

Lecture # 26

AC circuits

The more resistance there is in an LRC circuit, the more likely it is for the circuit to oscillate

(1) True

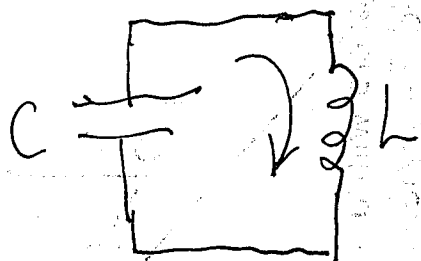
(2) False

(3) oscillation independent of resistance

Recall that in an LC circuit, the ^{radian} frequency of oscillation is $\omega_0 = \frac{1}{\sqrt{LC}}$. If

the maximum current in the inductor is I_{\max} , the maximum charge, Q_{\max} , across the capacitor is

$$I = I_{\max} \cos(\omega_0 t + \phi)$$



$$(1) \quad Q_{\max} = \sqrt{LC} I_{\max}$$

$$(2) \quad Q_{\max} = \frac{1}{\sqrt{LC}} I_{\max}$$

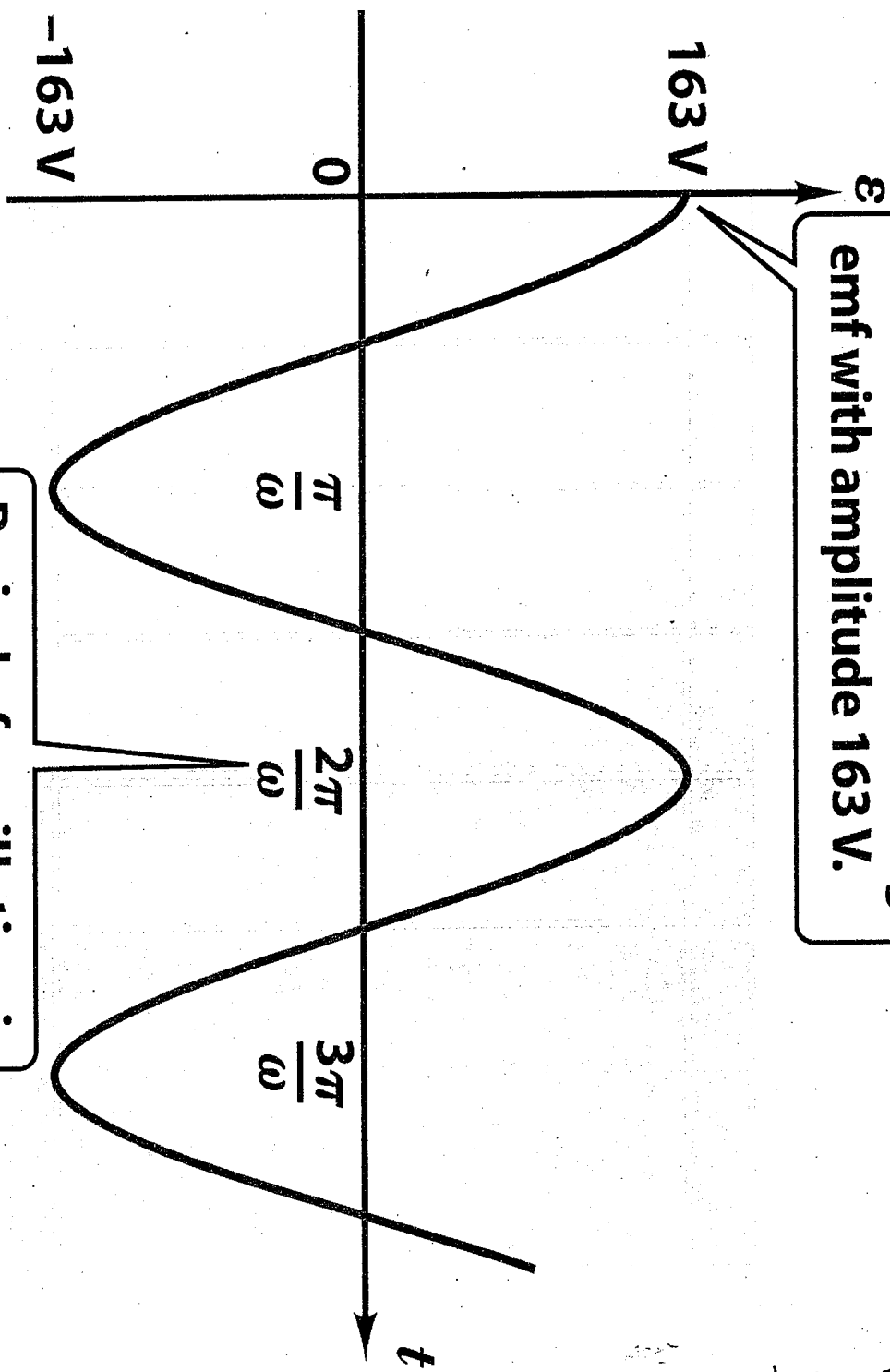
$$(3) \quad Q_{\max} = L I_{\max}$$

$$(4) \quad Q_{\max} = I_{\max} / C$$

Remember: oscillation transfers magnetic energy in inductor to electrical energy in capacitor. Total energy is always conserved $\frac{1}{2}(LI^2 + \frac{Q^2}{C}) = \frac{I_{\max}^2 L}{2} = \frac{Q_{\max}^2}{2C}$.

AC-currents: The Voltage cell around us

"115-V AC" voltage at electric outlets is oscillating emf with amplitude 163 V.



Period of oscillation is $T = 2\pi/\omega = 1/60 \text{ s}$.

