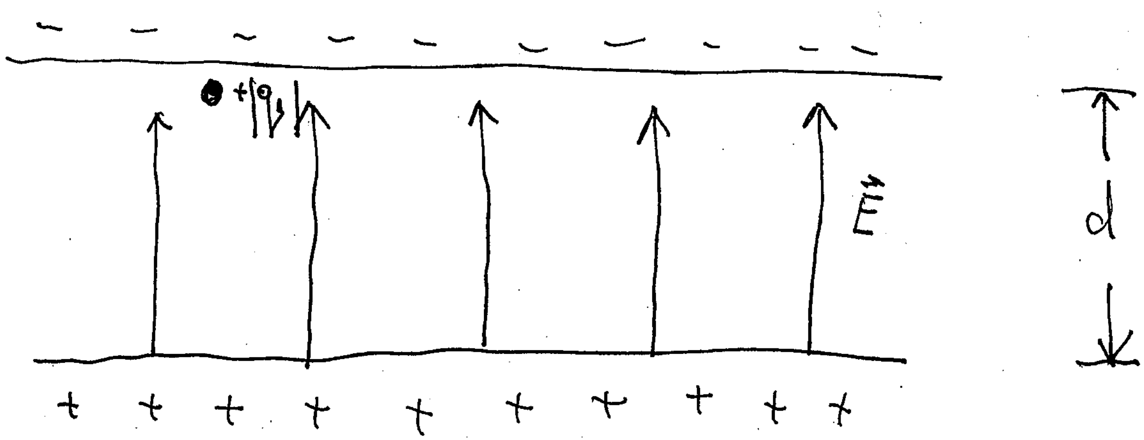
Lecture # 11

Review Electric Potential

Electric Potential

1. Electric Potential Energy, U

It is the minimum external work it takes to push a charge across a specific electric field



To take charge $|q_1|$ across capacitor gap requires minimum external work

$$W = |q_1| E d = U$$

Note minimum work means no change of kinetic energy of q_1

Electric field set up by charge Q: $\vec{E}_Q(r) = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$

Minimum Work it takes to bring charge q_1 from r_1 to r_2 is:

$$W = \int_{r_1}^{r_2} F_{ext}(r')\,dr' = -\int_{r_1}^{r_2} q_1 E_Q(r')\,dr'$$

$$= -\frac{q_1 Q}{4\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr'}{r'^2} = \frac{q_1 Q}{4\pi\epsilon_0}\frac{1}{r'}\Big|_{r_1}^{r_2}$$

$$W = \frac{q_1 Q}{4\pi\epsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Work to take r_1 from infinity ($r_1 = \infty$) to r ($r_2 = r$)

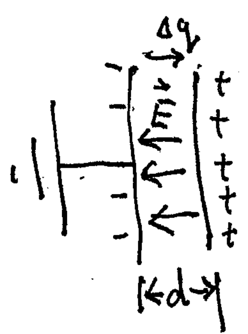
$$W = \frac{q_1 Q}{4\pi\epsilon_0}\frac{1}{r} = U$$

a. Frequently potential energy is measured with potential energy at ∞ set to zero

Potential energy of q is

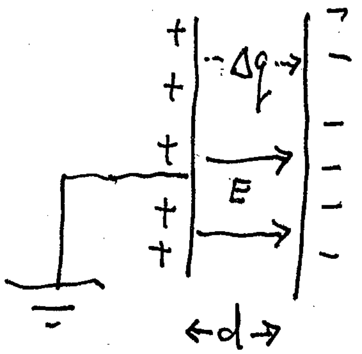
$$U = \frac{qQ}{4\pi\epsilon_0}\frac{1}{r}$$

b. Other-times potential energy is work required to move charge relative to some reference point (ground)



$PE = \Delta q E d$

What is potential energy of Δq relative to ground if Δq taken taken from left to right?



(1) $U = \Delta q E d$

(2) $U = -\Delta q E d$ ✓

(3) $U = 0$

Electric Potential,

Defined as electric potential energy of a particle / charge of particle

$$\boxed{V = U/q}$$

(or electric potential energy per ~~particle~~ unit charge)

Electric potential independent of a charge!

