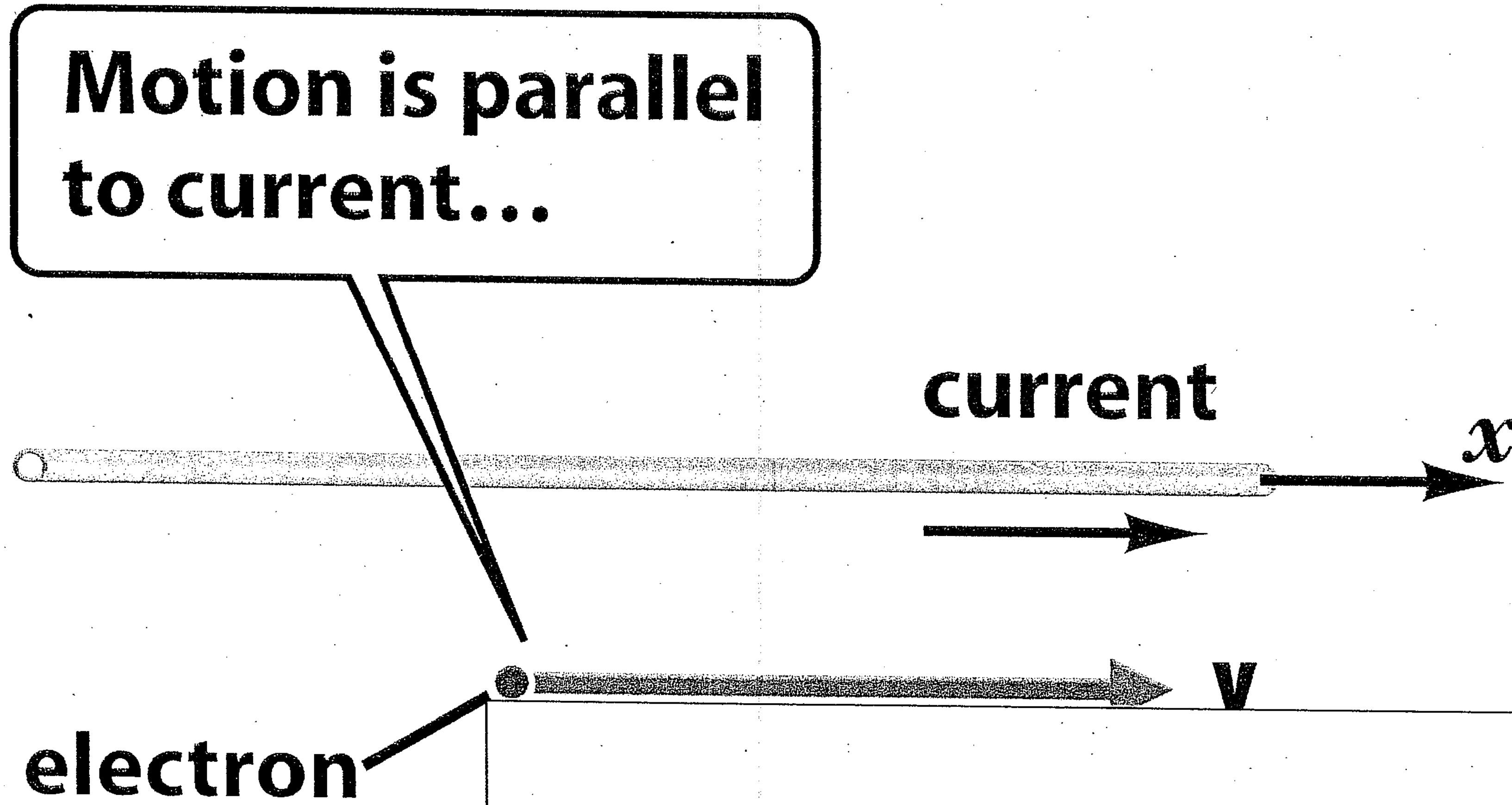
Lecture # 20

Biot Savart Law

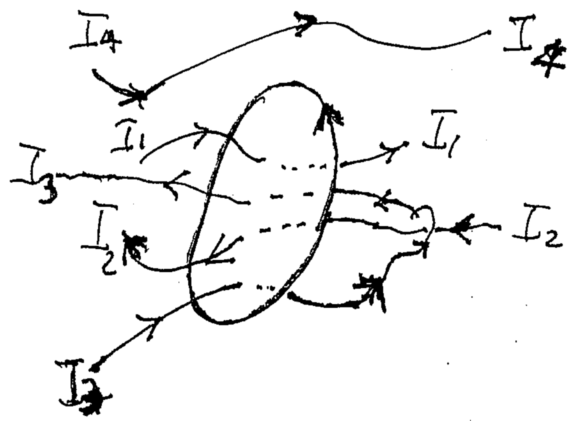


The direction of the force on the electron is:

