

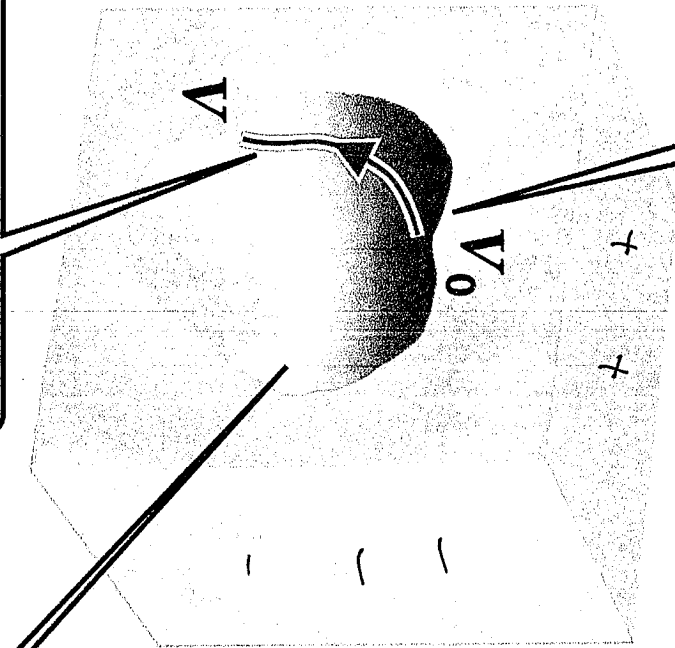
Lecture # 12

Electric Potential

&

Capacitance

For empty cavity, any field line would have to begin and end at surface.



Path parallel to field line would imply a potential difference: impossible!

Electric field is zero everywhere in cavity.

Potential inside metallic cavity is:
(a) 0 (zero charge inside cavity)
(b) V_0
(c) undetermined

Figure 25-19 Physics for Engineers and Scientists 3/e
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Equipotentials of a point charge

