Lecture # 26

AC circuits

The more resistance there is in an LRC circuit, the more likely it is for the circuit to oscillate

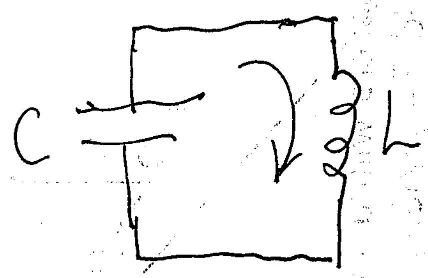
(1) True

(2) False

(3) oscillation independent of resistance

Recall that in an LC circuit, the radian frequency of oscillation is $\omega_0 = \frac{1}{\sqrt{LC}}$. If the maximum current in the inductor is I_{max}, the maximum charge, Q_{max}, across the capacitor is

$I = I_{max} \cos(\omega_0 t + \phi)$



(1) $Q_{max} = \sqrt{LC}\, I_{max}$

(2) $Q_{max} = \frac{1}{\sqrt{LC}} I_{max}$

(3) $Q_{max} = L\, I_{max}$

(4) $Q_{max} = I_{max}/C$

Remember: oscillation transfers magnetic energy in inductor to electrical energy in capacitor. Total energy is always conserved $\frac{1}{2}\left(LI^2 + \frac{Q^2}{C}\right) = \frac{I_{max}^2 L}{2} = \frac{Q_{max}^2}{2C}$.

(1)

AC-currents: The Voltage all around us

$T = \frac{2\pi}{\omega}$

$V_{RMS} = \left(\frac{1}{T}\int_0^T dt\, V^2(t)\right)^{1/2} = V_{max}/\sqrt{2}$

