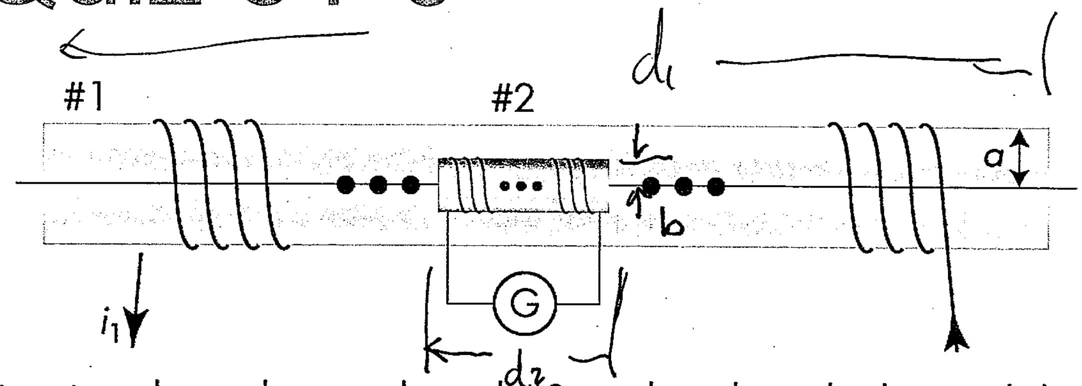
Lecture 26

L-R, L-C, RLC circuits

PhysiQuiz 31-6



In the situation shown here, solenoid #2 is placed inside the much larger solenoid #1. Solenoid #1 has N_1 turns, radius a, and length d_1. Solenoid #2 has N_2 turns, radius b, and length d_2. Determine the mutual inductance of the system:

A	$\mu_0 N_1 N_2 \pi a^2/d_1$
B	$\mu_0 N_1 N_2 \pi b^2/d_1$
C	$\mu_0 N_1 N_2 \pi a^2/d_2$
D	$\mu_0 N_1 N_2 \pi b^2/d_2$

Hint: $\varepsilon_{2,\text{ind}} = -N_2 \dfrac{d\phi}{dt} = M_{21} \dfrac{di_1}{dt}$, where ϕ is the magnetic flux at #2 due to i_1.

current through inductor cannot change instantaneously! $\tau = L/R$

Because of inductance current cannot change instantaneously, but it takes a characteistic time, $\tau_{L/R} = L/R$.

Use Khirchoff's Law again

$$-\mathcal{E} + IR + L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} + IR = \mathcal{E}$$

Solution

$$I(t) = \frac{\mathcal{E}}{R} + C\exp(-t/\tau_{L/R})$$

$$\boxed{\tau_{L/R} = \frac{L}{R}}$$

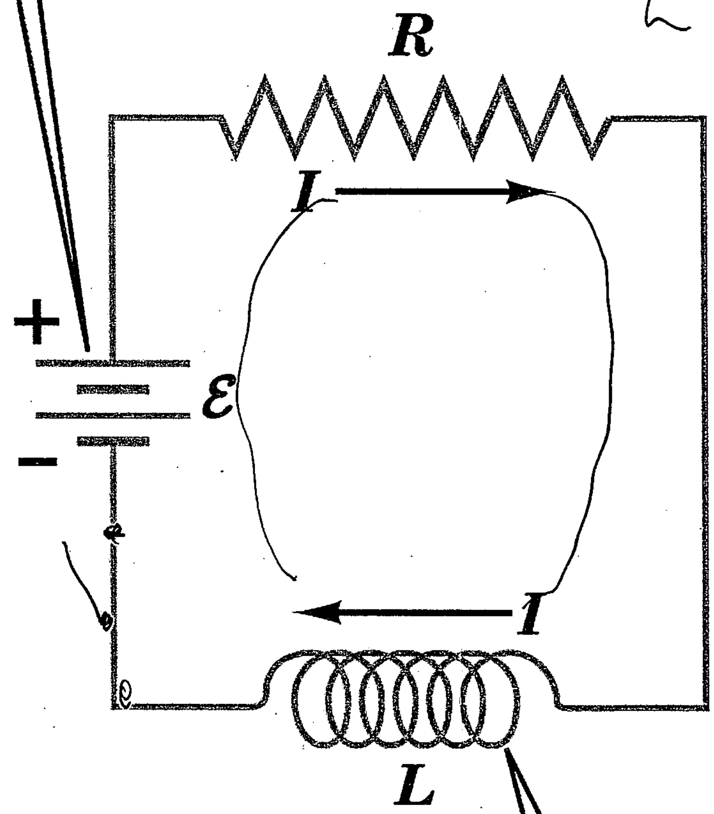
at $t = 0,\ I(t=0) = 0$, thus

$$I(0) = \frac{\mathcal{E}}{R} + C = 0, \quad \therefore\ C = -\frac{\mathcal{E}}{R}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - \exp(-t/\tau_{L/R})\right)$$