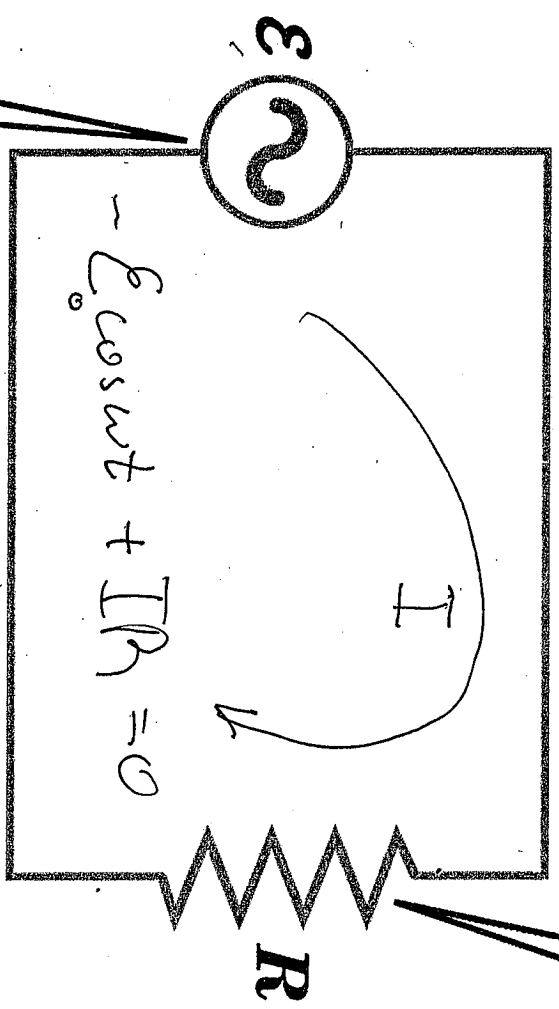
$$T = \frac{2\pi}{\omega}$$

$$I_{\text{RMS}} = \left(\frac{1}{T} \int_0^T V^2(t) dt \right)^{1/2} = V_{\text{max}} / \sqrt{2}$$

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Simplest AC circuit: oscillating emf supplies oscillating current through resistor.

Wave in circle is circuit symbol for source of oscillating emf.



$$I = \frac{\epsilon_0 \cos \omega t}{R}$$

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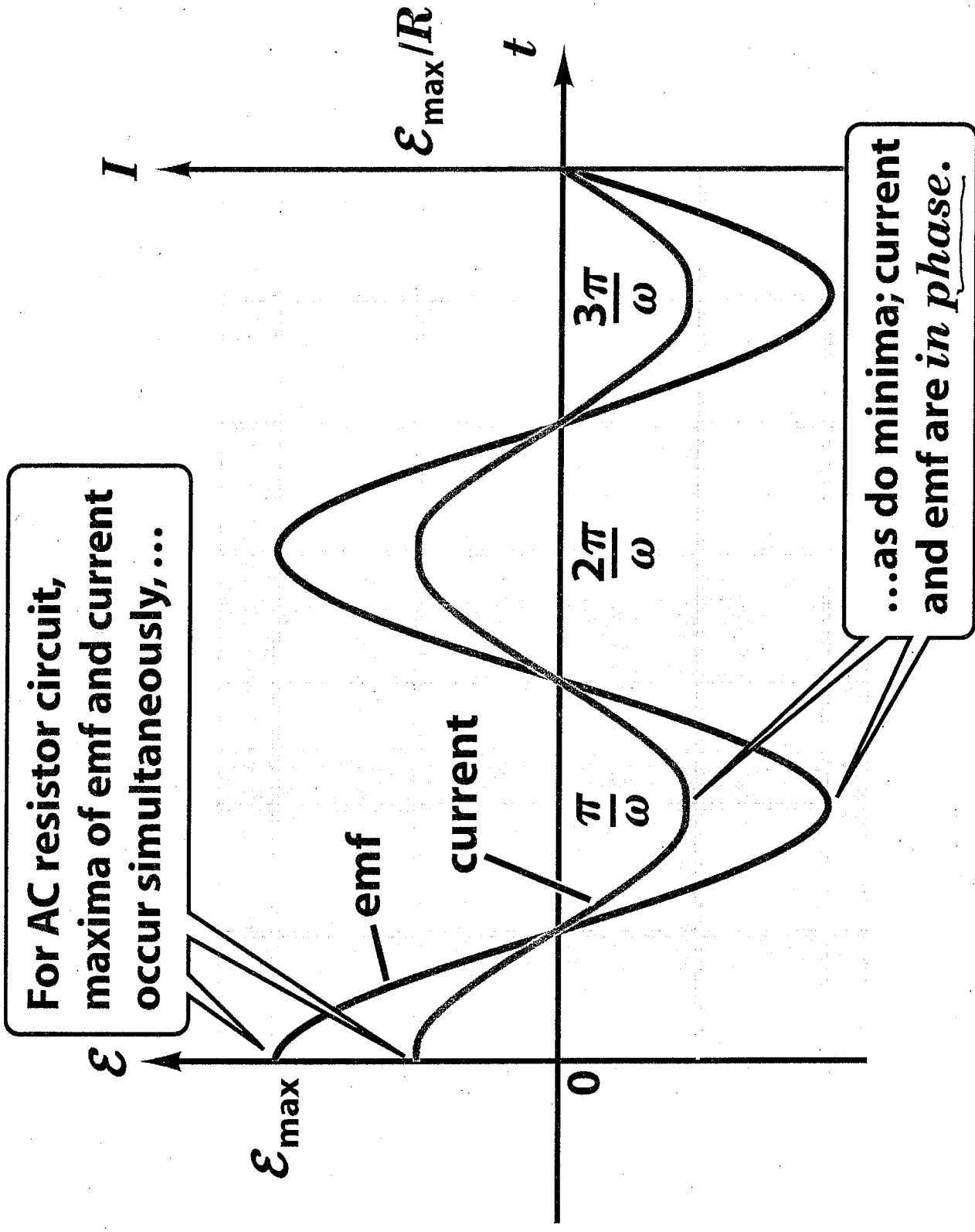


Figure 32-4. Physics for Engineers and Scientists 3/e
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$$T = \frac{2\pi}{\omega}$$

