Lecture 25

Mutual and Self Inductance

Faraday Law

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_m}{dt} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

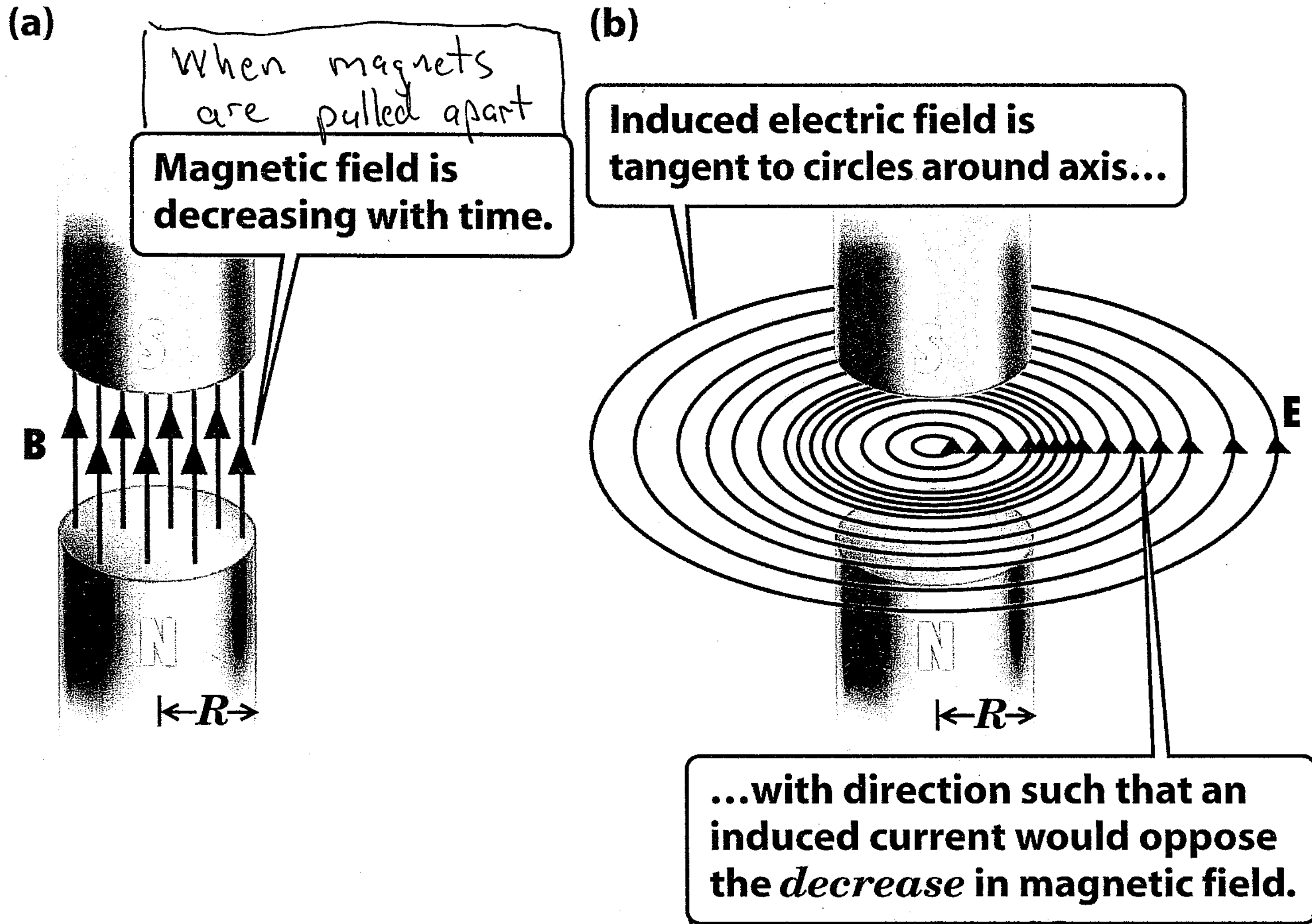
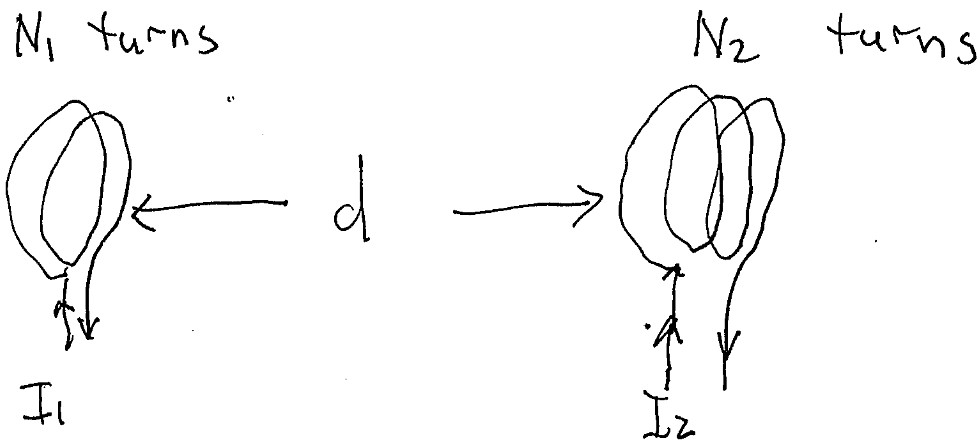


