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COLLECTIVE ION ACCELERATION BY A REFLEXING ELECTRON BEAM: MODEL AND SCALING

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ABSTRACT

Analytical and numerical calculations are presented for a reflexing electron beam type of collective ion accelerator. These results are then compared to those obtained through experiment. By constraining one free parameter to experimental conditions, the self-similar solution of the ion energy distribution agrees closely with the experimental distribution. Hence the reflexing beam model appears to be a valid model for explaining the experimental data. Simulation shows in addition to the agreement with the experimental ion distribution that synchronization between accelerated ions and electric field is phase unstable. This instability seems to further restrict the maximum ion energy to several times the electron energy.

I. INTRODUCTION

Experiments of collectively accelerating ions utilizing a reflexing intense relativistic electron beam in a plasma have recently been carried out.[1,2] These experiments began to teach us several characteristics of the acceleration mechanism albeit by bits and pieces. It is the purpose of this paper to present our latest understanding of the acceleration mechanism by closely comparing the experiments[1,2] and our analytical theory and numerical results.

One of the first explanations of the mechanism of collective ion acceleration from reflexing intense relativistic electron beam may be found in Ryutov and Stupakov.[3] In their model, an intense electron beam is injected through a thin metal anode into a pre-formed plasma density gradient, see Fig. 1(a). The plasma density abruptly drops to zero in a short distance beyond the anode. A calculation of the potential of a similar situation intended for study of ion collective acceleration was carried out in Ref. 4. This configuration forces the beam to turn around in its own space charge at the position of the virtual cathode [Fig. 1(b)]. With the anode voltage being applied to the returning electrons, the beam is forced into an oscillatory or reflexing state. In this model[3] an infinite magnetic field is applied along the direction of beam propagation, thus constraining the model to be one dimensional. The aspect of cylindrical geometry considered in Ref. 4 can be neglected here, since the collisionless skin depth is assumed to be shorter than the cylinder radius. Also, the beam does not have a bulk rotation[5] but can acquire Larmor rotation. Larmor rotation and electron thermalization are provided, in the optimal ion acceleration case, by elastic scattering of the beam as it passes through the anode foil.

Ion acceleration takes place by a mechanism due to the ambipolar field. The ions located at the plasma vacuum boundary are accelerated by the space charge electric field of the reflexing electrons which extend into the vacuum region. Ions are therefore accelerated in the direction of beam propagation. Since the ions move towards the virtual cathode and the virtual cathode cannot move without the ions, one might expect that the ions and virtual cathode move in a synchronous fashion. Thus, one would expect the ions to acquire a high final energy. The ion energy would of course be bounded above by the ion to electron mass ratio times the initial electron energy; that is, when the ions reach the initial electron velocity.

Ryutov et al.[3] argued that acceleration of ions ceases before ions acquire the electron velocity, which is in qualitative agreement with the experiments.[1,2] Ryutov's model proposed two factors which limit the ion velocity from becoming equal to the initial electron velocity. The first constraint is imposed by allowing electron thermalization with the anode. As the beam passes many times through the anode, the electron parallel energy is converted to perpendicular energy by elastic scattering in the anode. The final parallel electron velocity after thermalization is but a small fraction of its initial velocity. Thus, the final ion energy is limited by the final parallel electron velocity. The second constraint is that the diode is "turned off". That is, the cathode is no longer allowed to emit new electrons. This condition is imposed by the space charge of the reflexing beam in the diode region. Hence, without a "fresh" supply of the energetic electrons the final ion velocity is limited to the final parallel electron velocity after thermalization. In Ryutov et al.'s model these two elements limit the maximum ion energy. The maximum ion energy depends on the relativistic factor for the electron beam. The ratio of energy for an ion with a charge state of unity to the initial electron energy is predicted to be between two and five, depending on whether the beam is non-relativistic or ultrarelativistic, respectively.

It becomes evident, however, from the experiment[6] that the diode was in fact not "turned off", in contraction to Ryutov's model. In addition, it seems likely[6] that electron scattering by the anode is not as complete as assumed in Ryutov's model. These observations invalidate Ryutov's model as it is. We have to construct a physical model that is consistent with and constrained by the experiment.[1,6] Our method is yet phenomenological in a sense that we provide a (nonlinear) relation between the electron density and the potential suitable to and constrained by the experiment, replacing Ryutov's invalid relation between the density and the potential based on the above mentioned two factors.