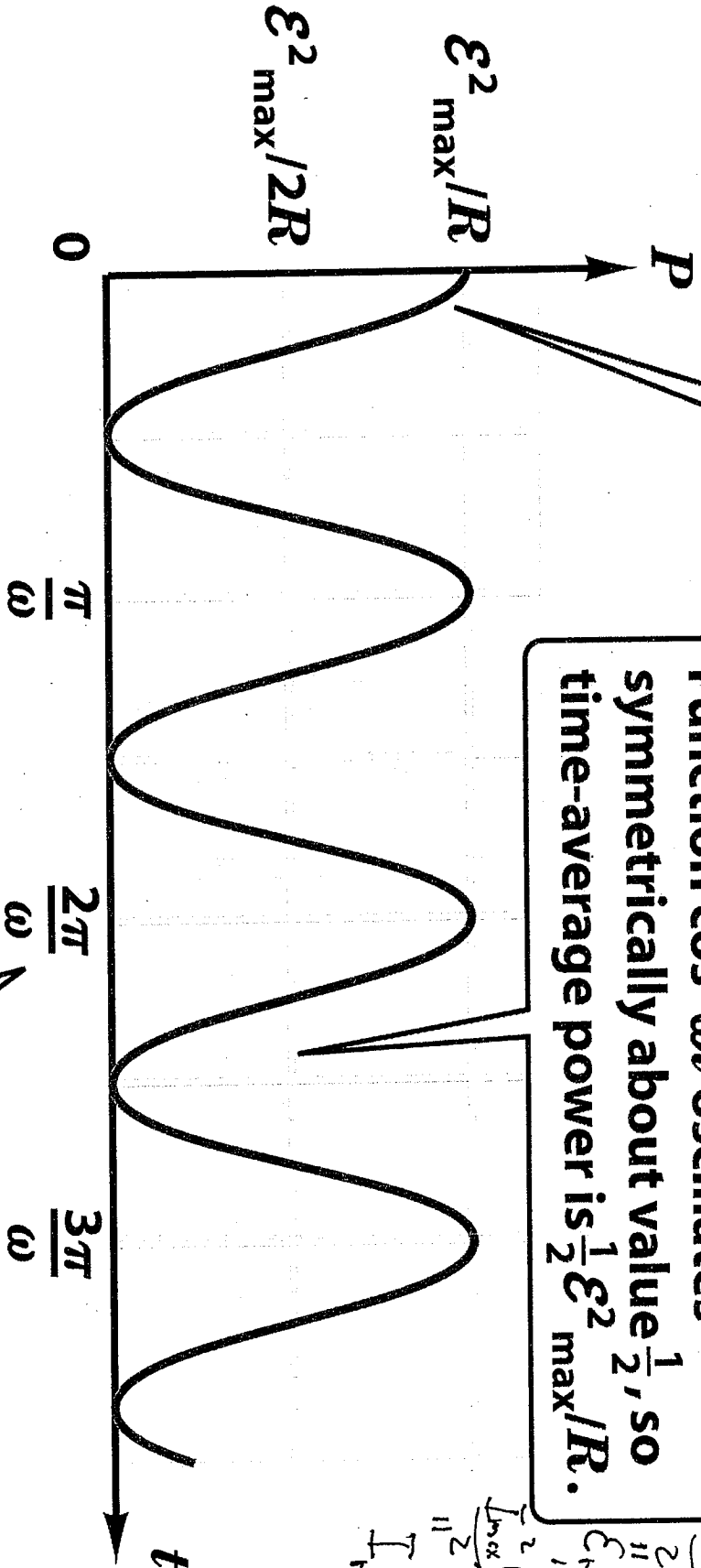
Average Power delivered

$$= \frac{1}{T} \int_0^T dt I(t) \mathcal{E}(t)$$

$$= \frac{\mathcal{E}_{\max} I_{\max}}{T} \int_0^T dt \cos^2 \omega t = \frac{\mathcal{E}_{\max} I_{\max}}{2}$$

Power dissipated in resistor is always positive.

Function $\cos^2 \omega t$ oscillates symmetrically about value $\frac{1}{2}$, so time-average power is $\frac{1}{2} \mathcal{E}_{\max}^2 / R$.



In one period $T = 2\pi/\omega$ of emf, power goes through two cycles.

$$\frac{\mathcal{E}_{\max}^2}{2R} = \frac{\mathcal{E}_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R$$

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(6)

AC capacitor circuit: oscillating emf produces oscillating charge on capacitor, $Q = C\mathcal{E}$.

