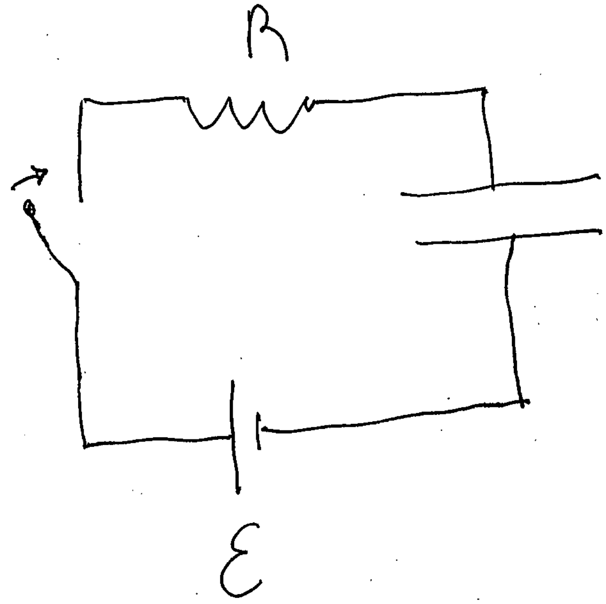
Lecture # 17

RC circuits



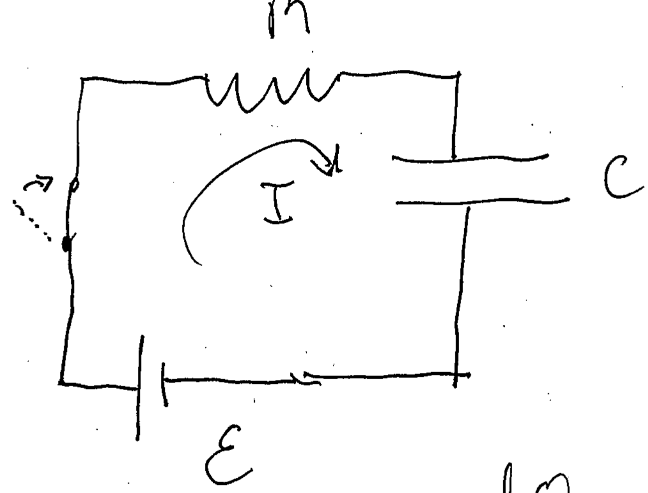
As soon as we close switch, Kirchoff's Voltage law holds:

Initially a switch is open and a capacitor is uncharged. The switch is suddenly closed. Immediately afterward, before any charge can flow through the battery, the voltage across the capacitor is:

(a) 0 (b) $\mathcal{E}$

If after we close the switch and wait long enough, the voltage across the battery is?

Khirchoff's voltage law applies to an RC circuit



$$I = \frac{dQ}{dt}$$

$$-\mathcal{E} + IR + \frac{Q}{C} = 0$$

or

$$-\mathcal{E} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

Standard Form

$$\boxed{R\frac{dQ}{dt} + \frac{Q}{c} = \mathcal{E}}$$

What is solution to $(\mathcal{E}=0)$ equation?

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\ln Q = -\frac{t}{RC} + \text{Const}$$

let $\text{Const} = \ln Q_0$

$$\ln Q = -\frac{t}{RC} + \ln Q_0$$

$$\ln Q - \ln Q_0 = \ln(Q/Q_0)$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$\boxed{Q = Q_0 \exp(-t/RC)}$$

Mathematically: $\ln Q_0$ constant of integration of an integral

Special solution, Q_I, to equation:

$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}$$

$Q_I = C\mathcal{E}$ (or inhomogeneous solution)

Homogeneous solution, Q_H

$$R\frac{dQ_H}{dt} + \frac{Q_H}{C} = 0$$

$$Q_H(t) = Q_0 \exp\left(-\frac{t}{RC}\right) \equiv Q_0 \exp\left(-\frac{t}{\tau}\right)$$