When battery is connected, current in circuit will gradually start to increase.

L-R circuit



Circuit symbol for inductor is a coiled line.

Figure 31-24 Physics for Engineers and Scientists 3/e

$I(t) = \frac{\mathcal{E}}{R}\left[1 - \exp\left(\frac{tR}{L}\right)\right]$

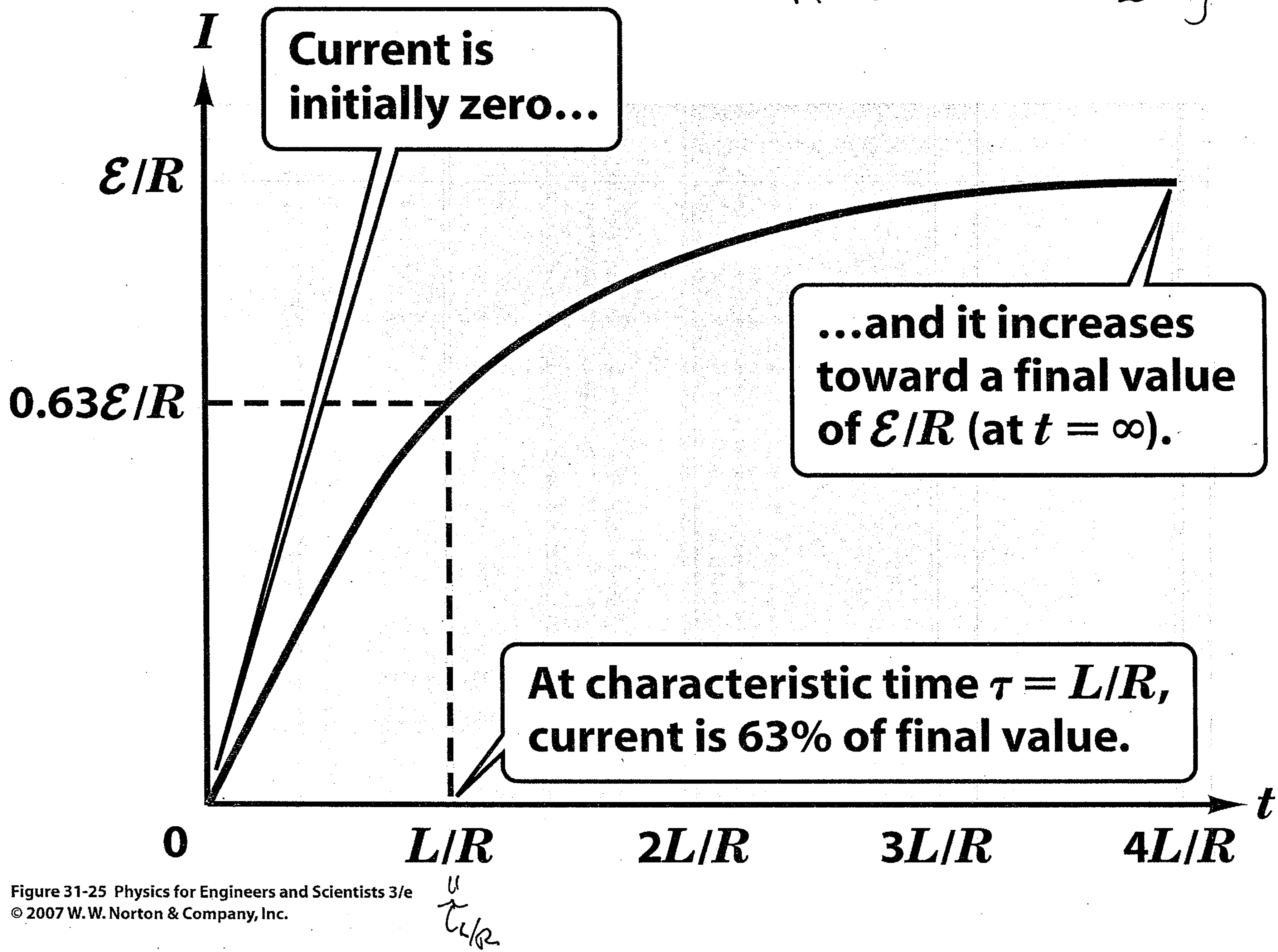


Figure 31-25 Physics for Engineers and Scientists 3/e

$I_a = \dfrac{\mathcal{E}}{R_a}$