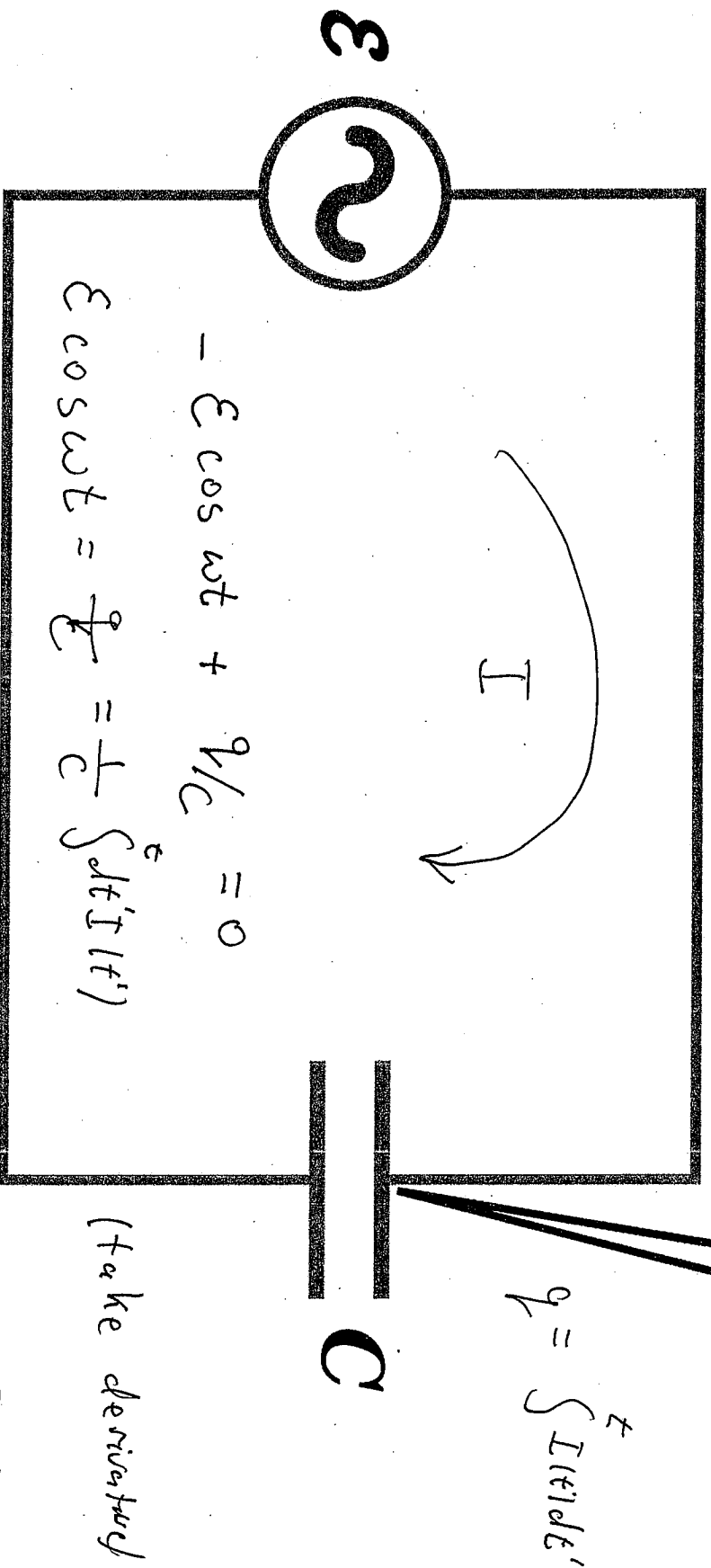


Figure 32-7 Physics for Engineers and Scientists 3/e
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$$-\omega \mathcal{E} \sin \omega t = \frac{I(t)}{C}; \quad I(t) = -\omega C \mathcal{E} \sin \omega t$$

Capacitive circuit [Current Leads Voltage]

$$\mathcal{E} = \mathcal{E}_{\max} \cos(\omega t)$$

$$I = \omega C \mathcal{E}_{\max} \cos(\omega t + \frac{\pi}{2}) = -\omega C \mathcal{E}_{\max} \sin(\omega t)$$

Current Leads Voltage in capacitive circuit

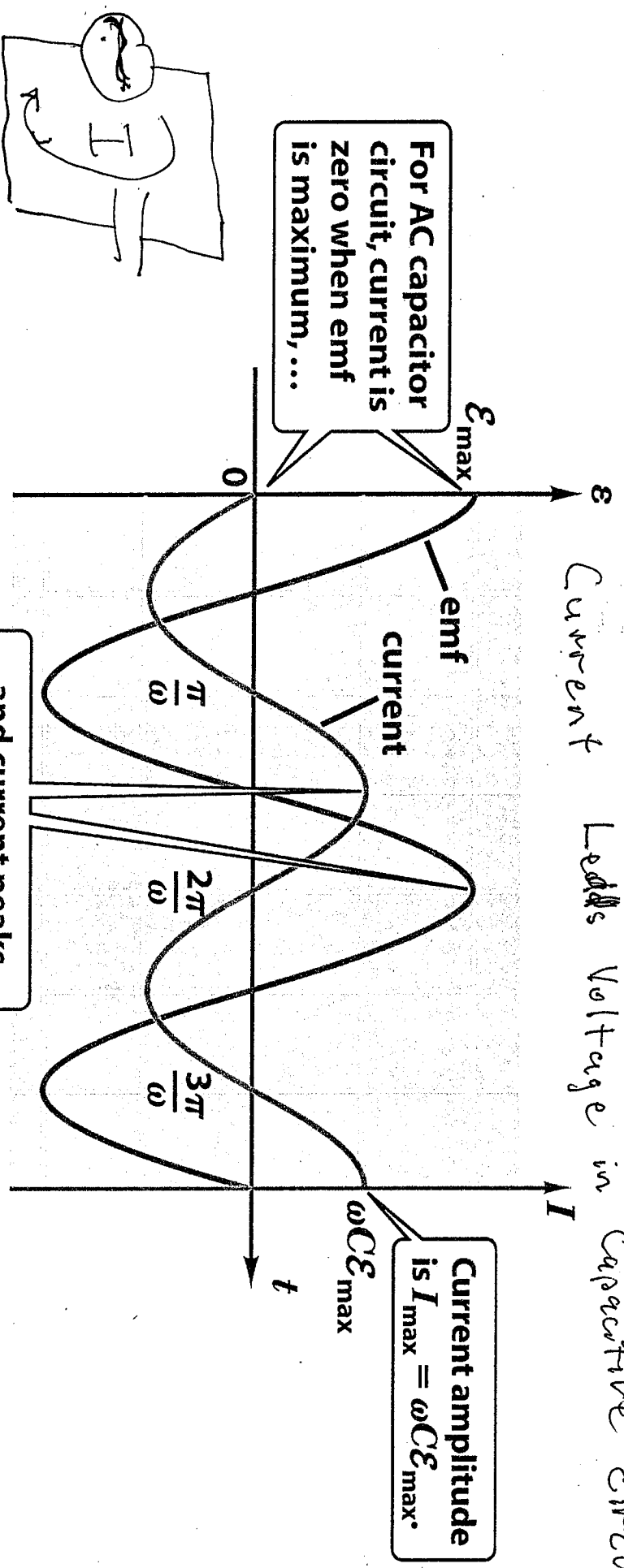


Figure 32-8 Physics for Engineers and Scientists 3/e © 2007 W. W. Norton & Company, Inc.