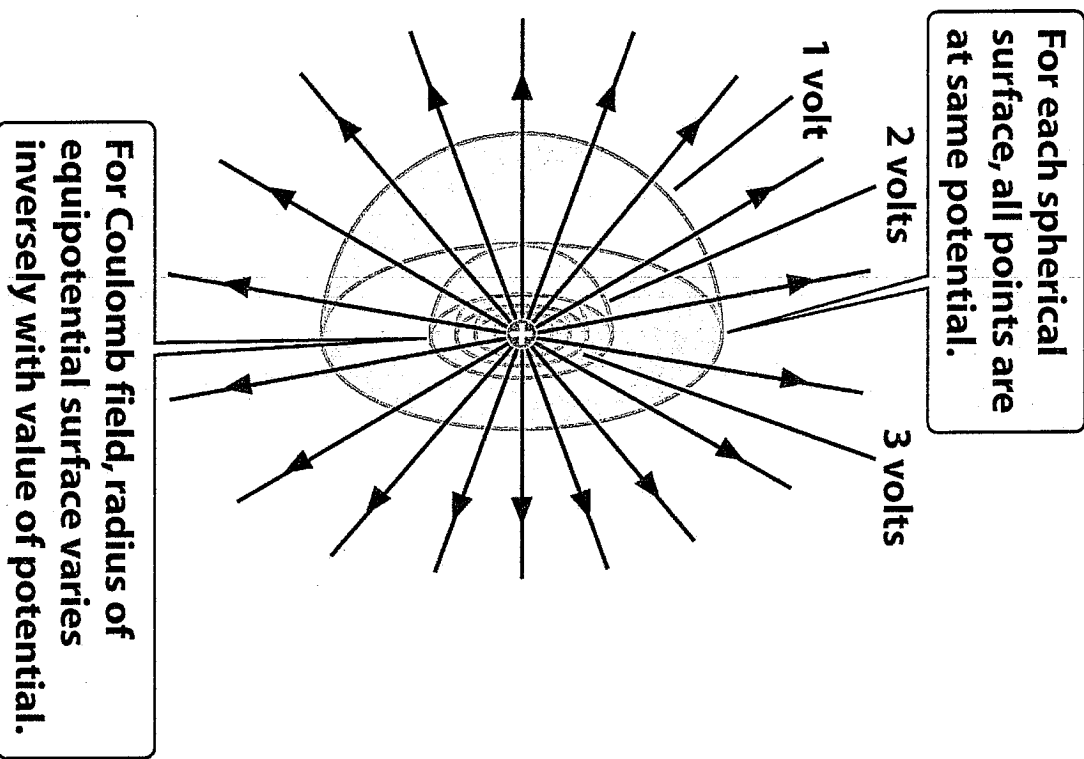
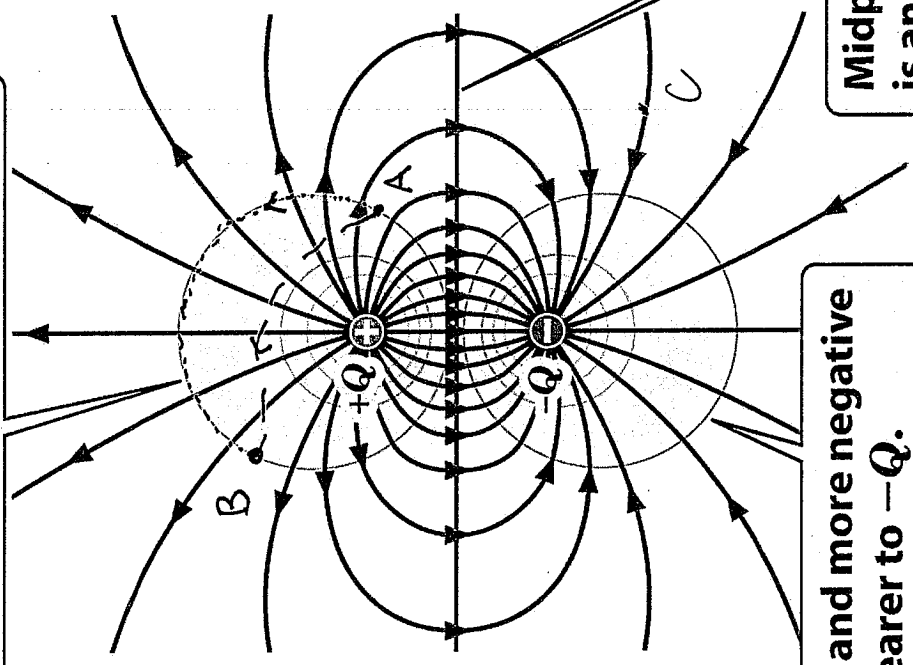


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Equipotential Surface \perp to E -field

Equipotential surfaces become more positive nearer to $+Q$...



...and more negative nearer to $-Q$.

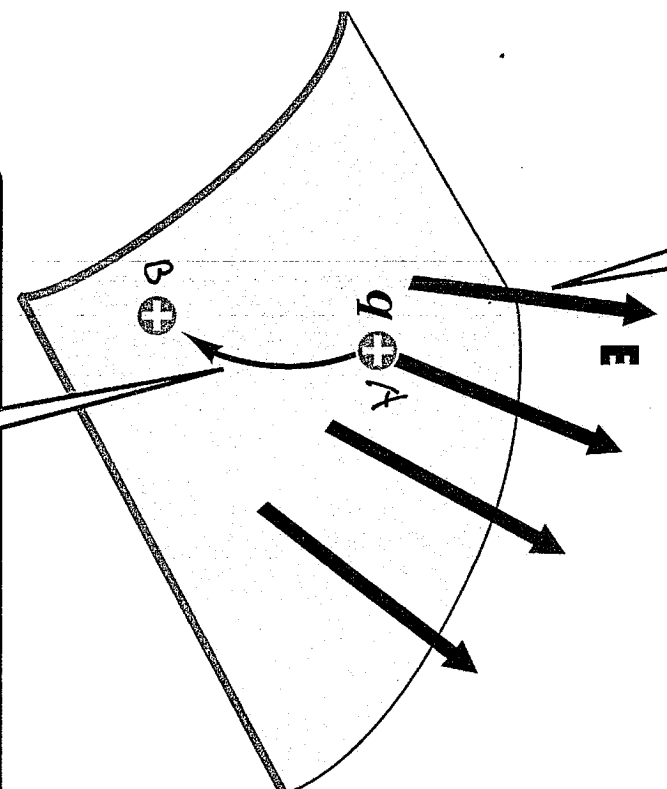
Midplane between charges is an equipotential.