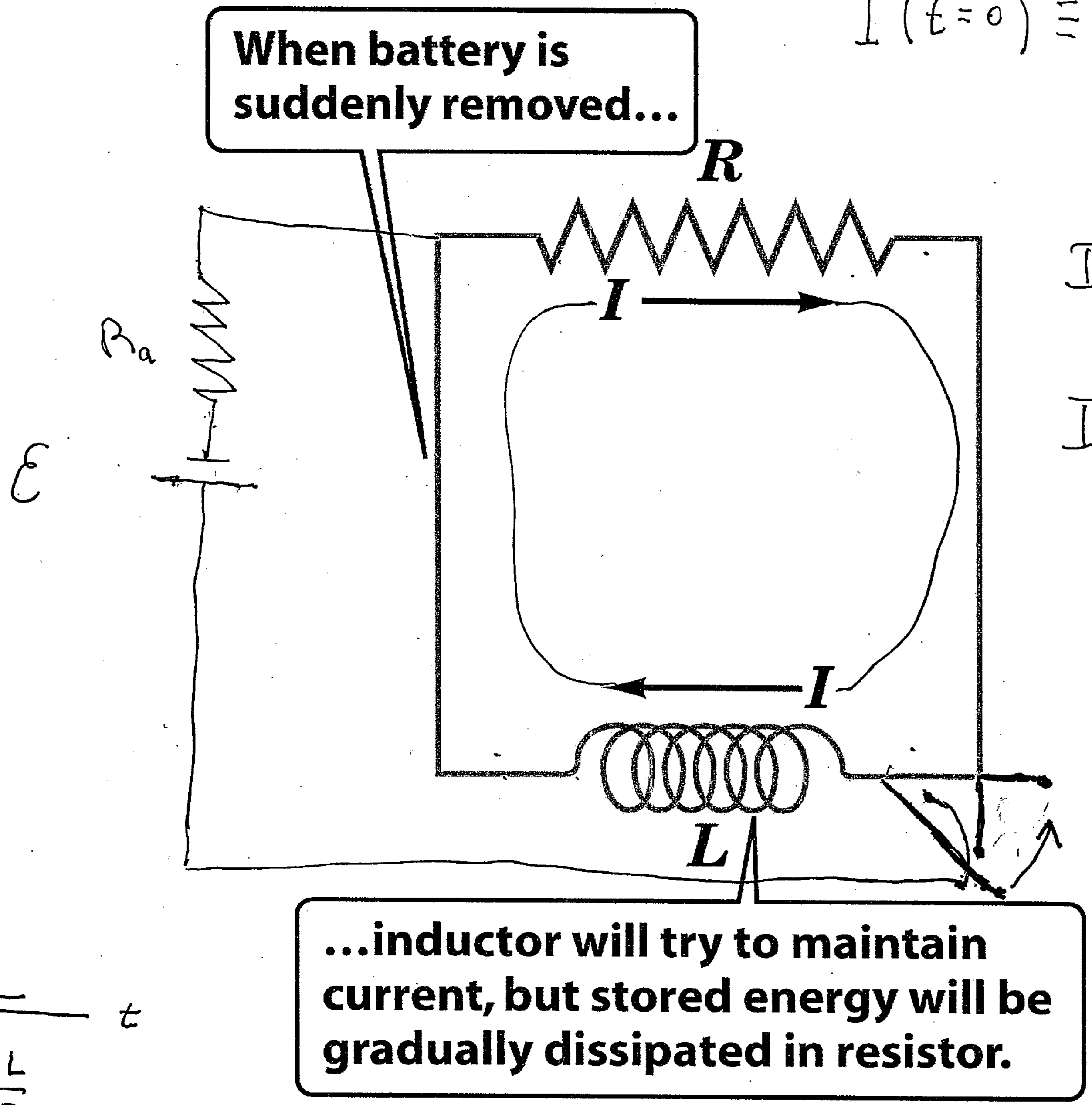


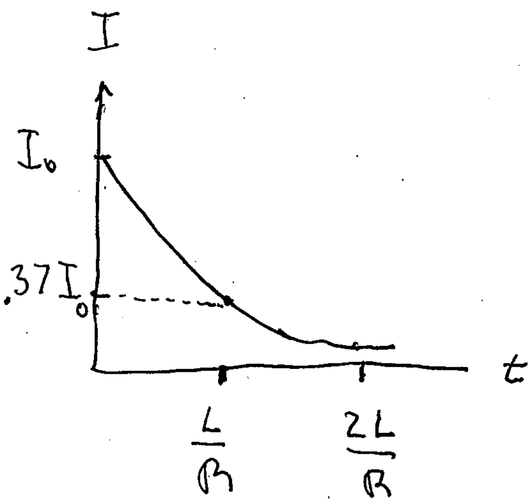
Figure 31-26 Physics for Engineers and Scientists 3/e

$I(t=0) \equiv I_0 =$

$IR + L\dfrac{dI}{dt} = 0$

$I = I_0 e^{-t/\tau_{L/R}}$

$\tau_{L/R} = \dfrac{L}{R}$



If at time t=0 the switch is suddenly turned on, (1) what is the initial current through the battery? (2) the final battery current?

