Coupling of η_i and Trapped Electron Modes in Plasmas with Negative Magnetic Shear

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Abstract

In toroidal collisionless plasmas, the ion temperature gradient (ITG or η_i) and the trapped electron (TE) modes are shown to be weakly (strongly) coupled when both the temperature gradients and the driving mechanism of the trapped electrons are moderate or strong (weak but finite). In the regime of strong coupling, there is a single hybrid mode, unstable for all η_i in plasmas with positive magnetic shear. For the weak coupling case, two independent unstable modes, one in the ion and the other in the electron diamagnetic direction, are found to coexist. In either situation, the negative magnetic shear exerts a strong stabilizing influence; the stabilizing effect is considerably enhanced by the presence of trapped particles. It is predicted that for a given set of plasma parameters, it will be much harder to simultaneously excite the two modes in a plasma with negative shear. The results of this study are significant for tokamak experiments.

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I. Introduction

Transport of energy, momentum and particles in magnetically confined plasmas is often observed to be much higher than predicted by interparticle collisions. Turbulence induced by plasma instabilities is generally believed to be the cause of this anomaly. However, quantitative theories with both predictive ability and a sound footing in first principles physics are still under development. Recent results from experiments on Tokamak Fusion Test Reactor (TFTR),[1] Joint European Torus (JET),[2] and DIII-D[3] have demonstrated that particle and energy confinements are significantly improved in reversed magnetic shear regions of tokamak plasmas. These experiments have sparked great interests in the investigation of instabilities peculiar to tokamak plasmas with negative magnetic shear.

That negative shear could exert stabilizing influence on ideal ballooning modes was known but not appreciated in the early work of Greene and Chance.[4] Around the same time, there were indications in Kadomtsev and Pogutse[5] that negative shear tends to stabilize non-ideal microinstabilities. Kessel *et al.*[6] demonstrated in a recent work that the reversed shear, indeed, has a surprisingly strong stabilizing influence on these non-ideal instabilities. In addition, it is found[6] that the ion temperature gradient (ITG or η_i) modes are stabilized in a portion of the negative shear region for a proposed discharge with optimized plasma density, current and temperature profiles. A physical picture showing the effect of negative shear on curvature driven instabilities is given by Antonsen, Jr. *et al.*[7] and the influence of the negative shear on resistive ballooning modes has been recently studied by Drake *et al.*[8]

Motivated by the experiments mentioned above and by the common belief[9,10] that the ITG instability is likely to be the dominant mechanism for anomalous transport in tokamak plasmas, a systematic investigation of the ITG and PVS (parallel velocity shear) driven modes has been performed in plasmas with negative magnetic shear.[11] It was demonstrated

that in toroidal geometry, when the geodesic curvature drift of the ions is taken into consideration, the negatively sheared plasmas show not only lower growth rates but also higher thresholds (for the onset of the ITG instability) as compared to plasmas with positive shear. This should certainly contribute to the processes which cause the confinement improvement in negative shear regions. However, our theoretical estimates reveal that the differences between the strengths of the instabilities for the normal and reversed shears (for ITG) may not be sufficient to account for so dramatic an improvement in confinement. Additional mechanisms have to be invoked to understand the physics of confinement and of such severe transport reduction.

The search for instabilities (or lack thereof) must also contend with an extremely important experimental finding that the spectrum of density fluctuations in tokamaks has two distinct wings: one of these rotates in the ion diamagnetic drift direction while the other does it in the opposite direction of the electron drifts. These two spectra are detected simultaneously or separately in tokamak plasmas, depending on the discharge conditions and plasma parameters.[12,13] Definitive theoretical explanations for this important experimental result are quite rare.

In this paper we make an attempt to look for possible answers to the questions raised by earlier as well as recent experiments. For this purpose, we deal with a model of a collisionless toroidal plasmas with negative magnetic shear which includes both the ITG and the TE (trapped electron) modes. A recently developed comprehensive gyrokinetic dispersion equation[14] used for the study of low frequency drift-like instabilities is now extended to include the trapped particle dynamics. The conditions for the existence of these two instabilities, simultaneously or separately, are discussed, and their mutual coupling and interaction is strongly emphasized. The results are compared with similar results in plasmas with positive magnetic shear. The theoretical results are also compared with the experiments on confinement improvement, and on microturbulence spectra.

