Lecture # 12

Electric Potential
&
Capacitance

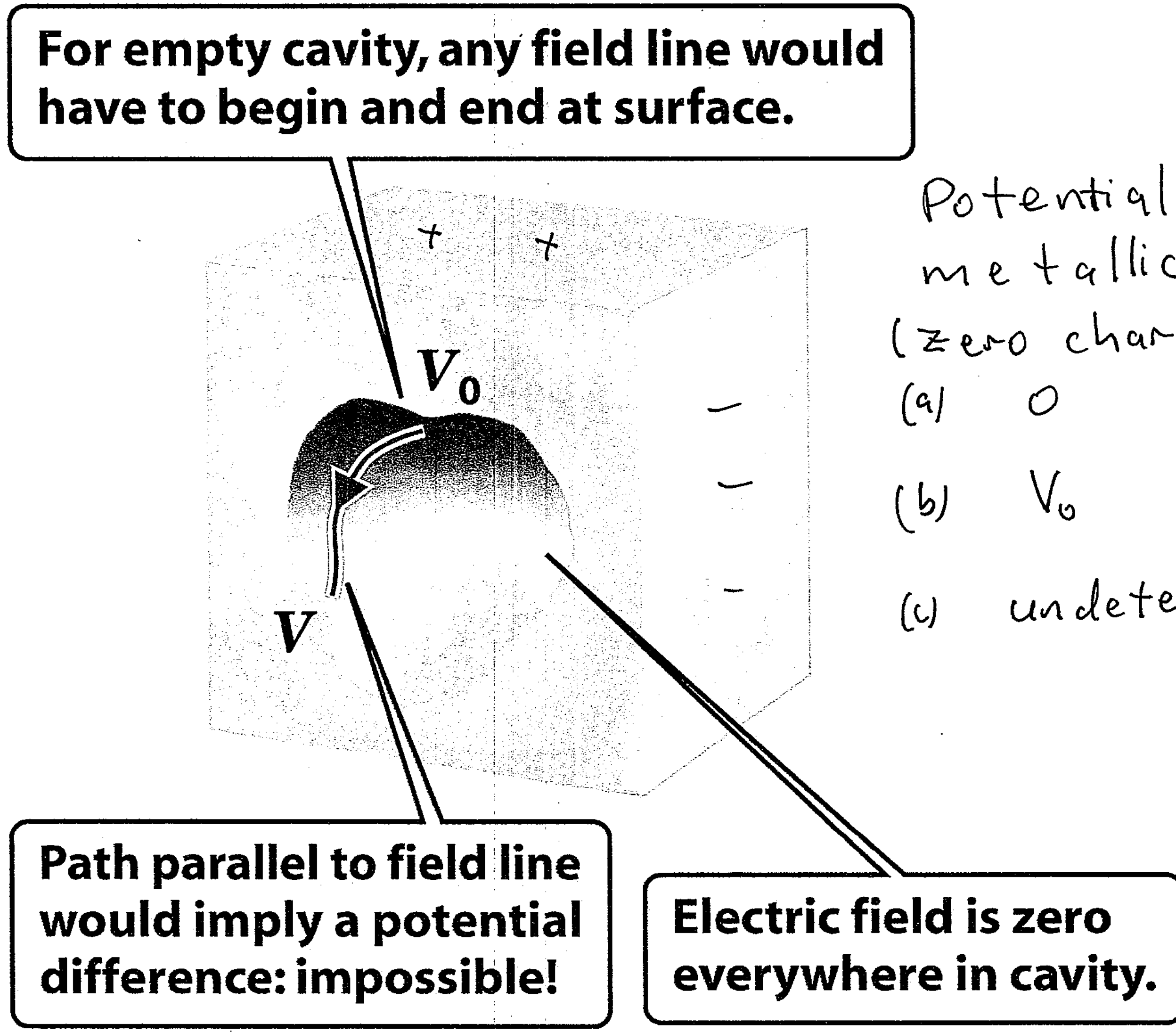

Figure 25-19 Physics for Engineers and Scientists 3/e

Potential inside metallic cavity is:
(zero charge inside cavity)

