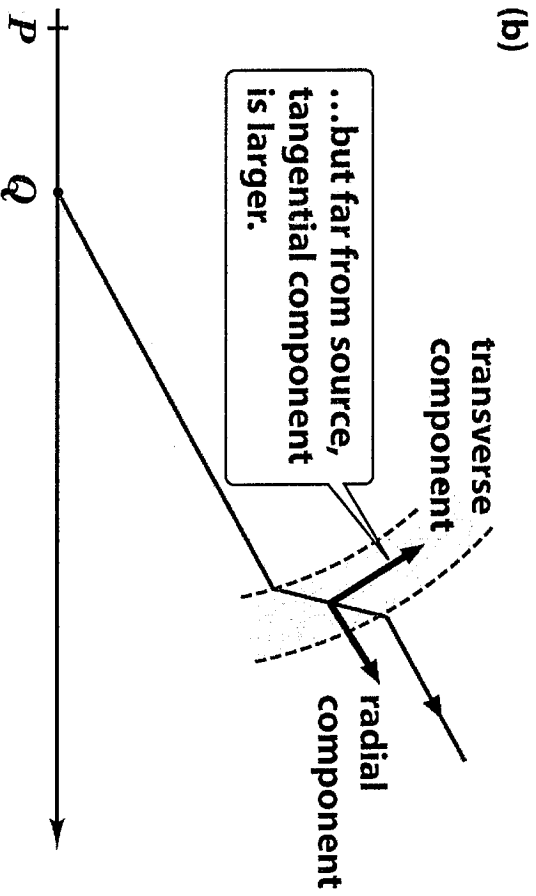
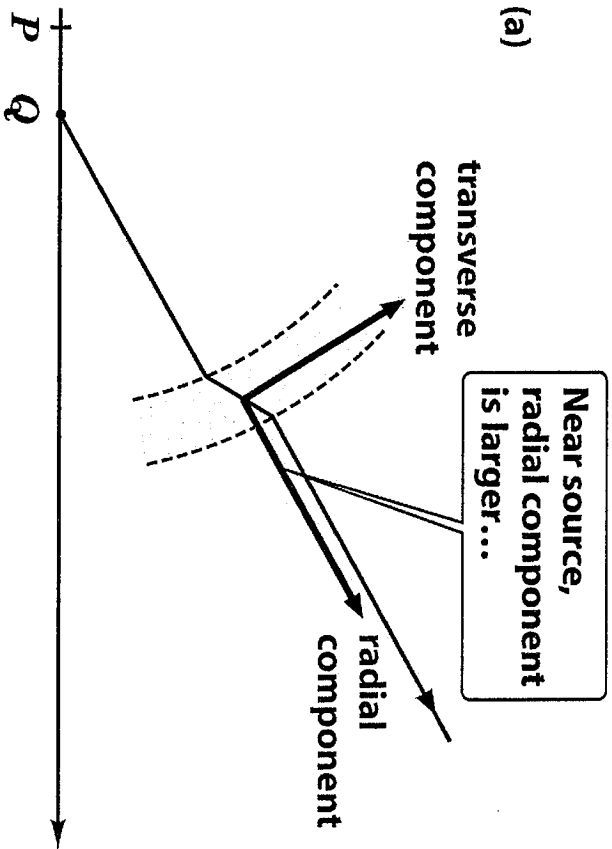


Figure 33-6 Physics for Engineers and Scientists 3/e
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Far from source, wave fronts of a spherical wave look like a plane wave.