The contents of this work are organized as follows. In Sec. II, the integral dispersion equation is displayed. In Sec. III, the additional contributions from the trapped particle dynamics to the dispersion equation are described. The numerical results and comparisons with experiments are presented in Sec. IV, and Sec. V is devoted to conclusions.

II. Integral Dispersion Equation

We begin this section with a brief description of the gyrokinetic integral equation[14] derived for the study of low frequency drift modes, such as the ITG mode. An impurity species is included and the curvature and magnetic gradient effects $\omega_D(v_\perp^2, v_\parallel^2, \theta)$ of both the hydrogenic and the impurity ions are retained. The ballooning representation is used so that the linear mode coupling due to the two dimensional character of the tokamak magnetic field is taken into account. The full ion transit $k_\parallel v_\parallel$ and the finite Larmor radius effects are retained. For simplicity, the passing electron response is assumed to be adiabatic. The response of the trapped particles will be discussed in the next section. The integral dispersion equation derived in Ref. 14 is easily written (after having been extended to include the second ion species) as

$$[1 + \tau_i(1 - f_z) + \tau_z Z f_z]\, \widehat{\phi}(k) = \int_{-\infty}^{+\infty} \frac{dk'}{\sqrt{2\pi}} K(k, k') \widehat{\phi}(k'), \tag{1}$$

where

$$K(k, k') = -i \int_{-\infty}^{0} \omega_{*e} d\tau \, \sqrt{2}\, e^{-i\omega\tau} \Bigg[(1 - f_z) \frac{\exp\left[-\frac{(k'-k)^2}{4\lambda}\right]}{\sqrt{a}\,(1+a)\sqrt{\lambda}}$$

$$\times \Bigg\{ \frac{\omega}{\omega_{*e}} \tau_i + L_{ei} - \frac{3}{2} \eta_i L_{ei} + \frac{2\eta_i L_{ei}}{(1+a)}$$

$$\times \left[1 - \frac{k_\perp^2 + k'^2}{2(1+a)\tau_i} + \frac{k_\perp\, k'_\perp}{(1+a)\tau_i} \frac{I_1}{I_0} \right] + \frac{\eta_i L_{ei} (k - k')^2}{4a\lambda} \Bigg\} \Gamma_0(k_\perp, k'_\perp) +$$

$$+ f_z \frac{\exp\left[-\frac{(k'-k)^2}{4\lambda_z}\right]}{\sqrt{a}_z\,(1+a_z)\sqrt{\lambda}_z} \Bigg\{ \frac{\omega}{\omega_{*e}} Z \tau_z + L_{ez} - \frac{3}{2} \eta_z L_{ez} + \frac{2\eta_z L_{ez}}{(1+a_z)}$$

$$\times \left[1 - \frac{(k_\perp^2 + k'^2)\mu}{2(1+a_z)Z^2\tau_z} + \frac{k_\perp\, k'_\perp \mu}{(1+a_z)Z^2\tau_z}\,\frac{I_{1z}}{I_{0z}}\right] + \frac{\eta_z L_{ez}(k-k')^2}{4a_z\lambda_z}\Bigg\}\Gamma_{0z}(k_\perp, k'_\perp)\Bigg] \tag{2}$$

with

$$\lambda = \frac{\tau^2\omega_{*e}^2}{\tau_i a}\left(\frac{\widehat{s}}{q}\epsilon_n\right)^2, \qquad \lambda_z = \frac{\tau^2\omega_{*e}^2}{\tau_z a_z \mu}\left(\frac{\widehat{s}}{q}\epsilon_n\right)^2,$$

$$a = 1 + \frac{i2\epsilon_n}{\tau_i}\omega_{*e}\tau\left(\frac{(\widehat{s}+1)(\sin\theta - \sin\theta') - \widehat{s}(\theta\cos\theta - \theta'\cos\theta')}{(\theta - \theta')}\right),$$