I_{RMS}

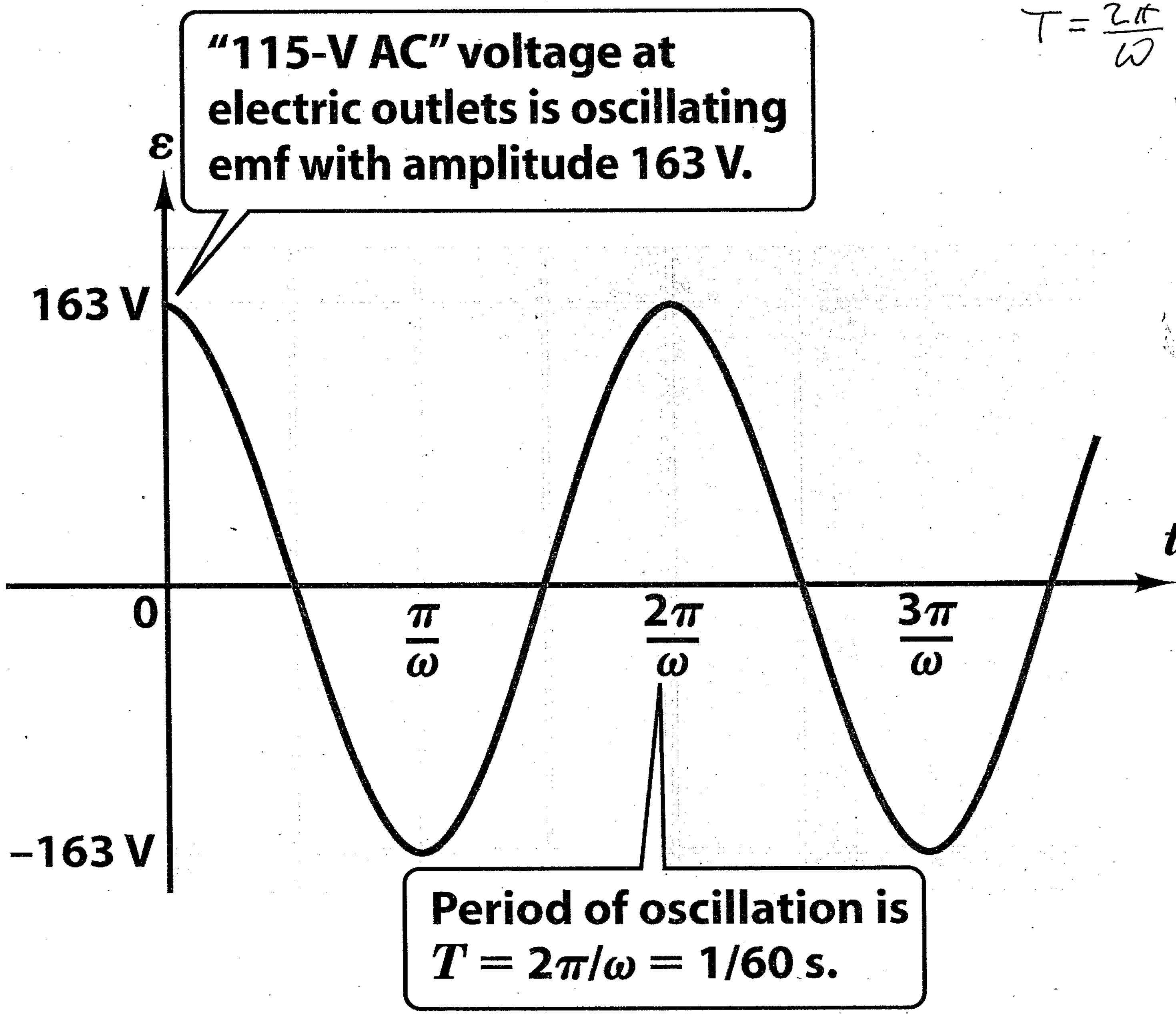


Figure 32-1 Physics for Engineers and Scientists 3/e

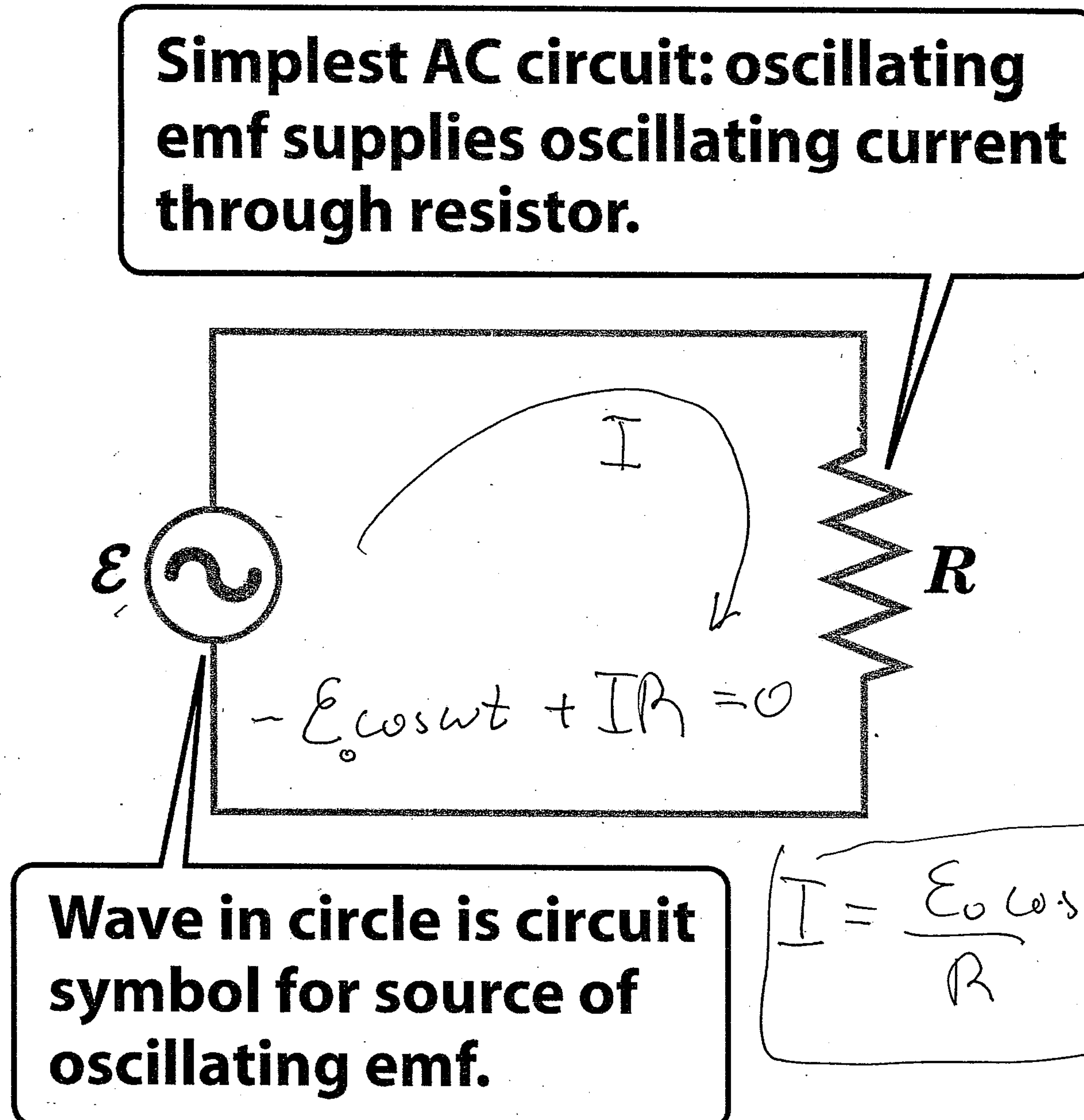


Figure 32-2 Physics for Engineers and Scientists 3/e

$$\boxed{I = \frac{\mathcal{E}_0 \cos\omega t}{R}}$$