Two objectives of this paper are as follows. The first objective is to establish our model that is experimentally constrained and consistent with our experimental and numerical observations. Our model is constructed in a spirit similar to but different from Ryutov's reflexing beam model. The second is to examine whether high ion energies are possible by a reflexing electron beam mechanism.

Many quantities can be calculated and compared to experimental values. However, the most important signature of an acceleration mechanism is the ion energy distribution it produces. Comparison of the ion energy distribution forms the basis for establishing the validity of our reflexing beam model. A nonlinear, self-similar model of Ryutov et al.'s type is introduced here to include a measured state of the electron beam. Our modified model or experimentally constrained model is then used to predict the ion energy distribution. Good agreement is found between the measured and the calculated ion distributions and, further, the numerical one. We establish scaleability of the reflexing beam model by using a particle code to solve the initial value problem. Simulation shows a similar distribution obtained in the experiment and our theoretical model. It shows that there is a limitation in the ion acceleration process. This limitation seems intrinsic to the present acceleration mechanism in addition to the model self-similarity limitation. The limitation is different from the external limitations considered in the Ryutov model. Synchronization between ion acceleration and the accelerating field is found to be phase unstable. This instability occurs because the virtual cathode is not locally neutralized at a rate sufficient enough for the virtual cathode to keep in step with the accelerated ions. The maximum ion energy is at most several times the electron energy.

In the next section the fluid equations are used to determine the ion response to the electron beam. The steady state electron beam density as a function of electrostatic potential is calculated using experimental constraints. The solution of a final state is found by the self-similarity method. The ion density and velocity are calculated as a function of position and time in the final state. Also, the electrostatic potential and field are calculated. We find the relation of these two is different from Ryutov's and consequently we show that the maximum ion energy is calculated differently from Ref. 3. Finally, our theoretical ion energy distribution is calculated and compared to experimental and numerical results in good agreement.

The third section contains results from the particle code simulation. A one-dimensional space and three-dimensional velocity electrostatic relativistic particle code is used to evaluate the ion acceleration mechanism. At the initial time step an intense electron beam begins to be injected normally through a grounded metal plane into a dense plasma. The plasma density abruptly drops to zero in a distance large compared to the plasma Debye length and short compared to the distance between ground planes. The above mentioned two factors that determine the maximum ion energy in the Ryutov model are not observed here. Simulation can be done in order to examine the accessibility of the obtained analytical solution and the scaleability of the mechanism, not subject to the external limitations discussed above nor the assumption of self-similarity. The ions at the plasma front reach a maximum energy after about ten plasma periods. After this time the ions lose synchronization with the accelerating field and drift past the virtual cathode. The virtual cathode is not neutralized rapidly enough for the accelerating field to keep in phase with the energetic ions. This completes the objective of this section, suggesting that the reflexing beam scheme does not seem a scaleable mechanism for achieving high ion energies.

The final section draws the conclusions.

II. THE REFLEXING BEAM MODEL

In this section we consider the electron beam driven expansion/-acceleration of a collisionless plasma. From the reference frame of the ions the beam electrons will appear to be in a steady state since the beam electron plasma period is short in comparison to the time scale for ion motion. The thermal energy of the ions will be neglected since this is small in comparison with the final accelerated ion energy. A one-dimensional treatment is sufficient since the ions are radially confined by the electrostatic field of the electron beam and the electron beam is forced to be one-dimensional by a large external magnetic field. Thus the fluid equations are used to describe the response of the ions to the electrostatic field due to the electron beam

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial z}(v_i n_i) = 0 \ , \tag{1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial z} = \frac{q}{M}\frac{\partial \phi}{\partial z} \ , \tag{2}$$

where v_i, n_i are the ion velocity and density, respectively. M and q are the ion mass and charge, respectively. ϕ is the electrostatic potential, z is the ion axial position coordinate with its origin at the plasma front and t is the time where t=0 corresponds to the beginning of the plasma explansion/acceleration. To close Eqs. (1) and (2) we will need to determine the electrostatic potential as a function of electron beam density and assume quasi-neutrality when the ions have evolved to a self-similar state.

We will now determine the experimentally constrained relation for the electron beam density as a function of electrostatic potential. In order to determine the above relation, we need an equation which relates the electron beam density and electrostatic potential to measurable properties of the electron beam. The following relation for the electron density that relates the density and velocity is used:

$$n_e = -\frac{2}{e}\int_0^{E_{max}} \frac{(dJ/dE)\ dE}{v(E,\phi)}\ , \tag{3}$$

where, J,E,v are the beam current density, electron total energy and electron velocity, all parallel to the magnetic field, respectively. The quantity e is the electronic charge. The factor of 2 accounts for total reflection of the electrons at the virtual cathode. This was very nearly the case for the experiment considered[6]. All that is needed to complete Eq. (3) is a measurement of the current density as a function of total electron energy.