$$a_z = 1 + \frac{i2\epsilon_n}{\tau_z Z}\omega_{*e}\tau\left(\frac{(\widehat{s}+1)(\sin\theta - \sin\theta') - \widehat{s}(\theta\cos\theta - \theta'\cos\theta')}{(\theta - \theta')}\right),$$

$$\theta = \frac{k}{\widehat{s}k_\theta}, \quad \theta' = \frac{k'}{\widehat{s}k_\theta},$$

$$\Gamma_0 = I_0\left(\frac{k_\perp\, k'_\perp}{(1+a)\tau_i}\right)\exp\left[-(k_\perp^2 + k_\perp'^2)/2\tau_i(1+a)\right]$$

$$\Gamma_{0z} = I_0\left(\frac{k_\perp\, k'_\perp \mu}{(1+a_z)\tau_z Z^2}\right)\exp\left[-(k_\perp^2 + k_\perp'^2)\mu/2Z^2\tau_z(1+a_z)\right],$$

$$k_\perp^2 = k_\theta^2 + k^2, \qquad k_\perp'^2 = k_\theta^2 + k'^2,$$

$$\epsilon_n = \frac{L_{ne}}{R}, \quad \eta_i = \frac{L_{ni}}{L_{Ti}}, \quad \eta_z = \frac{L_{nz}}{L_{Tz}}, \quad \tau_i = \frac{T_e}{T_i}, \quad \tau_z = \frac{T_e}{T_z},$$

$$f_z = \frac{Zn_{0z}}{n_{0e}}, \quad \mu = \frac{m_z}{m_i}, \quad L_{ei} = \frac{L_{ne}}{L_{ni}}, \quad L_{ez} = \frac{L_{ne}}{L_{nz}}.$$

The quantities k, k' and k_θ are normalized to $\rho_i^{-1} = \Omega_i/v_{ti} = eB/c\sqrt{2T_i m_i}$, x is normalized to ρ_i, and $I_j(j = 0, 1)$ is the modified Bessel function of order j. The symbols with the subscript "i" (or without any subscript) stand for the primary ion species (hydrogenic ions) while those with "z" and "e" stand respectively for the second ion species and the electrons.

All other symbols have their usual meanings such as the L_n's are the density scale lengths, n_0's are the unperturbed densities, L_T's are the temperature scale lengths, q is the safety factor, $\widehat{s} = rdq/qdr$ is the magnetic shear and $\omega_{*e} = ck_\theta T_e/eBL_{ne}$ is the electron diamagnetic drift frequency, Z is the charge number of impurity ions, m's and T's are the species' mass and temperature, respectively. The derivation and details of this equation are given in Ref. 14 and will not be repeated here.

It must be mentioned that not all of the above mentioned parameters are independent. For example, the quasi-neutrality condition imposes the relation

$$L_{ei} = \frac{1 - f_z L_{ez}}{1 - f_z}, \tag{3}$$

implying

$$\eta_z = \frac{\eta_i(\frac{L_{nz}}{L_{ne}} - f_z)}{1 - f_z}, \tag{4}$$

under the assumption $T_i(r) = T_z(r)$.

III. Contribution from Trapped Electrons

The calculation for the trapped electrons is straightforward and well documented[5,15–18] when the finite gyroradius effects are neglected. In the ballooning representation, the perturbation of the trapped electron density can be written as

$$\widetilde{n}_{et} = -\frac{en_e}{T_e}\sqrt{\frac{2\epsilon}{\pi}}\int_0^\infty dt\sqrt{t}e^{-t}\int_0^1 \frac{\omega - \omega_*^e}{\omega - \overline{\omega}_d^e}\cdot\frac{d\kappa^2}{4F(\kappa)}\sum_{j=-\infty}^{+\infty} g(\theta - 2\pi j, \kappa)\int_{-\infty}^{+\infty} d\theta' g(\theta', \kappa)\phi(\theta' - 2\pi j) \tag{5}$$

where

$$g(\eta,\ \kappa) = \int_{-\theta_r}^{\theta_r} \frac{\delta(\eta - \theta')d\theta'}{\sqrt{\kappa^2 - \sin^2\frac{\theta'}{2}}},$$