The work it takes to push a charge from A to B along dotted curve, compared to dashed curve is

- (a) zero along dotted curve non-zero along dashed curve
- (b) non-zero along dotted curve, zero along dashed curve
- (c) zero along both curves

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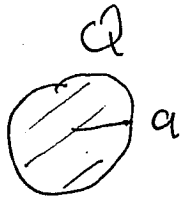
Electric field is everywhere perpendicular to any equipotential surface.



It takes no energy to move a charge along a surface of constant potential!

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Potential of a spherical conductor



$$dV = -\vec{E} \cdot d\vec{r}$$

$$\vec{E} = \begin{cases} \frac{Q}{4\pi\epsilon_0} \frac{\hat{r}}{r^2} & , r > a \\ 0 & , r < a \end{cases}$$

$$dV = -\frac{Q}{4\pi\epsilon_0} \frac{dr}{r^2} \quad \text{if } r > a$$

$$V(r) - V(\infty) = -\frac{Q}{4\pi\epsilon_0} \int_{\infty}^r \frac{dr'}{r'^2}$$

$$= \frac{Q}{4\pi\epsilon_0} \frac{1}{r'} \Big|_{\infty}^r = \frac{Q}{4\pi\epsilon_0 r}$$

$$V(a) = \frac{Q}{4\pi\epsilon_0 a}$$

if $r \leq a$, $dV = -\vec{E} \cdot d\vec{r} = 0$, $\therefore \frac{dV}{dr} = 0$ $\Rightarrow V(r) = \text{Const}$
 $= V(a) = \frac{Q}{4\pi\epsilon_0 a}$