Electric field is perpendicular to the direction of propagation

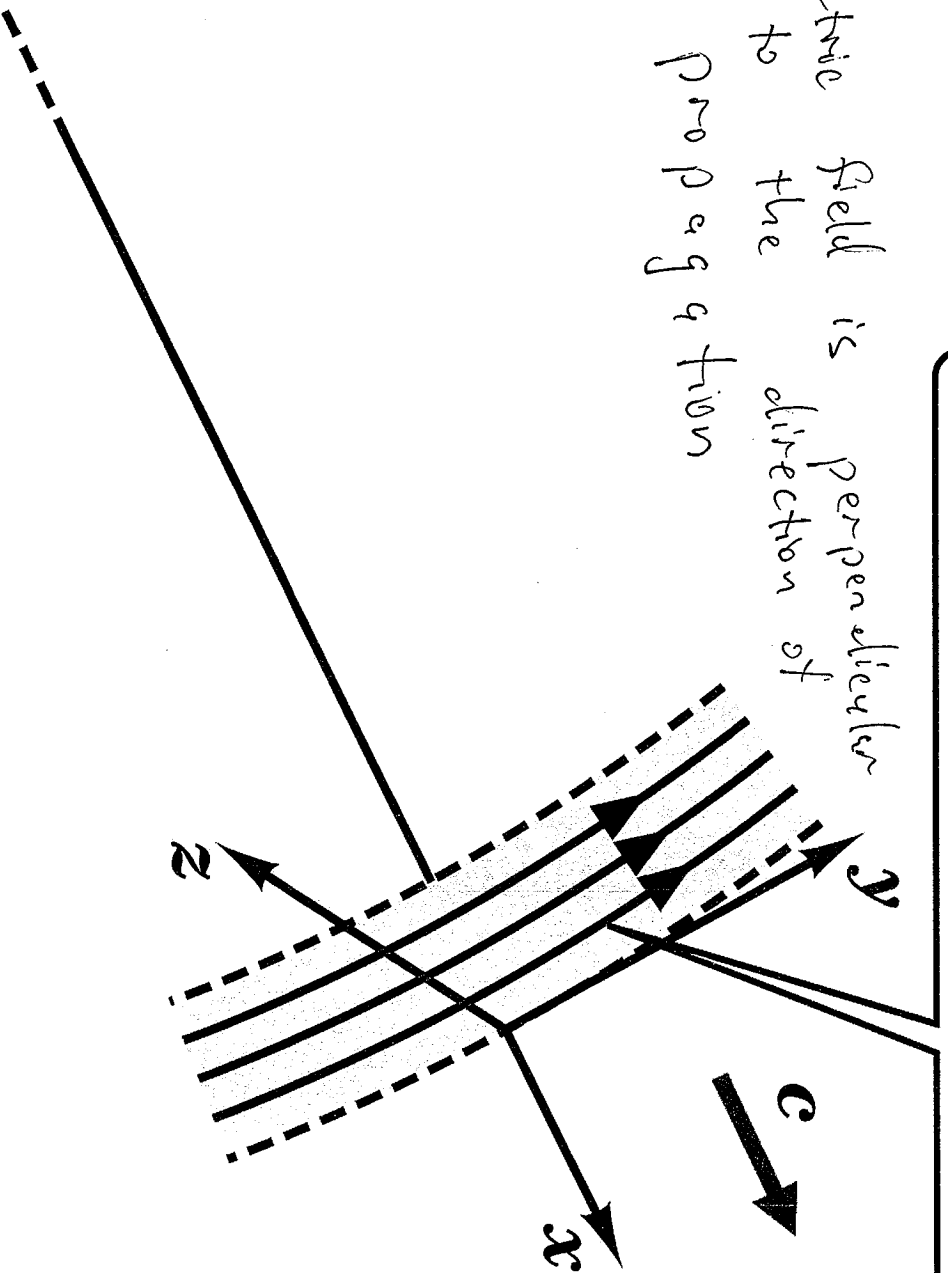


Figure 33-7 Physics for Engineers and Scientists 3/e
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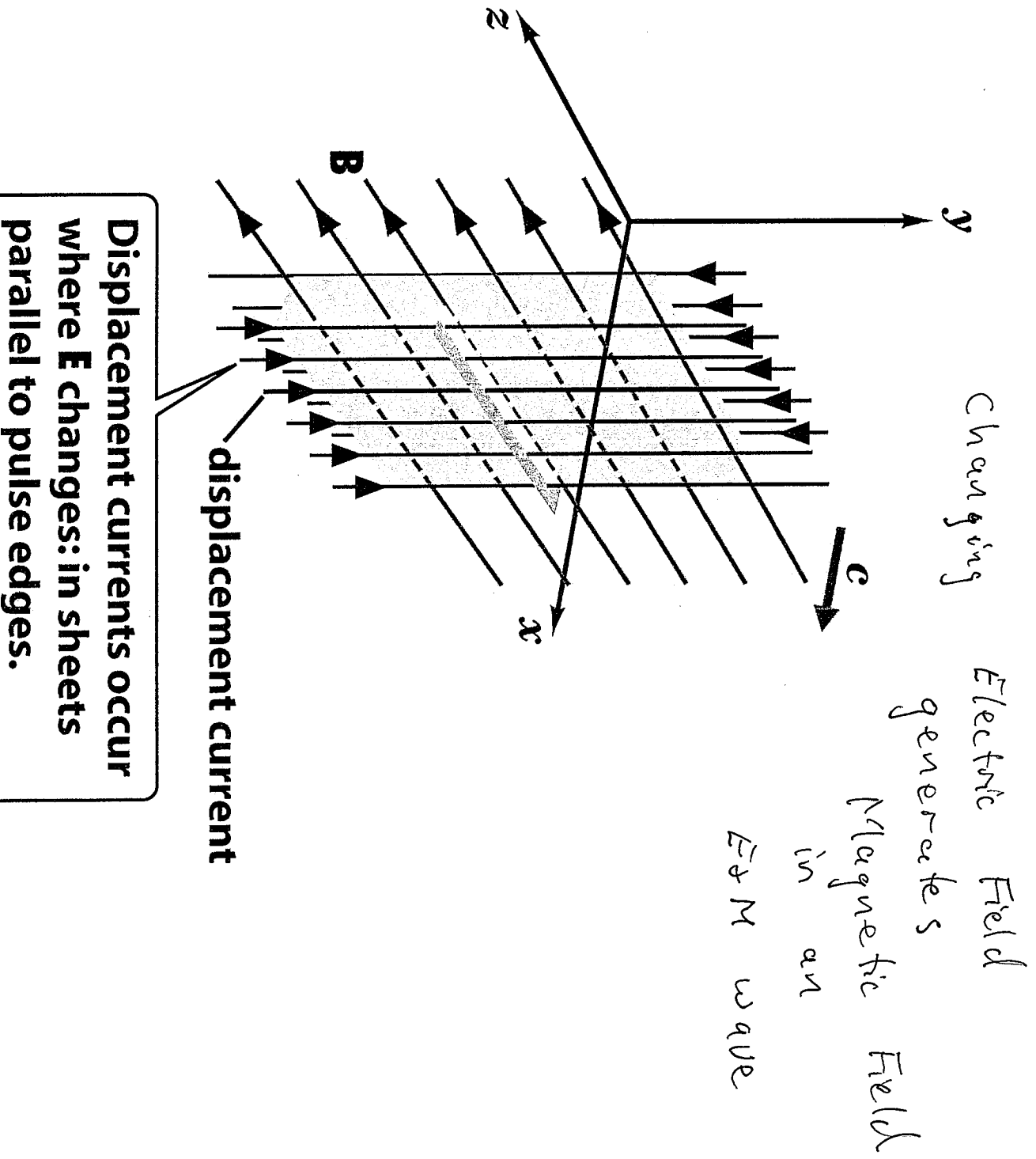
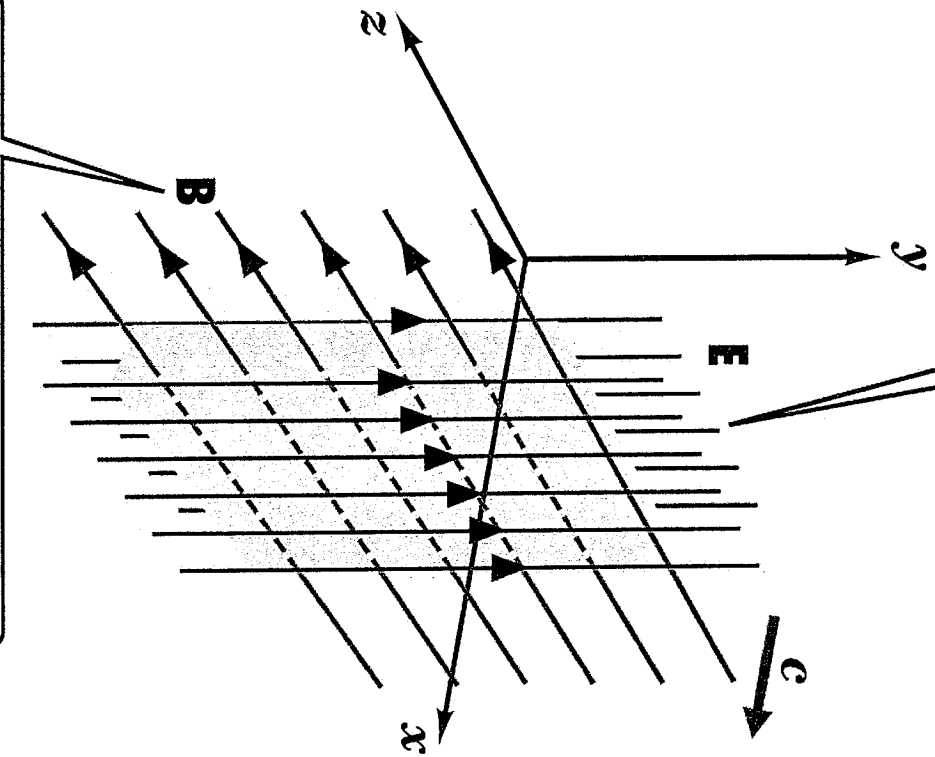