$\boxed{\tau \equiv RC \equiv \text{time constant of RC circuit.}}$

General Solution

$$Q(t) = Q_0 \exp\left(-\frac{t}{RC}\right) + \mathcal{E}C$$

$$0 = Q(t=0) = Q_0 + \mathcal{E}C\,; \quad Q_0 = -\mathcal{E}C$$

Charging a capacitor

Suppose initially there is no charge on capacitor:

General Solution to circuit equation is:

$$Q(t) = Q_0 \exp\left(-\frac{t}{RC}\right) + \mathcal{E} C$$

$Q(t) \equiv$ charge on capacitor:

What must Q_0 be?

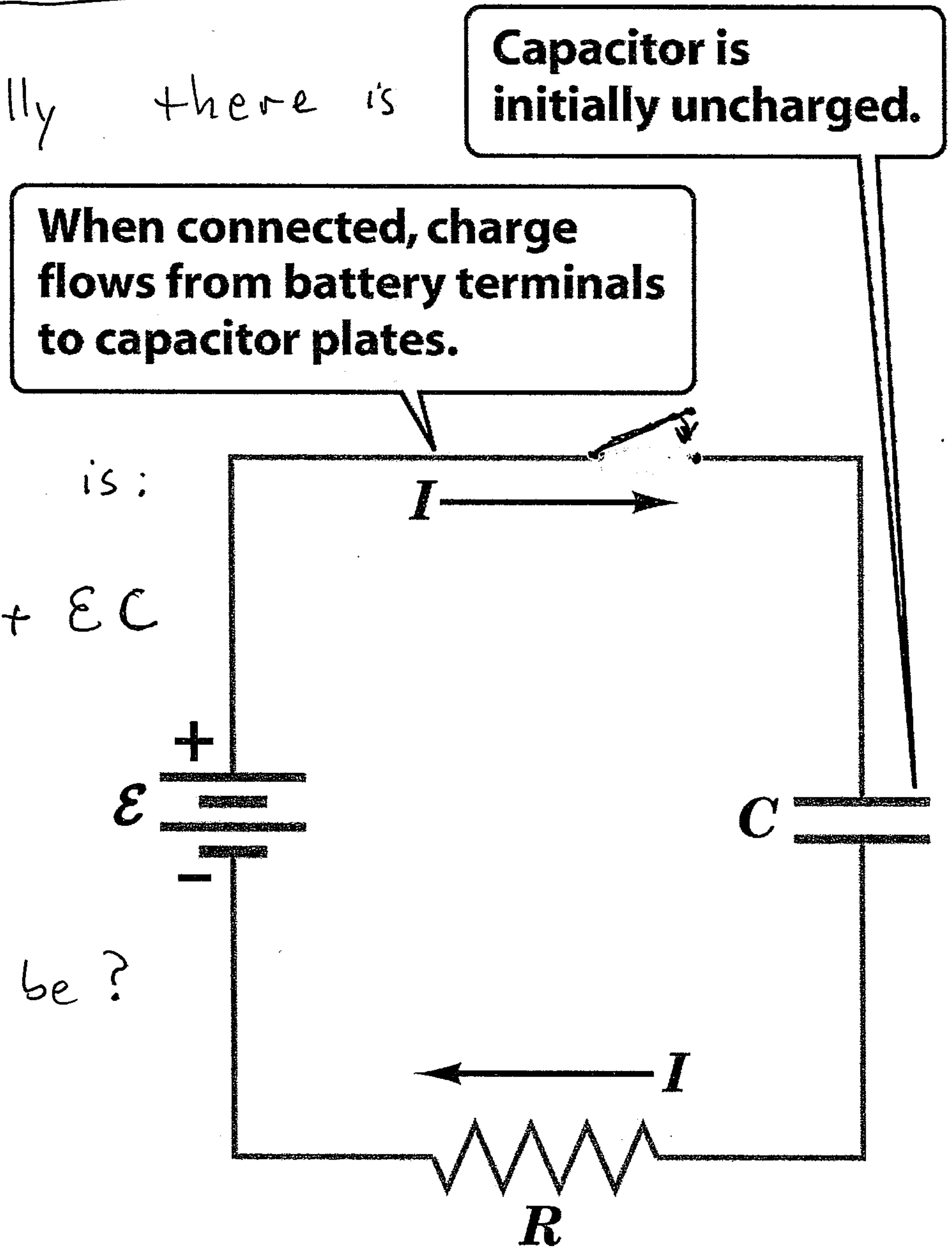
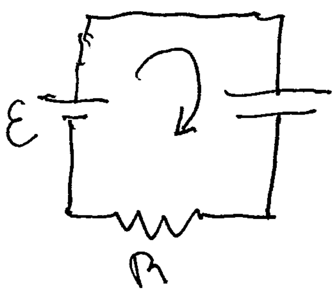