(a) 0

(b) V_0

(c) undetermined

Equipotentials of a point charge

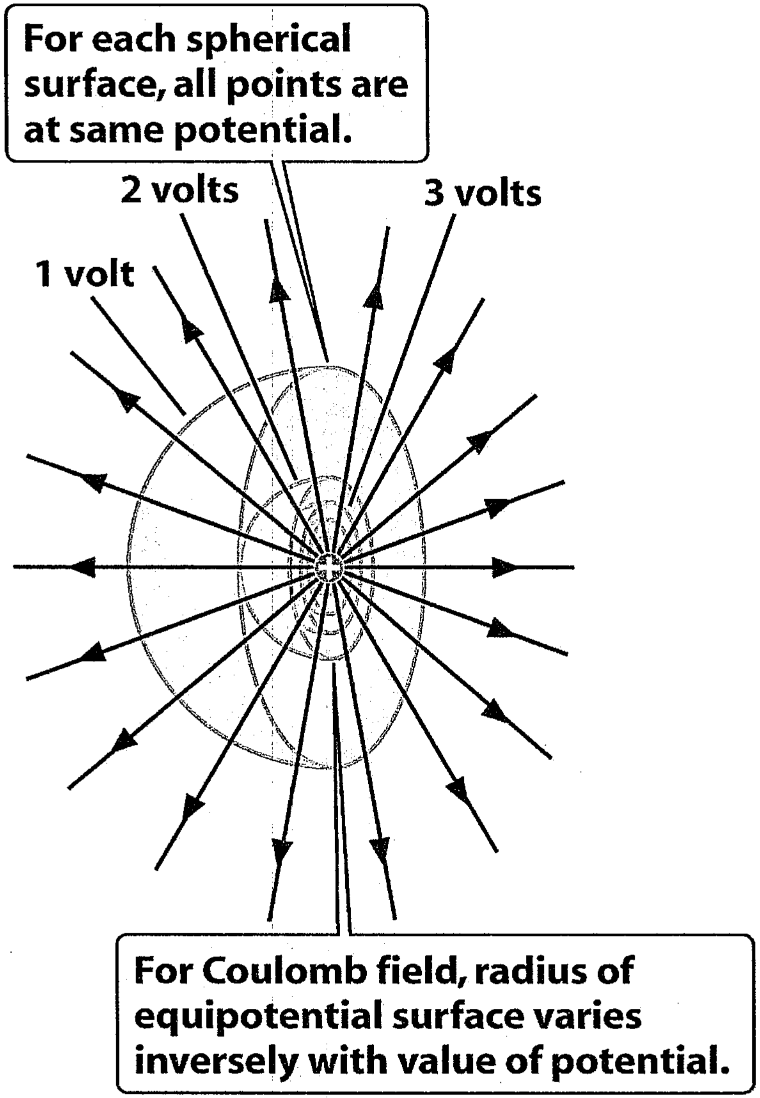

Figure 25-24 Physics for Engineers and Scientists 3/e

Equipotential surface ⊥ to $\vec{E}$-field