take derivative

$$-\mathcal{E} \cos \omega t + \int_0^t I dt' / C = 0$$

$$+\omega \mathcal{E} \sin \omega t + \frac{I}{C}$$

$$I = -\omega C \mathcal{E} \sin \omega t$$

$$X_C \equiv \text{capacitive impedance} = \frac{1}{\omega C} \text{ (unit of resistance)}$$

$$I = -\frac{\mathcal{E}_{\max}}{X_C} \sin \omega t$$

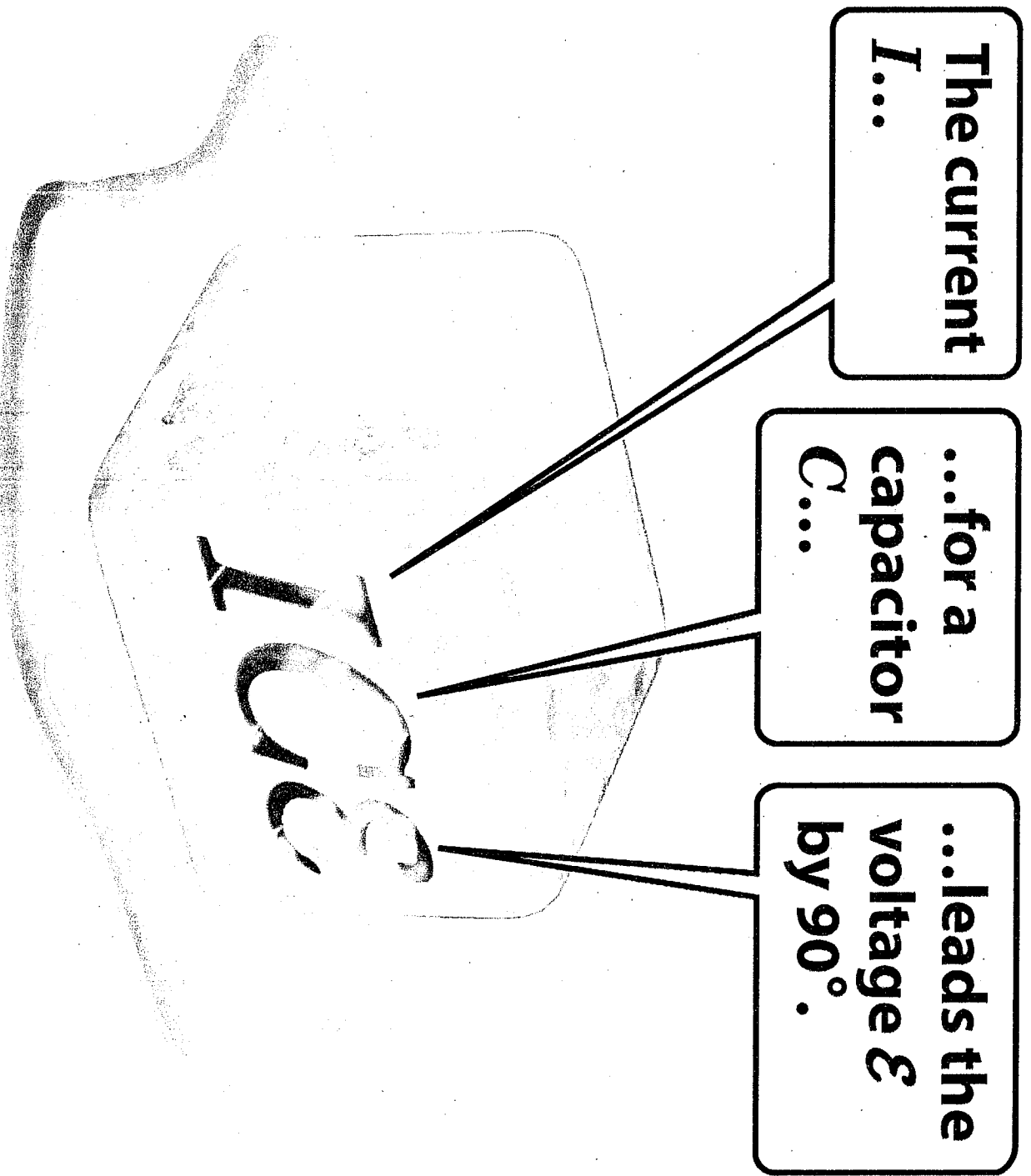


Figure 32-9 Physics for Engineers and Scientists 3/e
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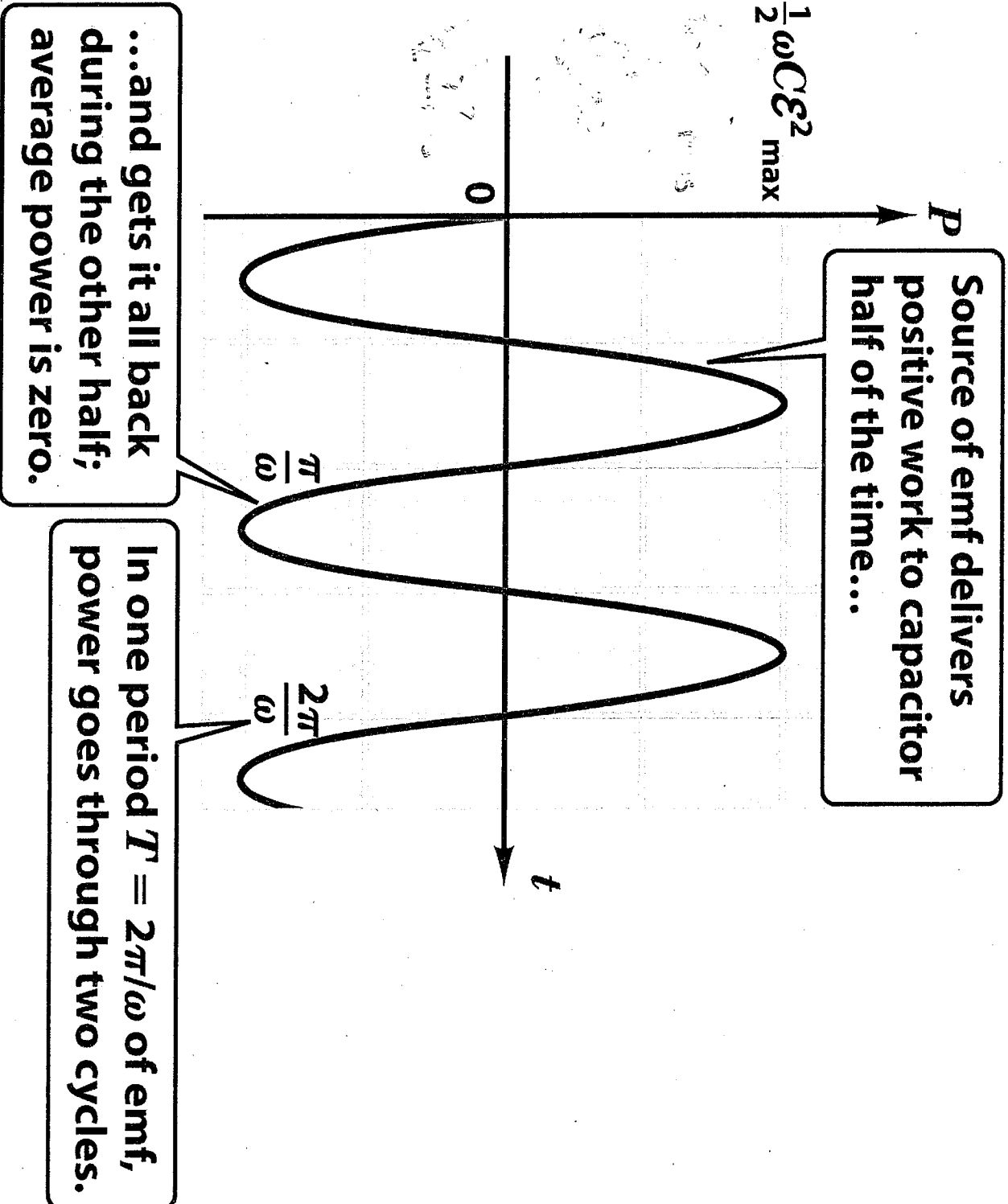


Figure 32-10 Physics for Engineers and Scientists 3/e
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$$\mathcal{E} = \mathcal{E}_{\max} \cos \omega t$$

