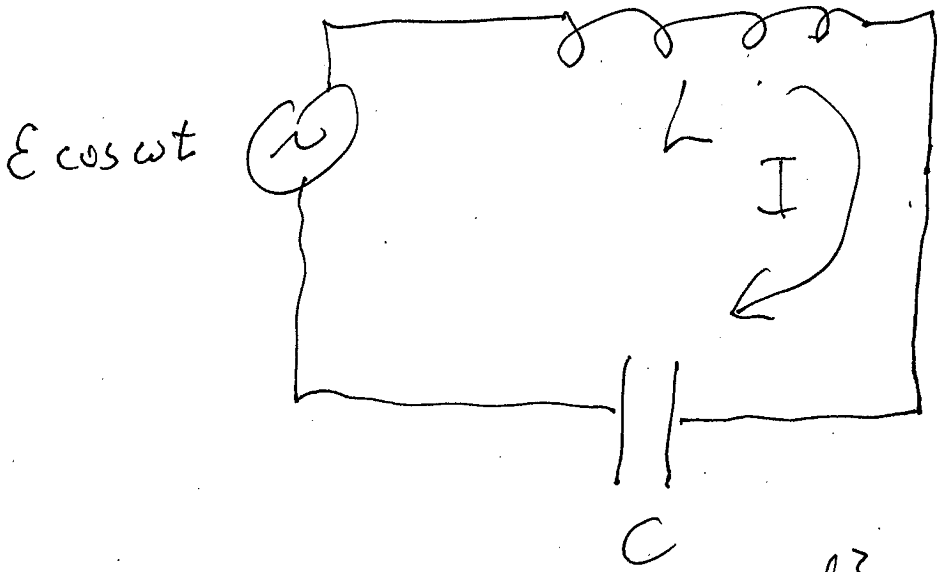
Lecture # 27

LC & RLC AC circuits

LC ~ series circuit



$$-\mathcal{E}\cos\omega t + L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$I = \frac{dq}{dt} = \frac{\mathcal{E}\sin\omega t}{\omega L - 1/\omega C} = \frac{\mathcal{E}\sin\omega t}{X_L - X_c}$$

$$\boxed{\begin{aligned} X_L &\equiv \omega L \equiv \text{inductive impedance} \\ X_c &= \frac{1}{\omega C} \equiv \text{capacitive impedance} \end{aligned}}$$

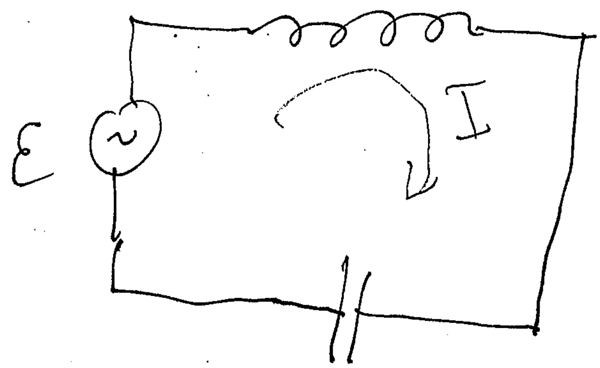
$$= \frac{\mathcal{E}\cos(\omega t - \frac{\pi}{2})}{\omega L - \frac{1}{\omega C}}$$

$$= \frac{\mathcal{E}\cos(\omega t + \frac{\pi}{2})}{\frac{1}{\omega C} - \omega L}$$

If $\omega L > \frac{1}{\omega C}$ (high frequency), I lags V

$\frac{1}{\omega C} > \omega L$ (low frequency), V lags I

$$I = \frac{\mathcal{E} \sin \omega t}{\omega L - 1/\omega C}$$



$$\mathcal{E} = \mathcal{E}_0 \cos \omega t$$

$$I = \frac{\mathcal{E}_0 \sin \omega t}{\omega L - 1/\omega C}$$

(a) What is voltage across the inductor?

(1) $\dfrac{L \quad \mathcal{E}_0 \sin(\omega t)}{\omega L - 1/\omega C}$ (b) $\dfrac{\mathcal{E}_0 \cos(\omega t)}{\omega\left(\frac{1}{\omega C} - \omega L\right)C}$

(c) $\dfrac{\mathcal{E}_0 \sin(\omega t)}{\omega\left(\frac{1}{\omega C} - \omega L\right)C}$ (d) $\dfrac{\omega L\, \mathcal{E}_0 \cos(\omega t)}{\omega L - 1/\omega C}$

(b) What is voltage across the capacitor?