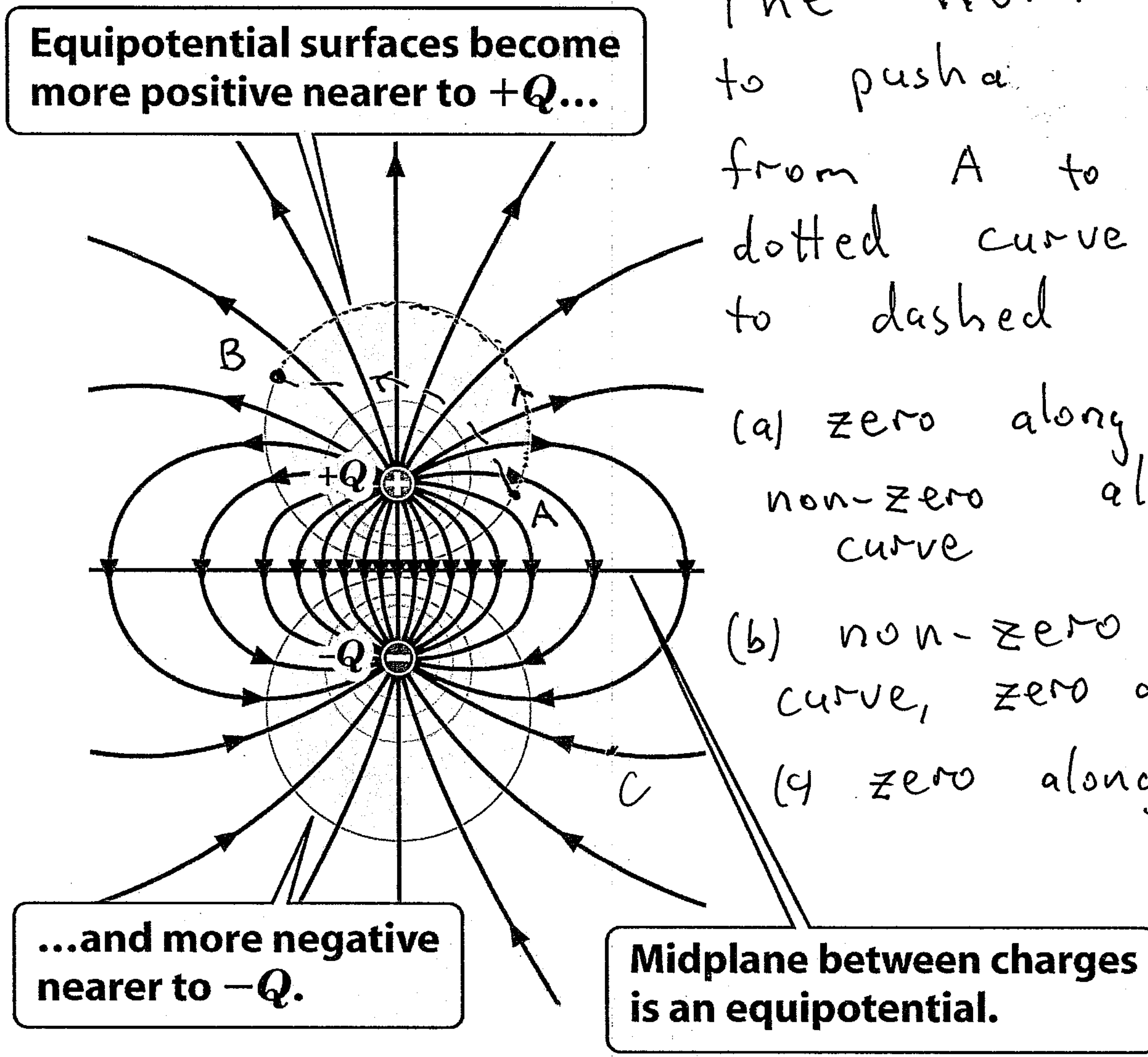

Figure 25-25 Physics for Engineers and Scientists 3/e

The work it takes to push a charge from A to B along dotted curve, compared to dashed curve is