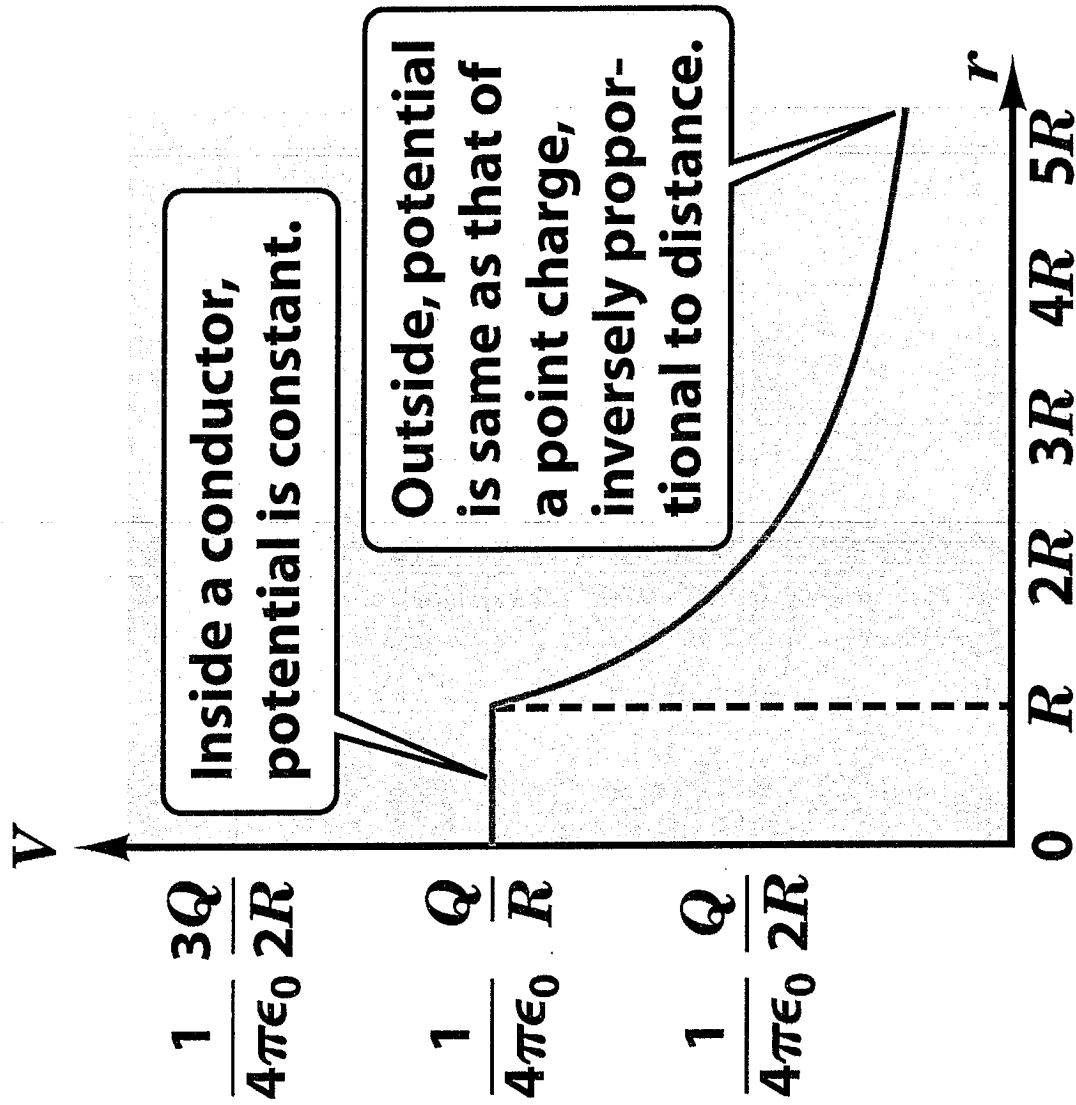


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Potential of a uniformly charged spherical insulator

[find $V(r=0)$]

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}, \quad r > a$$



$$4\pi r^2 E_r = \rho \frac{4\pi r^3}{3\epsilon_0}, \quad r < a$$

$$= \frac{Q}{4\pi\epsilon_0 a^3/3} \quad \frac{4\pi r^3}{3} = \frac{Q r^3}{\epsilon_0 a^3}$$

$$\therefore E_r = \frac{Q r}{4\pi a^3 \epsilon_0} \quad (r < a)$$

$$V(r) = - \int_{\infty}^r E_r dr$$

$$= - \frac{Q}{4\pi\epsilon_0} \int_{\infty}^r \frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0 r} \quad r \rightarrow a$$

$$V(r) = - \frac{Q}{4\pi\epsilon_0} \left[\int_{\infty}^a \frac{dr}{r^2} + \int_a^r \frac{dr r}{a^3} \right]$$

$$= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{a} + \frac{1}{2} \left(\frac{a^2 - r^2}{a^3} \right) \right]$$

$$V(0) = \frac{Q}{4\pi\epsilon_0 a} \left[1 + \frac{1}{2} \right] = \frac{3}{8} \frac{Q}{4\pi\epsilon_0 a}$$