and

$$\kappa^2 = \sin^2\frac{\theta_r}{2},$$

$$\epsilon = \frac{r}{R},$$

$$\overline{\omega}_d^e = \omega_{*e}\epsilon_n t G(\widehat{s}, \kappa) = \omega_{*e}\epsilon_n t\left[\frac{2F(\kappa)}{K(\kappa)} - 1 + 4\widehat{s}\left(\frac{F(\kappa)}{K(\kappa)} - (1 - \kappa^2)\right)\right],$$

$$\omega_*^e = \omega_{*e}\left[1 + \eta_e\left(t - \frac{3}{2}\right)\right],$$

$$\eta_e = \frac{L_{ne}}{L_{Te}},$$

$\delta(x)$ is Dirac's delta function, r is the minor radius associated with the rational surface, R is the major radius of the plasma column, and $K(\kappa)$ ($F(\kappa)$) is the complete elliptic integral of the first (second) kind.

IV. Numerical Results

After adding the trapped electron contribution [Eq. (5)] to Eq. (1), we numerically solve the eigenvalue problem. Special attention must be paid to the logarithmic singularity at $\tau = 0$ when $k = k'$.[14] The numerical methods for solving Fredholm integral equations of the second kind are quite standard and well documented.[14]

A. η_i threshold in the presence of trapped electrons

We first neglect the complicating effects of the temperature gradient of the trapped electrons, $\eta_e = 0$ in our study of the ITG modes in the presence of trapped electrons. The normalized growth rate and real frequency versus η_i are plotted in Fig. 1. The other parameters are $q = 1.5$, $\epsilon_n = 0.05$, $\tau = 1$, $f_z = 0$ and $\epsilon = 0.1$. The solid and the dashed lines are for $\widehat{s} = -1.5$ and $\widehat{s} = 1.5$, respectively. For the positive shear $\widehat{s} = 1.5$ (the dashed lines), there is no threshold η_i value for the ITG mode to be unstable: the mode, referred to as the trapped electron-η_i [17] mode, is unstable even for $\eta_i = 0$. The rotation direction of the mode changes from the ion direction to that of the electrons when η_i decreases. This has the implication that when η_i decreases, an effective transformation from an ITG to a predominantly TE mode takes place.

These results are similar to what has been previously reported.[17]

For the negative shear case (for the same plasma parameters including the magnitude of the magnetic shear), the instability threshold is around $\eta_i \sim 1$. For lower values of η_i, the stabilizing effect of the negative shear overcomes the combined driving forces of the TE and ITG, and the mode is stabilized completely. In the still unstable region, i.e., $\eta_i \gtrsim 1$, the

negative shear brings down the growth rate to less than a half of its value for the positive shear. The stabilizing effects of negative shear are clearly shown in Fig. 1.

The situation changes if we switch on a constant temperature gradient η_e for the trapped electrons and then study the mode stability as a function of η_i. Graphs in Fig. 2 pertain to the same plasma parameters as in Fig. 1 except that now, we have $\eta_e = 2$ and $k_\theta \rho_i = 0.3$. As long as one is concerned with the ITG mode alone, the temperature gradient of the trapped electrons has a strong stabilizing effect for $\eta_i \leq 1$. The mode remains stable up to an $\eta_{ic} \sim 1$ even for positive magnetic shear ($\widehat{s} = 1.5$). This is to be contrasted with the results displayed in Fig. 1 ($\eta_e = 0$) for which there exists no η_i threshold. A finite trapped electron η_e leads to similar consequences for negative magnetic shear: the growth rate is significantly decreased to less than a half of that for $\eta_e = 0$, and the threshold value η_{ic} is dramatically increased from $\eta_{ic} \sim 1$ when $\eta_e = 0$ to $\eta_{ic} \sim 2$ when $\eta_e = 2$.

Again, the growth rate of the ITG mode in plasmas with negative shear is less than half of that with positive shear when $\eta_e = 2$ and $\eta_i > \eta_{ic}$. Comparing with the results obtained previously,[6] it may be concluded that the effects of the sign of magnetic shear are much more significant when the trapped electron dynamics is taken into account than it is when it is neglected.

An important consequence of the finite trapped electron η_e, in addition to enhancing the driving mechanism of the TE mode, is to push its real frequency away from that of the ITG mode decoupling it from the latter. This issue will be discussed in the next subsection in detail.