$$-\mathcal{E} + L \frac{dI}{dt} = 0$$

$$\frac{dI}{dt} = \frac{\mathcal{E}_{\max}}{L} \cos \omega t$$

$$I(t) = \frac{\mathcal{E}_{\max}}{\omega L} \sin \omega t$$

$$\omega L = X_L$$

$$I(t) = \frac{\mathcal{E}_{\max}}{X_L} \sin(\omega t)$$

$$= \frac{\mathcal{E}_{\max}}{X_L} \cos(\omega t - \frac{\pi}{2})$$

AC inductor circuit: oscillating emf produces oscillating current in inductor.

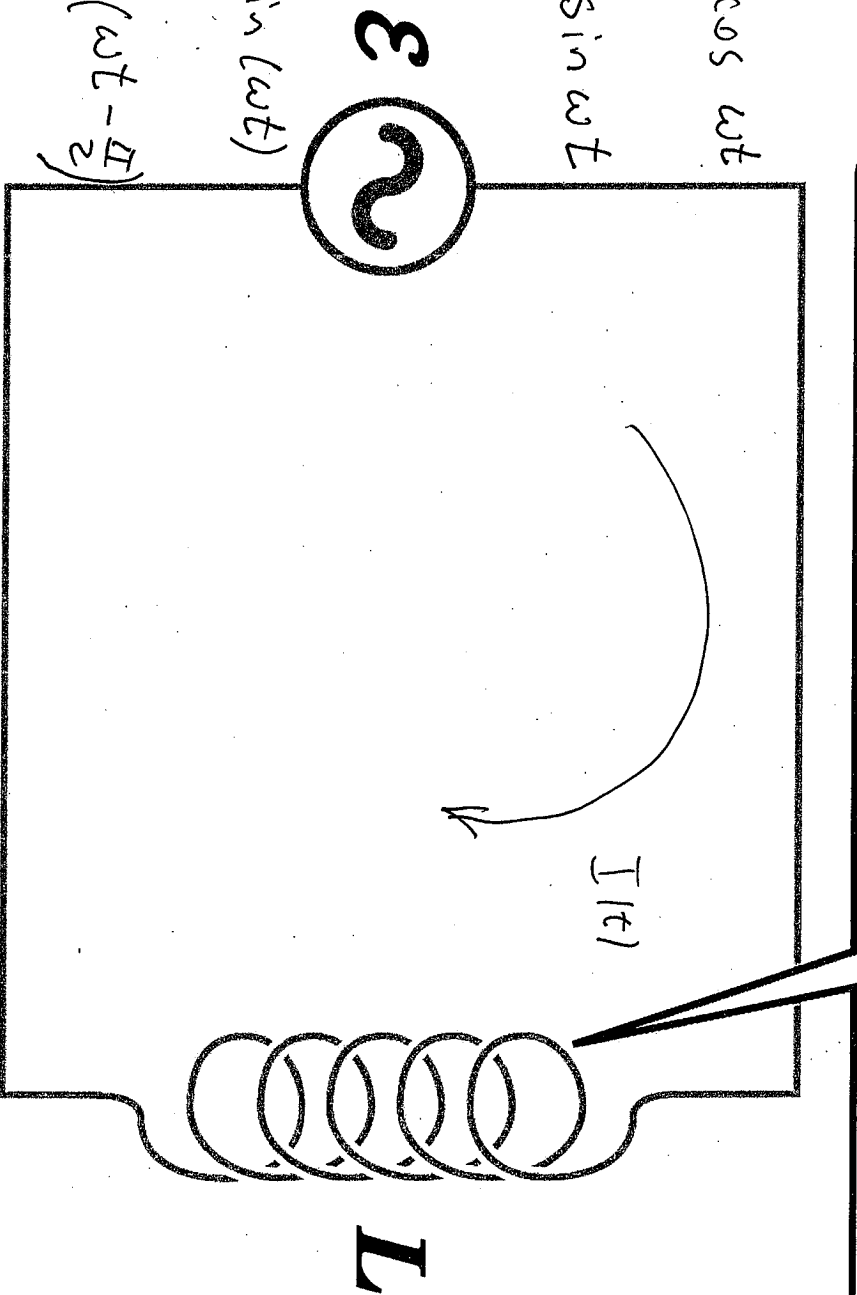


Figure 32-11 Physics for Engineers and Scientists 3/e
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$X_L = \omega L \equiv$ inductive impedance