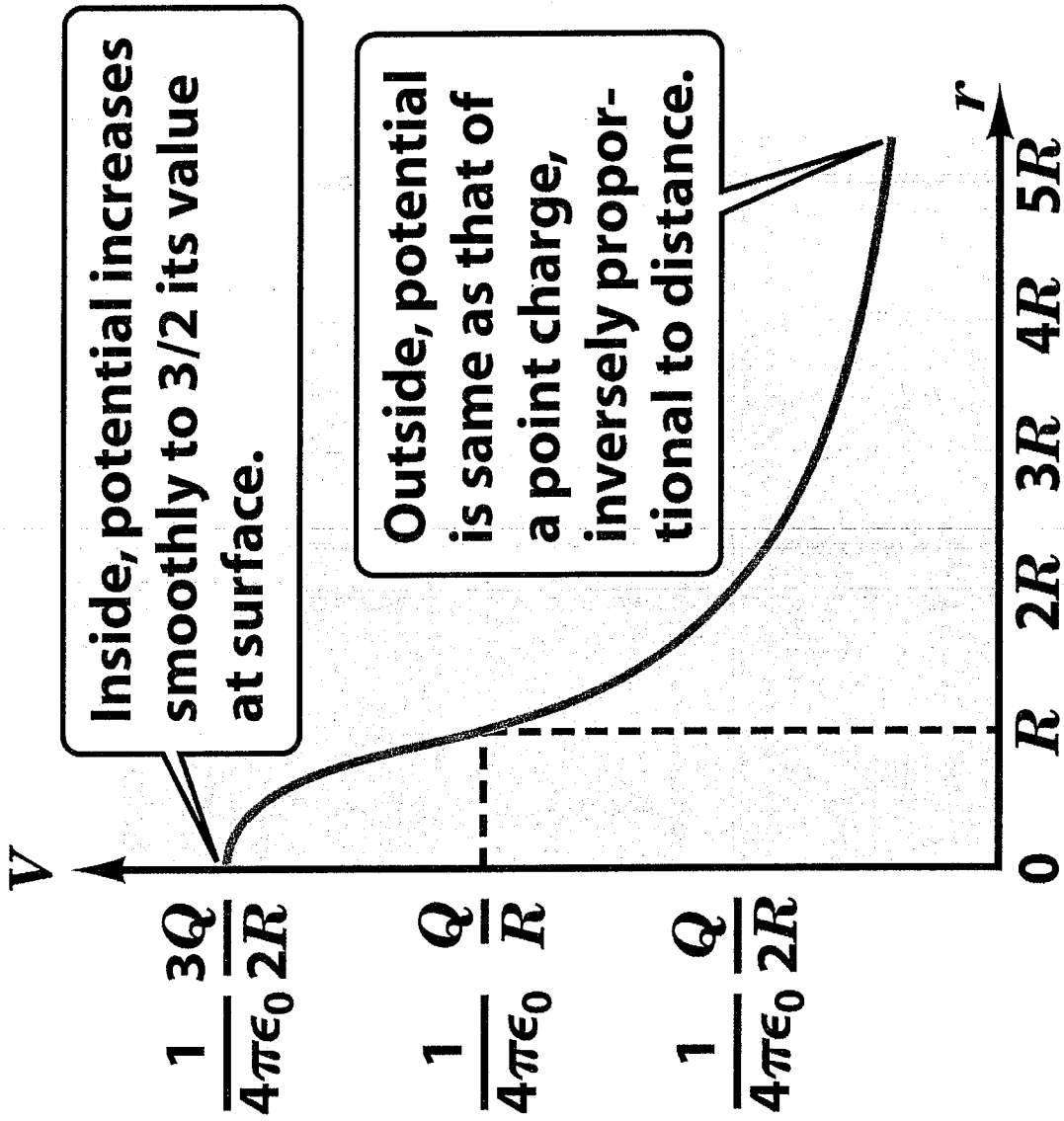


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Find Potential at P if charge Q is uniformly distributed