From Ref. 6 the experimentally constrained relation for J(E) is given as

$$J(E) = J_o(1-E/E_{max})^{3.42}\ ,\ 0 \leq E \leq E_{max} \tag{4}$$

where $J_o = J(E=0)$. Equation (4) includes the fact that the diode was still "turned on" during the beam pulse length. This condition was mentioned in the Introduction as a contradicting factor in Ryutov's model which prevents ions from reaching higher energies. The curvature of J(E) is consistent with that expected by scattering theory of electrons through a thin foil.[7]

The integration of Eq. (3) is now readily completed with the result

$$n_e = n_o(1 + e\phi/E_{max})^{5/2} , \tag{5}$$

where $n_o = \frac{16\ J_o}{5\ e}\sqrt{\frac{2m}{E_{max}}}$ and m = electron mass. Here the non-relativistic energy conservation equaion was used for $v(E,\phi)$, since most of the electrons are not relativistic. In addition, in Eq. (4) the power factor of 3.42 was reduced to 3 to simplify integration. Equation (5) differs from Ryutov's expression[3] which was based on two invalid assumptions, as we discussed earlier: Ryutov et al.'s expression has a power exponent of 1/2 instead of 5/2.

Our next concern is the solution of Eqs. (1) and (2) for ion response to the electron beam. The electron beam density and the electrostatic potential are related through Eq. (5). Since the final state of the ions is what we need for making a comparison with the experimental results, we will then solve Eq. (1) and (2) in the self-similar state. In the self-similar state the charge density will be quasi-neutral. Hence, the self-similar form of Eqs.(1) and (2) along with Eq. (5) form a closed set of equations. The self-similar condition is invoked that Eqs. (1), (2) and (5) are functions of the self-similar parameter ζ:

$$\zeta = z/(v_o t) , \tag{6}$$

where $v_o \equiv \sqrt{\frac{q\phi_o}{M}}$ and $e\phi_o = E_{max}$. With the following definitions

$$U \equiv v_i/v_o \ ,$$

$$N \equiv n_i/n_o \ , \tag{7}$$

$$\psi \equiv \phi/\phi_o \ ,$$

and Eq. (6), Eqs. (1), (2) and (5) become

$$N'(U-\zeta) + N\,U' = 0 \ , \tag{8}$$

$$U'(U-\zeta) + \frac{d\psi}{dN} N' = 0 \ , \tag{9}$$

$$N = (1+\psi)^{5/2} \ , \tag{10}$$

where primes denote derivatives with respect to ζ.

Now Eqs. (8)-(10) can readily be solved as

$$n_i = n_o \left\{ \sqrt{\frac{5}{6}} \left[1 - \frac{\sqrt{3}}{6} \left(\frac{z}{v_o t}\right) \right] \right\}^5 \ , \tag{11}$$

$$v_i = v_o \left[\frac{5}{6} \left(\frac{z}{v_o t}\right) + \frac{1}{\sqrt{3}} \right] \ , \tag{12}$$

$$\phi = \phi_o \left[\frac{5}{6} \quad 1 - \frac{\sqrt{3}}{6} \left(\frac{z}{v_o t}\right) \right]^2 - \phi_o \ . \tag{13}$$

The electric field is

$$E = \frac{\phi_o}{v_o t} \frac{5}{36} \left(\frac{6}{\sqrt{3}} - \frac{z}{v_o t} \right) \ , \tag{14}$$

where the conservation of energy was used as a boundary condition

$$U^2/2 + \psi = 0 \quad \text{at} \quad \zeta = 0 \; . \tag{15}$$

The maximum ion energy can now be obtained by setting $n_i = 0$, i.e.

$$E_{imax} = 6q\phi_o \quad \text{at} \quad \zeta = \frac{6}{\sqrt{3}} \; .$$

For doubly ionized helium[6] the maximum ion energy corresponds to

$$E_{imax} = 9.6 \text{ MeV} \quad \text{for} \quad \phi_o = 0.8 \text{ MV} \; .$$

The experimental result[6] for the maximum helium ion energy was 8 MeV. After making the adjustment $q\phi_o = 1.6$ MeV for the doubly ionized helium at $\phi_o = 0.8$ MV, the maximum energy calculation and measurement agree exactly. This adjustment is done since experimentally the detector was at ground potential.