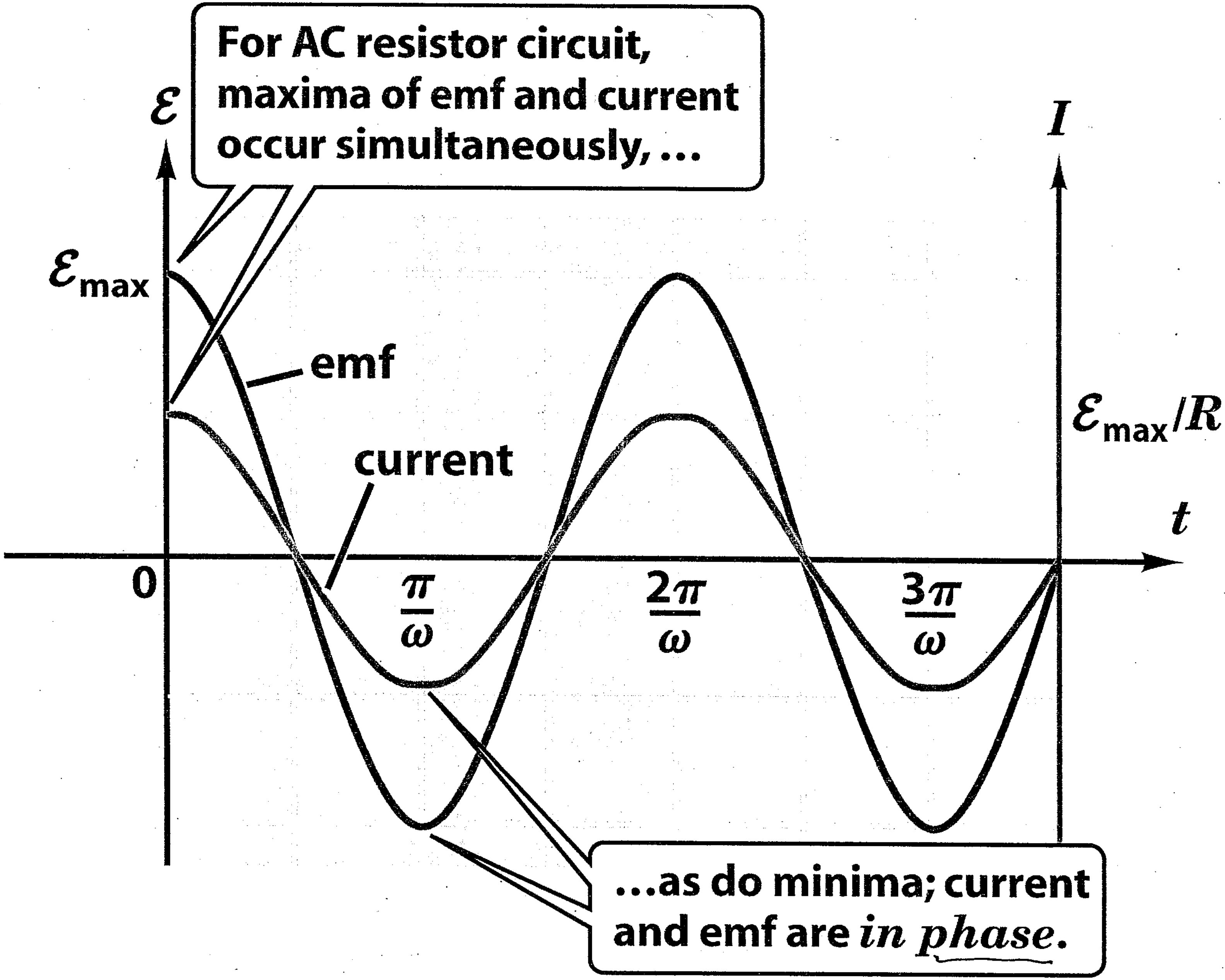


Figure 32-4 Physics for Engineers and Scientists 3/e

$T = \frac{2\pi}{\omega}$

Average Power delivered

$= \frac{1}{T}\int_0^T dt\, I(t)\,\mathcal{E}(t)$

$= \frac{\mathcal{E}_{max} I_{max}}{T}\int_0^T