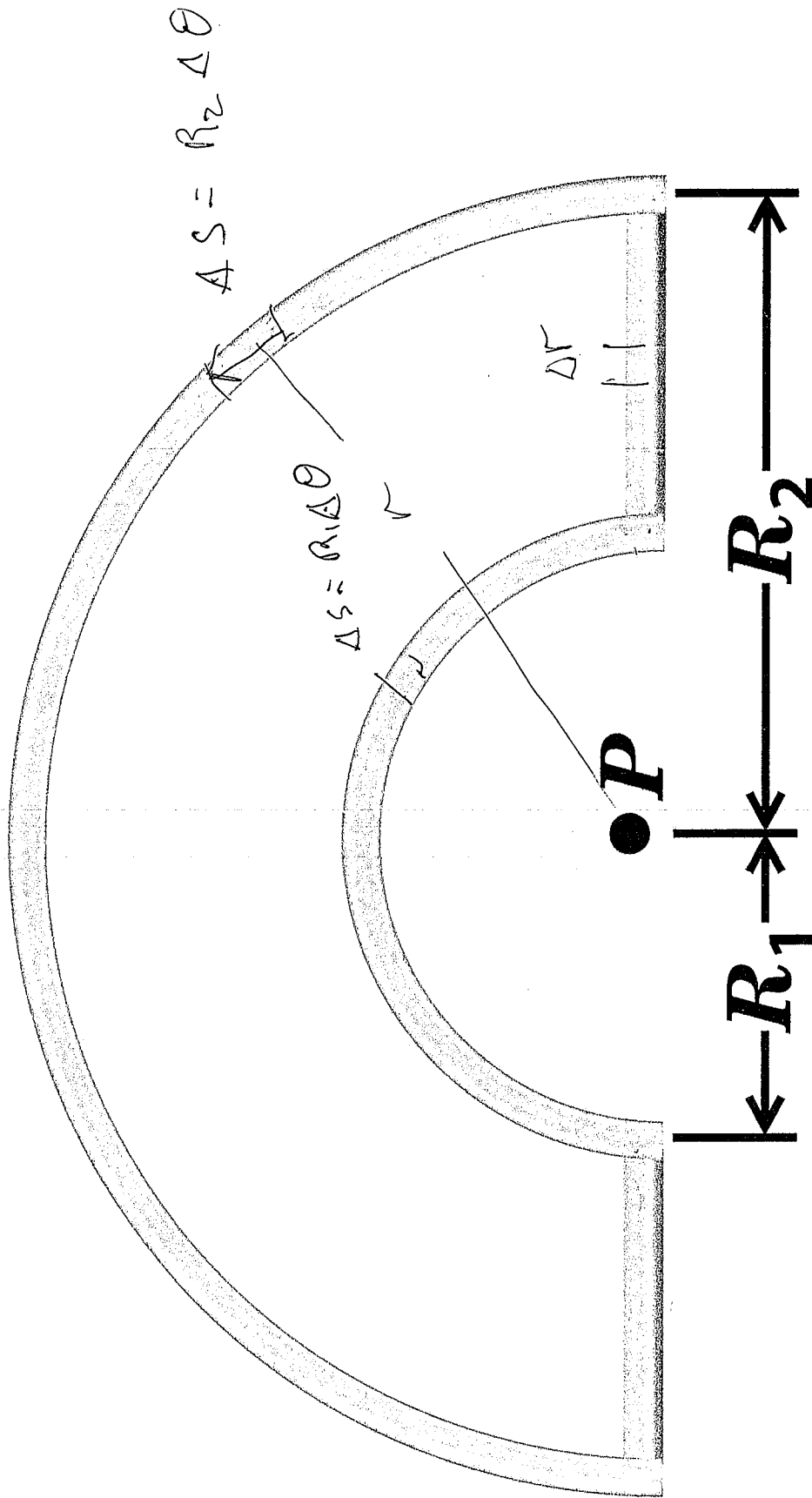


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$$Q = \lambda [\pi (R_2 + R_1) + 2 (R_2 - R_1)]$$

$$dV = \frac{\lambda ds}{4\pi\epsilon_0 r}, \quad V = \frac{\lambda}{4\pi\epsilon_0} \int \frac{ds}{r}$$