Figure 33-9 Physics for Engineers and Scientists 3/e
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While wave pulse travels past a point, E field is time-dependent there.



Changing
Electric Flux
Generates
Magnetic
Field

Changing electric flux produces a magnetic field.

Figure 33-8 Physics for Engineers and Scientists 3/e
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Without charges, the displacement current due to time varying electric flux, generates a magnetic field, while the changing magnetic flux generates an electric field.

Both $E(x,t)$ and $B(x,t)$ satisfy a

Wave equation

$$\mu_0 \epsilon_0 \frac{\partial^2 E(x,t)}{\partial t^2} - \frac{\partial^2 E(x,t)}{\partial x^2} = 0$$

$$\mu_0 \epsilon_0 = \frac{1}{c^2} = \frac{1}{(3 \times 10^8 \text{ m/s})^2}$$

$$\mu_0 \epsilon_0 \frac{\partial^2 B(x,t)}{\partial t^2} - \frac{\partial^2 B(x,t)}{\partial x^2} = 0$$

$c \equiv$ speed of light

further $\vec{B} \perp \vec{E} \perp \vec{B} \perp$ direction of wave propagation

solution of wave equation

$$E_y(x,t) = E_{y0} \cos(kx - \omega t)$$

HARMONIC TRAVELING WAVE SOLUTIONS OF THE WAVE EQUATION

$$c = 3 \times 10^8 \text{ m/s}$$

$$E_y = E_0 \cos [k(x - ct)]$$

$$B_z = B_0 \cos [k(x - ct)]$$

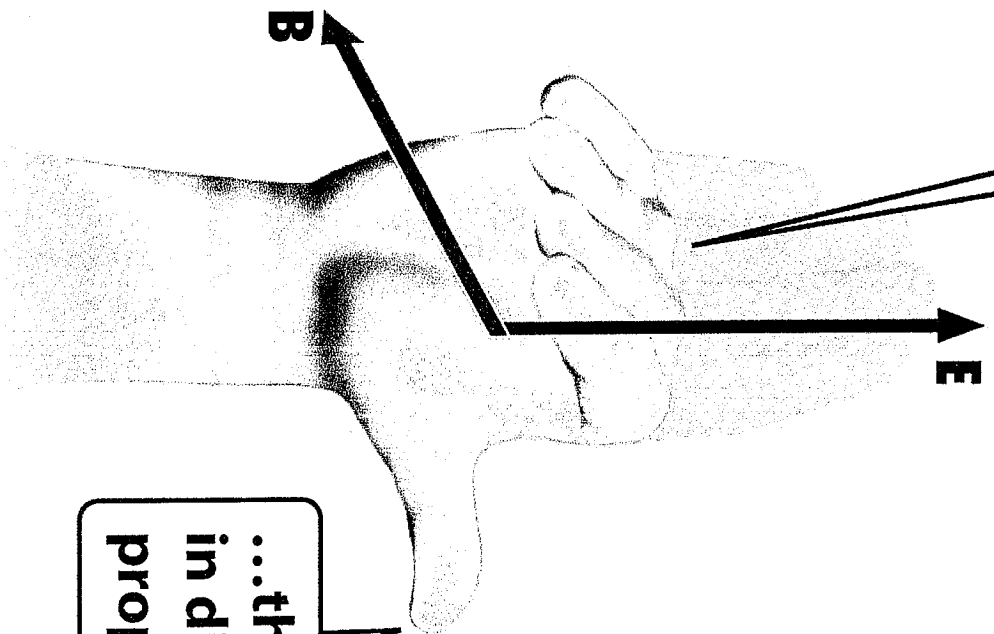
For EM wave $v = c$

$$k = \frac{2\pi}{\lambda}, \quad \omega = kc, \quad \omega = \frac{2\pi}{\lambda} f, \quad c = f/\lambda$$

$$\vec{E} = E_0 \cos \left[2\pi \left(\frac{x}{\lambda} - ft \right) \right]$$

Waves are travelling to the right

When fingers of right hand are curled from E to B, ...