dt \cos^2\omega t = \frac{\mathcal{E}_{max} I_{max}}{2}$

$= \frac{\mathcal{E}^2_{max}}{2R} = \mathcal{E}^2_{rms}/R = \frac{I^2_{max} R}{2} = I^2_{rms} R$

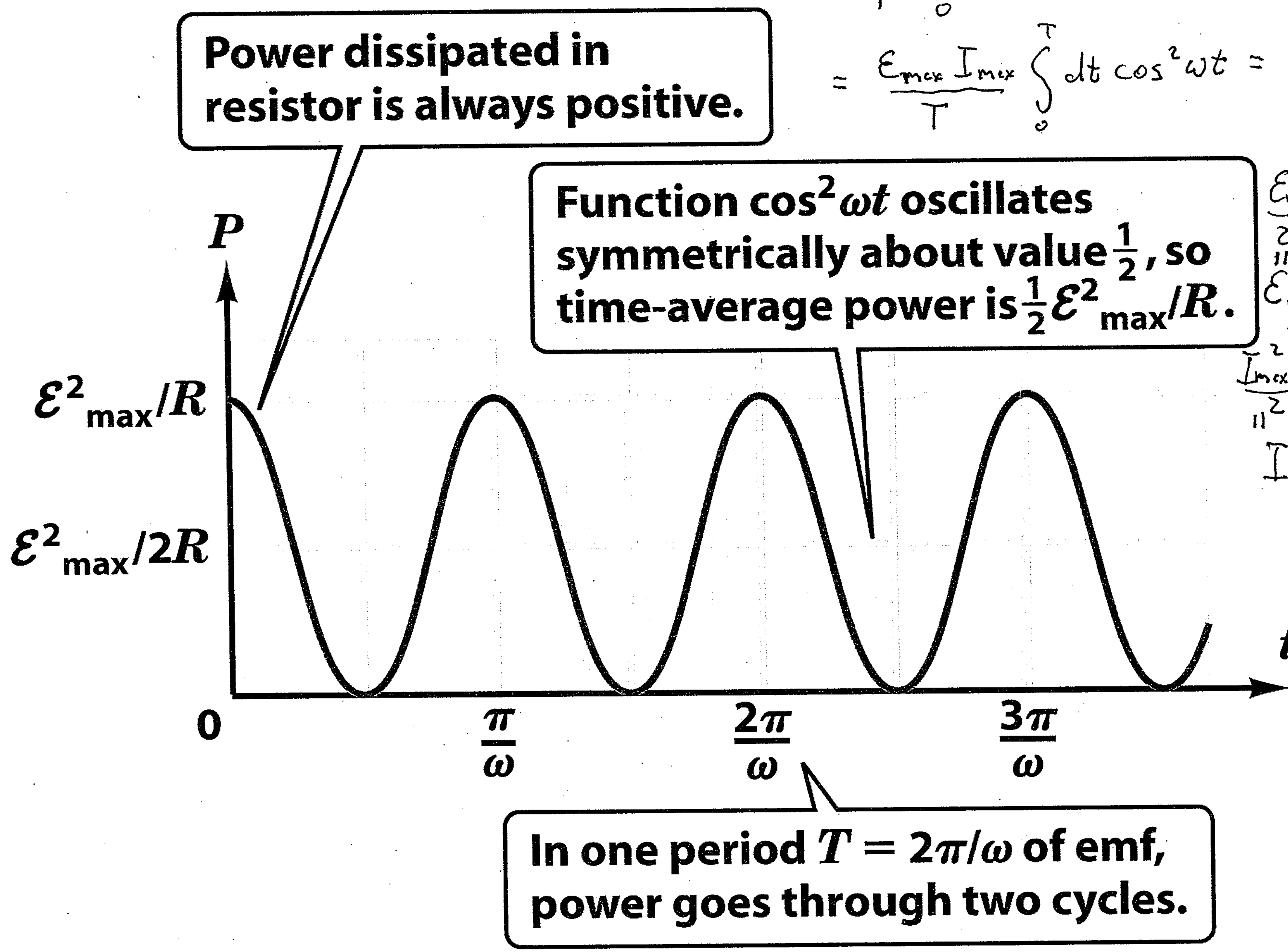
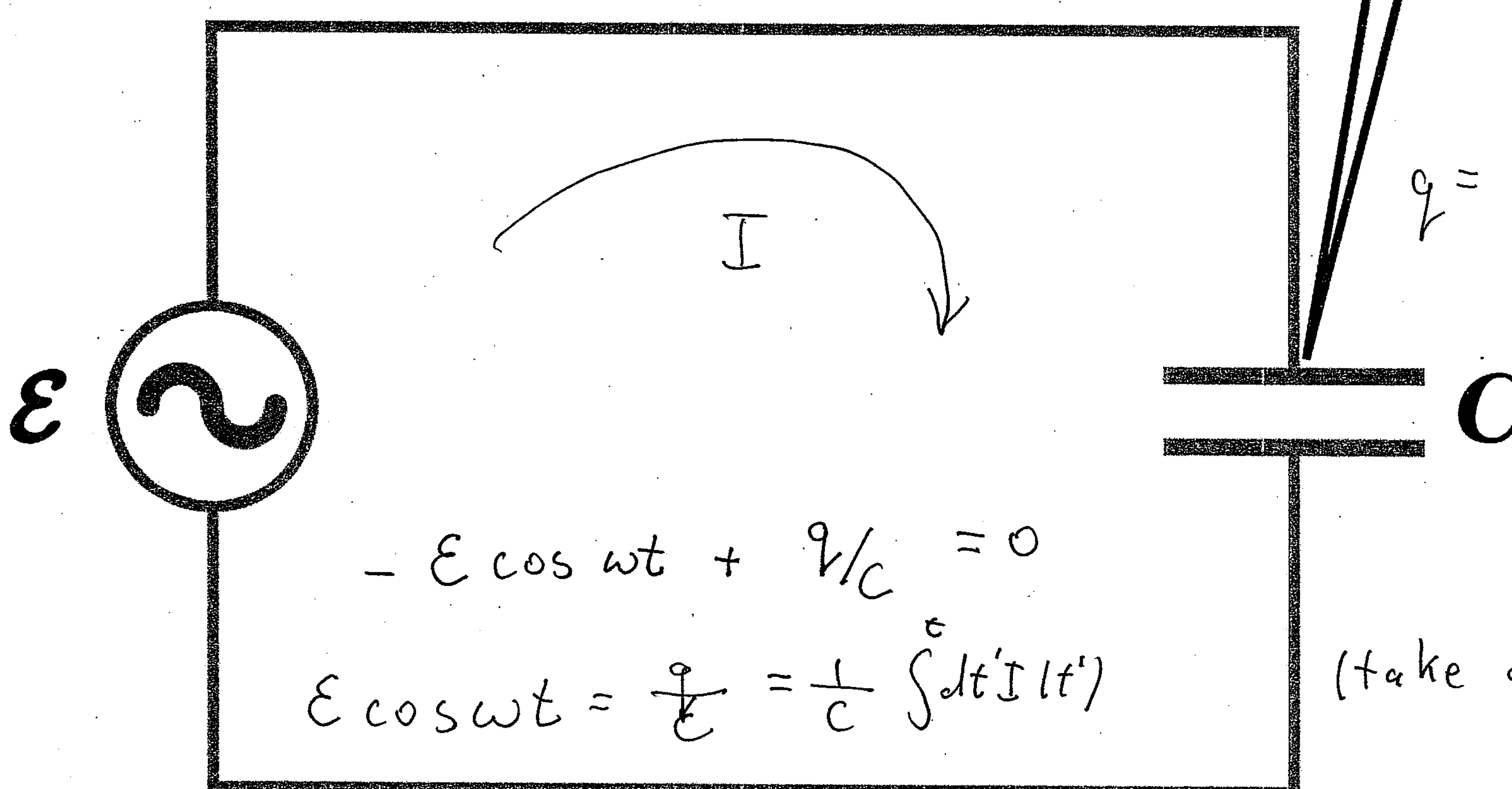