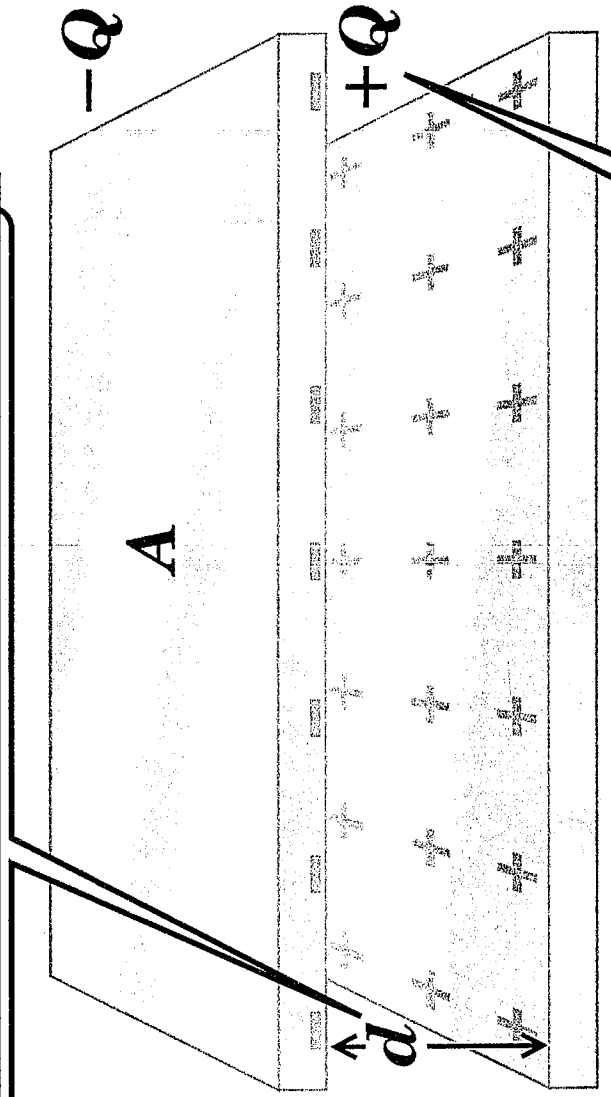
$$V = \frac{\lambda}{4\pi\epsilon_0} \left[\int_0^\pi \frac{R_2 d\theta'}{R_2} + \int_0^\pi \frac{R_1 d\theta}{R_1} + 2 \int_{R_1}^{R_2} \frac{dr'}{r'} \right]$$

$$= \frac{\lambda}{4\pi\epsilon_0} \left[2\pi + 2 \ln \left(\frac{R_2}{R_1} \right) \right]$$

$$V_p = \frac{Q}{4\pi\epsilon_0 [\pi (R_2 + R_1) + 2 (R_2 - R_1)]} \left[2\pi + 2 \ln \left(\frac{R_2}{R_1} \right) \right]$$

Capacitance between conductors:

$$C = Q/V \quad \text{Farads}$$



What is capacitance of above parallel capacitor plates?

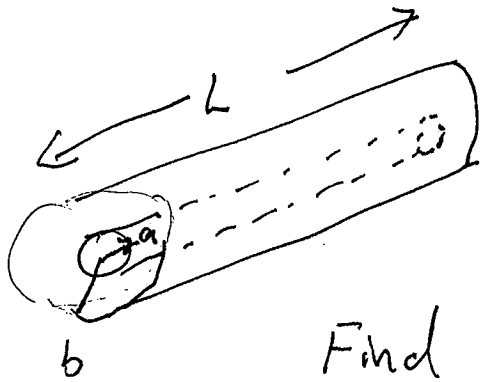
$$V = Ed, \quad E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0}, \quad V = \frac{Qd}{A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{A\epsilon_0}{d}$$

$$\text{If } A = 25\text{cm}^2, \quad d = 1\text{mm}, \quad C = \frac{A\epsilon_0}{d} = \frac{(25 \times 10^{-4})(10^{-11})}{1 \times 10^{-3}} = 25 \text{ picofarad} = 25 \times 10^{-12} \text{ F}$$

In Farads C is very small

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Find Capacitance

$$E \cdot 2\pi r L = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \cdot 2\pi r L}$$

$$|V| = \int_a^b \frac{dr}{r} \frac{Q}{2\pi \epsilon_0 L} = \frac{Q}{2\pi \epsilon_0 L} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{|V|} = \frac{2\pi \epsilon_0 L}{\ln(b/a)}$$