Figure 28-26 Physics for Engineers and Scientists 3/e

$$Q = C\mathcal{E}\left[1 - \exp(-t/RC)\right]$$

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}\exp(-t/RC)$$

Note, initially current flows as if capacitor is shorted



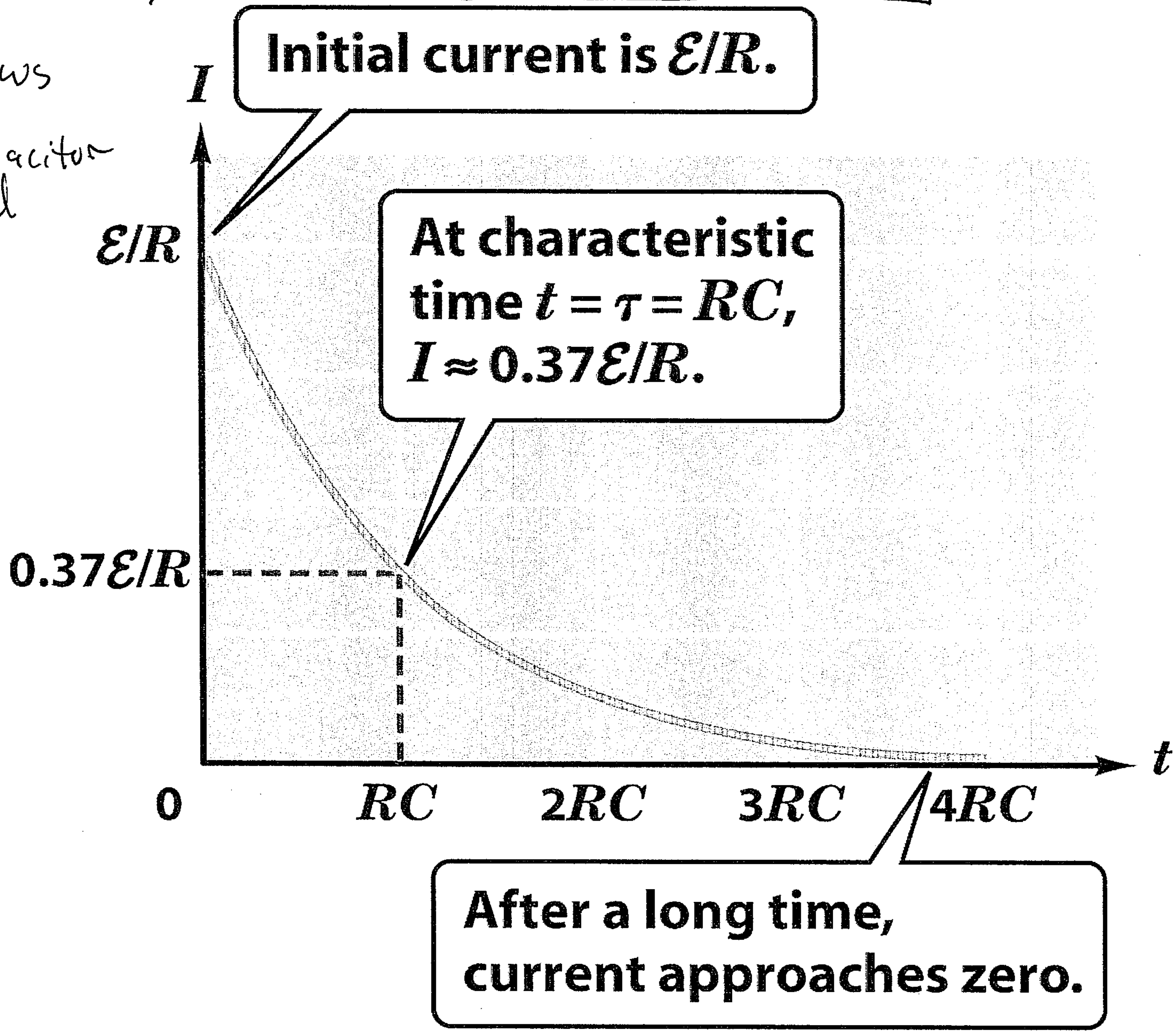