Figure 32-5 Physics for Engineers and Scientists 3/e

AC capacitor circuit: oscillating emf produces oscillating charge on capacitor, $Q = C\mathcal{E}$.



$$q = \int^t I(t')dt'$$

$$-\mathcal{E}\cos\omega t + q/C = 0$$

$$\mathcal{E}\cos\omega t = \frac{q}{C} = \frac{1}{C}\int^t dt' I(t')$$

(take derivative)

$$-\omega\mathcal{E}\sin\omega t = \frac{I(t)}{C}; \quad \boxed{I(t) = -\omega C\,\mathcal{E}\sin\omega t}$$

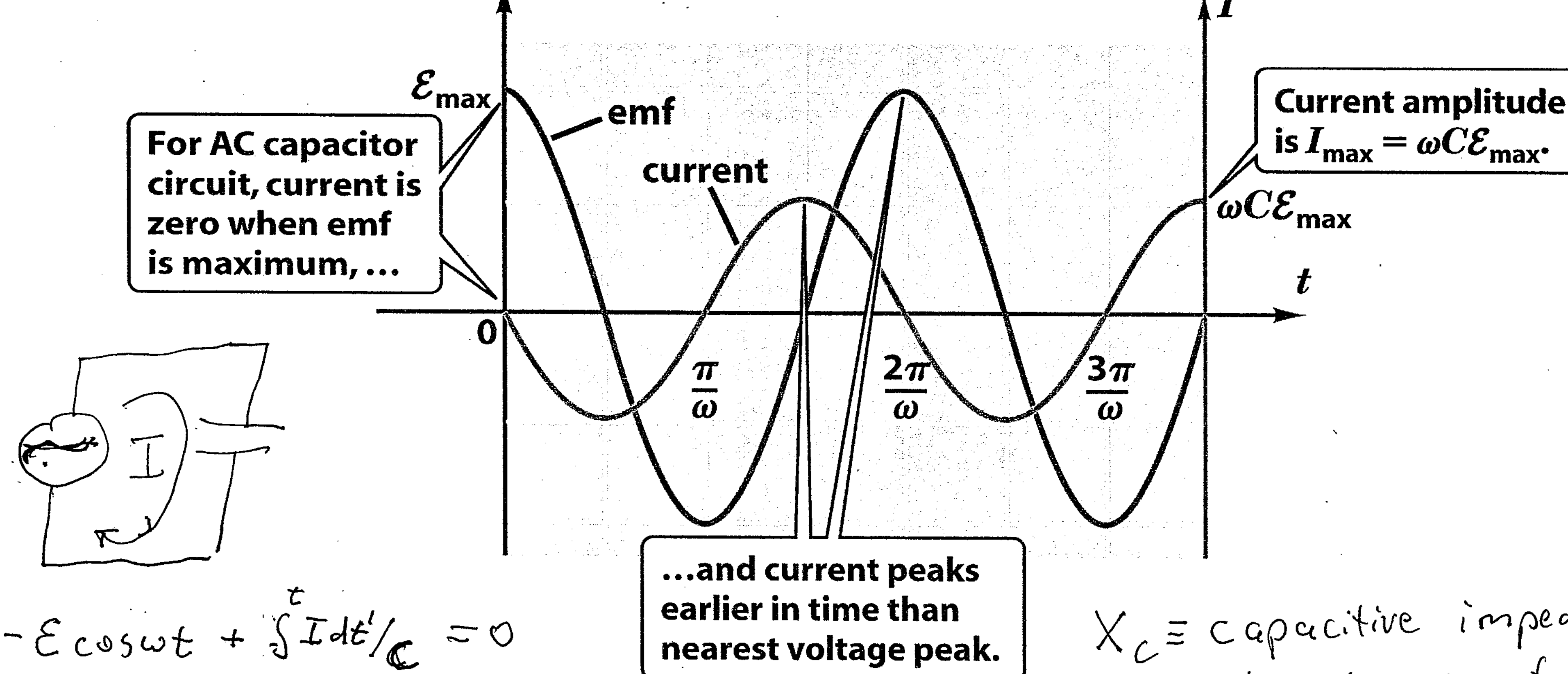
Figure 32-7 Physics for Engineers and Scientists 3/e