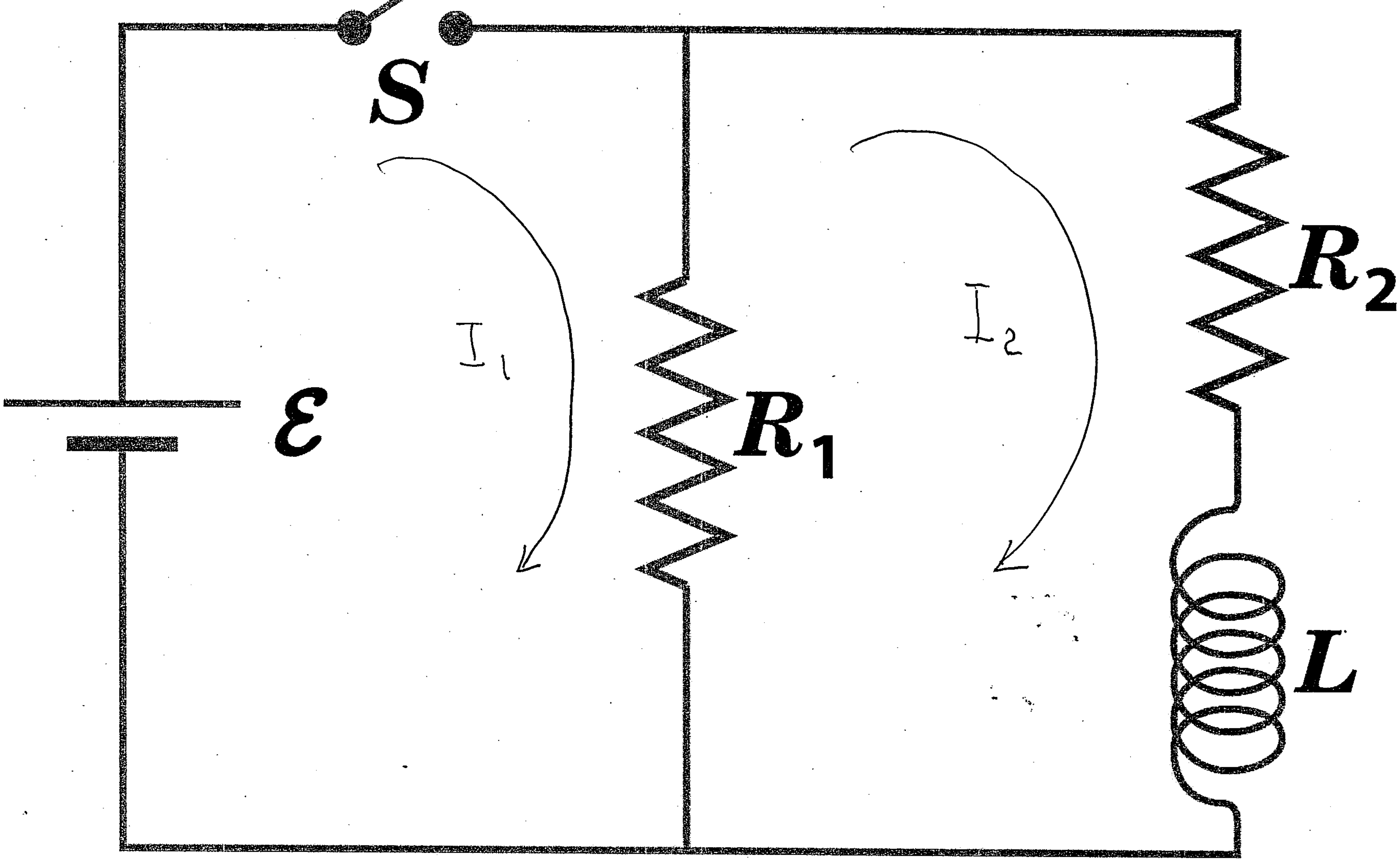


Figure 31-27 Physics for Engineers and Scientists 3/e

(*a*) $\mathcal{E} / R_1$ (b) $\mathcal{E} / R_2$ (c) $\mathcal{E}(R_2 + R_1) / R_2 R_1$