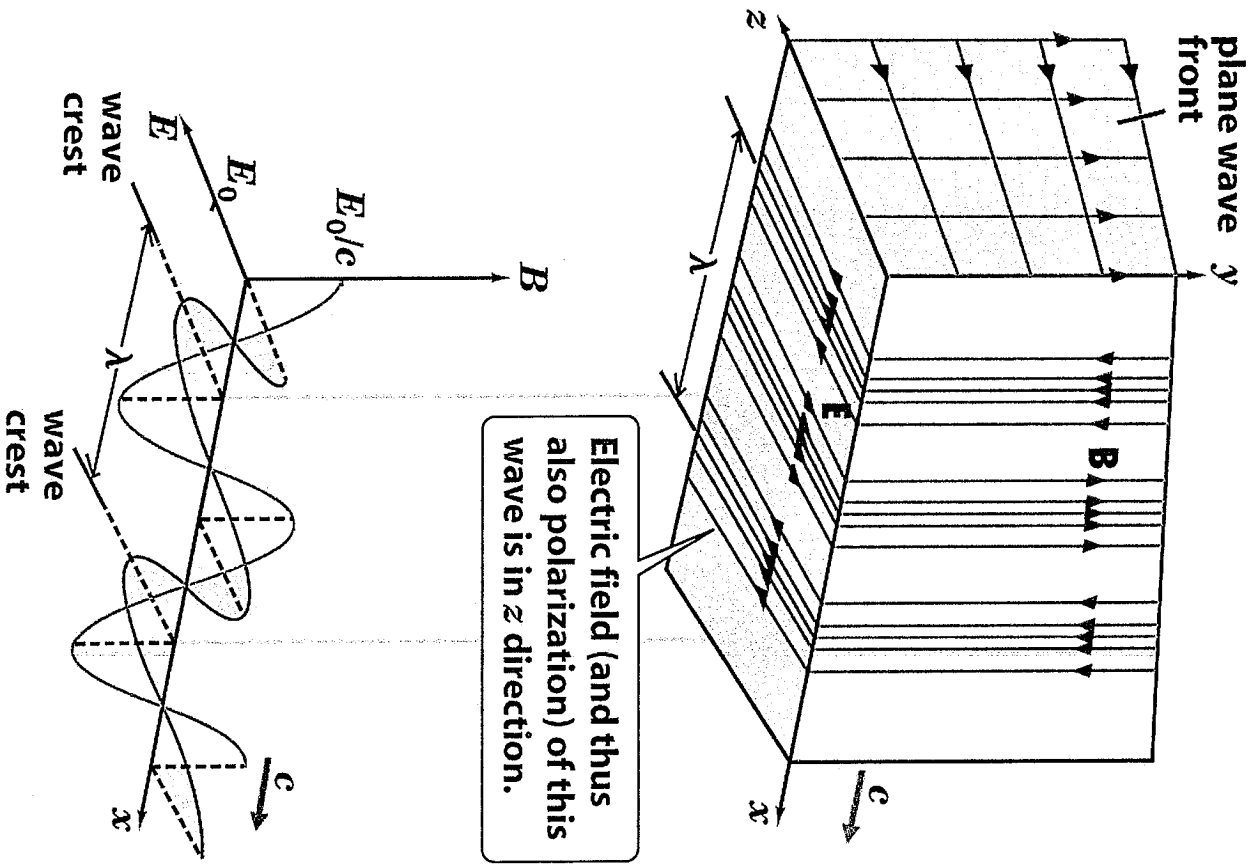


direction of propagation

...thumb points in direction of propagation.