Figure 28-27b Physics for Engineers and Scientists 3/e

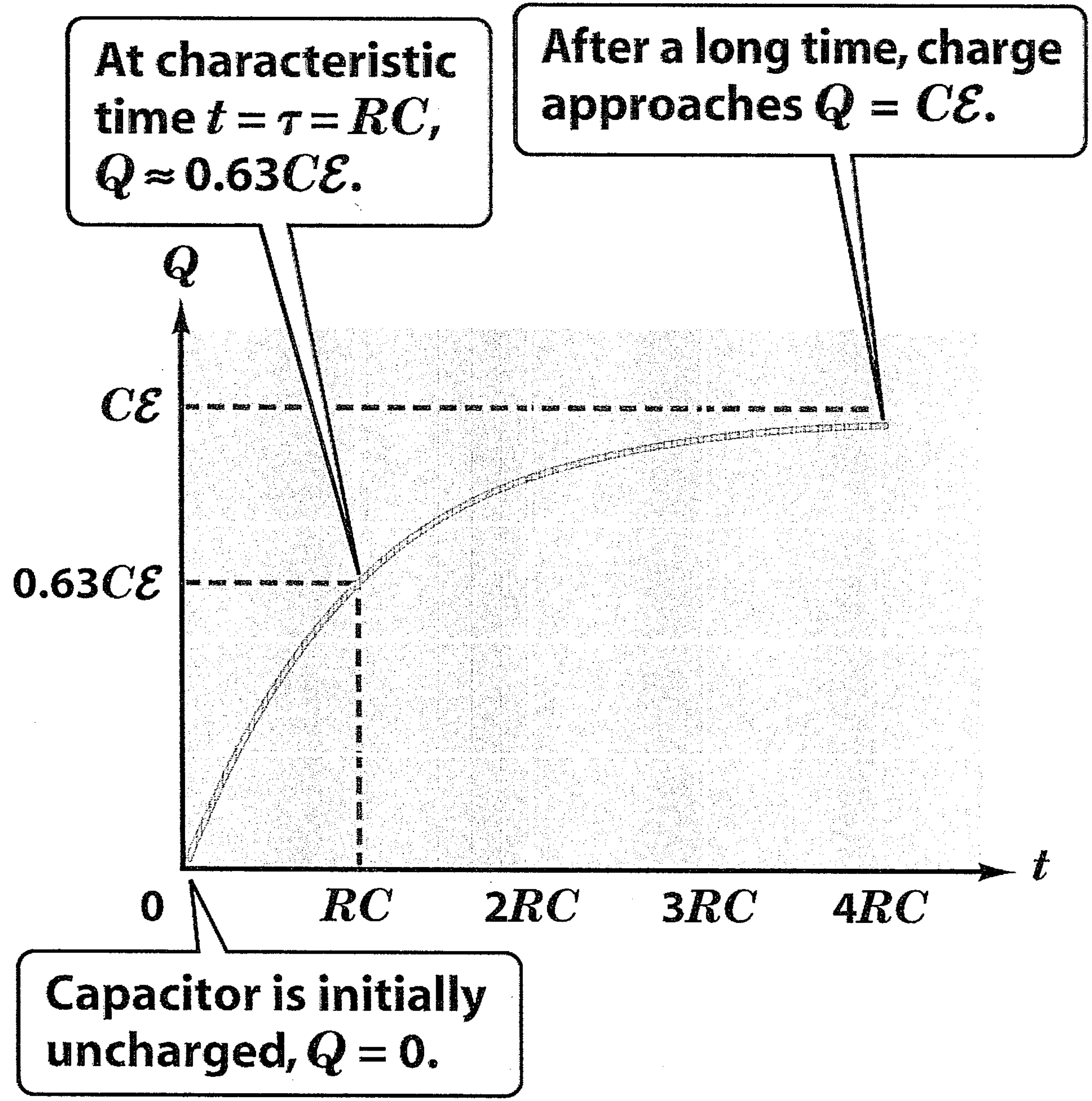


Figure 28-27a Physics for Engineers and Scientists 3/e

The initial voltage across resistor after circuit is closed is $\Delta V = ?$

(a) 0, (b) $\frac{Q_0}{C}$

The initial current, I, through the resistor after circuit is closed is