L - C circuit

Inductor stores B - field energy

$W_I = \frac{1}{2} L I^2$

$W_C = \frac{Q^2}{2C} \equiv$ E-field Energy

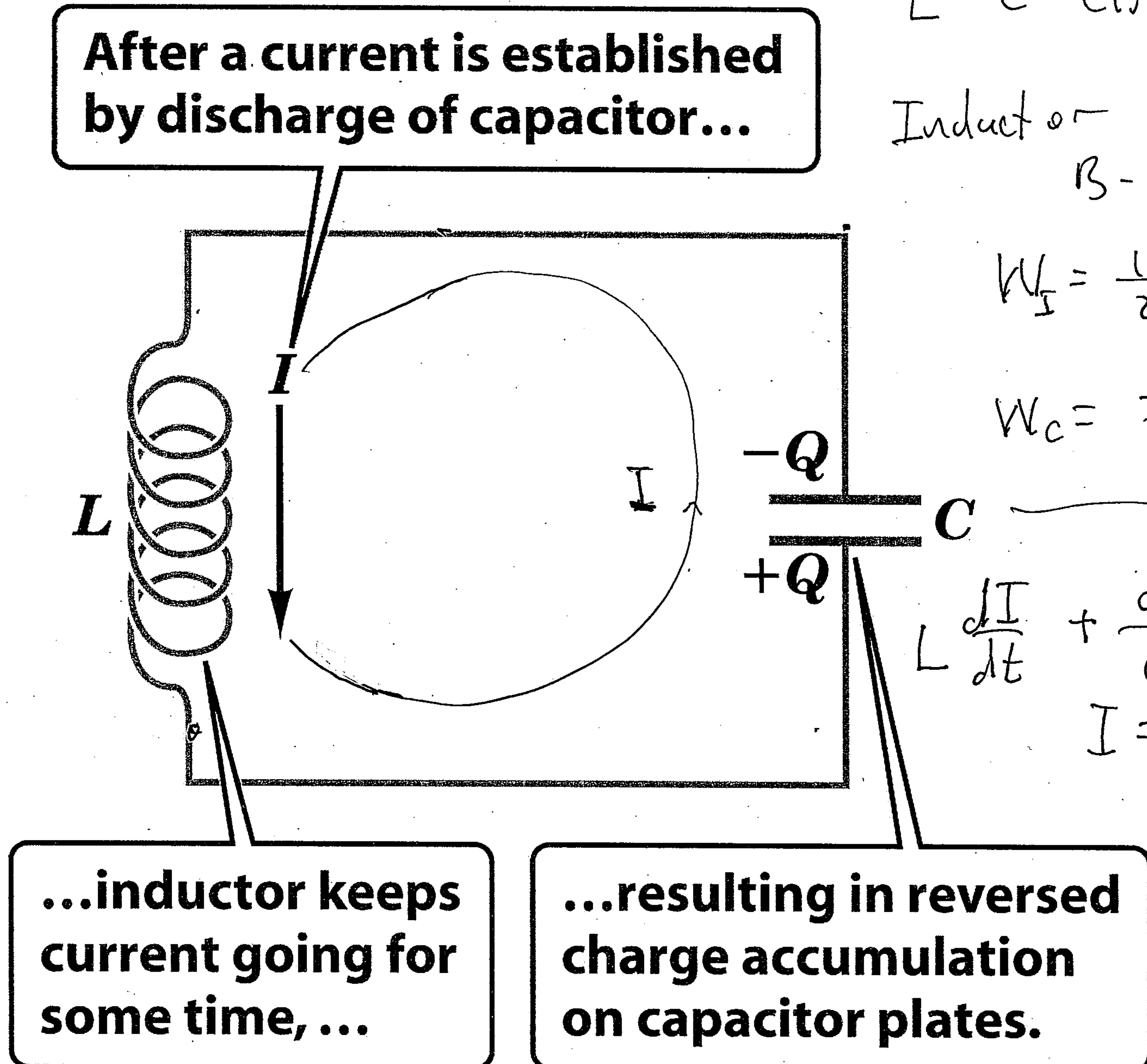


Figure 32-15 Physics for Engineers and Scientists 3/e

$L \frac{dI}{dt} + \frac{q}{C} = 0$

$I = \frac{dq}{dt}$

$\boxed{L \frac{d^2 q}{dt^2} + \frac{q}{C} = 0}$

Mechanical analogy of LC circuit

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

$$\frac{1}{2}LI^2 = \frac{1}{2}L\left(\frac{dq}{dt}\right)^2 \equiv \text{magnetic field energy}$$

$$\frac{q^2}{2C} \equiv \text{electric field energy}$$

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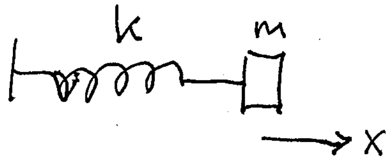
compare with motion of harmonic oscillator



$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = KE$$

$$U = \frac{1}{2}kx^2 = PE$$

$$KE + PE = \text{const}$$

$$\cancel{m}\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \qquad \omega_0^2 = \frac{k}{m}$$

$$x = x_0\cos(\omega_0 t + \phi)$$

$m \to L$, $k \to \frac{1}{C}$, $x \to q$

equations are identical

The sum of magnetic field energy plus electric field energy remains constant in time

$$\frac{1}{2}L\left(\frac{dq}{dt}\right)^2 + \frac{q^2(t)}{2C} = \text{total Energy}$$