B. Coexistence of ITG and TE modes

The possibility for the co-existence of ITG and TE modes in collisionless toroidal plasmas is explored in this subsection. For these studies, the parameter ϵ, determining the ratio of the trapped electron, is increased to 0.3, and fully ionized carbon ions are added as an

impurity. The relevant parameters are: $f_z = 0.3$, $k_\theta \rho_i = 0.5$, $q = 3$, $\epsilon_n = 0.1$, $\tau = 1$, $\eta_z = \eta_i$, $L_{ei} = L_{ez} = 1$ and $T_i = T_z$. The results are shown in Fig. 3 (4) with magnetic shear $\widehat{s} = -1(1)$.

Let us first analyse the results for $\eta_e = 0$ in Figs. 3 and 4 (the lines with squares). The results in Fig. 4 with positive magnetic shear $\widehat{s} = 1$ is similar to what is shown in Fig. 1 for $\widehat{s} = 1.5$ (the dashed lines in Fig. 1). The so-called trapped electron-η_i mode is unstable even for $\eta_i = 0$ in both these cases. However, the behaviour of the mode growth rate versus η_i for negative shear $\widehat{s} = -1$, displayed respectively in Fig. 3 (the line with squares) and 1 (the solid line with circles) is quite different. Now, the mode is unstable for the negative shear ($\widehat{s} = -1$) even when η_i=0, unlike its counterpart in Fig. 1 where the mode is stable for $\eta_i \lesssim \eta_{ic} \sim 1$. This dramatic change is due to the competition between the strong stabilizing effect of the negative shear and the driving mechanism due to the trapped electron dynamics represented by the parameter ϵ. The suppression overcomes the comparatively weaker ($\epsilon = 0.1$) driving force and the mode becomes stable for $\eta_i \leq 0.8$ for the Fig. 1 example. In contrast, in the Fig. 3 example, the strong TE drive ($\epsilon = 0.3$) assures instability even for no ($\eta_i = 0$) ion gradient drive. Nevertheless, the mode growth rates for $\widehat{s} = -1$ are much lower than they are for a similar plasma with $\eta_i = 1$. For example, when $\eta_i = 0$, the growth rate for $\widehat{s} = -1$ (Fig. 3) is less than a half of that for $\widehat{s} = 1$ (Fig. 4).

Next, let us consider the cases when $\eta_e \neq 0$. We shall study both the ITG and the TE modes simultaneously. For $\eta_e = 2$, it is clearly demonstrated in Figs. 3 and 4 that the ITG (the curves with open circles) and the TE (the curves with closed circles) modes do not couple, and evolve independently as η_i varies. There are two modes co-existing in a narrow η_i interval, $1.5 \lesssim \eta_i \lesssim 1.75$ for $\widehat{s} = -1$, and $1.3 \lesssim \eta_i \lesssim 1.9$ for $\widehat{s} = 1$, respectively. It is also clear from the data [Figs. 3(b) and 4(b)] that with increasing η_e, the difference between the real frequency of the TE and the ITG mode becomes larger. This surely must contribute to the decoupling between the two instabilities near the η_i threshold.

We now make a small digression to remind the reader of an important experimental result: the observed density fluctuation spectrum shifts from the electron to the ion direction when the plasma parameters change.[13] Depending on the parameters, either a single wing unidirectional or a double winged bi-directional (with both the ion and the electron features are present simultaneously) spectrum is detected.[12] It is quite obvious that in the theoretical results obtained in this work, there could lie a possible explanation for these experimental observations.