(analog to electric field to electric force)

$$V(x, y, z) = -\vec{E} \cdot d$$

Relation between electric potential and electric field

$$\vec{E} = -\hat{x}\,\frac{\partial V(x,y,z)}{\partial x} - \hat{y}\,\frac{\partial V(x,y,z)}{\partial y} - \hat{z}\,\frac{\partial V(x,y,z)}{\partial z}$$

Gradient

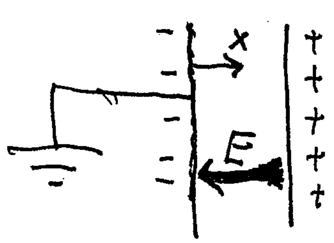
$\frac{\partial}{\partial x}$ = partial derivative with respect to x ; with other variables constant

For example point charge

$$V(\vec{r}) = \frac{Q}{4\pi\epsilon_0}\,\frac{1}{r}$$

$$\vec{E}(r) = -\frac{dV(r)}{dr}\,\hat{r} = \frac{Q}{4\pi\epsilon_0}\,\frac{1}{r^2}\,\hat{r}$$

Potential between two plates



$$V(x) = Ex + C$$

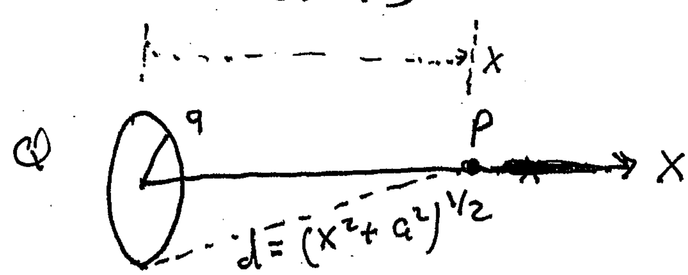
$$\vec{E} = -\frac{dV(x)}{dx} = -E\,\hat{x}$$

$$V(x, y) = V_0 \left[(x-x_0)^2 + (y-y_0)^2 \right]$$

$$\vec{E}(x, y) = -\frac{\partial V}{\partial x}(x, y)\,\hat{x} - \frac{\partial V}{\partial y}(x, y)\,\hat{y}$$

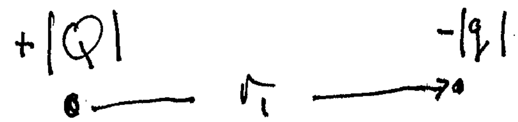
$$= -2V_0 \left[(x-x_0)\,\hat{x} + (y-y_0)\,\hat{y} \right]$$

Electric Field along axis of a ring



$$V(x) = \frac{Q}{4\pi\epsilon_0} \frac{1}{(x^2 + a^2)^{1/2}}$$

$$\vec{E}(x) = -\frac{dV(x)}{dx}\,\hat{x} = \frac{Q}{4\pi\epsilon_0} \frac{x}{(x^2 + a^2)^{3/2}}\,\hat{x}$$



What is the minimum kinetic energy required for charge of mass m_1, at r_1, to escape from charge $+|Q|$

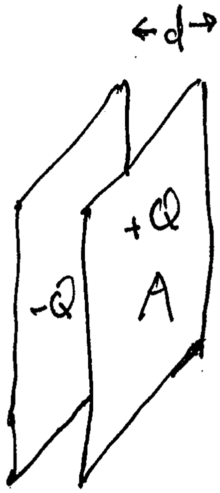
$$E_i = E_f$$

$$\| $$

$$+ m_1 \frac{v^2}{2} - \frac{|q|\,Q}{4\pi\epsilon_0}\frac{1}{r_1} = \underbrace{\frac{m_1 v_f^2}{2}}_{=0} - \underbrace{\frac{|q|\,Q}{4\pi\epsilon_0\, r_f}}_{=0}$$

$$= 0$$

$$KE = \frac{m_1 v_i^2}{2} = \frac{|q|\,Q}{4\pi\epsilon_0\, r_1}$$



What is electric Potential Across plate?

(a) $V = \frac{Q}{\epsilon_0} d$

(b) $V = \frac{Q}{\epsilon_0 A} d$ ✓ $= Ed$

$EA = \frac{Q}{\epsilon_0}$

$E = \frac{Q}{A\epsilon_0}$

(c) $V = \frac{Q}{2\epsilon_0 A} d$

(d) $V = \frac{Q}{2\epsilon_0} d$

How much energy does it take to have charged particle, Δq, to cross plate?

(a) $\Delta q E d$

(b) $\frac{\Delta q Q d}{\epsilon_0 A}$

(c) both of previous ~~are~~ correct ✓

Energy Required to charge Plates

$$\Delta W = \Delta q \, V(q)$$

$$dW = dq \, V = dq \left(\frac{q d}{A \epsilon_0} \right)$$

$$W = \frac{d}{A \epsilon_0} \int_0^{Q} dq \, q = \frac{d}{A \epsilon_0} \frac{q^2}{2} \Big|_0^{Q}$$

$$= \frac{d}{2 A \epsilon_0} Q^2 = \frac{Q V}{2}$$

Capacitance : $C = \dfrac{Q}{V}$

For capacitor plate, $V = \dfrac{Q d}{A \epsilon_0}$

$$C = \frac{Q}{Q d / A \epsilon_0} = \frac{A \epsilon_0}{d} \quad (\text{Farad})$$

Capacitance : Inpendent of charge