In an RC AC circuit, what is the ratio of the average stored electric to magnetic energy?

stored in the circuit

electric energy stored in capacitor

$$W_E = \frac{\overline{Q^2}}{2C} = \frac{\overline{(\int^t dt\, I)}}{2C}$$

$\omega_0^2 = \frac{1}{LC}$

$$= \frac{\mathcal{E}^2\, \overline{\cos^2(\omega t - \varphi)}}{2\omega^2 C\left[\omega L - \frac{1}{\omega C}\right]^2} = \frac{\mathcal{E}^2}{\omega^2 4C\left[\omega L - \frac{1}{\omega C}\right]^2}$$

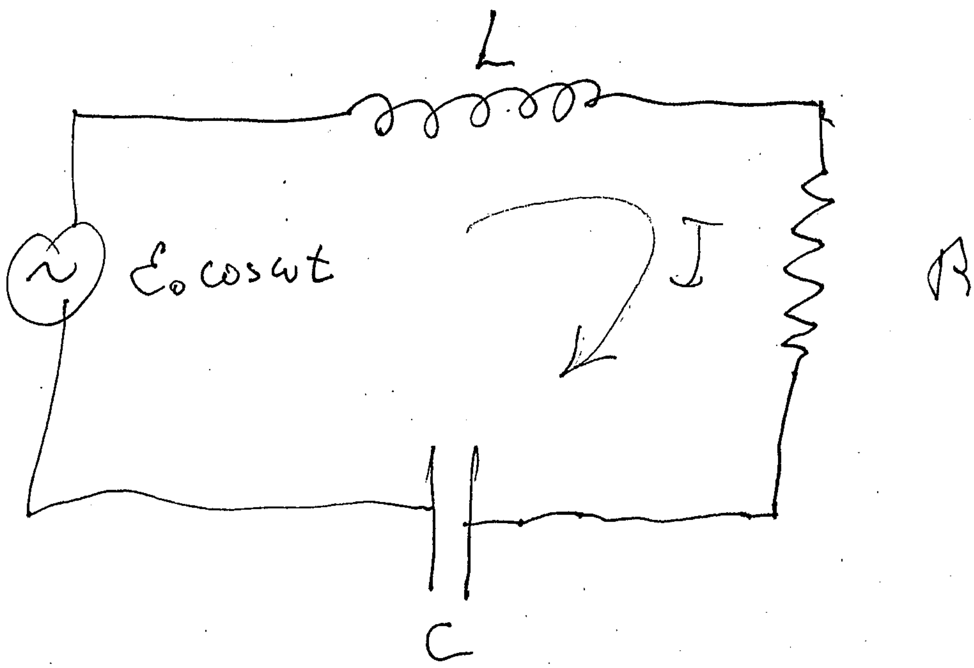
$$W_B = \frac{\overline{L I^2}}{2} = \frac{L}{2}\,\frac{\mathcal{E}^2\, \overline{\sin^2(\omega t - \varphi)}}{\left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \frac{L}{4}\,\frac{\mathcal{E}^2}{\left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\frac{W_E}{W_B} = \frac{\frac{\mathcal{E}^2}{4\omega^2 C}}{\frac{L}{4}\mathcal{E}^2} = \frac{1}{\omega^2 LC} = \frac{\omega_0^2}{\omega^2} \quad ; \quad \omega_0^2 = \frac{1}{\sqrt{LC}}$$

$W_E > W_B$ if $\omega < \omega_0$, $\underset{W_L}{\overset{\shortparallel}{W_B}} > \underset{W_C}{\overset{\shortparallel}{W_E}}$ if $\omega > \omega_0$

RLC circuit



$$-\mathcal{E}_0 \cos\omega t + L\frac{dI}{dt} + IR + \frac{\int^t dt'\, I(t')}{C} = 0$$

solution

$$I = \frac{\mathcal{E}_0 \cos(\omega t - \varphi)}{Z}$$

$$Z = \left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{1/2} \equiv \text{total impedance}$$

$$\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad X_L = \omega L \qquad X_C = 1/\omega C$$