(a) $I = 0$, (b) $I = \frac{Q_0}{RC}$

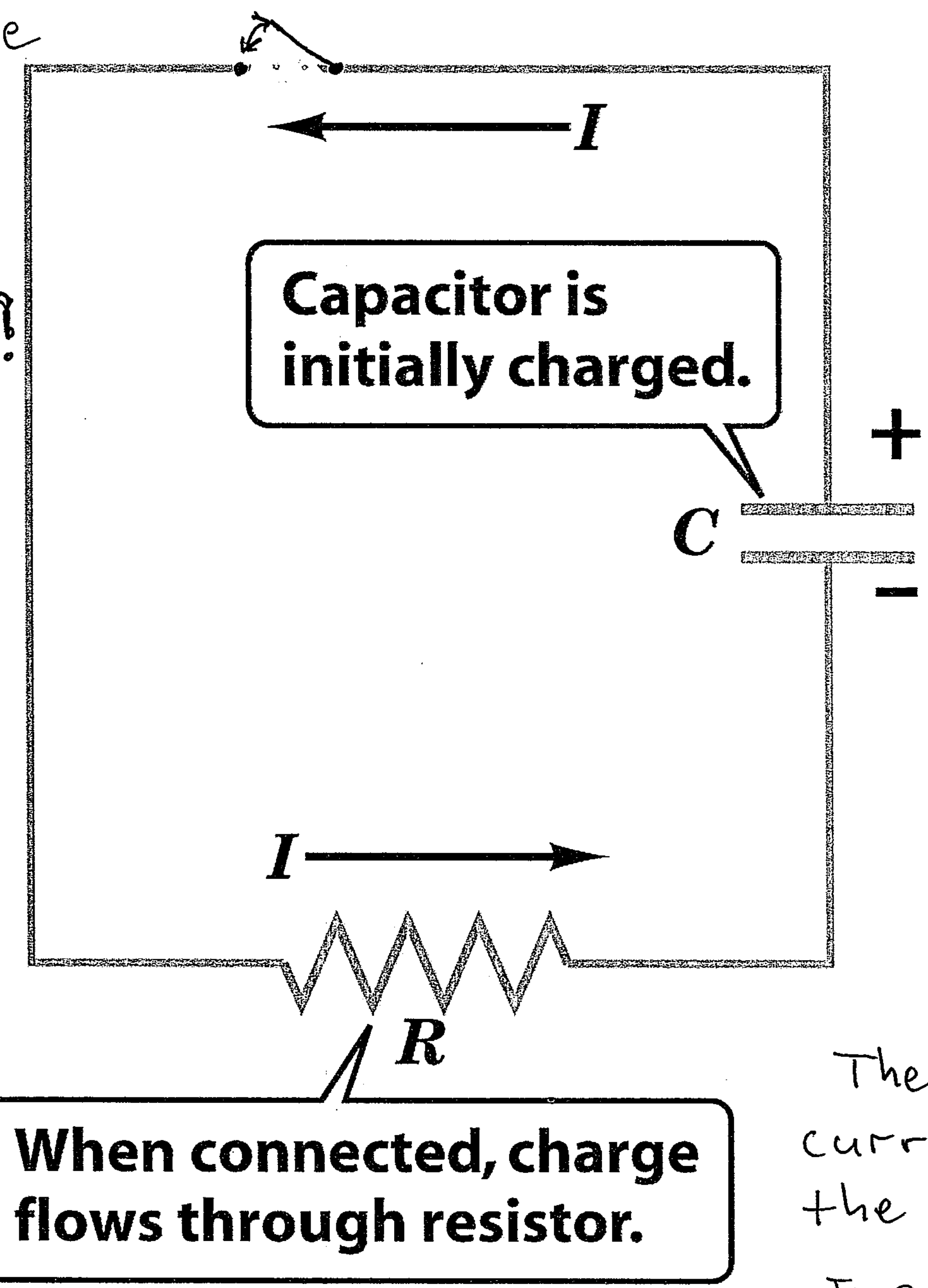


Figure 28-28 Physics for Engineers and Scientists 3/e

Q_0 = initial charge

The final current I through the resistor is

(a) $I = 0$ (b) $I = \frac{Q_0}{RC}$

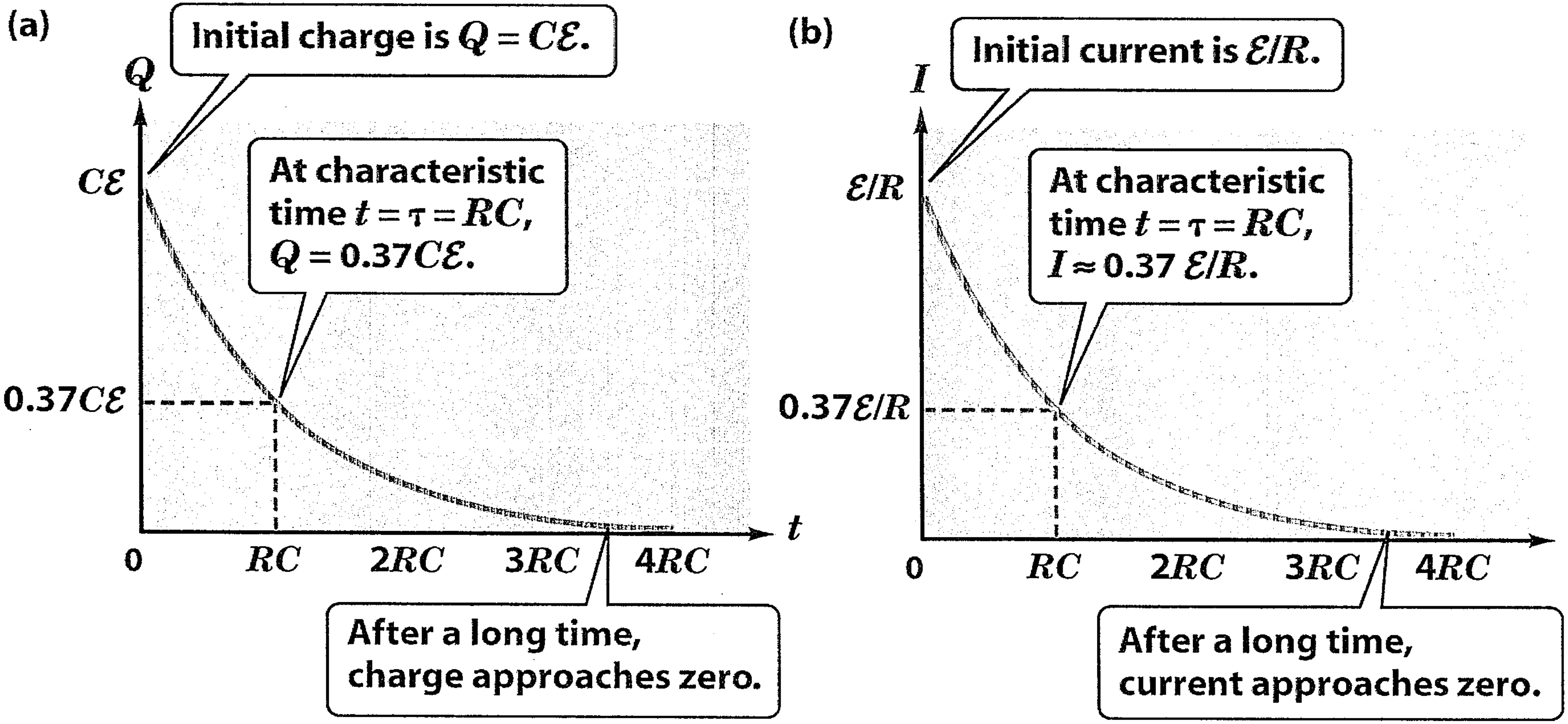


