Lecture # 16

DC circuits

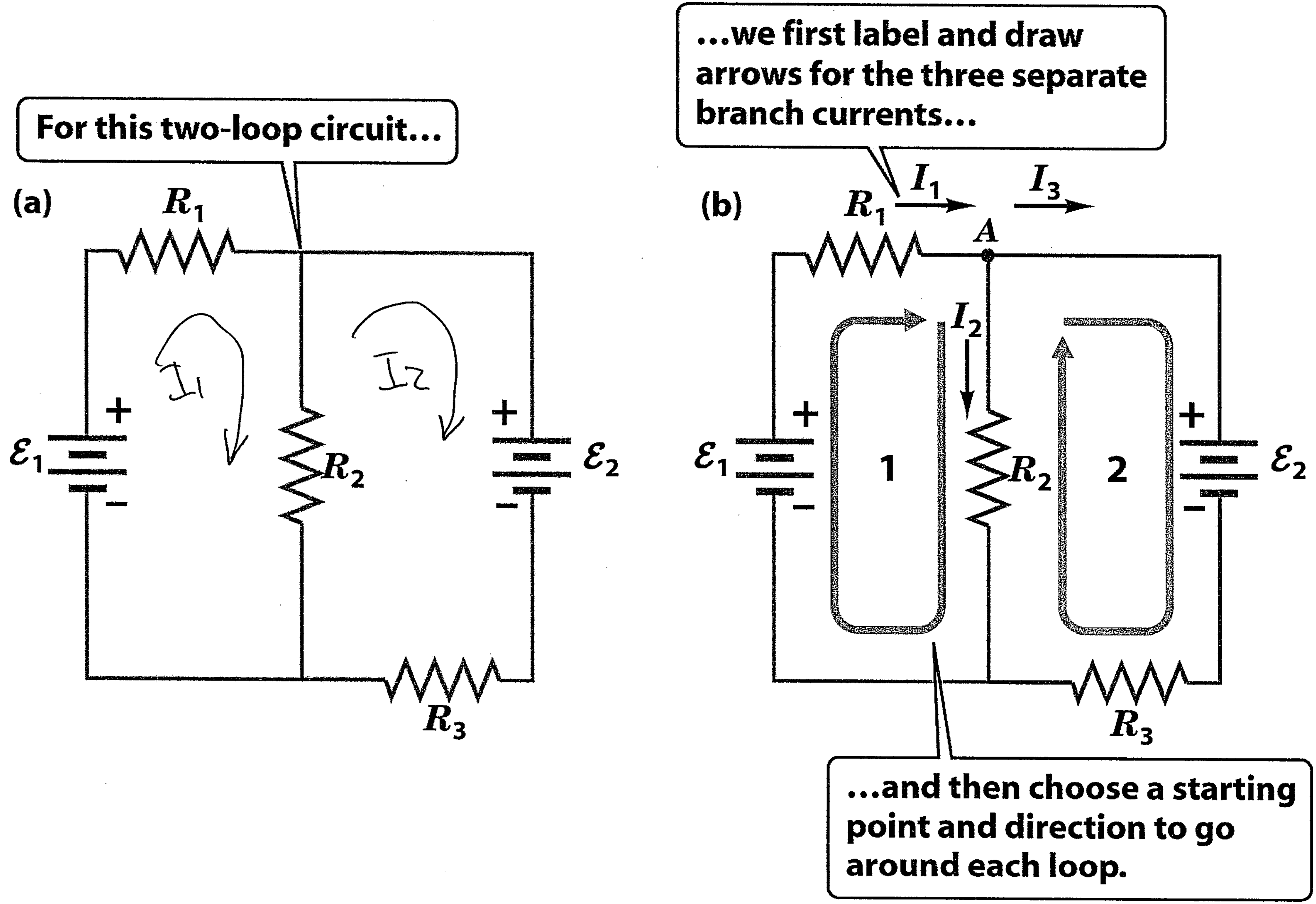


Figure 28-17 Physics for Engineers and Scientists 3/e

<u>Two loop active circuit</u>

$$-\mathcal{E}_1 + I_1 R_1 + (I_1 - I_2) R_2 = 0$$
$$\mathcal{E}_2 + I_2 R_3 + (I_2 - \cancel{I_1}) R_2 = 0$$