(a) zero along dotted curve non-zero along dashed curve

(b) non-zero along dotted curve, zero along dashed curve

(c) zero along both curves

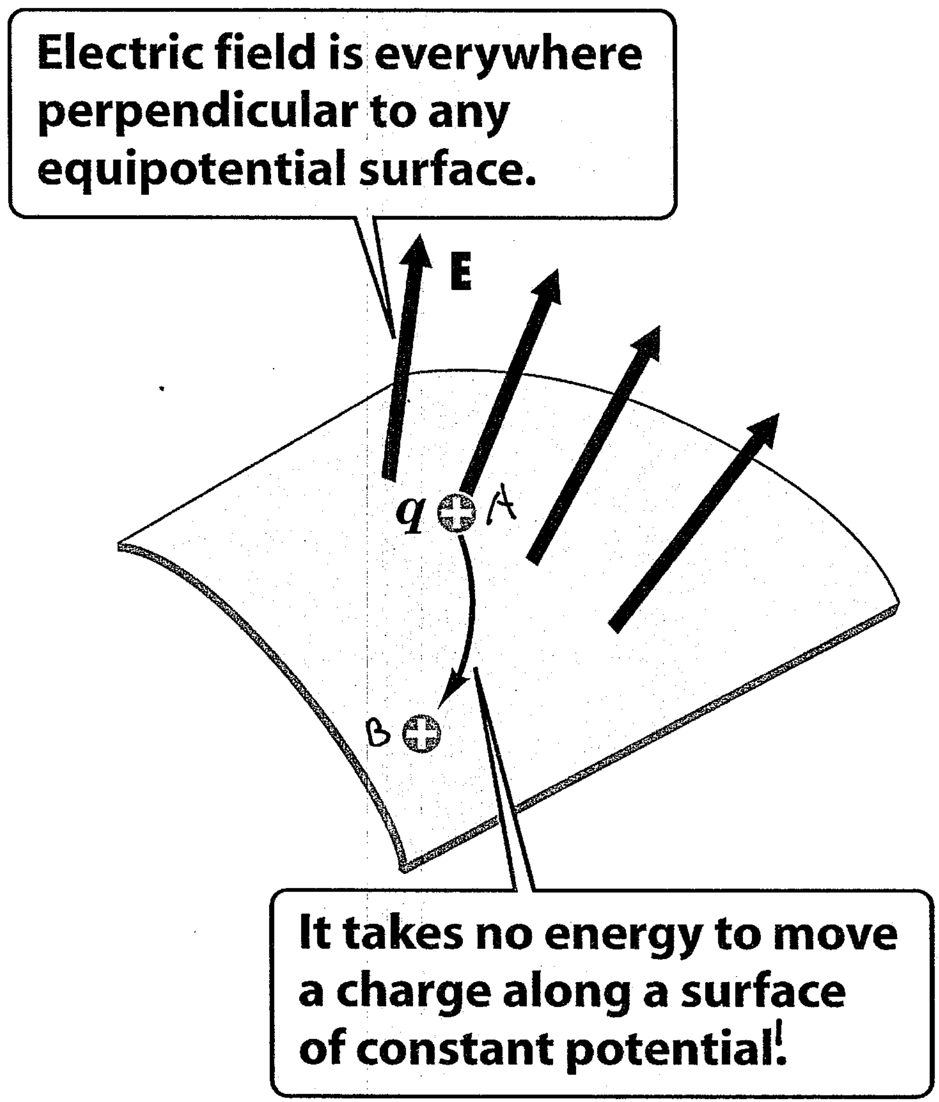

Figure 25-26 Physics for Engineers and Scientists 3/e

Potential of a spherical conductor

$$dV = -\vec{E}\cdot d\vec{r}$$

$$\vec{E} = \begin{cases} \dfrac{Q}{4\pi\epsilon_0}\dfrac{\hat{r}}{r^2}, & r > a \\ 0, & r < a \end{cases}$$

$$dV = -\frac{Q}{4\pi\epsilon_0}\frac{dr}{r^2} \quad \text{if } r > a$$

$$V(r) - \underset{0}{\underset{\shortparallel}{V(\infty)}} = -\frac{Q}{4\pi\epsilon_0}\int_\infty^r \frac{dr'}{r'^2}$$

$$= \frac{Q}{4\pi\epsilon_0}\frac{1}{r'}\Big|_\infty^r = \frac{Q}{4\pi\epsilon_0 r}$$

$$V(a) = \frac{Q}{4\pi\epsilon_0 a},$$

if $r \leq a$; $dV = -E_r\,dr = 0$, $\therefore \frac{dV}{dr} = 0$) $V(r) = \text{Const} = V(a) = \frac{Q}{4\pi a\epsilon_0}$