Figure 31-19 Physics for Engineers and Scientists 3/e

Do you understand how Lenz's law is illustrated?

Mutual Induction



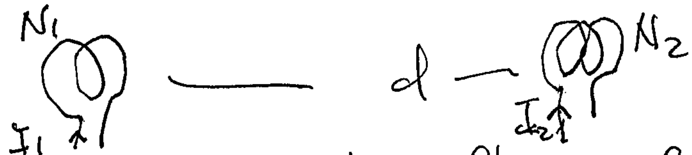
If a current flows in coil 1, the magnetic flux through coil 2, from coil 1, is proportional to

(a) I_1

(b) N_1

(c) N_2

(d) all of the above



The magnetic flux from coil 1 at coil 2 is proportional to

$$\Phi_{21} \propto N_1 N_2 I_1$$

$$\Phi_{21} = M_{21} I_1$$

$M_{21} \equiv$ mutual induction coefficient

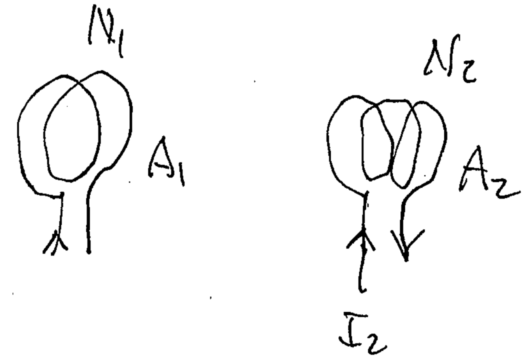
Now if I_1 changes in time there will be an EMF generated in coil 2.

$$\mathcal{E}_{MF2} = -\frac{d\Phi_{21}}{dt} = -M_{21}\frac{dI_1}{dt}$$

What then is electric field in coil 2 if it is a circle with N_2 turns?

$$\mathcal{E}_{MF2} = \oint \vec{E}\cdot d\vec{l} = N_2 (2\pi R_2) E_2 = -M_{21}\frac{dI_1}{dt}$$