I_m inductor current Lags Voltage

$I(t) = \frac{\mathcal{E}_{max}}{X_L} \cos(\omega t - \frac{\pi}{2}) ; X_L = \omega L$

Current amplitude is $I_{max} = \mathcal{E}_{max} / \omega L$.

$\frac{\mathcal{E}_{max}}{X_L}$

...and voltage peaks earlier in time than nearest current peak.

For AC inductor circuit, current is zero when emf is maximum, ...

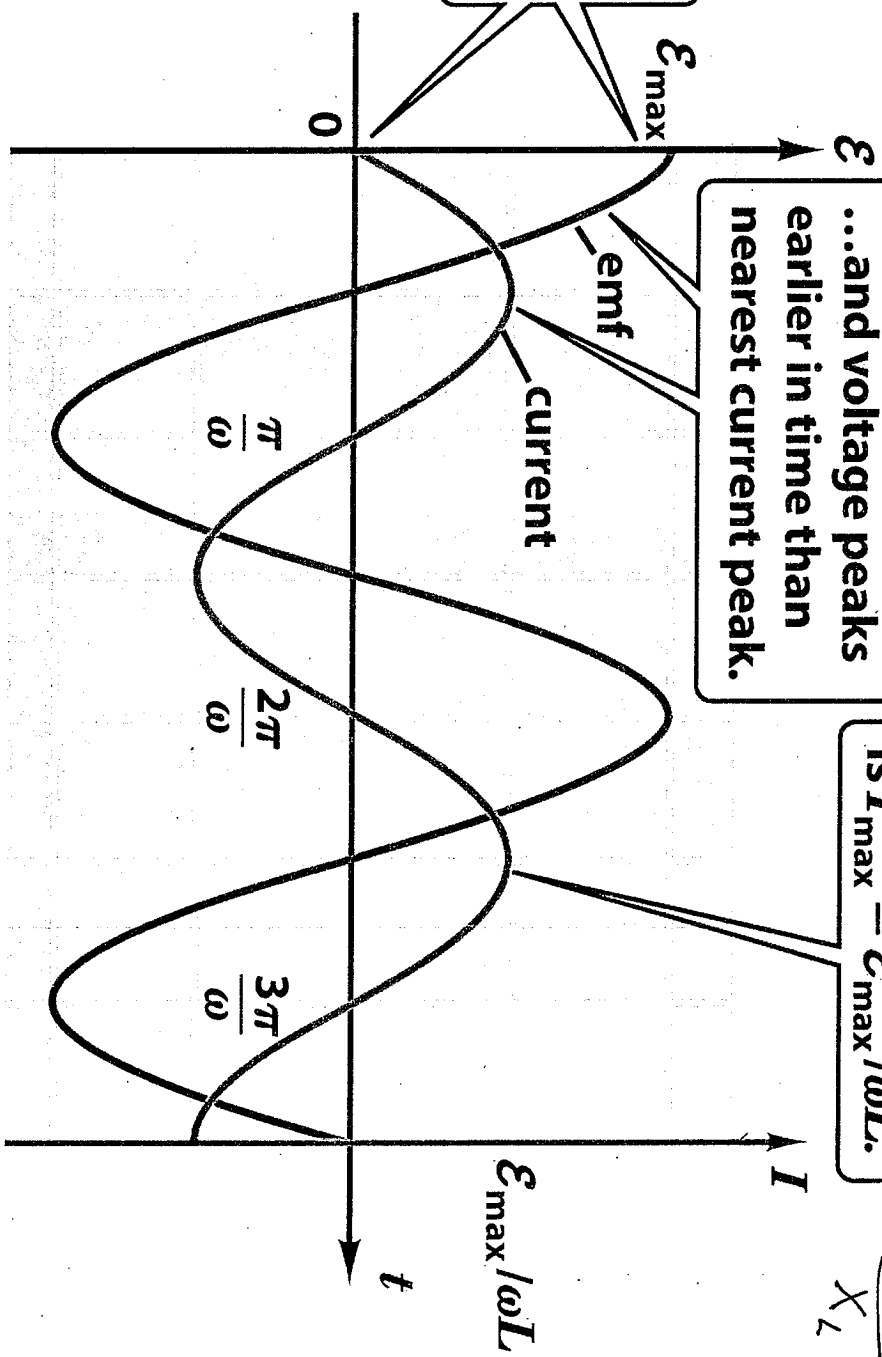


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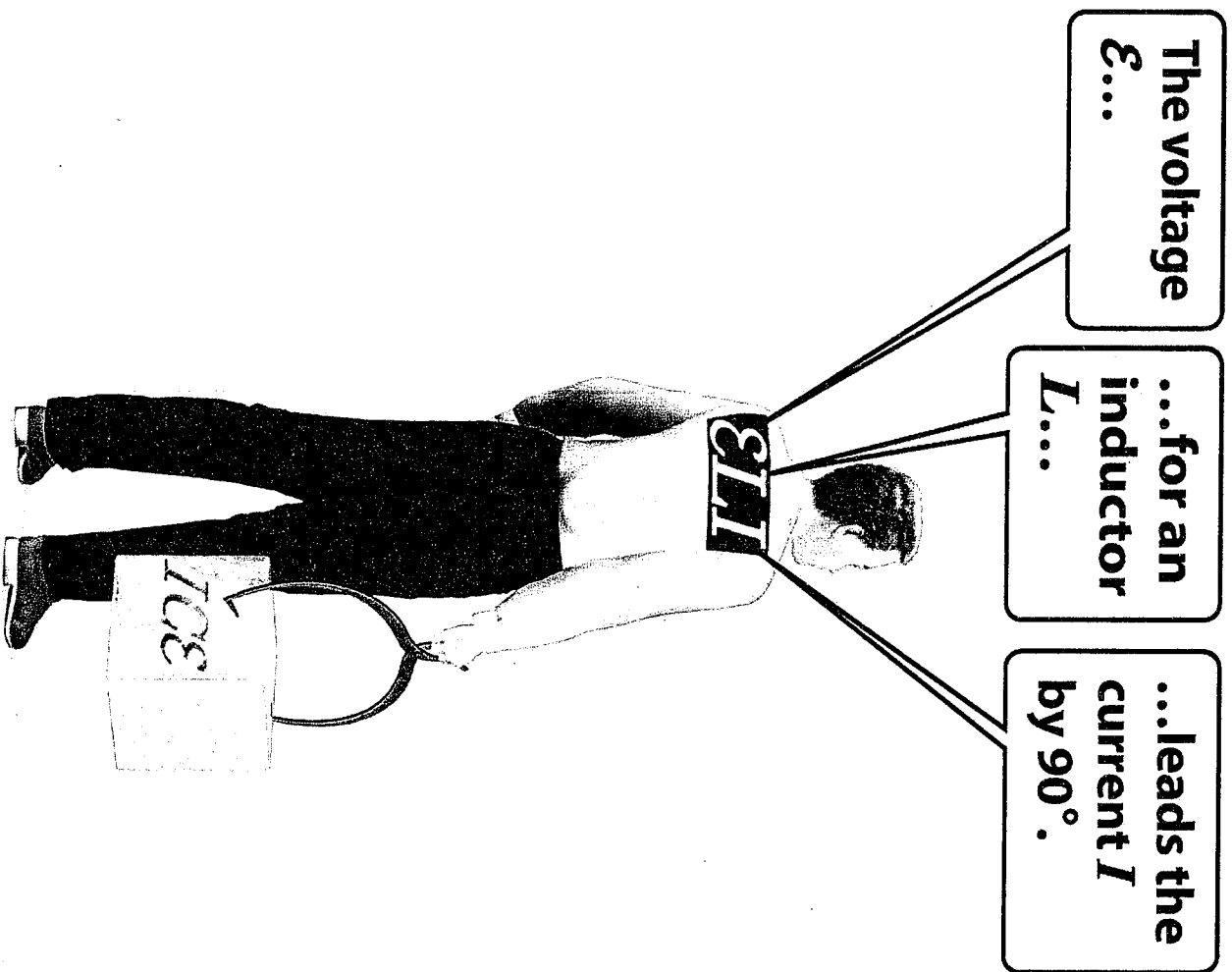


Figure 32-13 Physics for Engineers and Scientists 3/e