The ion number as a function of energy is calculated to be

$$N_i(E_i) = \frac{n_o A}{\beta}\left(\frac{6}{\sqrt{5}} - \sqrt{E_i/5q\phi_o}\right)^6 , \tag{16}$$

where

$$n_o A = \frac{16}{5}\,\frac{J_o A}{e}\sqrt{\frac{2m}{e\phi_o}} ,$$

$$\beta \equiv \sqrt{\frac{5}{2}}\,\frac{1}{v_o t} ,$$

$$A = \pi r_b^2 \ , \ r_b = \text{electron beam radius},$$

and

$$v_o = \sqrt{\frac{q\phi_o}{M}} \ .$$

Equation (16) is our main result. The natural logarithm of Eq. (16) is plotted in Fig. 2 along with the experimental data (corrected for the 1.6 MeV energy loss at the detector). The following experimental values were used: $J_o = 40$ kA, $\phi_o = 0.8$ MV, $q = 2e$ (doubly ionized helium), $t = 100$ ns and $r_b = 2.5$ cm. The agreement between Eq. (16) and the experiment[6] is reasonable. The relation in Ref. 3 does not provide such a good fit: it has a too weak slope.

III. SCALING AND ACCESSIBILITY OF THE MODEL

In the preceding section, the analysis assumed that a self-similar state could be reached. To address the question of whether a self-similar state can be attained, it requires a detailed analysis of the initial value problem (the accessibility question). This detailed analysis should include a self-consistent treatment of the dynamics of all particle species. The problem must be solved from the origin of time when the beam is first injected into the plasma and must include the effects of a finite geometry. Therefore, a particle code has been selected to study this initial condition problem.

Since the point here is to determine whether the accelerated ions can evolve to a self-similar state, the parameters used in this study were selected for convenience. The distance between the beam injection and absorption planes was 15 cm. The initial spatial extent of plasma from the injection plane was 0.5 cm. A beam density of 1.3×10^{10} cm^{-3} was typically used with a plasma density of 1×10^{12} cm^{-3}. A cyclotron frequency of about three times the plasma frequency was used. The beam electron velocity was 0.67c with an electron thermal velocity of 0.1c. The electron and ion temperature were initially equal and an ion to electron mass ratio was 20 or 40. From simulation we obtained a qualitative agreement with experiment (seen in Fig. 2) and in agreement with Eq. (16). Thus the self-similar solution Eq. (16) seems to be an accessible one in the dynamical sense.

Figure 3(a) shows the beam and plasma electrons in phase space shortly after the beam is injected through the plasma (at $t = 5\omega_{pe}^{-1}$). The beam front is accelerated forward by the space charge behind it and the image charges at the grounded absorption plane (which is located at the far right). Just behind the beam front the electrons are slowed down by both the space charge in front of it and the ions in the plasma (since plasma electrons are expelled into the injection plane). Figure 3(b) is the corresponding ion phase space at a time of $5/\omega_{pe}$ after the beam was injected into the volume. At this time the ions have gained very little momentum. Figure 3(c) is the electric field as a function of position at a time of $5/\omega_{pe}$. The electric field is zero at all points beyond 2 cm from the injection plane, since there is no charge located in that region. (The polarity of the electric field is due to the electrons carrying a positive charge and the ions a negative charge for convenience of the code.) By comparing Figs. 3(b) and 3(c), it can be seen that the maximum value of the electric field is in spatial phase with the ion front at this time.

Figure 4(a) shows the electron phase space at $60/\omega_{pe}$. Now the flow of beam particles has fully developed into a space-charge-limited condition. This condition is characterized by the return flow of beam electrons from the plane of minimum energy to the injection plane. The plane of minimum energy being defined where the beam velocity goes to zero. In this case, it is located at about 5.75 cm from the injection plane. The magnitude of current that propagates beyond the plane of minimum energy is found to be approximately the Child-Langmuir current.[8,9] This result is expected, since a virtual cathode is formed at the plane of minimum energy.

In Fig. 4(b) it is clear that the ions have been accelerated. At this time ($60/\omega_{pe}$) the ions have reached their maximum energy, which is about three times the electron energy. The main point of this section is now established. By comparing the position of the maximum electric field in Fig. 4(c) with the position of the maximum ion momentum, it is clear that the ion acceleration process has become phase unstable.