$\uparrow$

phase or current

$$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$$

current lags $X_L > X_C$

voltage lags $X_C > X_L$

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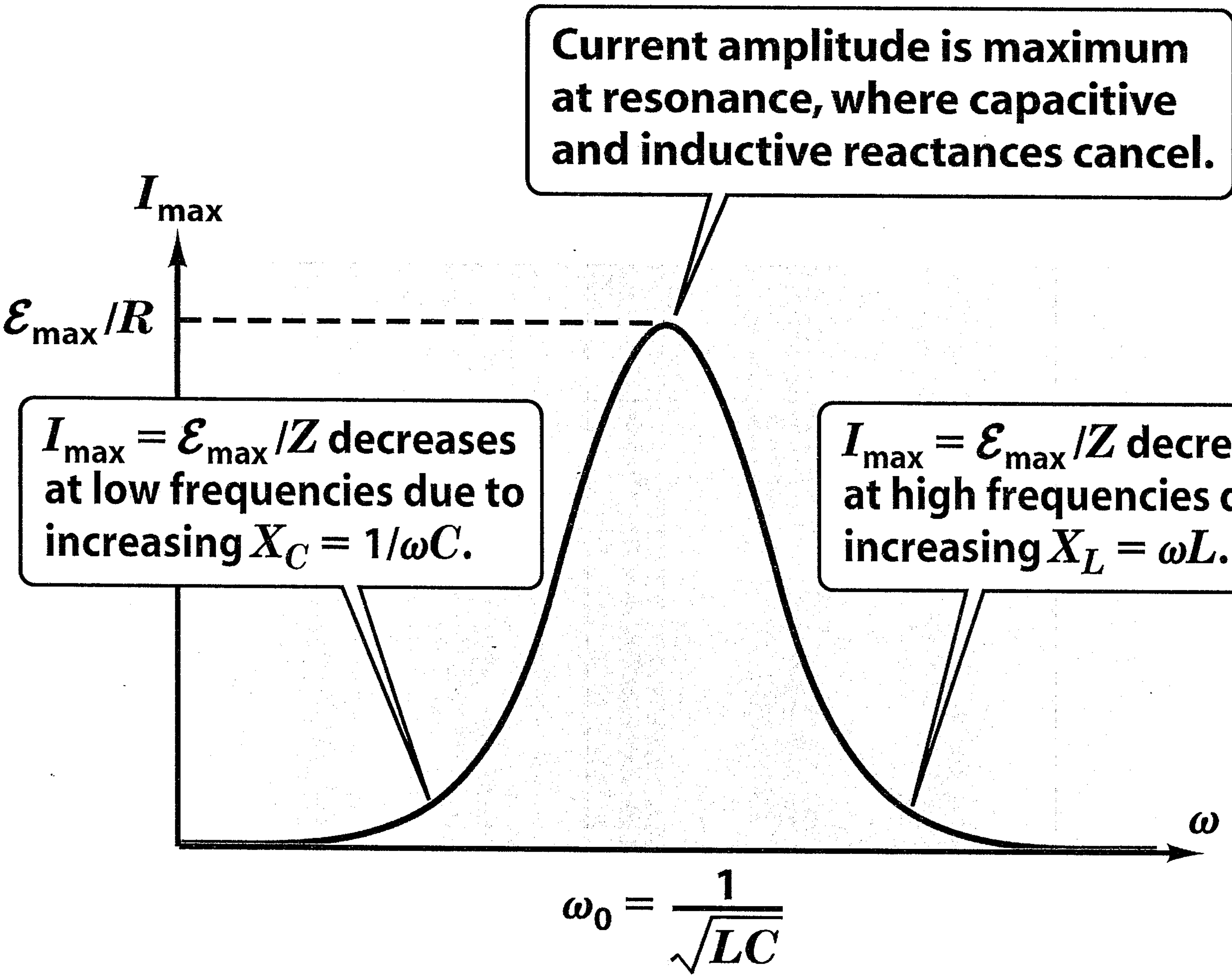

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Note Impedance is smallest, and current is largest at frequency $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$

$$L\frac{I_{rms}}{\mathcal{E}_{rms}} = \frac{1}{\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{1/2}}$$

RMS Power Delivered by Alternator

$$\overline{P} = \frac{1}{T}\int_0^T dt\ \mathcal{E}(t)\, I(t)$$

$$= \frac{\mathcal{E}_0^2}{T}\int_0^T dt\ \cos\omega t\ \frac{\cos(\omega t - \varphi)}{Z(\omega)}$$

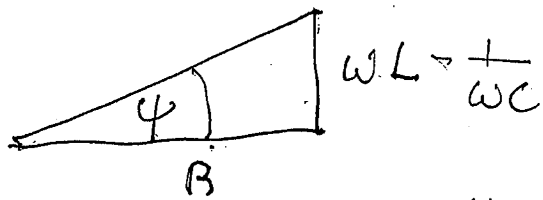
$$Z(\omega) = \left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{1/2}$$

$$= \frac{\mathcal{E}_0^2}{Z(\omega)}\left[\frac{1}{T}\int_0^T dt\ \cos\omega t\left(\underbrace{\cos\omega t\cos\varphi}_{=\frac{1}{2}\cos\varphi} + \underbrace{\sin\omega t\sin\varphi}_{=0}\right)\right.$$

$$= \frac{\cos\varphi}{2Z(\omega)}\,\mathcal{E}_0^2 = \overline{P}$$

Power Delivered

$$\bar{P} = \frac{\cos\varphi \; \mathcal{E}_0^2}{2\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{1/2}}$$



$$\cos\varphi = \frac{R}{\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]^{1/2}}$$

$$\boxed{\bar{P} = \frac{R\,\mathcal{E}_0^2}{2\left[\left(\omega L - \frac{1}{\omega C}\right)^2 + R^2\right]}}$$