Similarly mutual induction produces B-flux through coil 1 due to current in coil 2



$$\Phi_{12} \propto N_1 N_2 I_2 = M_{12} I_2$$

It can be shown that

$$M_{12} = M_{21}$$

EMF in circuit "1" arises if I_2 changes

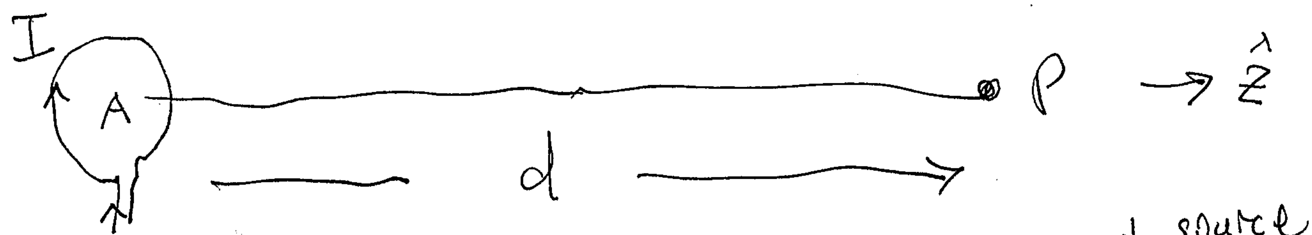
$$\mathcal{E}_{MF1} = M_{12} \frac{dI_2}{dt} = M_{21} \frac{dI_2}{dt}$$

Let us give a calculation that demonstrates $M_{12} = M_{21}$

Recall Magnetic field from a current loop of area A and current I, on axis far from the loop at P is given by

$$B = \frac{\mu_0 IA}{2d}\hat{z}$$

$IA \equiv$ magnetic moment

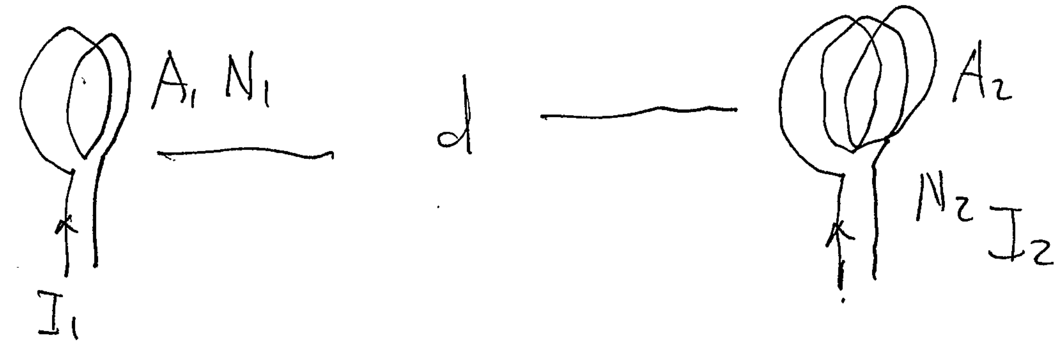


If ~~we~~ had N-loops at current source B field at P would the be

$$B_N = \frac{\mu_0 NIA}{2d}\hat{z}$$

magnet moment NIA

~~Now then consider two coils 1, & 2, with area A_1 & A_2 far apart, distance d and calculate mutual flux through one loop with current I_1 N_1 coils through loop 2 with N_2 loops~~



$$B_{21} = \frac{\mu_0 A_1 I_1 N_1}{2d}\,, \qquad B_{12} = \frac{\mu_0 A_2 I_2 N_2}{2d}$$