Several ideas were tried in an attempt to break up this instability. All ideas that were tried failed to increase the ion energy. First, a large temperature spread was given to the beam in order to smear the electric field over the ions. The thermal energy was comparable to the depth of the potential wells in the vicinity of the ions. Next the beam density was increased in order to add additional "pressure" to the ions. The increase in beam density was accomplished by re-cycling the beam electrons that returned to the injection plane from the plane of minimum energy. This method of increasing the beam density was also used to represent reflexing of the beam electrons. When the ion mass was doubled, the ion velocity decreased inversely as the square root of the mass. Thus, the acceleration process appears to be momentum limited, since the number of accelerated ions remained constant.

Another thought was that the phase instability was only temporary or periodic in time. However, the simulation was run up to a time of $200/\omega_{pe}$ with little change in the ion energy. The ion velocity reached a maximum at $60/\omega_{pe}$ and oscillated to a maximum every ion plasma period. This ion velocity oscillations correlated with the position of the peak electric field and the virtual cathode oscillating at the ion plasma period. When the position of the electric field peak or the virtual cathode reached a maximum, the ion velocity reached a minimum value. A virtual anode formed when the ion velocity was a minimum and it disappeared at the ion velocity maximum. The appearance of a virtual anode was observed experimentally by the first author.[6] In another simulation[10,11] virtual anodes were also observed. However, their ion energy reached a maximum when the virtual anode appeared. In spite of this contradiction in details their ion energy gain was approximately the same. One remaining task to be explored is a scaling of energy with respect to the system length of simulation.

The essential problem with the reflexing beam mechanism is first that the peak electric field forms between the highest electron and ion densities. Secondly, a low density of initially accelerated ions can drift force-free by forming a charge neutral region with the transmitted electron beam. Thus, it appears, within the context of this mechanism, that a phase instability is irrevocable. This further limits the maximum ion energy obtainable based on the self-similarity solution of Eq. (16) that was determined with the quasi-neutrality.

IV. CONCLUSIONS

We explored the mechanism of collective ion acceleration done in the experiment[1,6] and theoretically established the validity of the reflexing beam model. We derived the ion population as a function of energy and compared with the experiment and with the simulation. In addition, the reflexing beam mechanism was found to be unsuitable for scaling to high ion energies, since the accelerating mechanism appears to be phase unstable. The theoretical expression for the energy spectrum of ions Eq. (16) may be useful in other applications as well (such as in astrophysical settings and beam injection experiments).

Although it does not seem possible to alter the internal aspects of the acceleration mechanism for removing the instability, it may be possible to circumvent the problem. Since the rate of charge neutralization of the potential well is too slow for the well to keep in phase with the ions, a method of increasing the rate of charge neutralization must be externally invoked. This can be accomplished by adding or creating more plasma at the virtual cathode when the phase instability appears. One way of adding plasma would be from sequentially timed plasma sources, though this approach may turn out to be technically difficult. A continuous version of this idea has been explored by Olson.[12]

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FIGURE CAPTIONS

Fig. 1 -

Electron beam injected through an anode into a plasma that extends only a short distance beyond the anode. $B_z = \infty$.

Fig. 2 -

Comparison between theory, experiment and simulation, of the natural logarithm of the ion number vs. energy.

Fig. 3 -

Simulation phase space at early time $t = 5/\omega_{pe}$

(a) Electron Phase Space (Beam and Plasma).

(b) Ion Phase Space.

(c) Electron Field vs. Position.

Figure 4 -

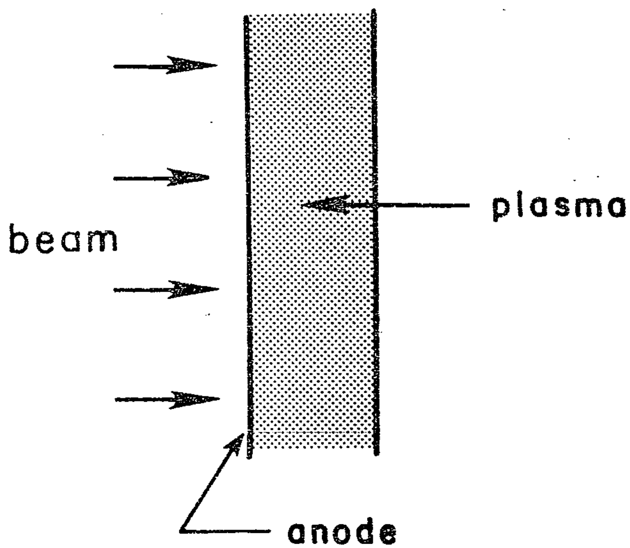
Phase space from simulation in later time $t = 60/\omega_{pe}$.