Capacitive Circuit (Current Leads Voltage)

$\mathcal{E} = \mathcal{E}_{max} \cos(\omega t)$

$I = \omega C \mathcal{E}_{max} \cos(\omega t + \frac{\pi}{2}) = -\omega C \mathcal{E}_m \sin(\omega t)$

Current Leads Voltage in Capacitive circuit



$-\mathcal{E} \cos \omega t + \int^t I dt' / C = 0$

Figure 32-8 Physics for Engineers and Scientists 3/e

take derivative

$+\omega \mathcal{E} \sin \omega t + \frac{I}{C}$

$I = -\omega C \mathcal{E} \sin \omega t$

$X_C \equiv$ capacitive impedance

$= \frac{1}{\omega C}$ (unit of resistance)

$I = -\frac{\mathcal{E}_m}{X_C} \sin \omega t$

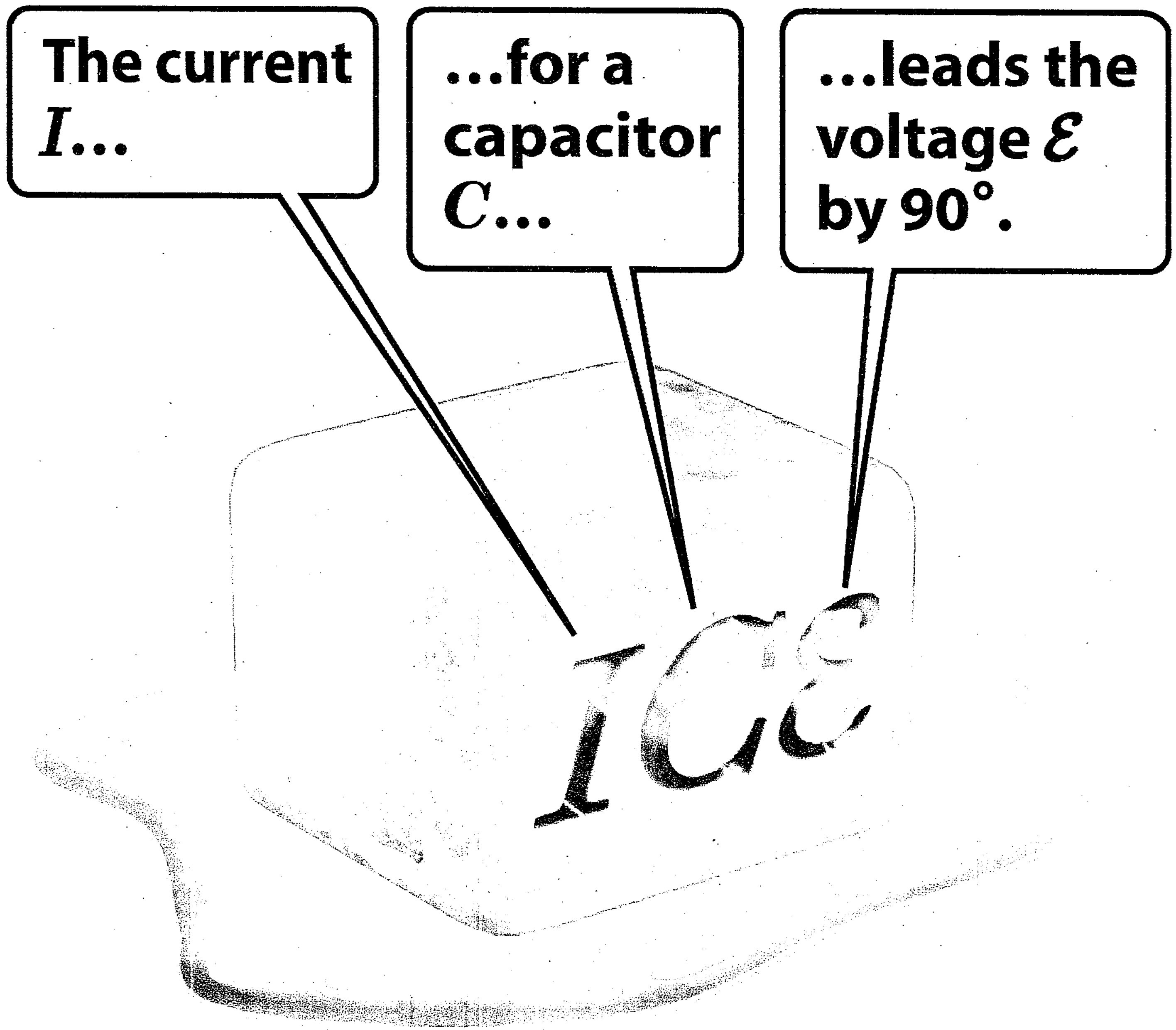


Figure 32-9 Physics for Engineers and Scientists 3/e

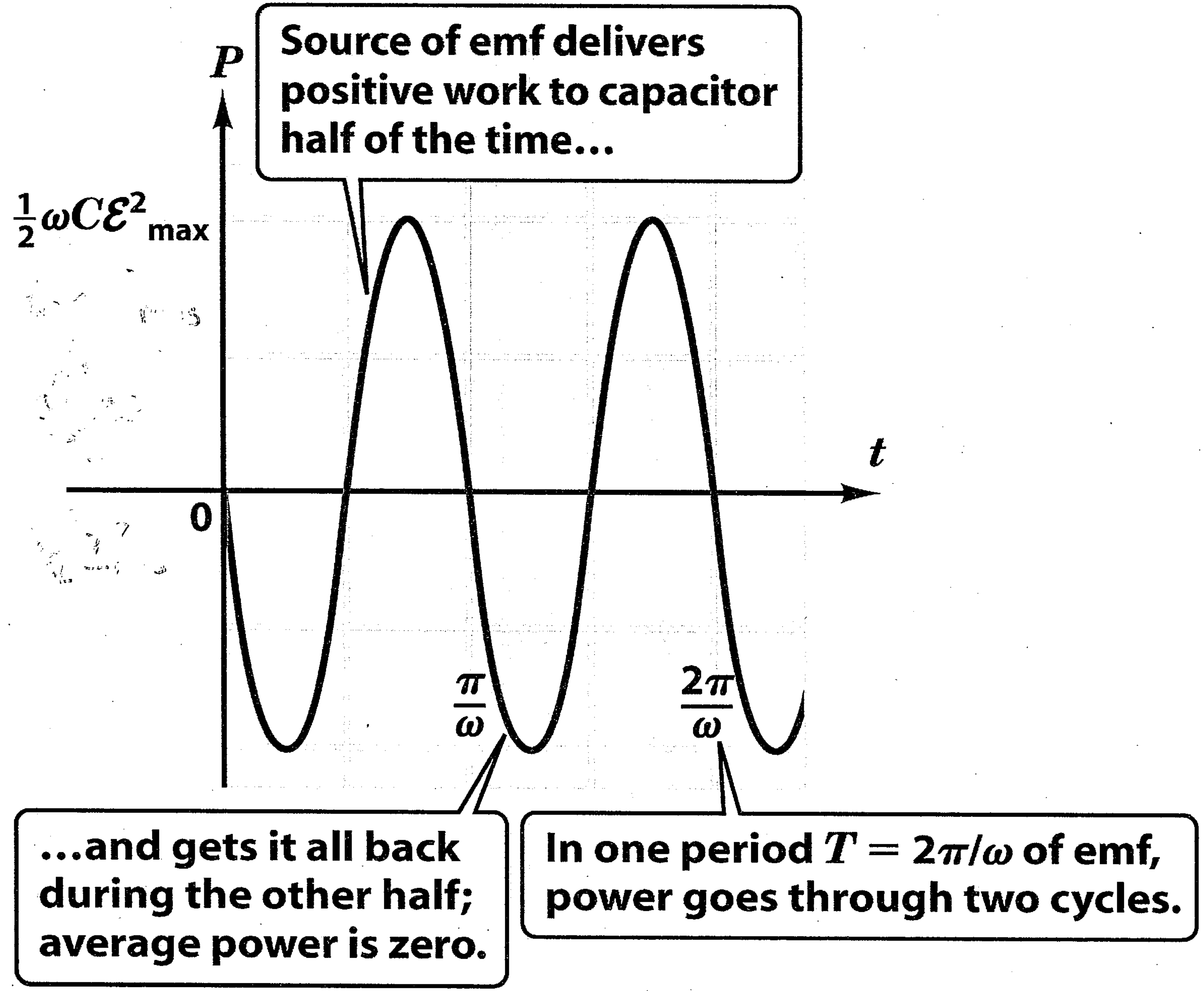