Flux through loop 1, from coil 2,

$$\Phi_{12} = B_{12} A_1 N_1 = \frac{\mu_0 A_2 I_2 N_2}{2d} \cdot A_1 N_1$$

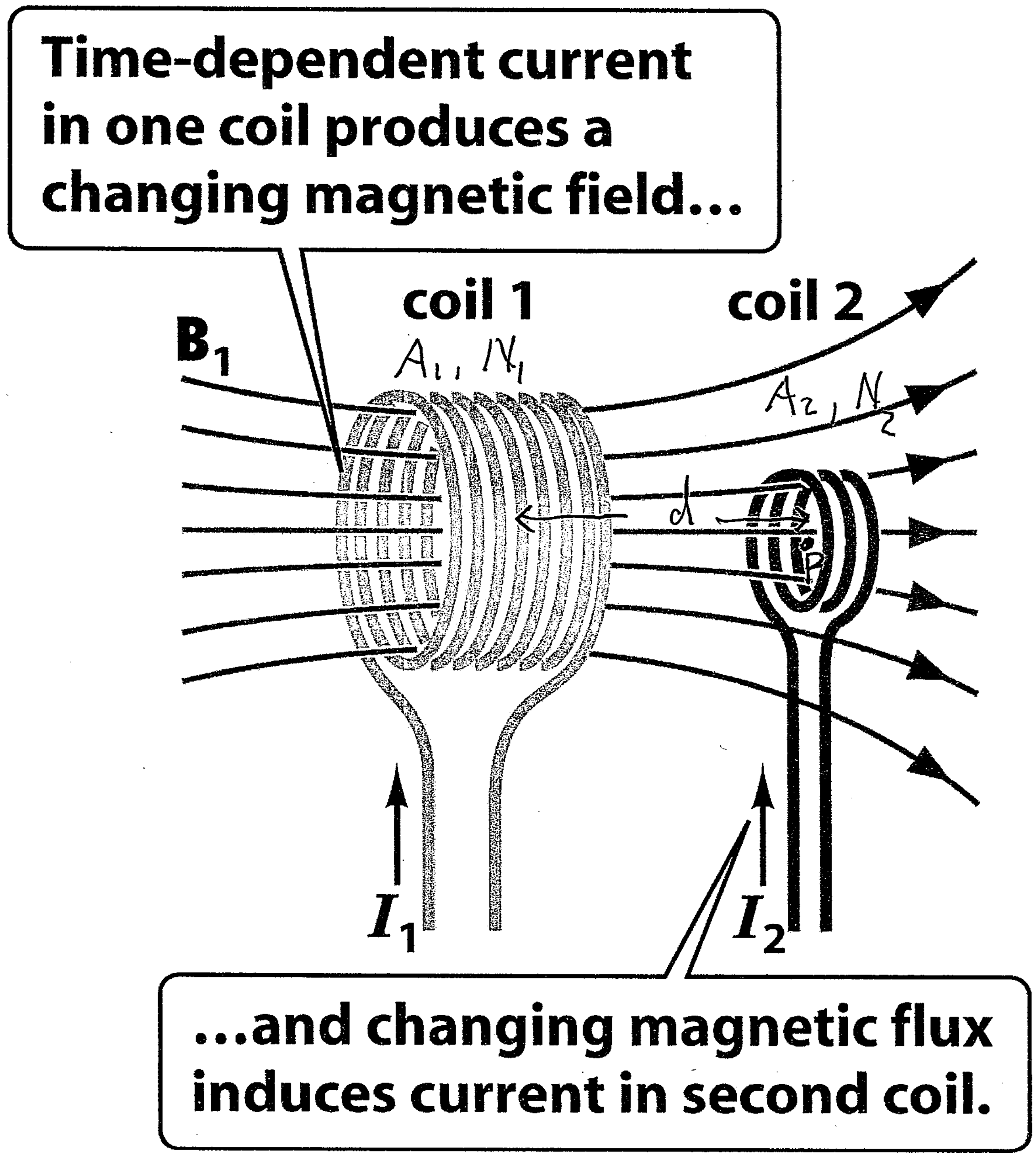
$$= \left(\frac{\mu_0 A_2 A_1 N_1 N_2}{2d}\right) I_2 \equiv M_{12} I_2$$