(a) Electron Phase Space.

(b) Ion Phase Space.

(c) Electron Field.

(a) $t = 0$. The beam is injected into dense plasma. ($n_p \gg n_b$)



(b) The beam is space charge limited.

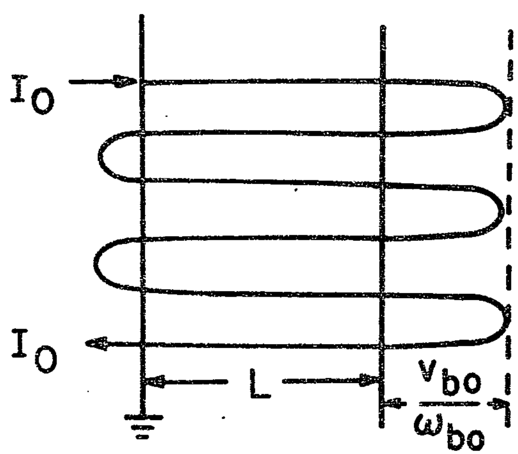
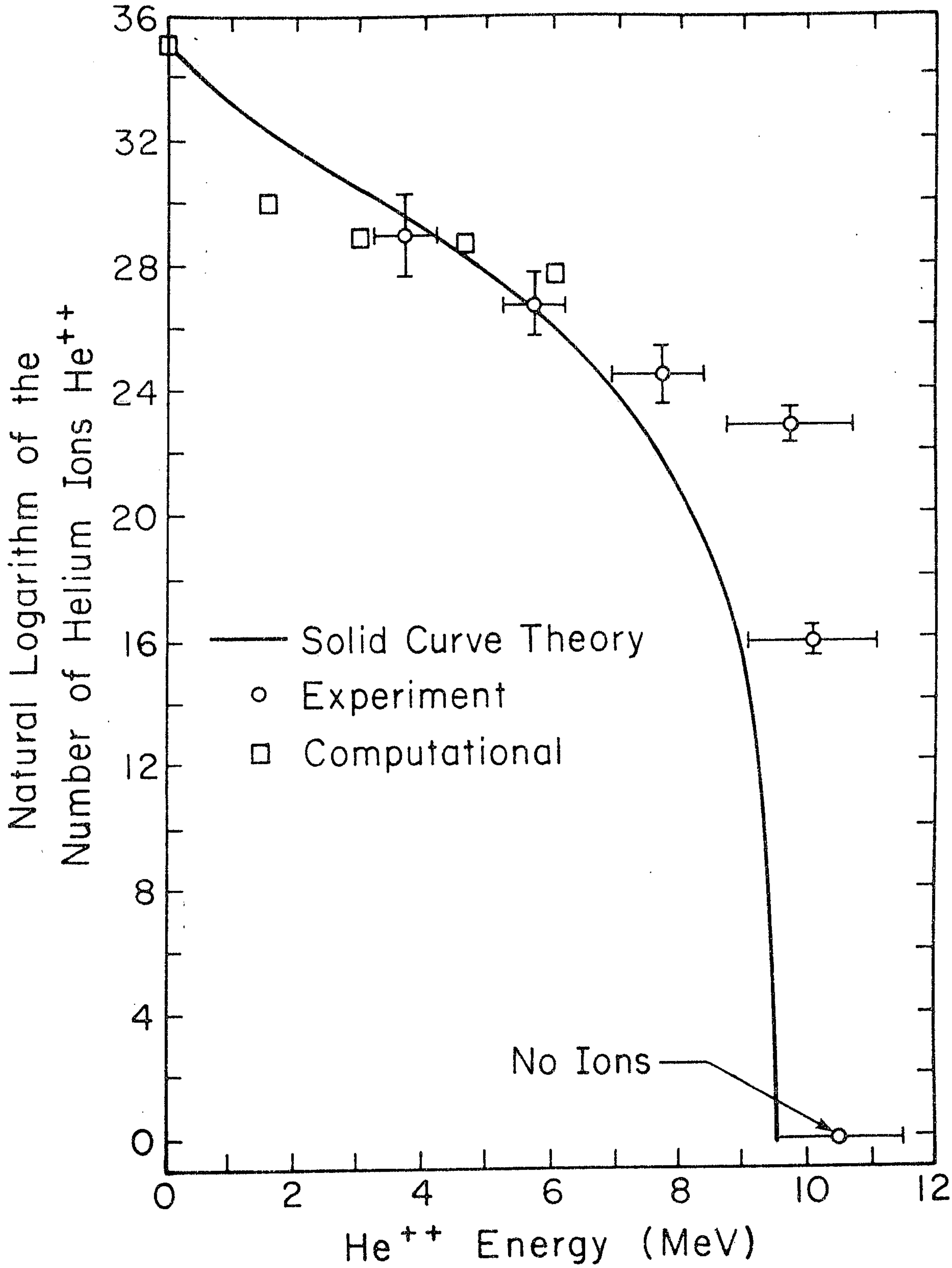
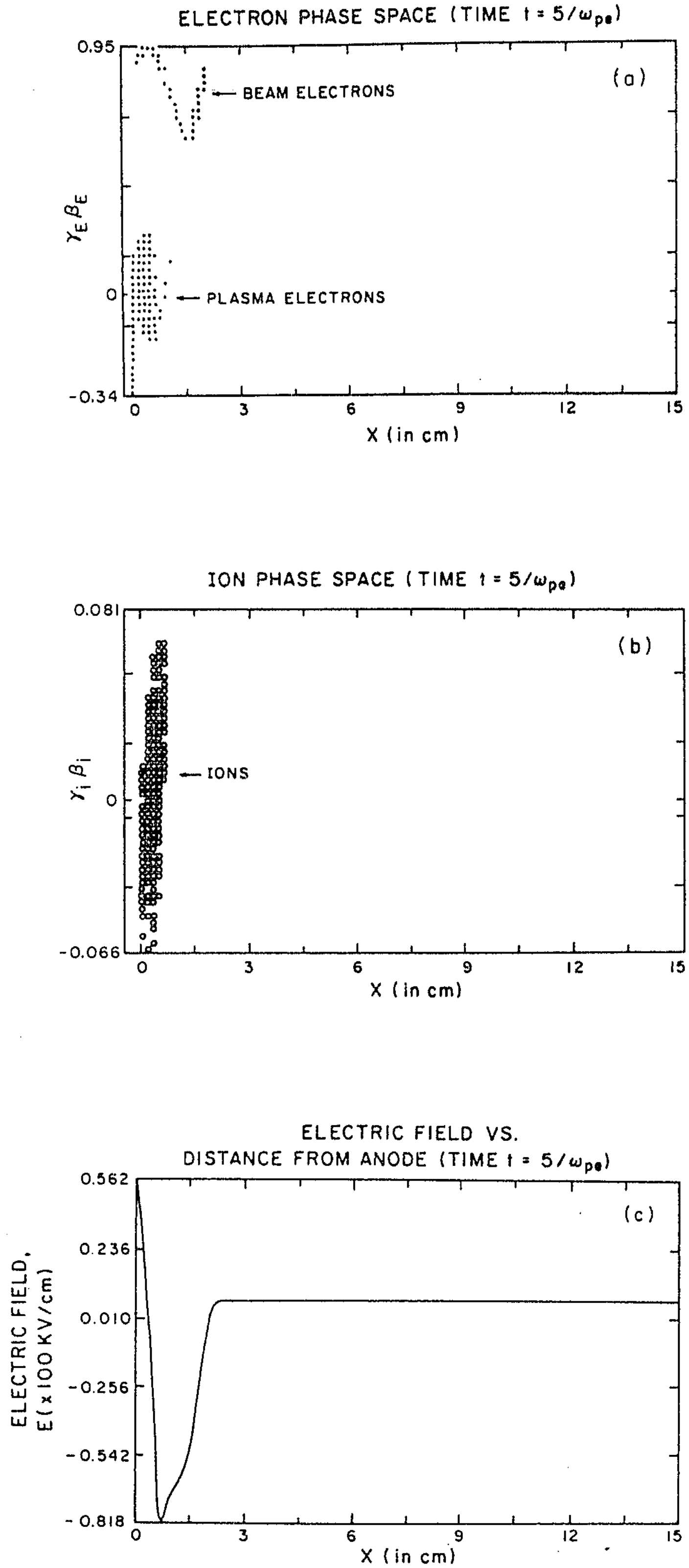