Figure 32-10 Physics for Engineers and Scientists 3/e

AC inductor circuit: oscillating emf produces oscillating current in inductor.

$\mathcal{E} = \mathcal{E}_{max} \cos \omega t$

$-\mathcal{E} + L \frac{dI}{dt} = 0$

$\frac{dI}{dt} = \frac{\mathcal{E}_{max}}{L} \cos \omega t$

$I(t) = \frac{\mathcal{E}_{max}}{\omega L} \sin \omega t$

$\omega L = X_L$

$I(t) = \frac{\mathcal{E}_{max}}{X_L} \sin(\omega t)$

$= \frac{\mathcal{E}_{max}}{X_L} \cos\left(\omega t - \frac{\pi}{2}\right)$

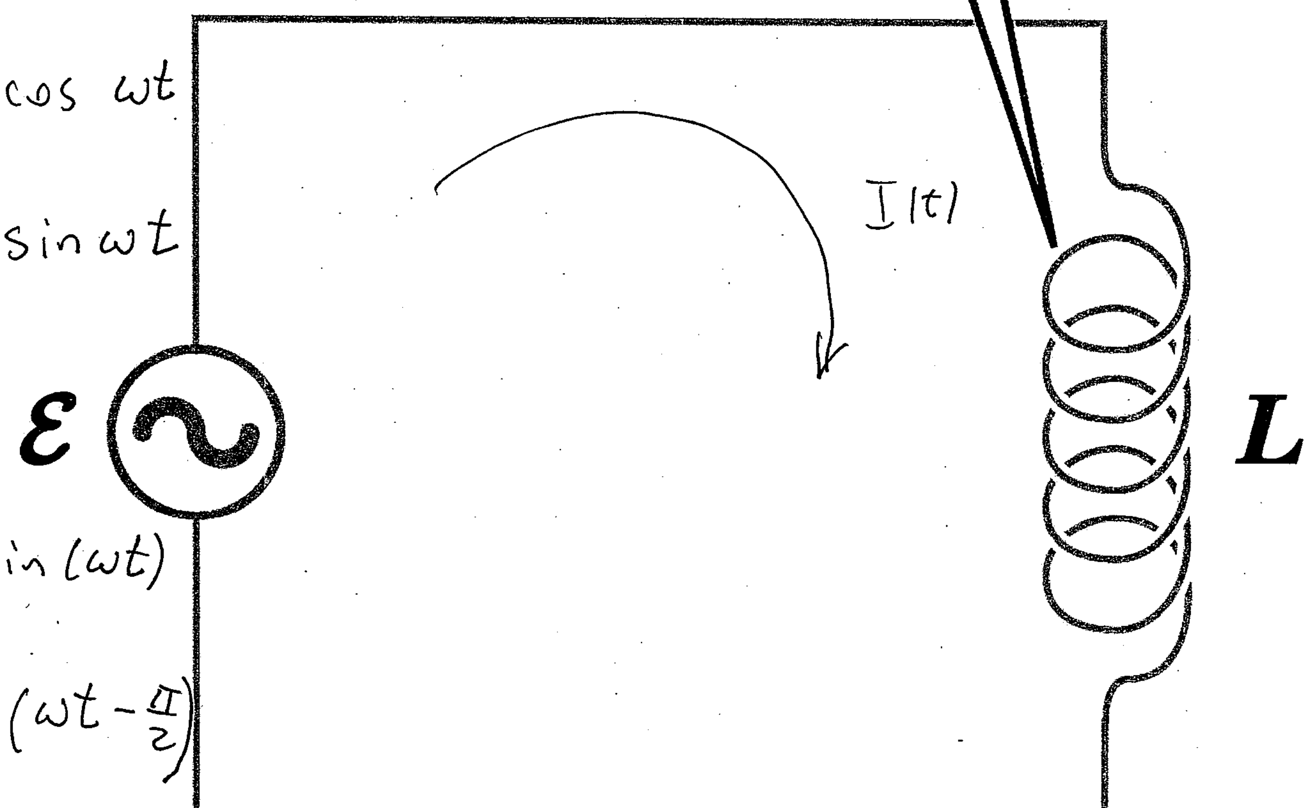


Figure 32-11 Physics for Engineers and Scientists 3/e

$X_L = \omega L \equiv$ inductive impedance

In inductor current Lags Voltage

$I(t) = \frac{\mathcal{E}_{max}}{X_L} \cos(\omega t - \frac{\pi}{2})$; $X_L = \omega L$