$$(R_1 + R_2) I_1 - R_2 I_2 = \mathcal{E}_1$$
$$-R_2 I_1 + (R_3 + R_2) I_2 = -\mathcal{E}_2$$

$$I_1 = \frac{\begin{vmatrix} \mathcal{E}_1 & -R_2 \\ -\mathcal{E}_2 & R_3 + R_2 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_3 + R_2 \end{vmatrix}}$$

$$I_2 = \begin{vmatrix} R_1 + R_2 & \mathcal{E}_1 \\ -R_2 & -\mathcal{E}_2 \end{vmatrix} \Big/ \begin{vmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_3 + R_2 \end{vmatrix}$$

$$I_1 = \frac{(R_3 + R_2)\mathcal{E}_1 - R_2 \mathcal{E}_2}{(R_1 + R_2)(R_3 + R_2) - R_2^2}, \quad I_2 = \frac{R_2 \mathcal{E}_1 - \mathcal{E}_2 (R_1 + R_2)}{(R_1 + R_2)(R_3 + R_2) - R_2^2}$$

note: $(R_1 + R_2)(R_3 + R_2) - R_2^2 = R_1 R_3 + R_2 R_1 + R_2 R_3$

For a complicated circuit with several loops and branches…

(a)

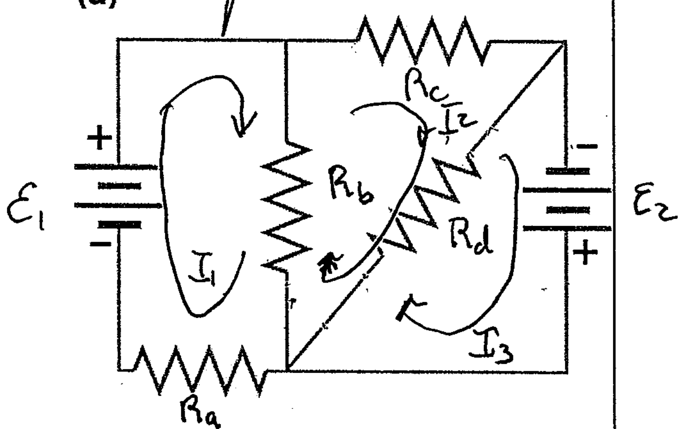


Figure 28-16 Physics for Engineers and Scientists 3/e

$$-\mathcal{E}_1 + R_b(I_1 - I_2) + R_a I_1 = 0$$

$$I_2 R_c + R_d(I_2 - I_3) + R_b(I_2 - I_1) = 0$$

$$-\mathcal{E}_2 + R_d(I_3 - I_2) = 0$$

unknowns I_1, I_2, I_3

$$(R_b + R_a) I_1 - R_b I_2 + 0 I_3 = \mathcal{E}_1$$

$$-R_b I_1 + (R_b + R_d + R_c) I_2 - R_d I_3 = 0$$

$$0 I_1 - R_d I_2 + R_d I_3 = \mathcal{E}_2$$

Can be solved by methods of linear algebra

For a complicated circuit with several loops and branches...