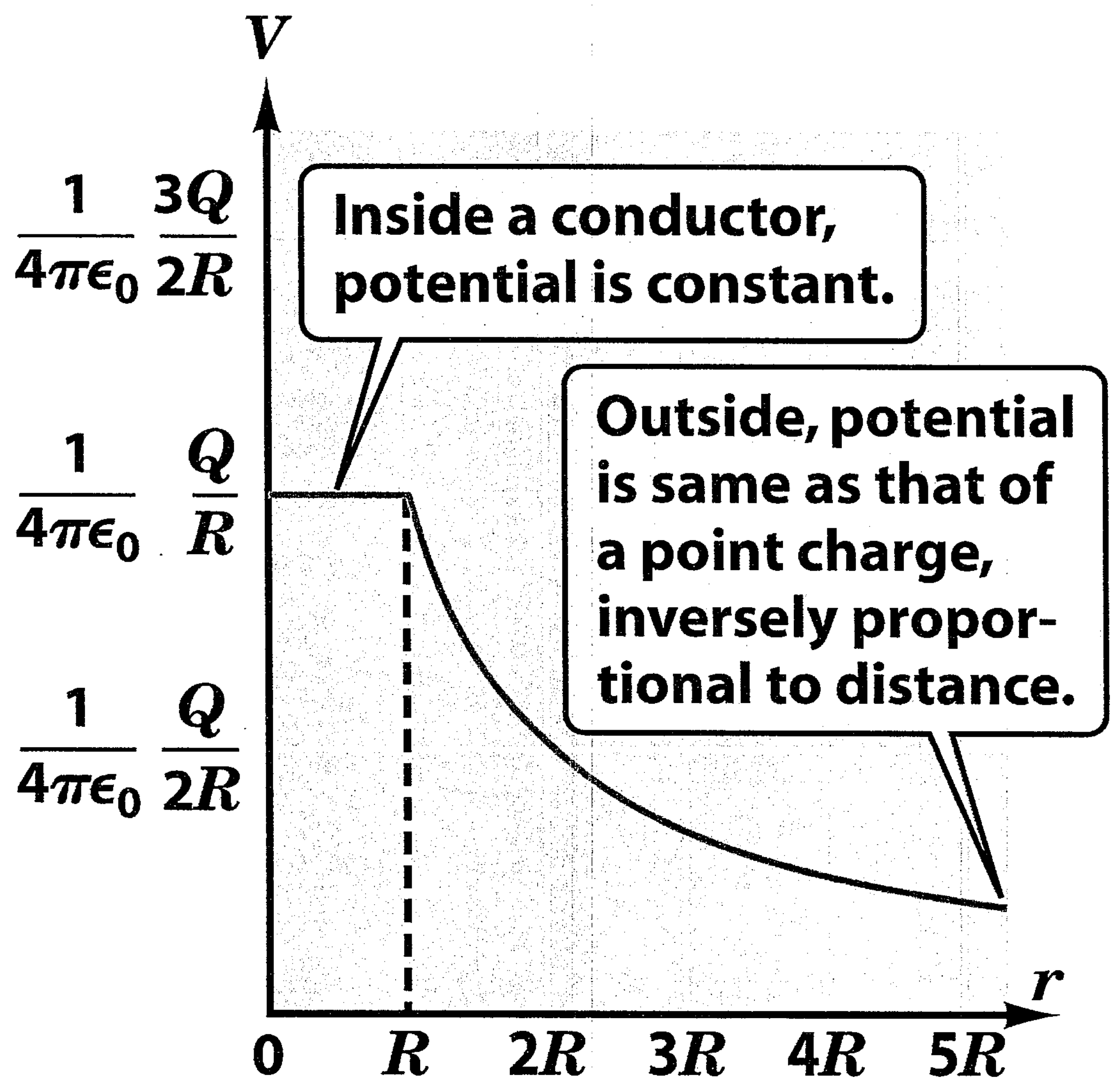

Figure 25-18 Physics for Engineers and Scientists 3/e

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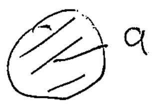
Potential of a uniformly charged spherical insulator

[find $V(r=0)$]

$$E = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} , \quad r > a$$



$$4\pi r^2 E_r = \rho \frac{4\pi r^3}{3\epsilon_0} , \quad r < a$$

$$= \frac{Q}{4\pi\epsilon_0 a^3/3} \frac{4\pi r^3}{3} = \frac{Q r^3}{\epsilon_0 a^3}.$$

$$\therefore \quad E_r = \frac{Q r}{4\pi a^3 \epsilon_0} \quad (r < a)$$

$$V(r) = -\int_{\infty}^{r} E_r \, dr$$

$$= \frac{-Q}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0 r} \quad r \geq a$$

$$V(r) = -\frac{Q}{4\pi\epsilon_0}\left[\int_{\infty}^{a} \frac{dr}{r^2} + \int_{a}^{r} \frac{dr\, r}{a^3}\right]$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} + \frac{1}{2}\left(\frac{a^2 - r^2}{a^3}\right)\right]$$

$$V(0) = \frac{Q}{4\pi\epsilon_0 a}\left[1 + \frac{1}{2}\right] = \frac{3}{8}\frac{Q}{4\pi\epsilon_0 a}$$