Figure 28-29 Physics for Engineers and Scientists 3/e

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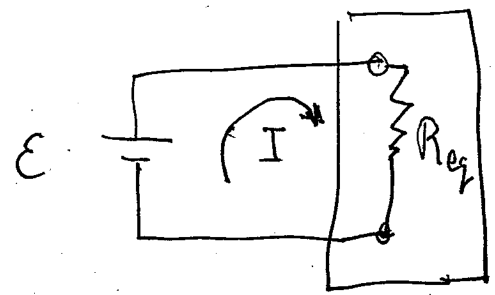
POWER DELIVERED BY A SOURCE OF emf



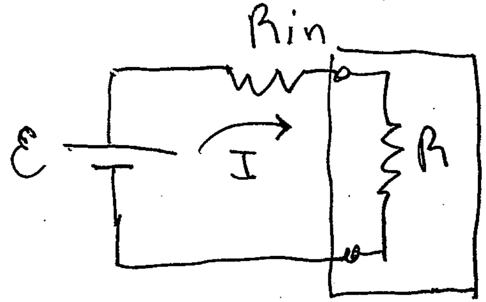
$$P = \mathcal{E} I$$

Equation 28-22 Physics for Engineers and Scientists 3/e

POWER DISSIPATED BY A RESISTOR
(JOULE HEAT)

$\Delta V = I R$

$$P_{diss} = \Delta V I = I^2 R = \frac{(\Delta V)^2}{R}$$



$$I = \frac{\mathcal{E}}{R_{in} + R}$$

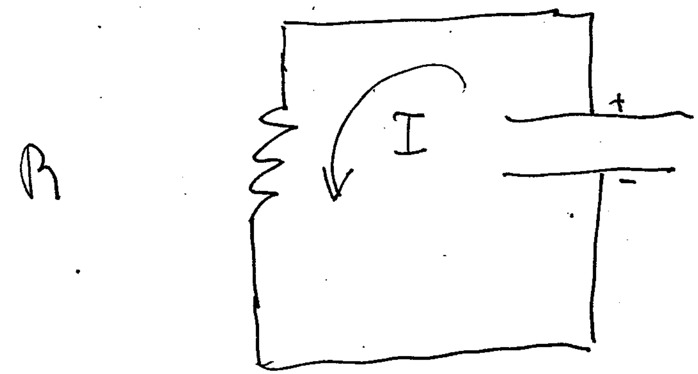
$$P_{diss} = I^2 R = \frac{\mathcal{E}^2 R}{(R_{in} + R)^2}$$