$\vec{E} \times \vec{B}$ // to direction of propagation

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Another
polarization

Figure 33-12 part 1 Physics for Engineers and Scientists 3/e
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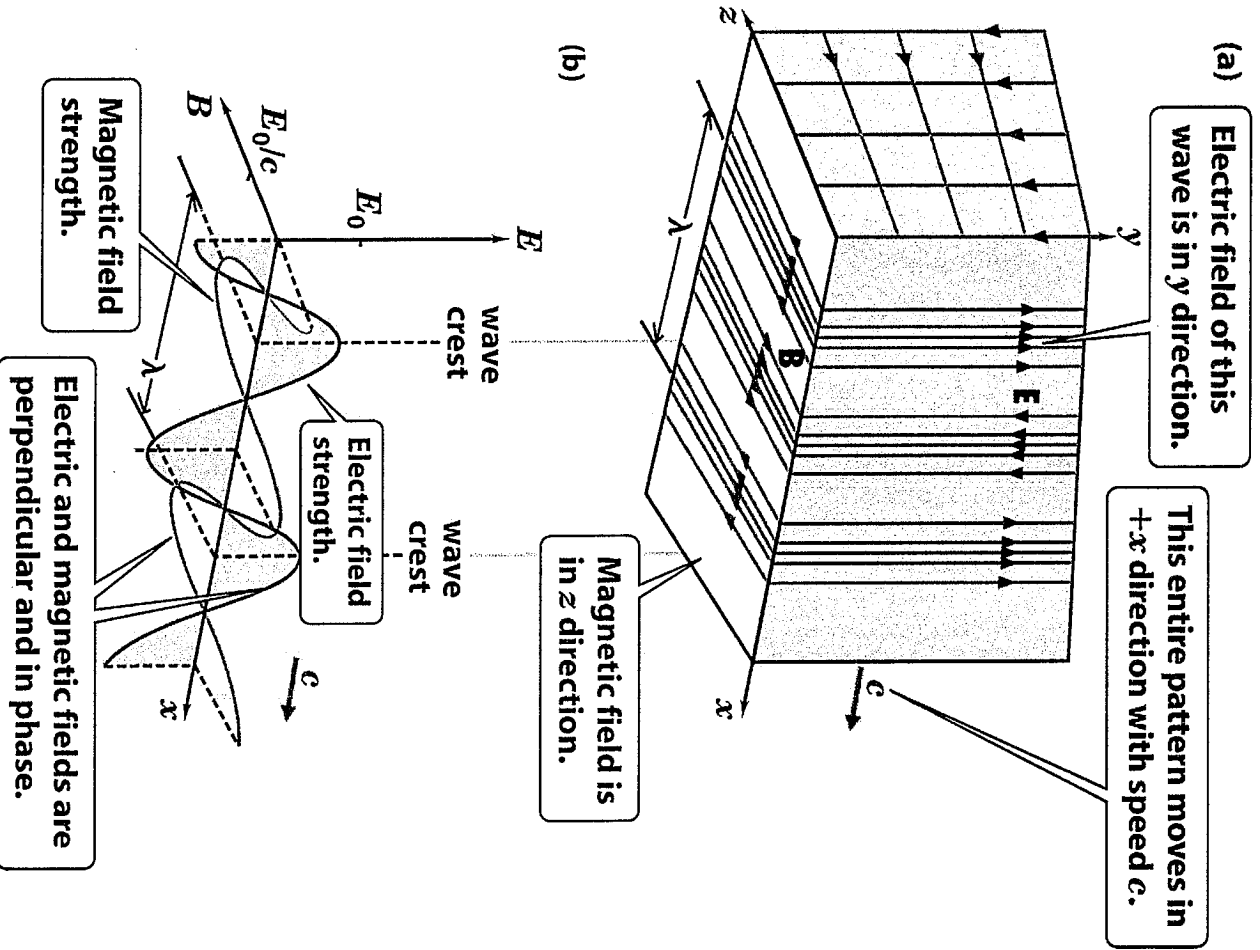


Figure 33-11 part 1 Physics for Engineers and Scientists 3/e
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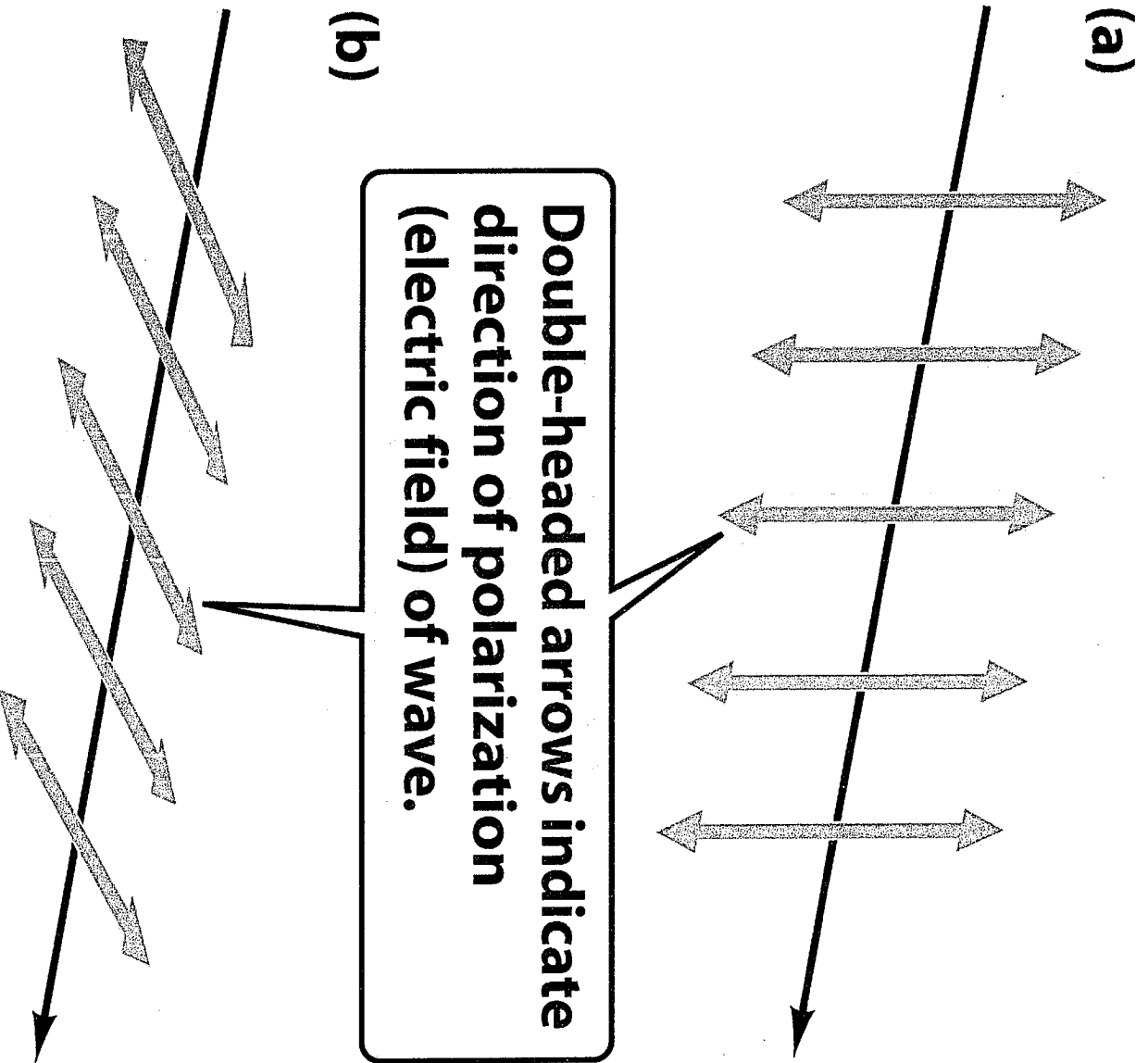