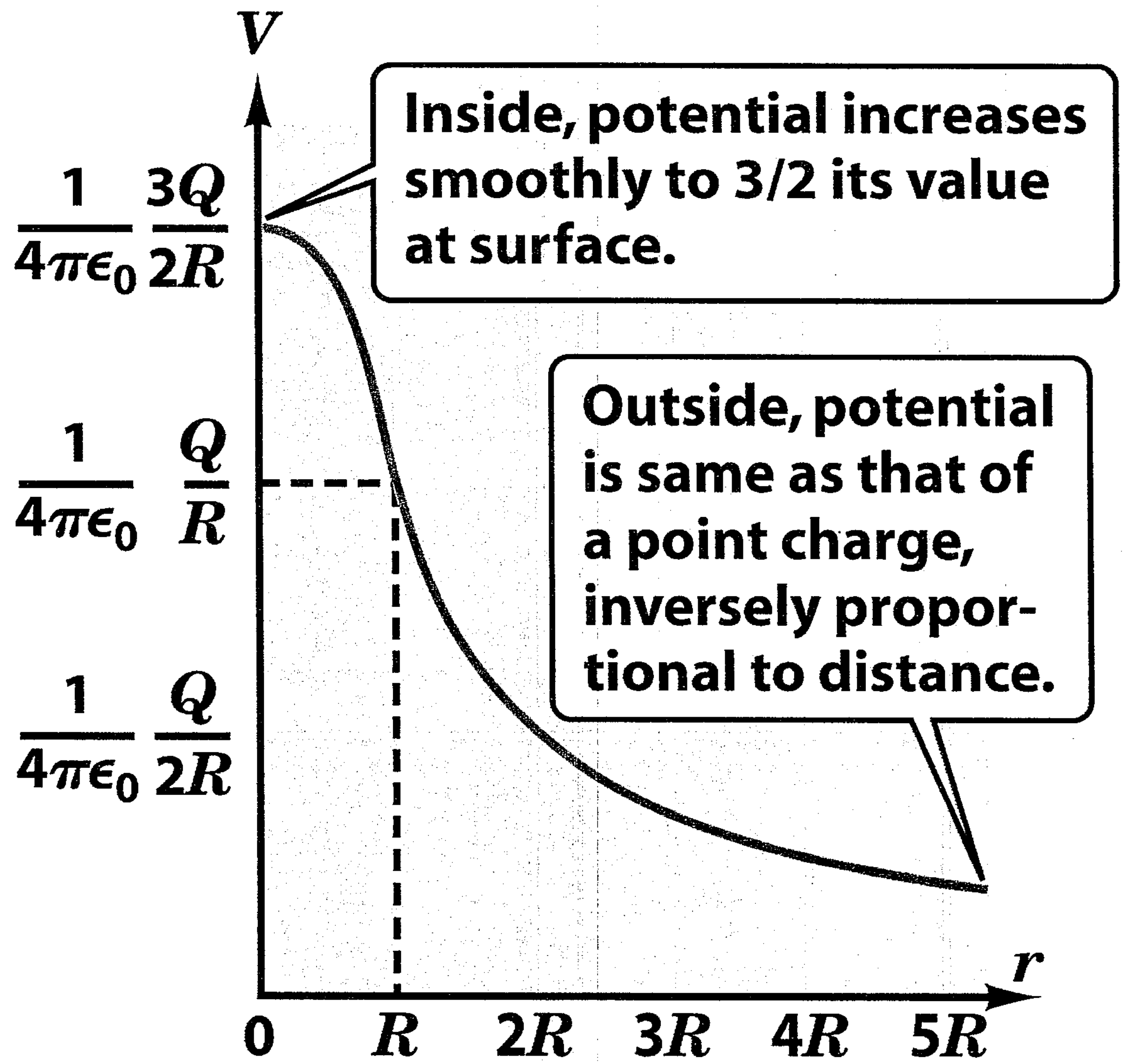


Figure 25-13 Physics for Engineers and Scientists 3/e

Find Potential at P if charge Q is uniformly distributed

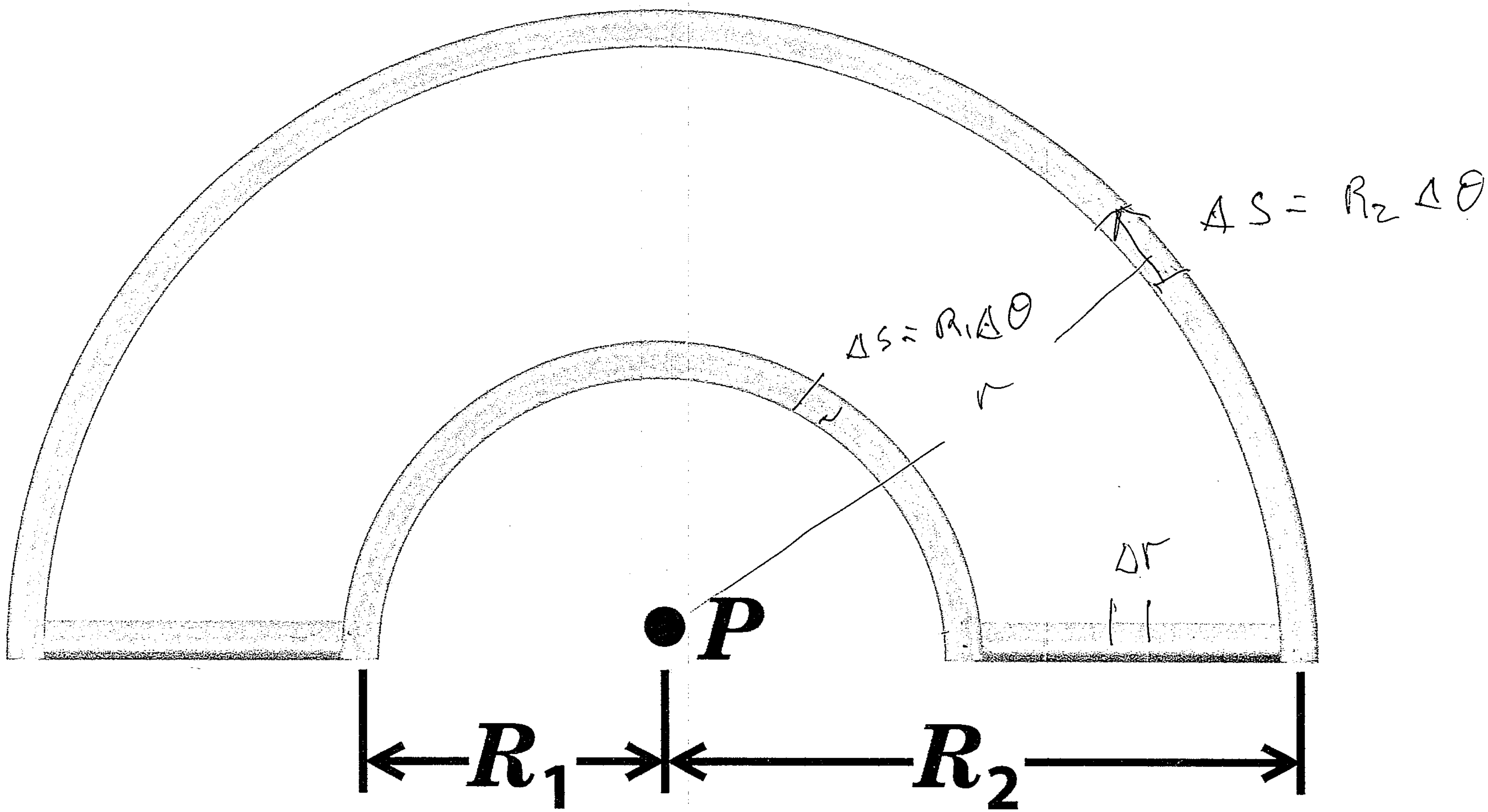


Figure 25-37 Physics for Engineers and Scientists 3/e

$$Q = \lambda\left[\pi(R_2+R_1) + 2(R_2-R_1)\right]$$

$$dV = \frac{\lambda\, ds}{4\pi\epsilon_0 r}, \qquad V = \frac{\lambda}{4\pi\epsilon_0}\oint \frac{ds}{r}$$

$$V = \frac{\lambda}{4\pi\epsilon_0}\left[\int_0^{\pi} \frac{R_2\, d\theta'}{R_2} + \int_0^{\pi} \frac{R_1\, d\theta}{R_1} + 2\int_{R_1}^{R_2} \frac{dr'}{r'}\right]$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[2\pi + 2\ln\left(\frac{R_2}{R_1}\right)\right]$$

$$V_p = \frac{Q}{4\pi\epsilon_0\left[\pi(R_2+R_1) + 2(R_2-R_1)\right]}\left[2\pi + 2\ln\left(\frac{R_2}{R_1}\right)\right]$$