(a) up (b) down (c) out of the screen (d) into screen

Figure 29-7 Physics for Engineers and Scientists 3/e

Ampere's Law

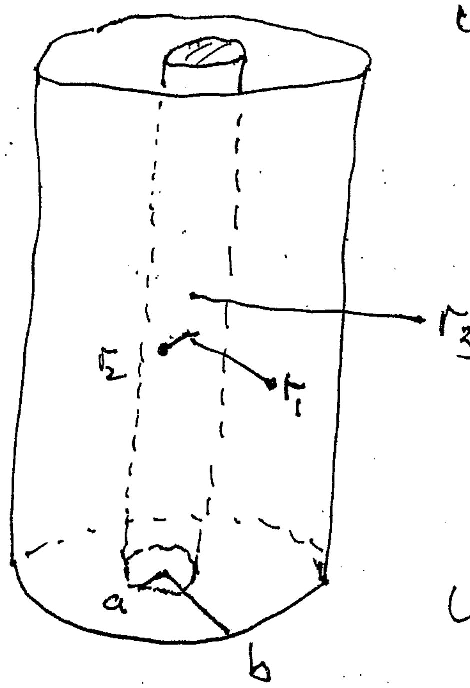


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{total}$$

In the above figure we have, $\oint \vec{B} \cdot d\vec{s}$, around the red boundary, equal to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \begin{cases} I_1 + I_2 + I_3 + I_4 & (a) \\ -I_1 + I_2 & (b) \\ I_1 + I_2 + 2 I_3 & (c) \end{cases}$$

Magnetic field of a Coaxial-cable



upward vertical current I on inner shell at radius $r = a$

downward vertical current on outer shell at radius $r = b$

Uniform surface current density on shells

Find Magnetic Field at r_1, r_2, r_3

$a < r_1 < b$; $r_2 < a$, $r_3 > b$

(a) 0 , (b) $\frac{\mu_0 I}{2\pi r}$, (c) $\frac{\mu I}{2\pi a}$

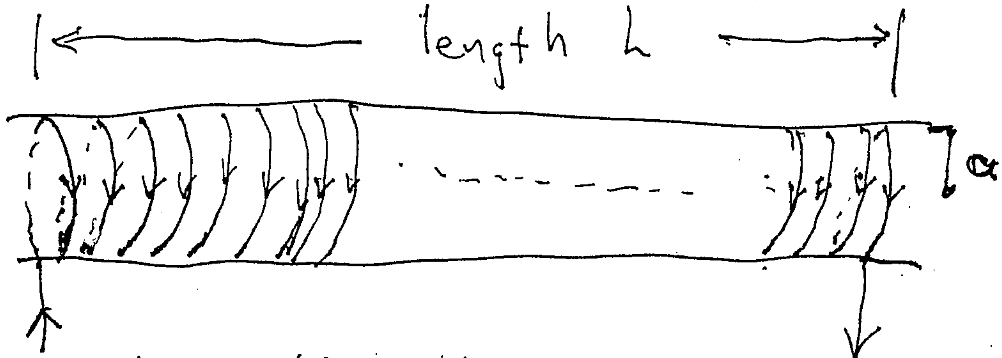
For, $r = r_1$

$r = r_2$

$r = r_3$

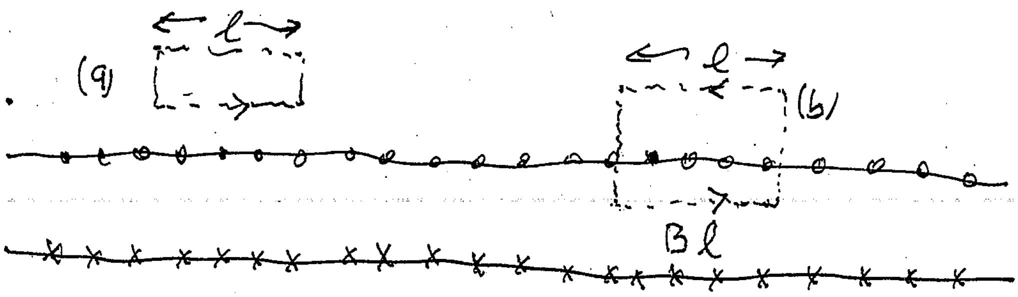
How would the above magnetic field change if the inner vertical current, I, was uniformly distributed throughout inner wire? (3)

Solenoid



Wire has N - turns, with current I through wire.
$n = N/L \equiv$ # turns / length

What is B - field in wire?