(a)

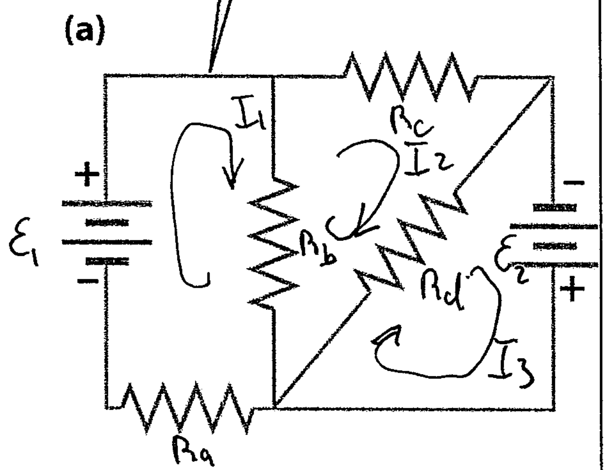


Figure 28-16 Physics for Engineers and Scientists 3/e

$$a_{11} X_1 + a_{12} X_2 + a_{13} X_3 = Y_1$$
$$a_{21} X_1 + a_{22} X_2 + a_{23} X_3 = Y_2$$
$$a_{31} X_1 + a_{32} X_2 + a_{33} X_3 = Y_3$$

$$X_1 = \frac{Y_1 \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - Y_2 \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} + Y_3 \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$$X_2 = \frac{-Y_1 \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + Y_2 \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} - Y_3 \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

$X_3 =$ ...

$$X_1 = \frac{\begin{vmatrix} Y_1 & a_{12} & a_{13} \\ Y_2 & a_{22} & a_{23} \\ Y_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \ldots \end{vmatrix}}, \quad X_2 = \frac{\begin{vmatrix} a_{11} & Y_1 & a_{13} \\ a_{21} & Y_2 & a_{23} \\ a_{23} & Y_3 & a_{33} \end{vmatrix}}{\mathrm{Det} \left| a_{ij} \right|}$$

$$\mathrm{Det} \left| a_{ij} \right| \equiv \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{21} & a_{23} \\ \ldots \end{vmatrix}$$

(3)

Let us solve for I_1

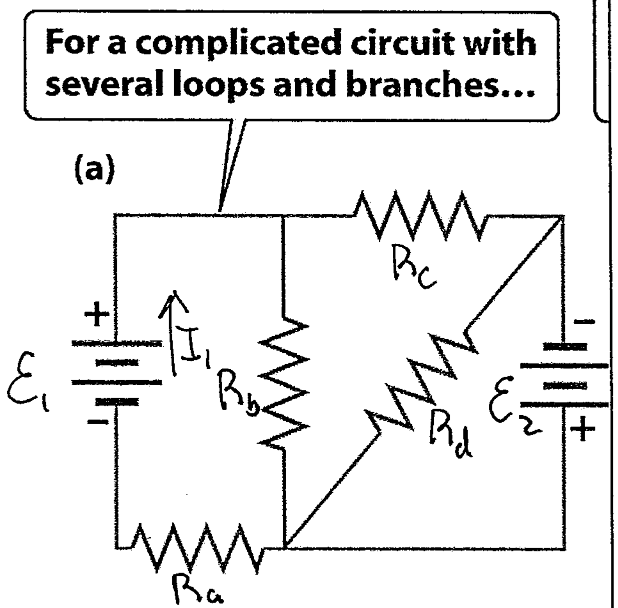
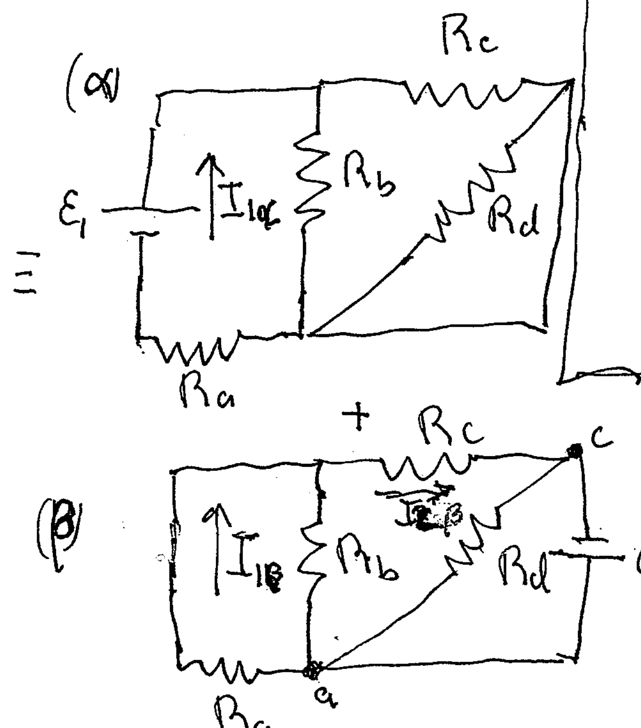