$$\boxed{M_{12} = \frac{\mu_0 A_2 A_1 N_1 N_2}{2d}}$$

Interchange 1 & 2 and you reach the conclusion that

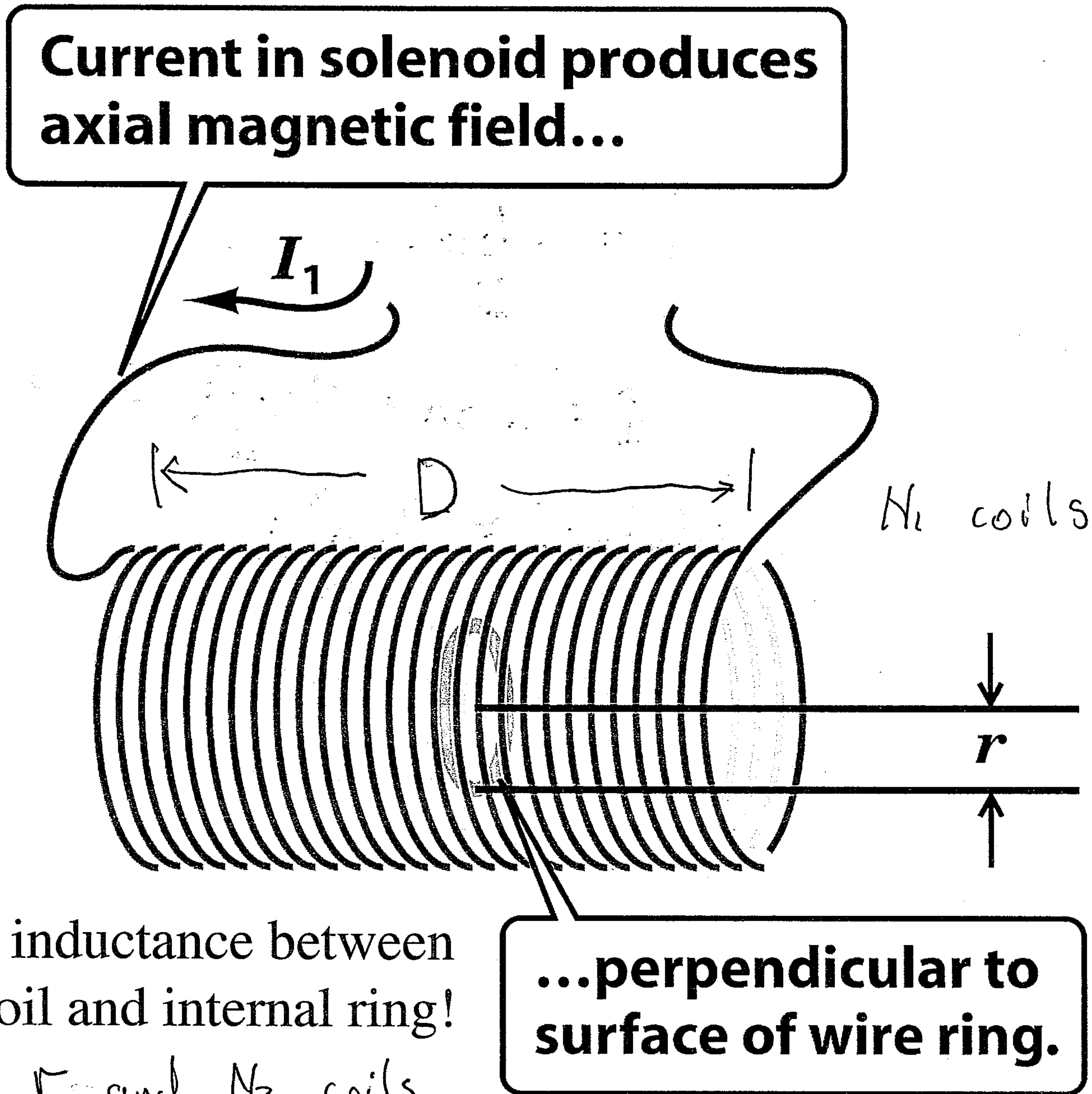
$$M_{21} = M_{12}\,, \qquad \boxed{M_{ij} = M_{ji}}$$