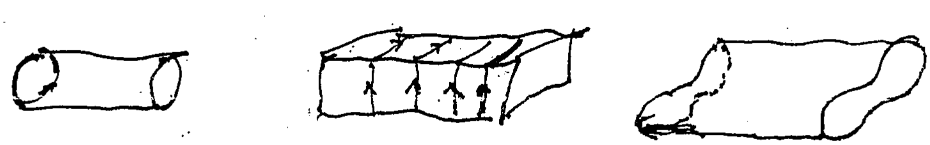
apply guass' law around path (a)

apply guass' law around path (b)

leads to $B = n\mu_0 I$ inside toroid

$B = 0$ outside toroid

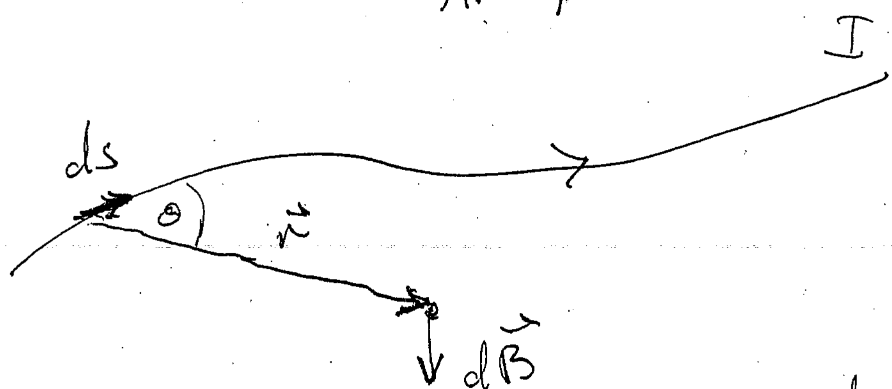
Inner field indepedndent of shape of cylindrical shell



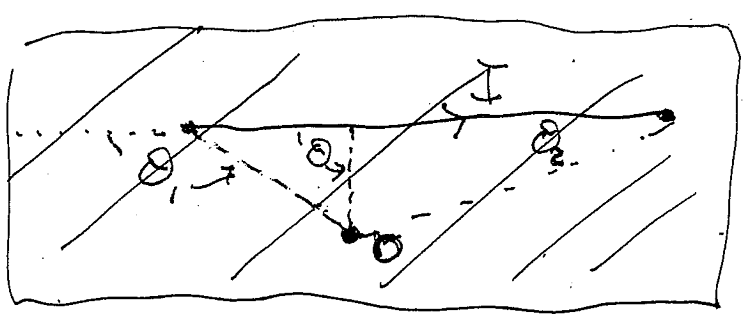
Biot · Savart Law

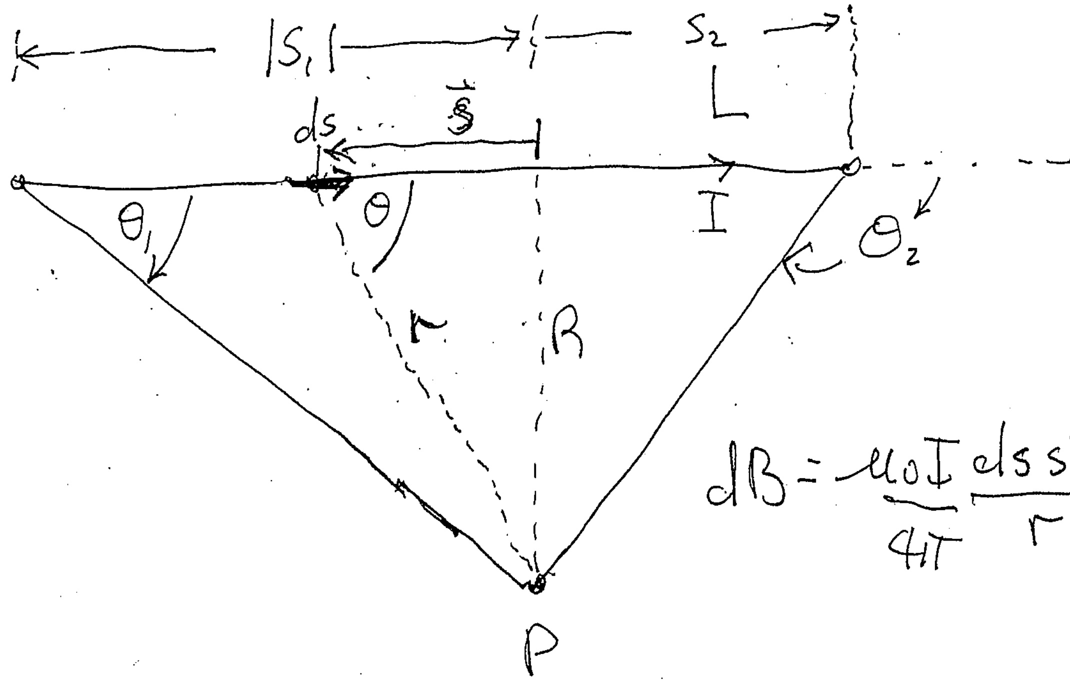
Most general way to calculate magnet field from a steady current