Figure 28-16 Physics for Engineers and Scientists 3/e



In (α) R_d is shorted, no ΔV across it and no current through it. Then R_c and R_b in parallel with $R_{eq} = R_b R_c / (R_b + R_c)$

R_{eq} & R_a in series to give current

$$I_{1\alpha} = \frac{\mathcal{E}_1}{R_a + \frac{R_b R_c}{R_b + R_c}}$$

$I_1 = I_{1\alpha} + I_{1\beta}$

In (β)
The voltage drop between a and c is $\Delta V = \mathcal{E}_2$
This produces a current through R_c given by

(4) $I_{2\beta} = \mathcal{E}_2 / \left(R_c + \frac{R_b R_a}{R_a + R_b} \right)$. Then $I_{1\beta} = \frac{R_b}{R_a + R_b} I_{2\beta} = \frac{\mathcal{E}_2 R_b}{R_c(R_a + R_b) + R_b R_a}$

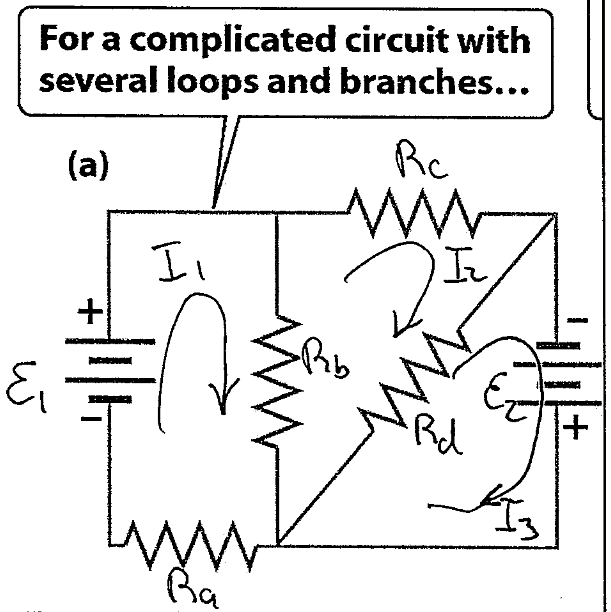
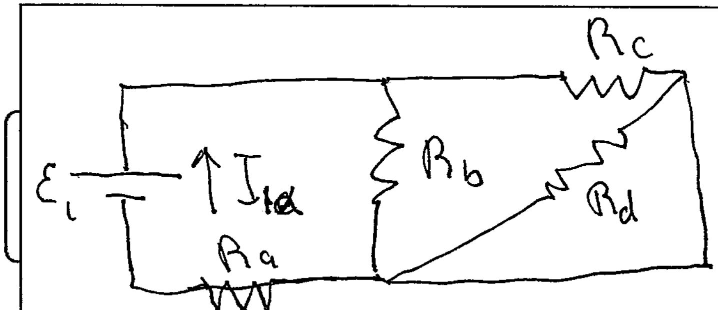
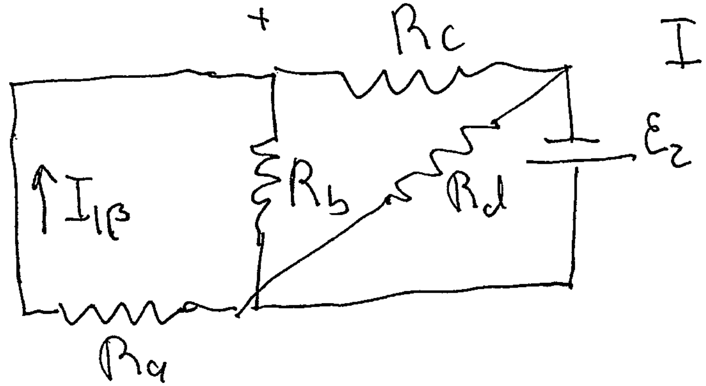