Figure 33-13 Physics for Engineers and Scientists 3/e
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A small window in the $E \times M$ spectrum

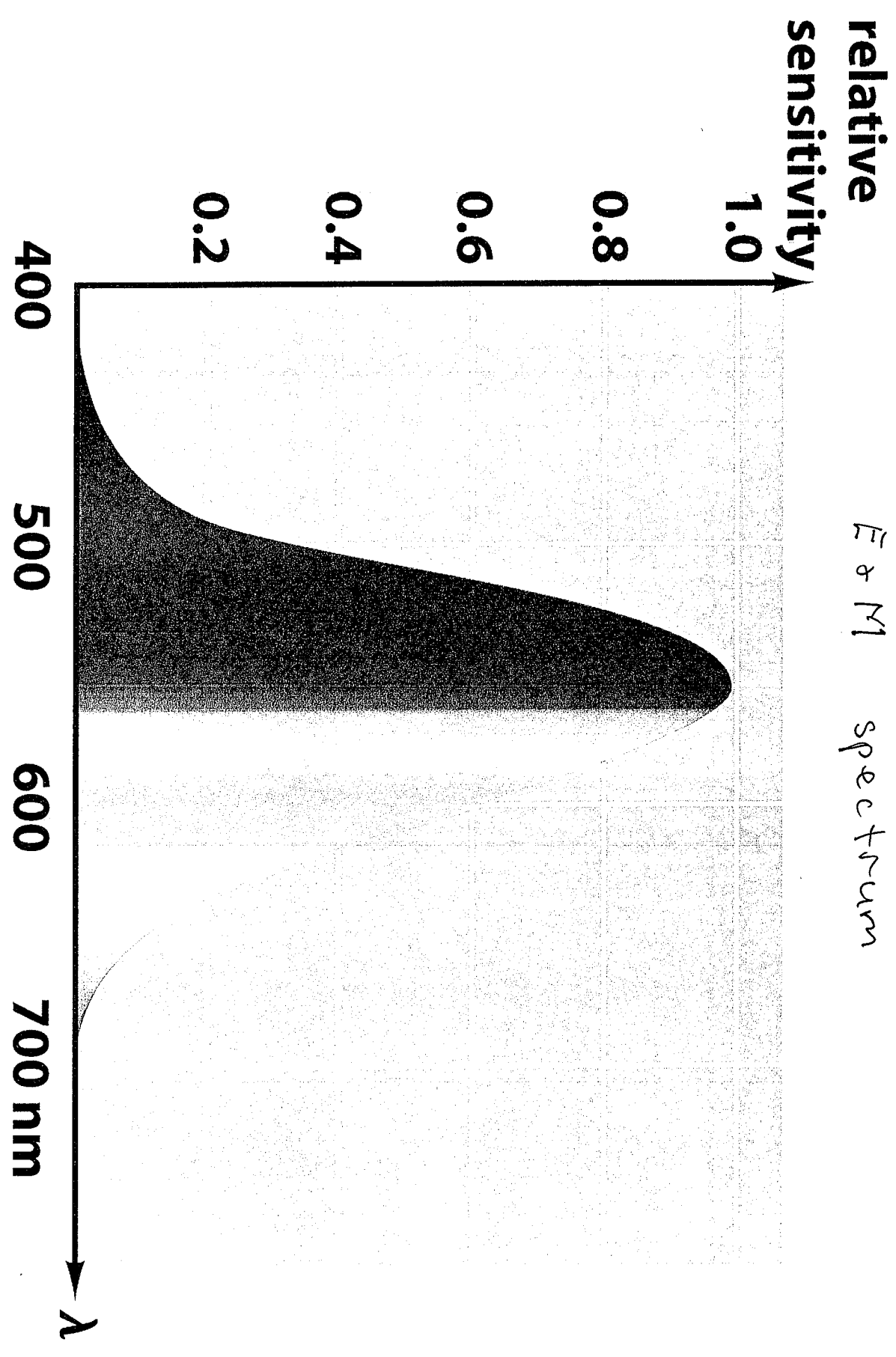


Figure 33-24 Physics for Engineers and Scientists 3/e
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EM Spectra Very Broad