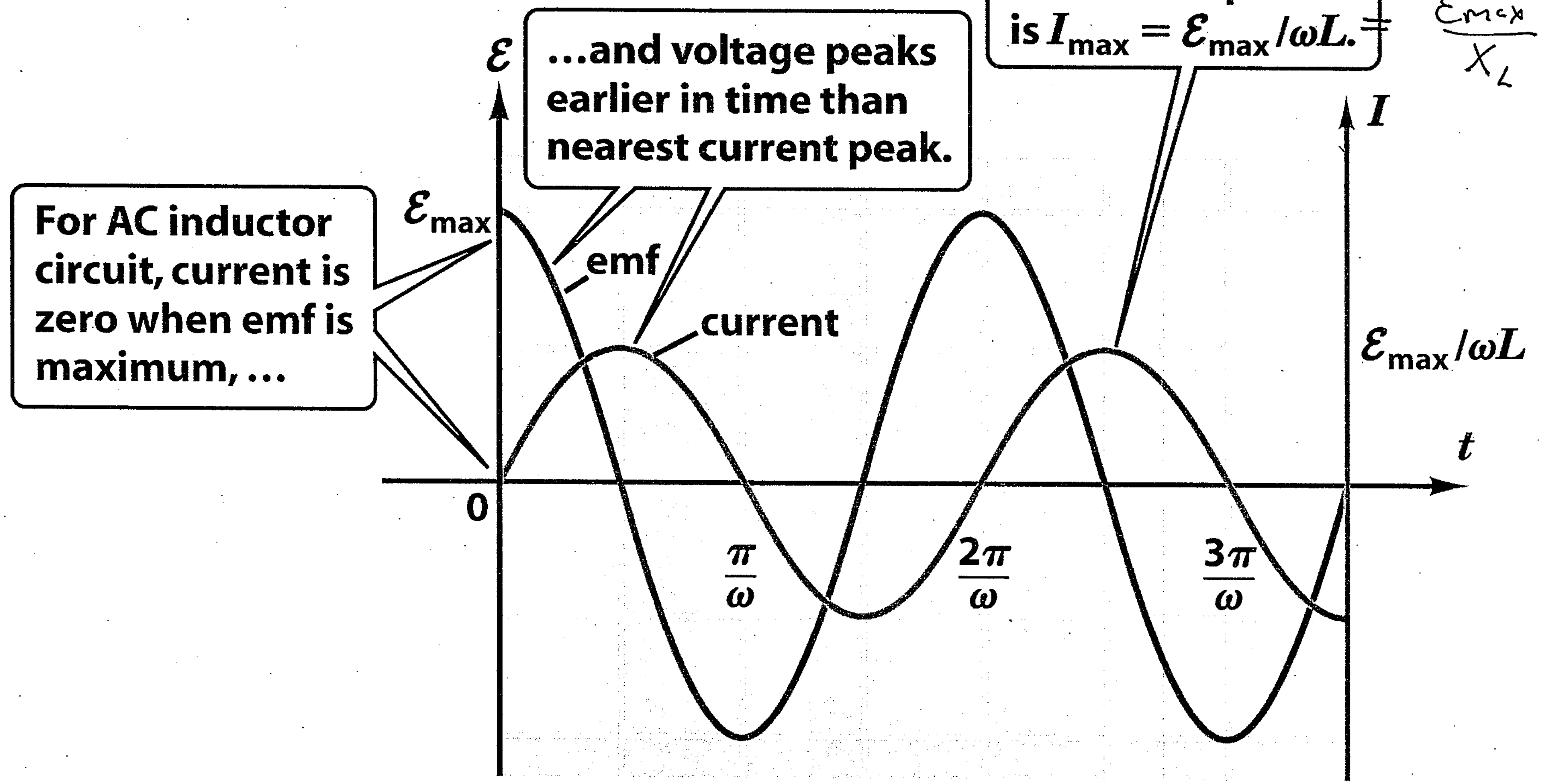


Figure 32-12 Physics for Engineers and Scientists 3/e

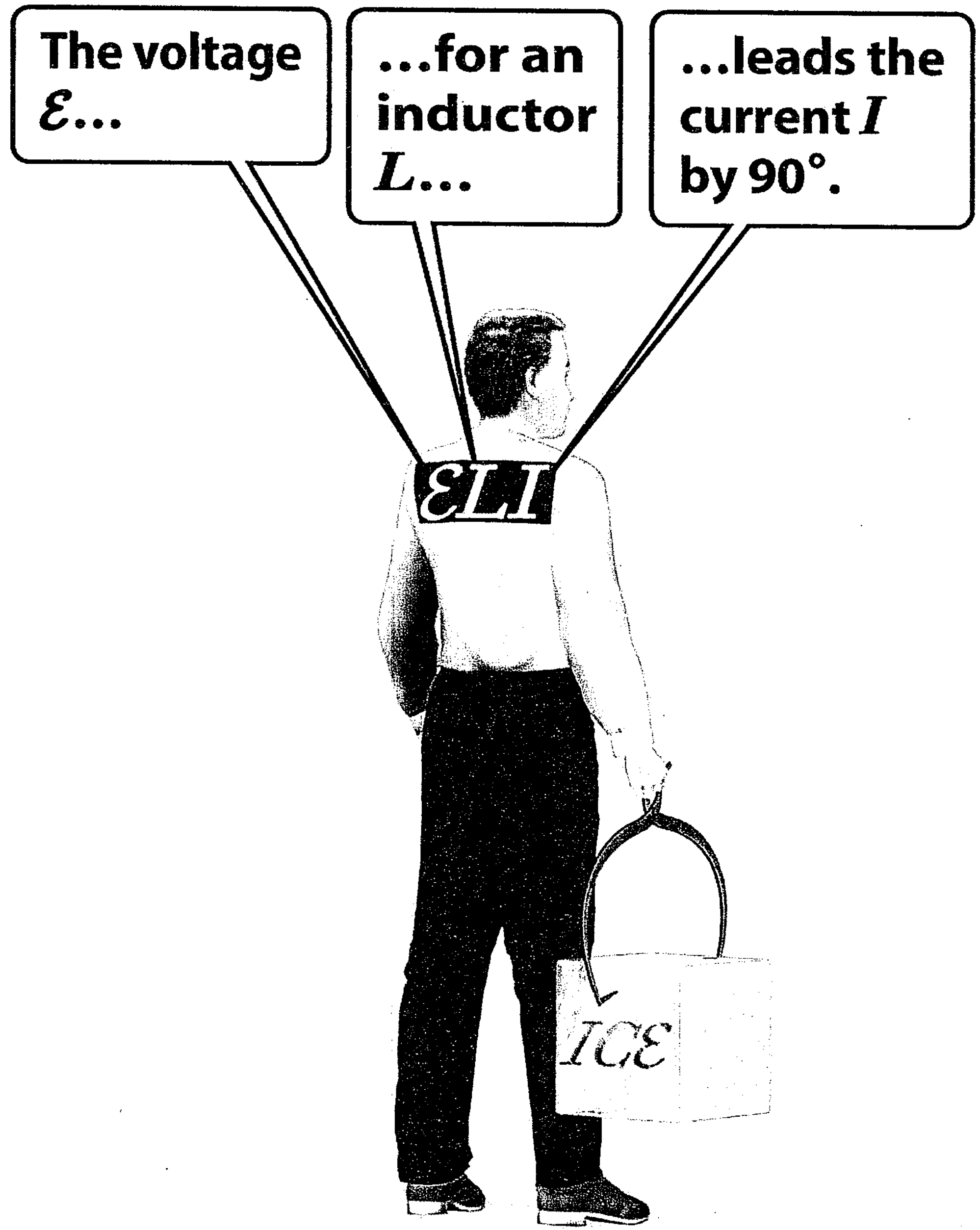


Figure 32-13 Physics for Engineers and Scientists 3/e

$$-\mathcal{E}_{max}\cos\omega t + L\frac{dI}{dt} + \frac{\int^{t} dt' I(t')}{C} = 0$$

$$I(t) = \frac{\mathcal{E}_{max}\sin\omega t}{\left(\omega L - \frac{1}{\omega C}\right)} = \frac{\sin\omega t\,\mathcal{E}_{max}}{(X_L - X_C)}$$

when $X_L = X_C$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$\omega_0 \equiv$ natural oscillation frequency

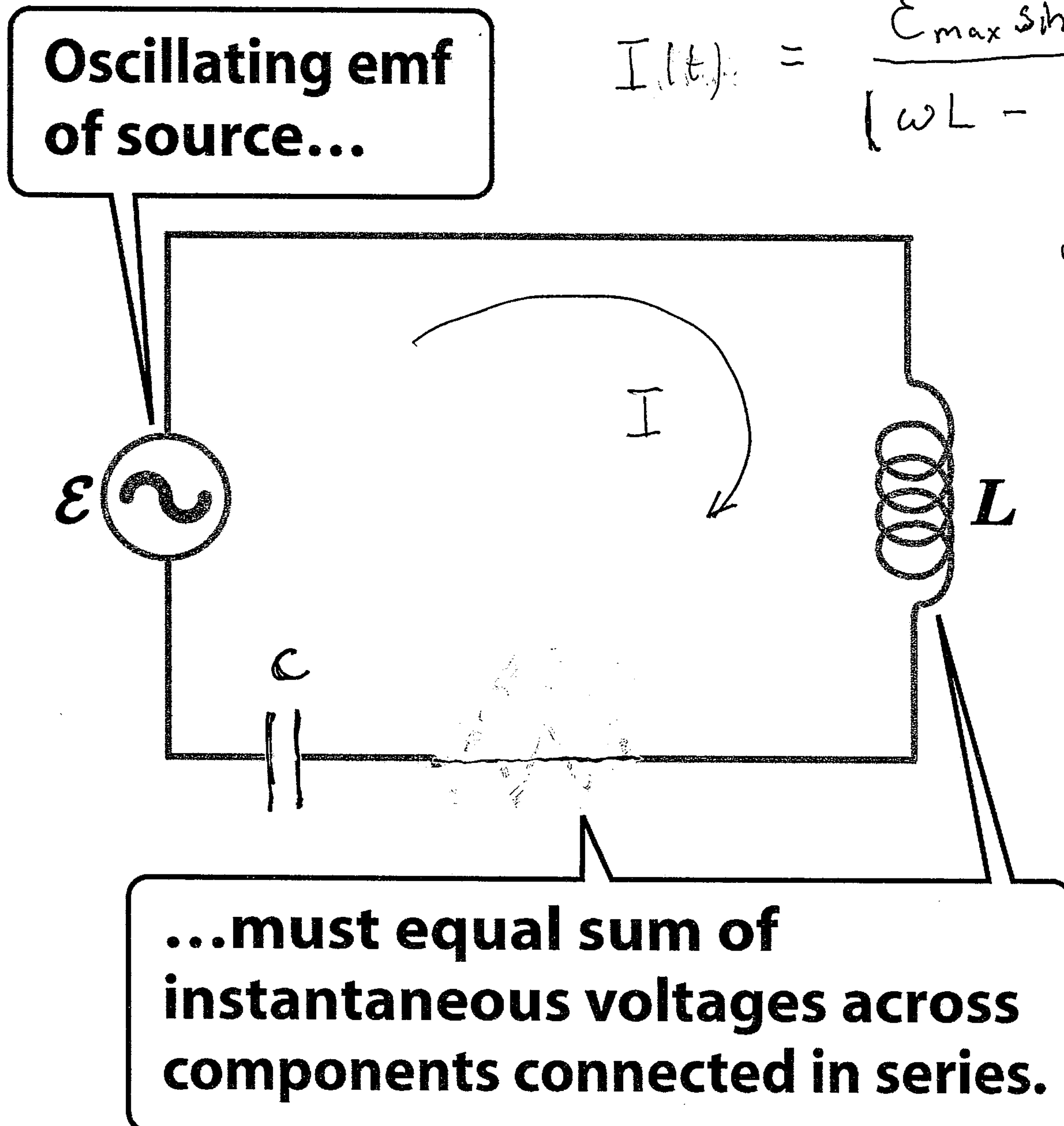


Figure 32-22 Physics for Engineers and Scientists 3/e