Figure 28-16 Physics for Engineers and Scientists 3/e



$$I_{1\alpha} = \frac{\mathcal{E}_1}{R_a + \frac{R_b R_c}{R_b + R_c}}$$

+



$$I_{1\beta} = \frac{\mathcal{E}_2 R_b}{R_c R_a + R_c R_b + R_b R_a}$$

$$I_1 = I_{1\alpha} + I_{2\alpha} = \frac{\mathcal{E}_1 (R_b + R_c) + \mathcal{E}_2 R_b}{R_a R_c + R_c R_b + R_b R_a}$$

(5)

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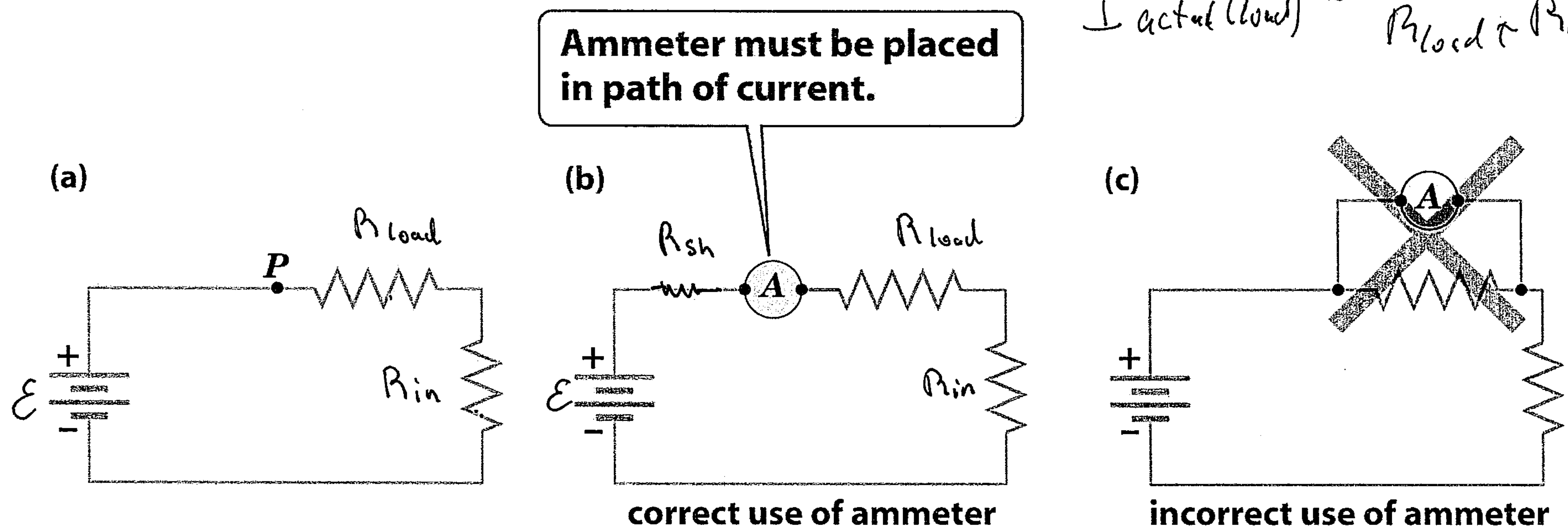
Loren Winters/Visuals Unlimited

(6)