$$P_{del} = \mathcal{E} I = \frac{\mathcal{E}^2}{R_{in} + R}$$

$$P_{diss} = P_{del} \frac{R}{R_{in} + R}$$

$$Eff = \frac{P_{dis}}{P_{del}} = \frac{R}{R_{in} + R}$$

Example of energy conservation



$$Q(t=0) = Q_0$$

$$\Delta V = Q(t)/C$$

Initial stored energy

$$W = \frac{1}{2}\frac{Q_0^2}{C} \qquad \Delta V(t=0) = \frac{Q_0}{C}$$

Power dissipated

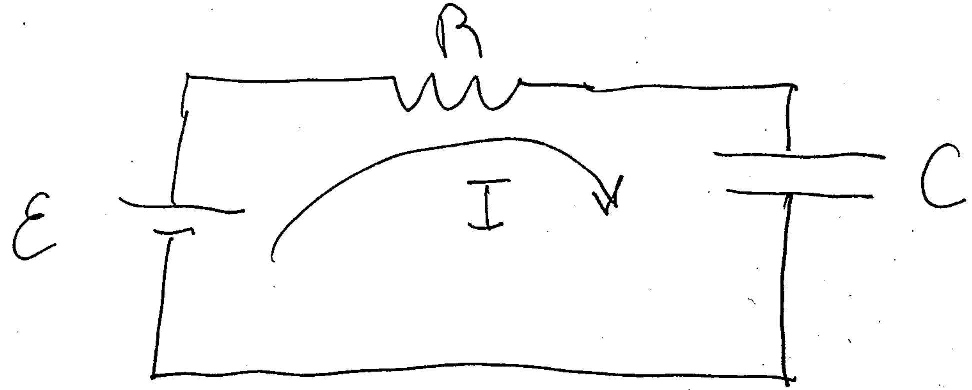
$$P = I^2 R, \qquad I(t) = \frac{\Delta V(t=0)}{R} e^{-t/RC} = \frac{Q_0}{RC} e^{-t/RC}$$

$$\text{Energy dissipated} = \int_0^\infty P\,dt = \int_0^\infty I^2(t) R\,dt$$

$$= \frac{R Q_0^2}{R^2 C^2}\int_0^\infty dt\, e^{-2t/RC} = \frac{Q_0^2}{2C}\int_0^\infty dx\, e^{-x} \qquad x = \frac{2t}{RC}$$

$$= \frac{Q_0^2}{2C}$$

How efficiently do we charge up a capacitor?



$$Q(t) = C\mathcal{E}\left[1 - \exp\left(-t/RC\right)\right]$$

$$W_C = \frac{Q^2}{2C} = \frac{\mathcal{E}^2}{2C}$$

Energy S : supplied by battery

$$\text{Energy } S = \int_0^\infty \mathcal{E}\, I(t)\,dt = \mathcal{E}\int_0^\infty dt\,\frac{dQ}{dt} = \mathcal{E}\big(\underbrace{Q(t=\infty)}_{=\,\mathcal{E}/C} - \underbrace{Q(t=0)}_{=\,0}\big)$$

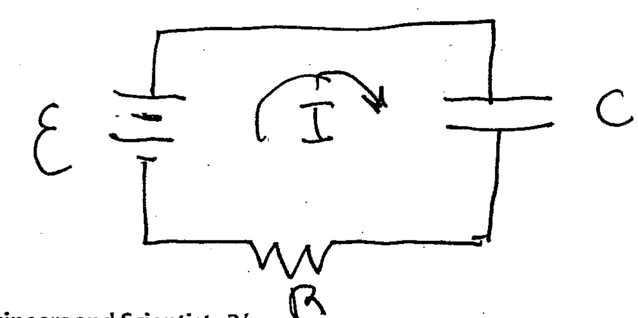
$$\text{Energy } S = \mathcal{E}^2/C$$

$\therefore$ Efficiency $= 50\%$

CHARGE ON CAPACITOR AND CURRENT IN *RC* CIRCUIT

when charging a capacitor

$$Q = C\mathcal{E}\left(1 - e^{-t/\tau}\right) \qquad I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-t/\tau}$$



$\tau = RC$