Mutual induction



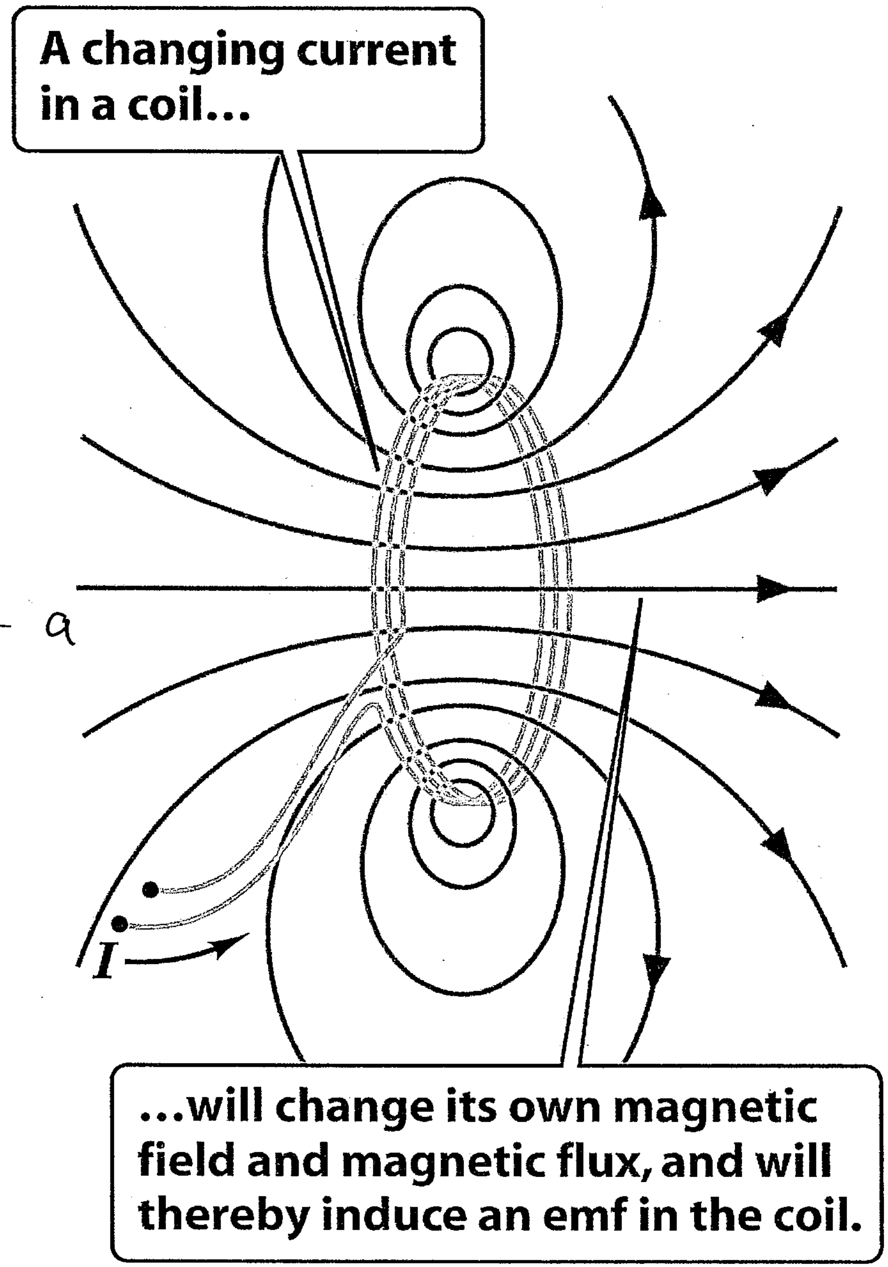
What is mutual induction between solenoid & inner coil?