The intervals where the ITG and TE modes coexist (both of them are unstable simultaneously) pertaining to the cases shown in Figs. 3 and 4 are too narrow to warrant comparison with experiments. After a careful parametric search, we have uncovered a much broader domain for their simultaneous existence. For simplicity, we show in Fig. 5 the results for a positive magnetic shear $\widehat{s} = 1.5$. The other parameters are $\eta_e = 2,\ q = 1.5,\ \tau = 1,\ k_\theta \rho_i = 0.3,\ \epsilon_n = 0.1$ and $f_z = 0$. The study is done for two values of the trapped electron parameter $\epsilon = 0.1$ (the dashed lines) and $\epsilon = 0.3$ (the solid lines). It is clear that the co-existing intervals are $1 \lesssim \eta_i \lesssim 2.5$ and $1 \lesssim \eta_i \lesssim 2$ for $\epsilon = 0.3$ and $\epsilon = 0.1$, respectively. The η_i values in these intervals are reasonable and common for tokamak plasmas. The scaling for the driving mechanism from the trapped electrons for the TE mode (the lines with squares) clearly emerges from Fig. 5(a); the growth rate for $\epsilon = 0.3$ is twice as high as that for $\epsilon = 0.1$. One can also infer from the figures that the trapped electrons have a marginal effect on the ITG mode. The ion temperature gradient (η_i), on the other hand, has a very strong stabilizing effect on the TE mode when $\eta_i \gtrsim 2$.

C. Spectrum study

The spectra of the ITG and TE modes are given in Fig. 6 for $\eta_i = 2,\ \widehat{s} = 1.5,\ \eta_e = 2,\ q = 1.5,\ \epsilon_n = 0.1,\ \tau = 1,\ \epsilon = 0.3,\ f_z = 0$. The lines with squares and circles are for the ITG and TE modes, respectively. The solid and dashed lines are respectively for the

real frequency and the growth rate. The maximum growth rate of the kinetic ITG mode in toroidal plasmas occurs at $k_\theta \rho_i \simeq 0.8$ when the trapped electron dynamics is neglected[14]; it is pushed to a higher value by the trapped electron effects. The growth rate of the ITG mode is always higher than the TE mode for all wavelengths and the plasma parameters considered in this study.

It is now worthwhile to examine the relevance of this effort to experimental observations. We begin by noting that the experimental frequencies are measured in the laboratory frame while the theoretical frequencies are calculated in the plasma frame. There may exist, for example, a Doppler shift between the two frequency spectra caused by a plasma rotation in the poloidal direction induced by a radial electric field. The calculation of the Doppler shift is rather troublesome due to the uncertainties in the measurement of the electric field. Fortunately, it is still possible to compare theory with experiment by dealing with frequency shifts between two typical frequencies rather than the frequencies themselves.

The first experimental observation of interest is the appearance, in the microturbulence spectrum, of a new peak[12] at positive frequency (in the ion diamagnetic drift direction) besides the peak in the electron direction when the plasma density increases. The frequency shift between these two peaks may be compared with the theoretical results given in Fig. 6(b) as follows. The experimental results (see Fig. 1 of Ref. 12) show that the shift between the mean frequencies of the two peaks (in the density fluctuation spectra) increases with the increase in the wavenumber. The results presented in Fig. 6(b), also show that the frequency shift between the ITG and TE modes increases with the wave number of the fluctuations. This similarity in scaling may suggest that the two-peak experimental feature may be viewed as the simultaneous existence of suprathermal levels of the ITG and the TE modes.

Naturally, a detailed or a quantitative comparison with experiments is beyond the scope of the present work. Nevertheless, a rough comparison may provide a stimulant for further studies. The typical measured fluctuation spectrum is in the 100 kHz range and the frequency

difference between the ion and the electron features is of the same order. It is shown in Fig. 6 that the theoretically calculated difference between the frequencies associated with the two instabilities is comparable to the electron diamagnetic drift frequency ω_{*e}. The electron drift frequency measured in the high density experiment is $\omega_{*e}/2\pi \sim 2 \times 10^5$ Hz.[12] The agreement between the theoretical calculations and the experimental observations is reasonably good.

Similar spectra for the negative magnetic shear, $\widehat{s} = -1.5$, are shown in Fig. 7. Here, the ITG mode is studied with $\eta_i = 2.5$ while the TE mode is investigated with $\eta_i = 0.75$. These η_i values are so chosen that both the TE and the ITG mode are optimally unstable. It should be noted that the TE mode becomes almost stable when $\eta_i = 2.5$ and so does the ITG when $\eta_i = 0.75$. In fact, for negative shear plasmas, it is quite difficult to find a broad enough parameter domain where both the TE and ITG are unstable. This shows, once again and from another point of view, that plasmas with negative magnetic shear are more stable than those with positive shear with respect to such micro-instabilities as ITG and TE modes.