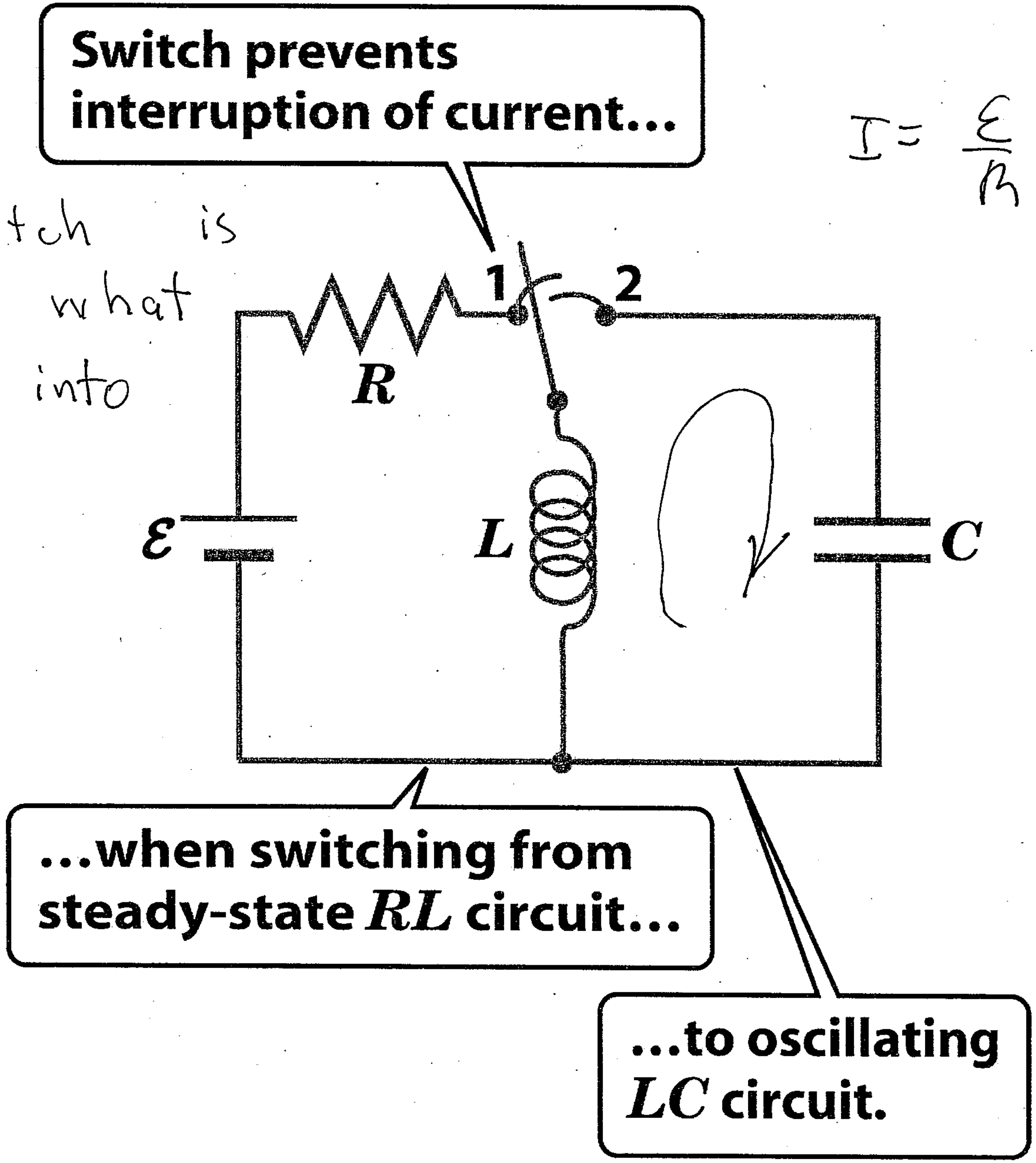
(total Electro-magnetic energy)

When switch is turned on what is current into capacitor?

(a) 0

(b) $\mathcal{E}/R$

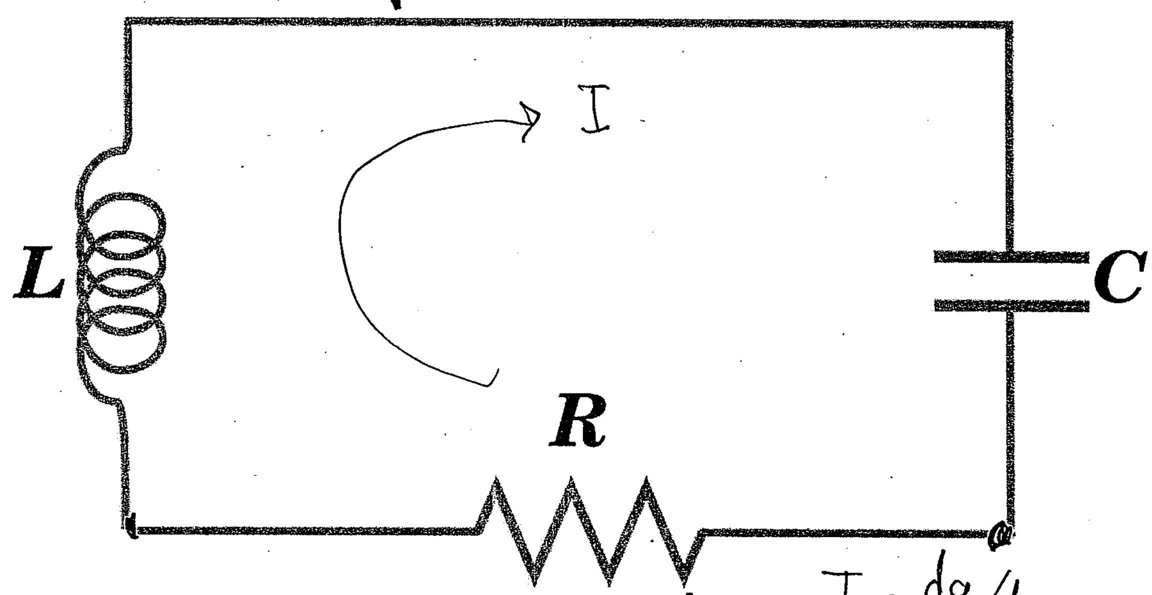
(c) $\frac{1}{\sqrt{LC}}\mathcal{E}$



$I = \frac{\mathcal{E}}{R}$

Figure 32-21 Physics for Engineers and Scientists 3/e

In *RLC* circuit without any source of emf, current and charge again oscillate…



...but now resistor causes damping.