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Capacitance between conductors:
$C=Q/V$ Farads

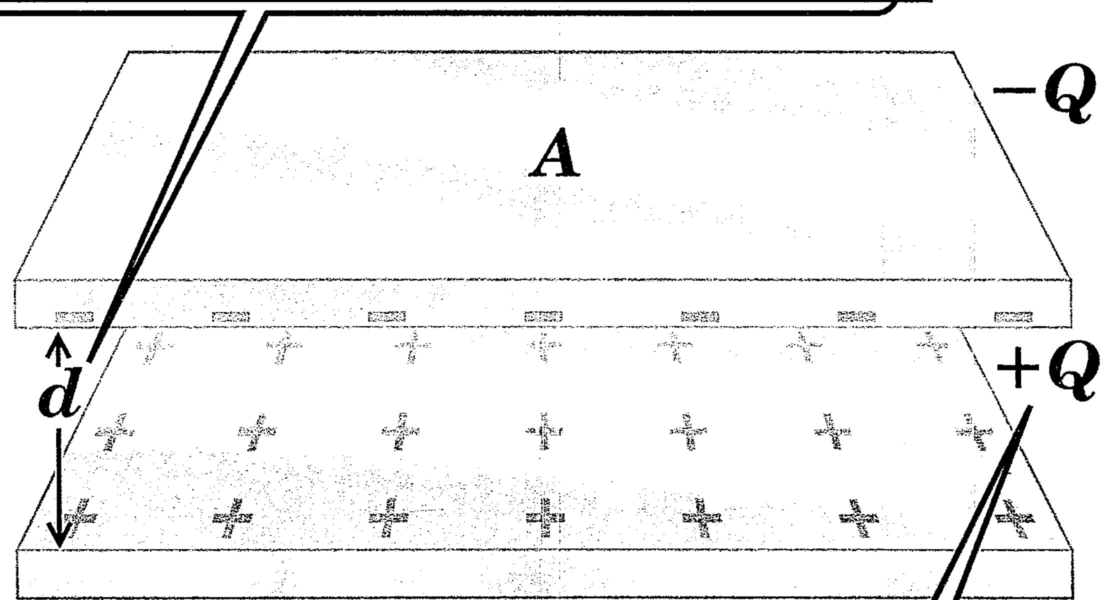

Figure 25-30 Physics for

What is capacitance of above parallel capacitor plates?

$$V = Ed, \quad E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0}, \quad V = \frac{Qd}{A\varepsilon_0}$$

$$C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

If $A = 25cm^2$, $d = 1mm$, $C = \frac{A\varepsilon_0}{d} = \frac{(25\times10^{-4})(10^{-11})}{1\times10^{-3}} = 25\,picofarad$ $= 25\times10^{-12}$ F

In Farads C is very small

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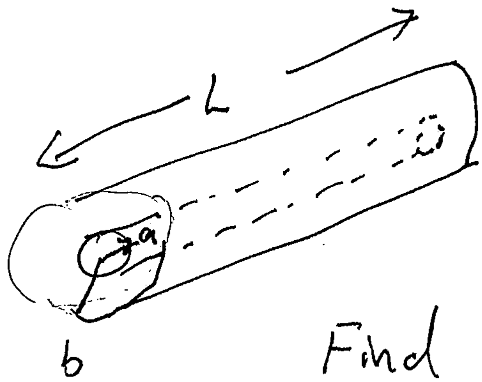

Find Capacitance

$$E \, 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 2\pi r L}$$

$$|V| = \int_a^b \frac{dr}{r} \frac{Q}{(2\pi\epsilon_0 L)} = \frac{Q}{2\pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{|V|} = \frac{2\pi \epsilon_0 L}{\ln(b/a)}$$