Internal Battery resistance, R_{in}

$I_{load} = \frac{\mathcal{E}}{R_{load}}$, $R_{in} = 0$

$I_{actual\,(load)} = \frac{\mathcal{E}}{R_{load} + R_{in}}$



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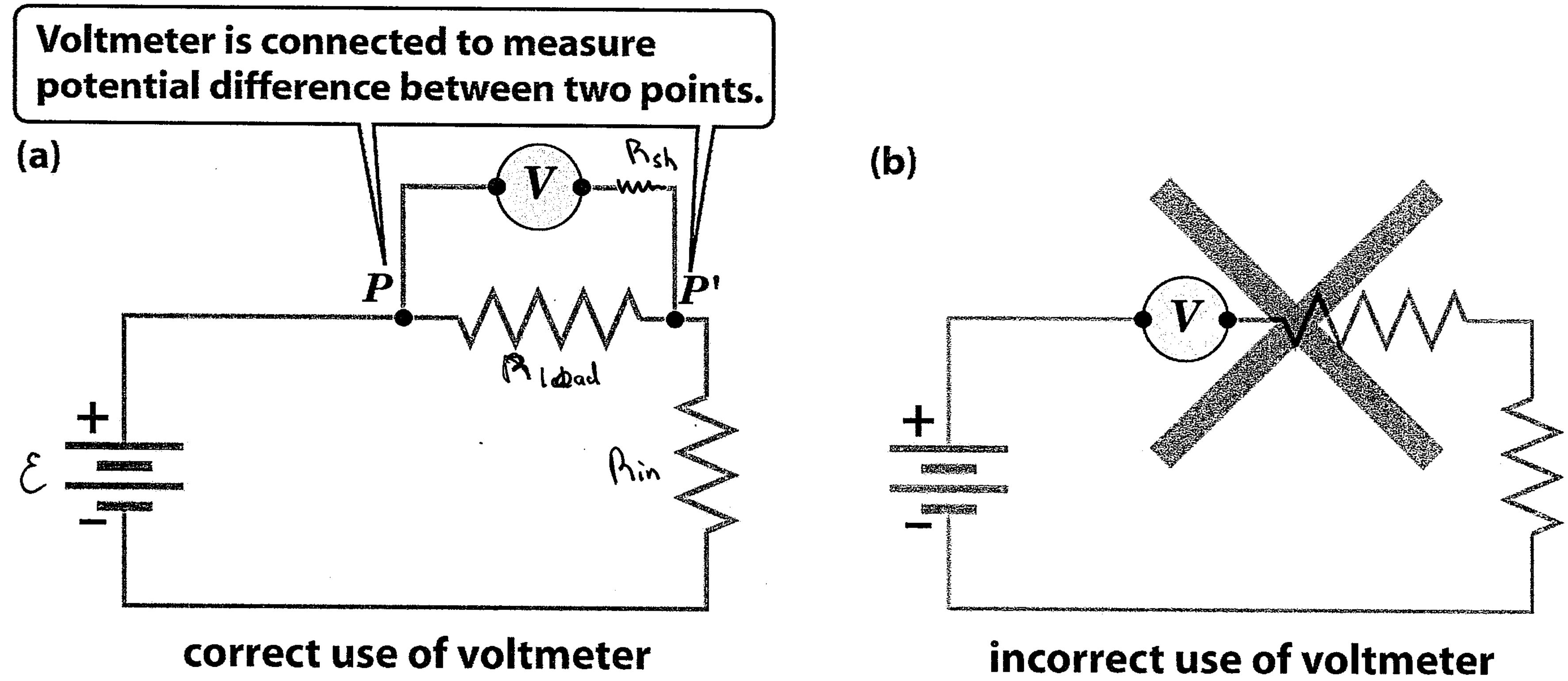
$R_{in} \equiv$ internal resistance of battery

$R_{sh} \equiv$ internal resistance of ammeter

We need to get an accurate reading of ammeter

(a) $R_{sh} \ll R_{load}$ (b) $R_{sh} \gg R_{load}$



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To get accurate reading on voltmeter, we need