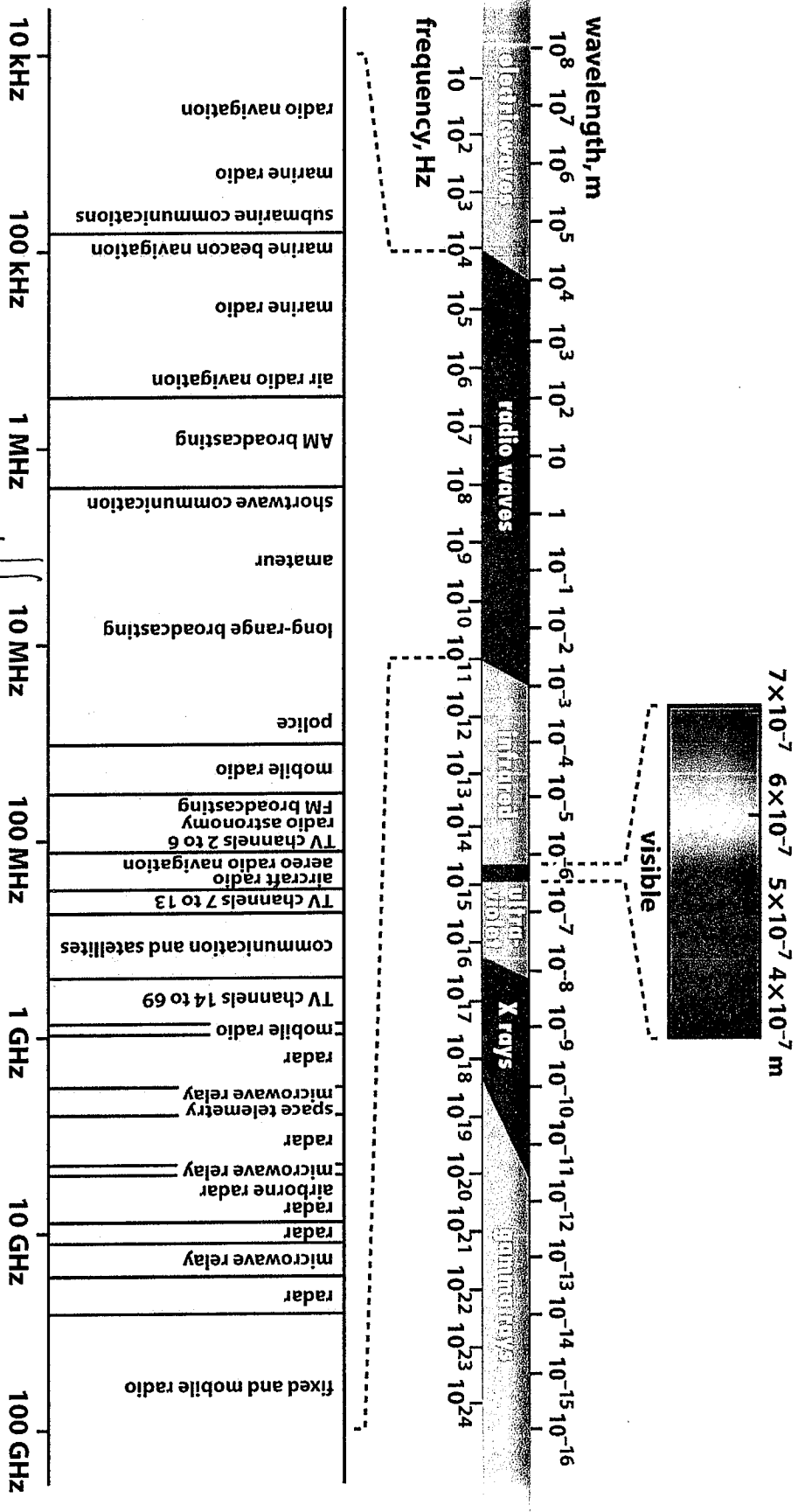


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Spectrum of Man Made Technology Today's

ENERGY DENSITY IN ELECTROMAGNETIC WAVE

$W_E = W_B$ in EM wave because $B = \frac{E}{c}$

$$W_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0} \frac{E^2}{c^2} = \frac{\epsilon_0 \mu_0 E^2}{2\mu_0} = \frac{\epsilon E^2}{2} = W_E$$

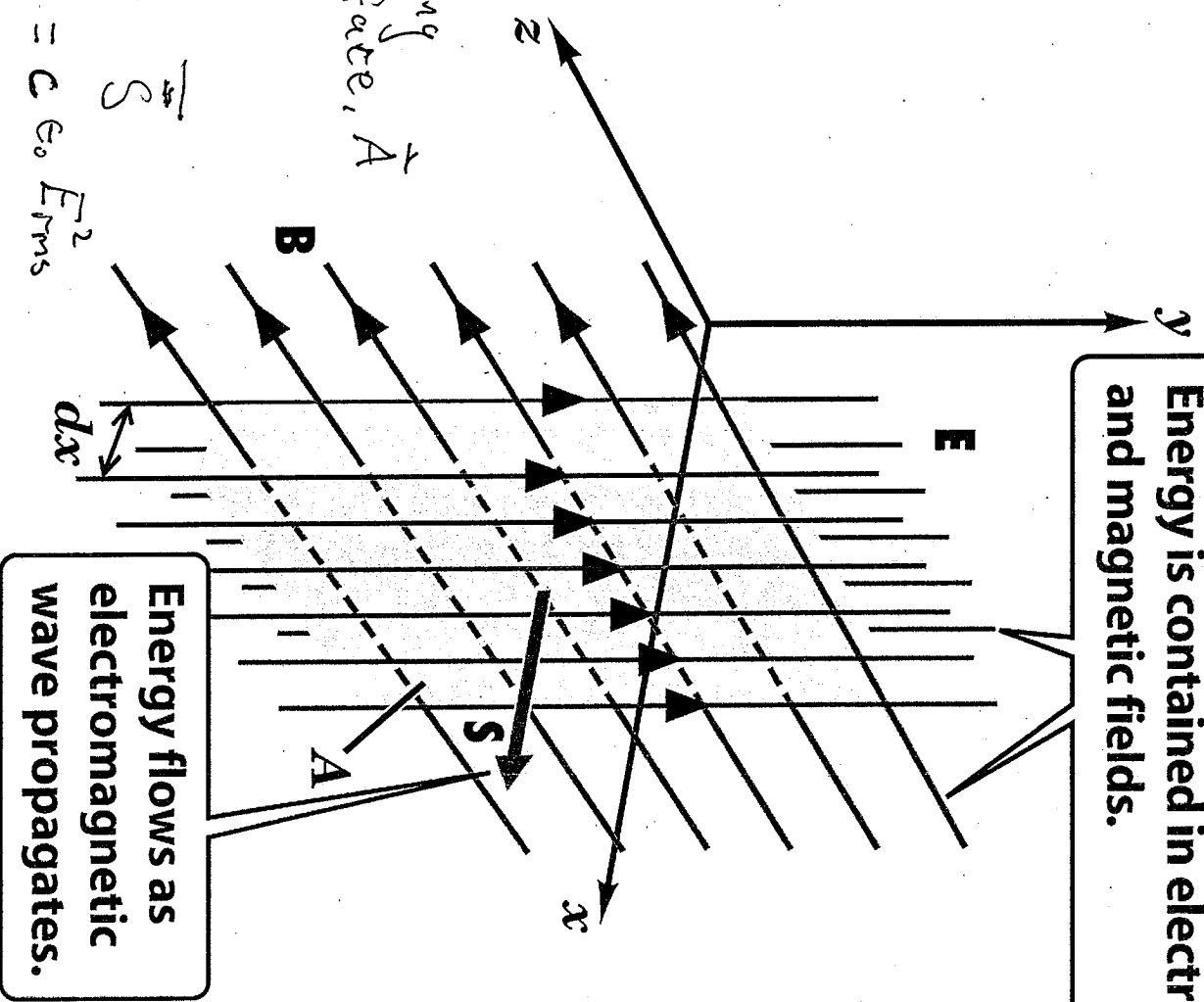
$$W_{EM} = \frac{\epsilon_0 E^2}{2} + \frac{1}{2\mu_0} B^2 = \epsilon_0 E^2$$