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \vec{r}}{r^3}$$



What is the B~field from a wire of current I of length L ?





$$dB = \frac{\mu_0 I}{4\pi} \frac{ds \sin\theta\, r}{r^3}$$

$$L = S_2 + |S_1|$$

$$r = \frac{R}{\sin\theta}$$

$$r \sin\theta = R \checkmark ;$$

$$\tan\theta = \frac{R}{-S}, \quad S = -\frac{R}{\tan\theta} = -R\cot\theta$$

$$\boxed{dB = \mu_0 I\, ds \frac{r\sin\theta}{4\pi r^3}}$$

$$ds = -R\, d(\cot\theta) = +\frac{R\, d\theta}{\sin^2\theta}$$

$$\therefore dB = +\frac{\mu_0 I R\, d\theta}{4\pi r^2 \sin\theta} \quad ; \quad r^2 = \frac{R^2}{\sin^2\theta}$$

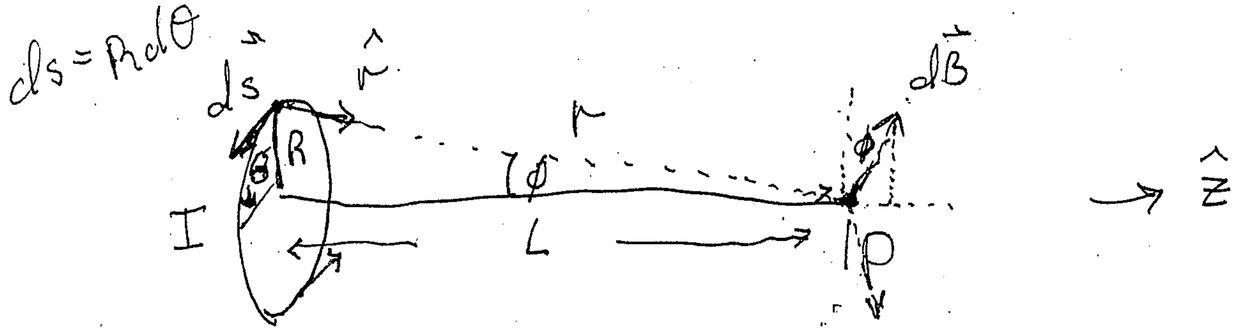
$$\therefore dB = +\frac{\mu_0 I R \sin\theta\, d\theta}{4\pi R^2}$$

$$B = +\frac{\mu_0 I}{4\pi R}\int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = -\frac{\mu_0 I}{R}(\cos\theta)\Big|_{\theta_1}^{\theta_2}$$

$$\boxed{B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 - \cos\theta_2)}$$

(7)

Magnetic Field on axis of loop



$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{r}}{4\pi r^2}$$

(only $d\vec{B}\cdot\hat{z}$ doesn't cancel on loop integration
$\hat{r} \perp d\vec{s} = R\,d\theta\,\hat{\theta}$
$d\vec{B}\cdot\hat{z} = dB \sin\phi$)

$$\hat{z}\cdot d\vec{B} = \frac{\mu_0 I \, R\,d\theta}{4\pi r^2} \sin\phi$$

$$\vec{B} = \hat{z}\frac{\mu_0 I R}{4\pi r^2} \sin\phi \; 2\pi$$

Now:

$$\sin\phi = \frac{R}{(R^2+L^2)^{1/2}}$$

$$r^2 = R^2 + L^2$$

$$\vec{B} = \frac{\mu_0 \hat{z} I}{2} \frac{R^2}{(R^2+L^2)^{3/2}}$$

$\vec{B} = \hat{z} \ldots$ $\boxed{\text{(a) for } L = 0 \text{ (center of loop)}; \; \vec{B} = \hat{z}\frac{\mu_0 I}{2R}}$

(b) for $L \gg R$ $\quad \vec{B} \approx \hat{z}\frac{\mu_0 I R^2}{2 L^3}$