(a) $R_{sh} << R_{load}$ (b) $R_{sh} >> R_{load}$

(8)

Energy ΔE dissipation in resistor in time Δt

$\Delta E = V \, \Delta q$

Power $P = \Delta E / \Delta t = V \, \Delta q / \Delta t$

or

$P = V dq/dt = VI = I^2R = V^2/R$

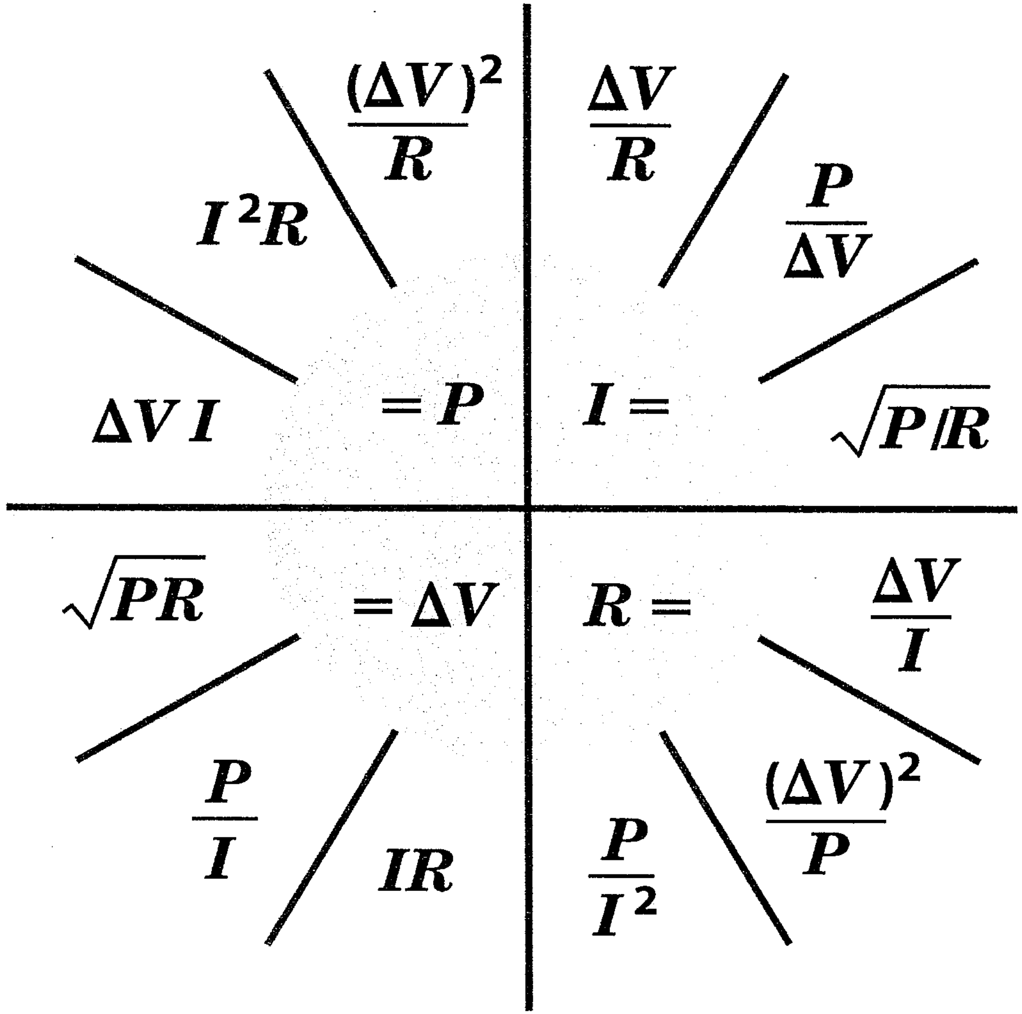

Figure 28-18 Physics for Engineers and Scientists 3/e

$P = VI$,

$V = IR$

$I = \frac{V}{R}$