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$$-\mathcal{E}_{\max} \cos \omega t + L \frac{dI}{dt} + \int \frac{dt' I(t')}{C} = 0$$

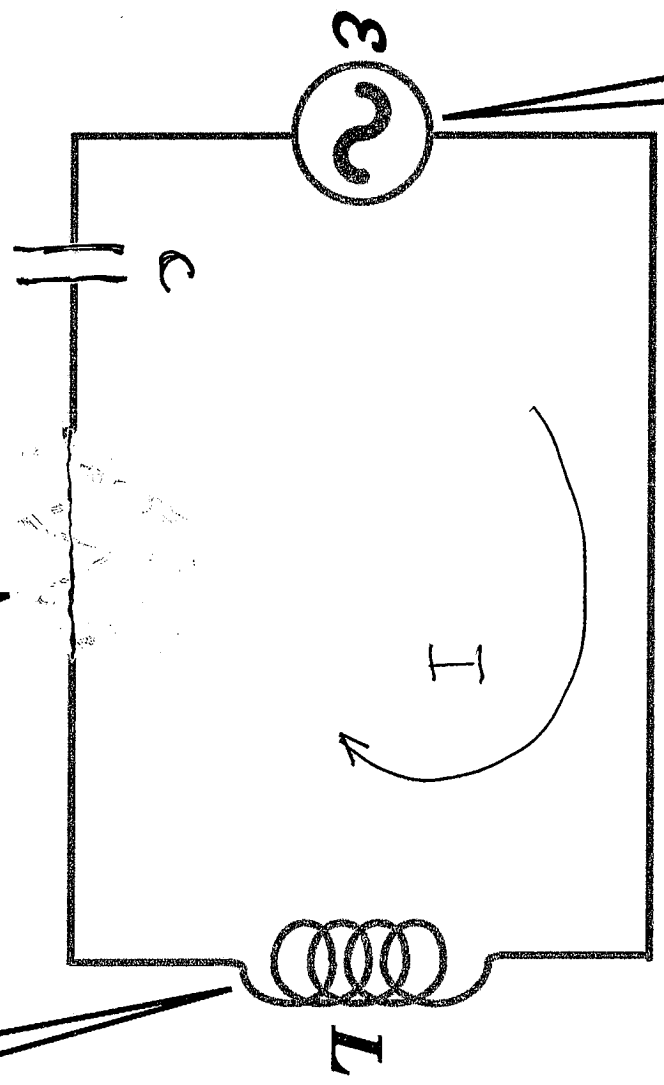
$$I(t) = \frac{\mathcal{E}_{\max} \sin \omega t}{\left(\omega L - \frac{1}{\omega C} \right)} = \frac{\sin \omega t \mathcal{E}_{\max}}{\left(X_L - X_C \right)}$$

when $X_L = X_C$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0 \equiv$ natural oscillation frequency

Oscillating emf of source...



...must equal sum of instantaneous voltages across components connected in series.

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