$P_L = RI^2$

$= R\left(\frac{dq}{dt}\right)^2$

$IR + L\frac{dI}{dt} + \frac{q}{C} = 0$

$\boxed{L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0}$

Figure 32-19 Physics for Engineers and Scientists 3/e

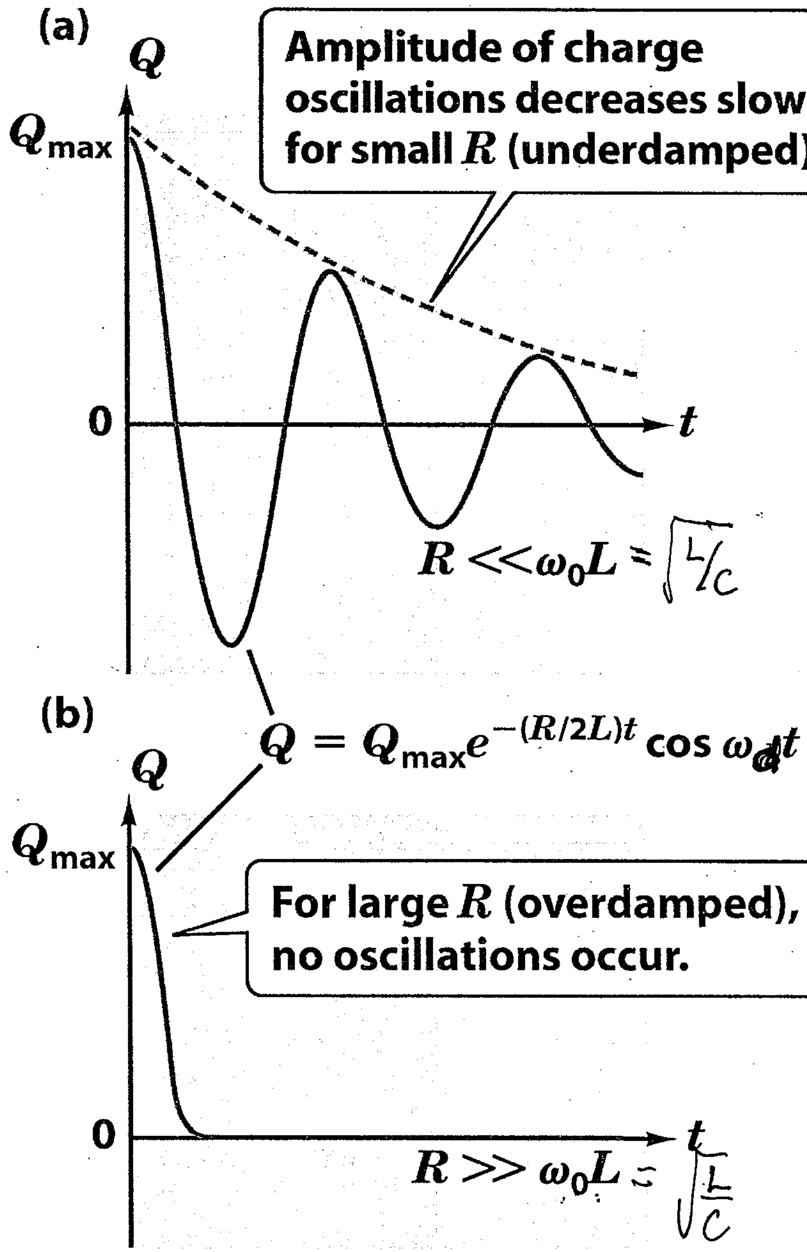


Figure 32-20 Physics for Engineers and Scientists 3/e

$\omega_0 = 1/\sqrt{LC}$

$\omega_d = \left[\frac{1}{LC} - \frac{R^2}{L^2}\right]^{1/2}$

critical damping

$R = \sqrt{L/C}$

ω