V. Summary and Conclusions

The gyrokinetic integral eigenvalue equation for the study of ITG modes in collisionless toroidal plasmas is extended to include the trapped electron dynamics. The study concentrates on delineating the following novel features: 1) the interactions and couplings between the standard TE and ITG instabilities, 2) the effect of negative magnetic shear on the stability of these modes. It is found that these two instabilities are strongly coupled when the temperature gradients as well as the driving mechanism of the trapped electrons are weak but not negligibly small. In this case, it is impossible to distinguish between them, and only one unstable mode, conveniently called the trapped electron-η_i mode is found to exist. In this regime, there is no η_i instability threshold for plasmas with positive magnetic shear. For negative shear, the mode is stable up to $\eta_{ic} \sim 1$, and for larger values of η_i, the growth rates

are considerably reduced.

When both the temperature gradient and the driving mechanism of the trapped electrons are moderate or strong, the coupling between these two instabilities is weak and they may coexist in plasmas. In this case, the threshold value for the ITG instability is $\eta_i \sim 1$ ($\eta_i \sim 2$) for $\widehat{s} = 1(-1)$. In addition, the driving mechanism of the TE mode is enhanced by the trapped electron temperature gradient for $\eta_i \lesssim 1$, and is suppressed by ITG when $\eta_i \gtrsim 2$. However, the effects of the trapped electrons on the ITG mode are negligibly small when $\eta_i > \eta_{ic}$.

For the same set of plasma parameters (including the magnitude of the shear), the growth rates of the ITG and TE modes in plasmas with positive magnetic shear are twice as high as the growth rates in plasmas with negative shear. The stabilizing effects of the negative shear are definitively shown to be much more significant when the TE dynamics is taken into account.

We believe that it is the first time that the coexistence of an ion (the ITG), and an electron mode (TE mode) in realistic collisionless toroidal plasmas is theoretically demonstrated. This highly desirable result should lend considerable experimental significance to our model. Finally, we make an important prediction: the possibility of a simultaneous detection of the ion and electron features in the density fluctuations in plasmas with negative magnetic shear is much lower than it is in plasmas in which the shear is positive.

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Figure Captions

1. (a) Mode growth rate and (b) real frequency versus η_i for $\eta_e = 0,\ q = 1.5,\ \epsilon_n = 0.05,\ \tau = 1,\ f_z = 0$ and $\epsilon = 0.1$. The solid and dashed lines are for $\widehat{s} = -1.5$ and $\widehat{s} = 1.5$, respectively.

2. The same as Fig. 1 except that $\eta_e = 2$ and $k_\theta \rho_i = 0.3$.

3. (a) Mode growth rate and (b) real frequency versus η_i for $k_\theta \rho_i = 0.5,\ q = 3,\ \epsilon_n = 0.1,\ \tau = 1,\ \epsilon = 0.3,\ f_z = 0.3,\ \eta_z = \eta_i,\ L_{ei} = L_{ez} = 1$, and carbon fully ionized as impurity. The magnetic shear $\widehat{s} = -1$.

4. The same as Fig. 3 except that $\widehat{s} = 1$.

5. (a) Mode growth rate and (b) real frequency versus η_i showing the co-existence of the ITG and TE modes. The parameters are $\eta_e = 2,\ \widehat{s} = 1.5,\ q = 1.5,\ \tau = 1,\ k_\theta \rho_i = 0.3,\ \epsilon_n = 0.1$ and $f_z = 0$. The squares and the circles are for the TE and ITG modes, respectively.

6. The spectra of TE (the lines with circles) and ITG (the lines with squares) modes for $\eta_i = 2,\ \widehat{s} = 1.5,\ \eta_e = 2,\ q = 1.5,\ \epsilon_n = 0.1,\ \tau = 1,\ \epsilon = 0.3,\ f_z = 0$. The solid (dashed) lines are for the real frequencies (growth rates).

7. The same as Fig. 6 except that $\widehat{s} = -1.5$, and $\eta_i = 0.75$ (TE mode) and 2.5 (ITG mode), respectively.