What is the EMF induced in wire ring if $I_1(t) = I_{10}t / T$



Find mutual inductance between solenoidal coil and internal ring!
of radius r and N_2 coils

Let us find self inductance of a solenoid



$$\mathcal{E}_{MF} = -L\frac{dI}{dt}$$

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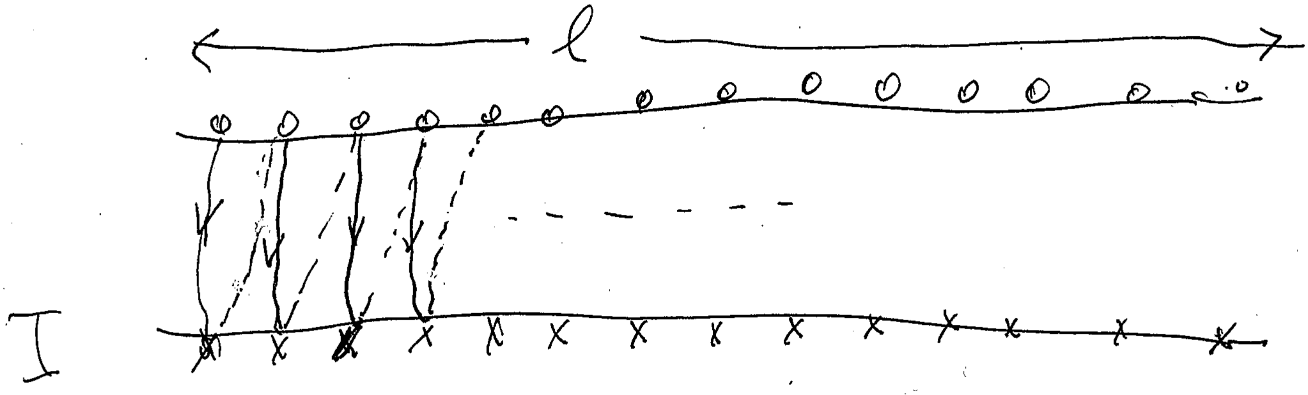
Self Inductance of a Solenoid



$$B = \mu_0 I \frac{N}{\ell}$$

$$\Phi_M = BAN = \frac{\mu_0 I N^2 A}{\ell} = LI$$

$$\boxed{L = \frac{\mu_0 N^2 A}{\ell}}$$

In general self-inductance of a coil $\propto N^2$ (i.e # of turns in the coil)

Energy stored in an inductor

Delivered Power $= \mathcal{E}_{MF} I$

$$= L \frac{dI}{dt} I$$

$$= \frac{L}{2} \frac{dI^2}{dt}$$

stored energy

$$W_L = \int_{-\infty}^{t} \text{Power}\, dt = \int_{-\infty}^{t} dt \frac{L}{2} \frac{dI^2}{dt} = \frac{L I^2}{2}$$

$$\boxed{W_L = \frac{L I^2}{2}}$$

In solenoid $L = \mu_0 \frac{A N^2}{D}$

$$W_L = \frac{\mu_0 (AD) N^2 I^2}{2 D^2} = \frac{V}{2\mu_0}\left(\frac{\mu_0 N I}{D}\right)^2 = \frac{B^2 V}{2\mu_0}$$

$AD = V =$ Volume

$\therefore$ Stored energy is B-field energy

$$W_L \equiv W_B = \frac{1}{2} L I^2$$

for solenoid

$$W_L = \frac{B^2 V}{2\mu_0}$$

$$\frac{B^2}{2\mu_0} \equiv \text{magnetic energy density}$$

In general the stored energy of inductors, can be expressed as

$$W_B = \frac{1}{2} L I^2 = \boxed{\cancel{\frac{1}{2\mu_0}}} \int dV \left(\frac{B^2(\underset{\sim}{\vec{r}})}{2\mu_0} \right)$$

Recall electric field energy is

$$W_E = \frac{q^2}{2C} = \boxed{\cancel{\frac{\epsilon_0}{2}}} \int dV \frac{\epsilon_0 E^2(r)}{2}$$

$\frac{1}{2} \epsilon_0 E^2 \equiv$ electric field energy density