(Response of a resonant circuit)

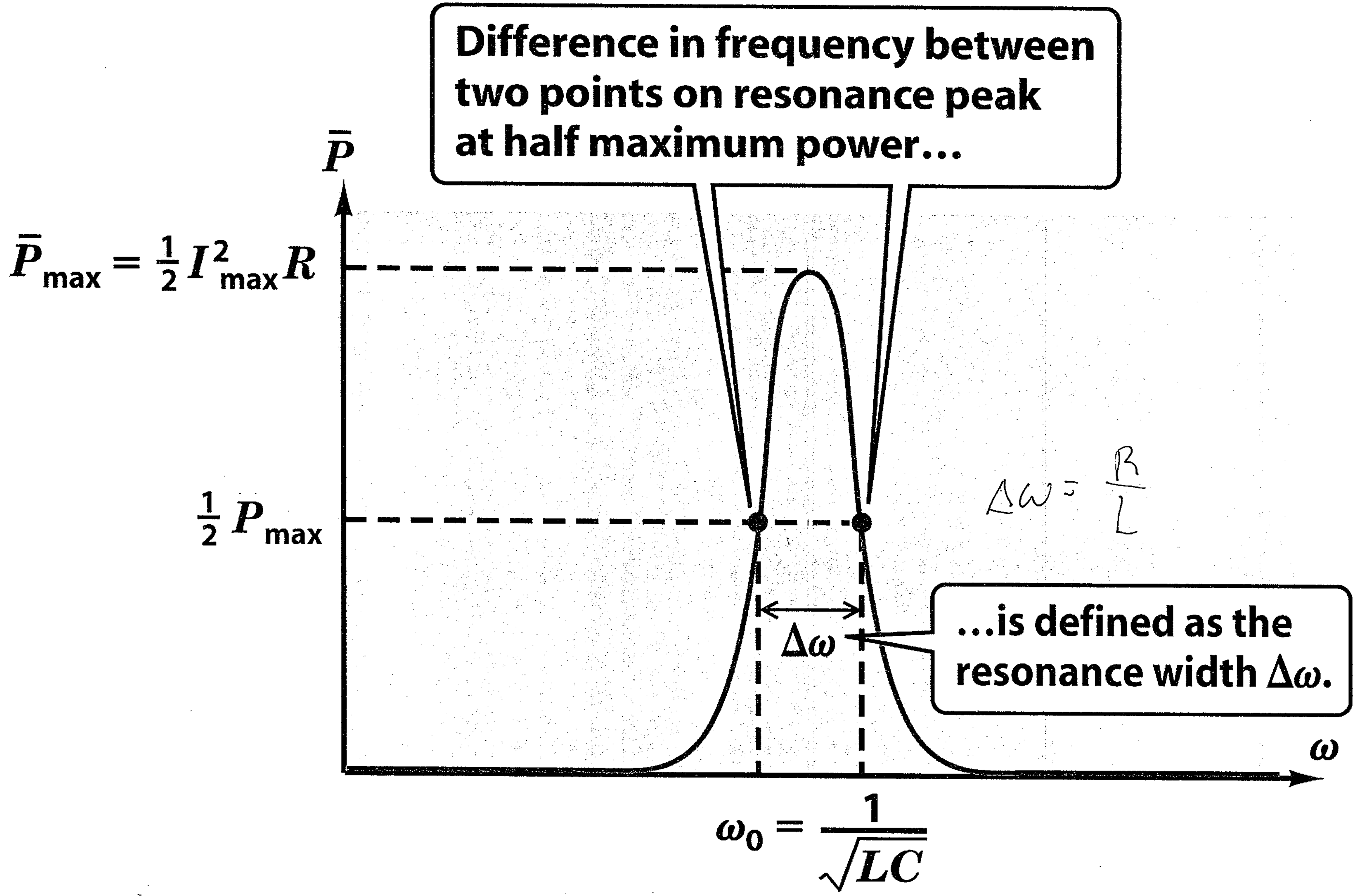
If $\frac{R^2}{L^2} \ll \omega^2$, $\omega - \omega_0 \ll \omega_0$

$$\bar{P} \approx \frac{\mathcal{E}_0^2}{2}\,\frac{R}{\left(4(\omega-\omega_0)^2 L^2 + R^2\right)}$$

$$\boxed{\bar{P} = \frac{\mathcal{E}_0^2}{2R}\,\frac{1}{\left(\frac{\omega-\omega_0}{\Delta\omega}\right)^2 + 1}}$$ Typical resonance function

Resonance Half width $\equiv 2\Delta\omega = \frac{R}{L}$

Quality Factor $\frac{\omega_0}{2\Delta\omega} = \left(\frac{L}{C}\right)^{1/2}\frac{1}{R}$



$\Delta\omega = \frac{R}{L}$

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