Energy is contained in electric and magnetic fields.

EM Power, P , impinging on a surface, A

$$P = \vec{S} \cdot \vec{A}$$

Intensity $I = \frac{P}{A} = c \epsilon_0 E_{rms}^2$



Energy flows as electromagnetic wave propagates.

Poynting's Vector

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$= \vec{c} W_{EM}$$

$$= \vec{c} \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right)$$

Watts / m²

Figure 33-25 Physics for Engineers and Scientists 3/e
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When wave propagates outward in all directions, its energy spreads over spheres of increasing area $A = 4\pi r^2$.

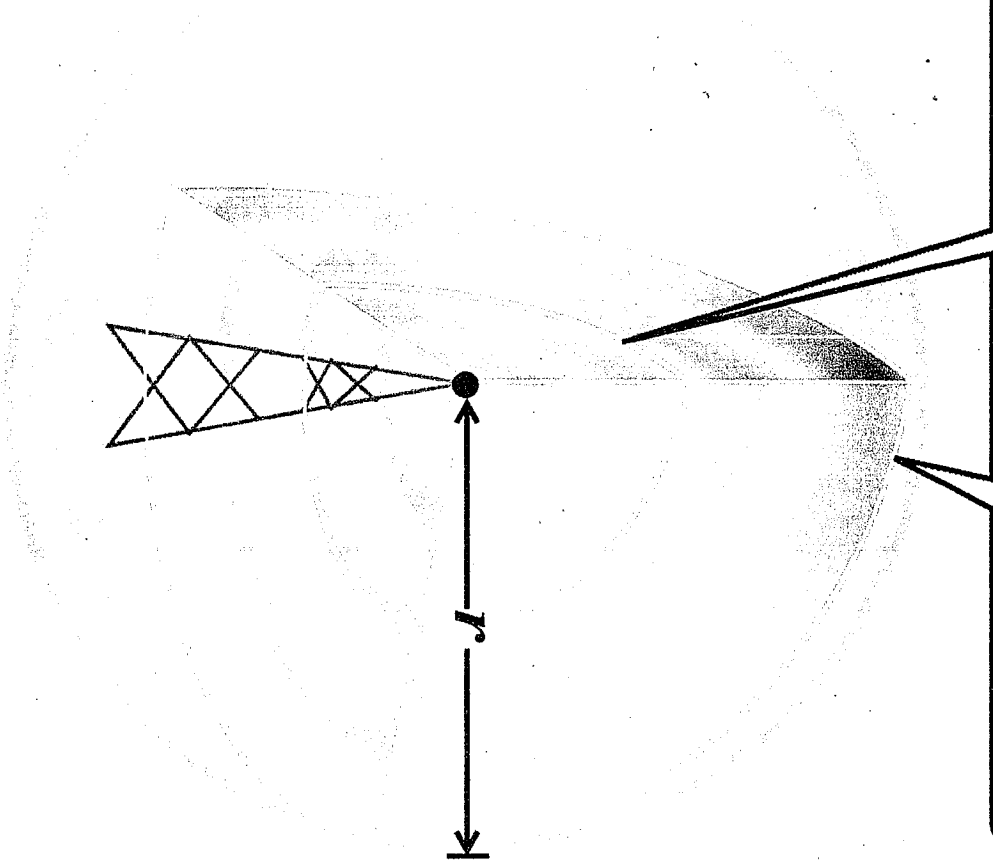


Figure 33-26 Physics for Engineers and Scientists 3/e
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PRESSURE OF ELECTROMAGNETIC WAVE

$$[\text{pressure}] = \begin{cases} \frac{S}{c} & \text{(absorption)} \\ \frac{2S}{c} & \text{(reflection)} \end{cases}$$