FIG. 1



36
32
28
24
20
16
12
8
4
0
0
2
4
6
8
10
12
Natural Logarithm of the Number of Helium Ions He++
He++ Energy (MeV)
Solid Curve Theory
Experiment
Computational
No Ions

FIG. 2



ELECTRON PHASE SPACE (TIME t = 5/ωpe)
0.95
(a)
BEAM ELECTRONS
γE βE
0
PLASMA ELECTRONS
-0.34
0
3
6
9
12
15
X (in cm)
ION PHASE SPACE (TIME t = 5/ωpe)
0.081
(b)
γi βi
IONS
0
-0.066
0
3
6
9
12
15
X (in cm)
ELECTRIC FIELD VS.
DISTANCE FROM ANODE (TIME t = 5/ωpe)
0.562
(c)
0.236
ELECTRIC FIELD,
E(x100 KV/cm)
0.010
-0.256
-0.542
-0.818
0
3
6
9
12
15
X (in cm)

FIG. 3

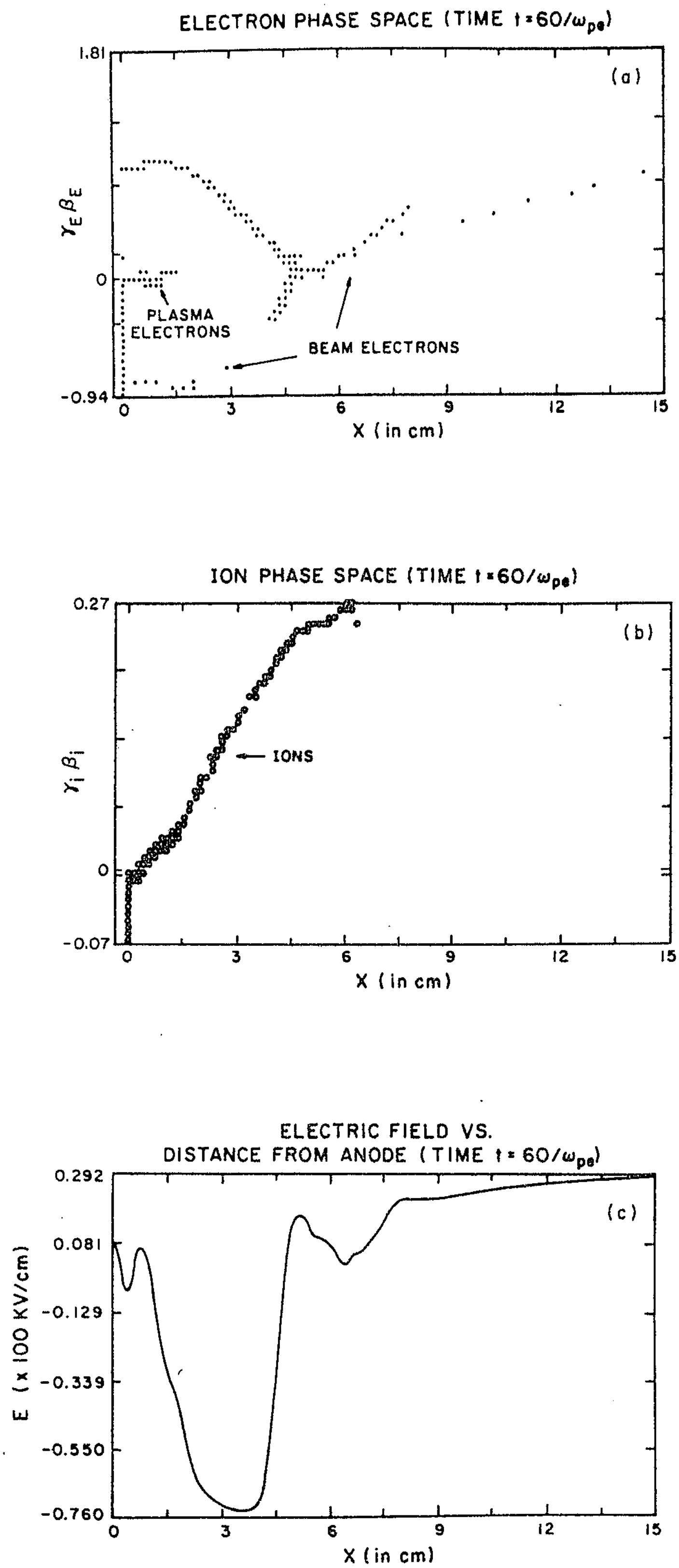

ELECTRON PHASE SPACE (TIME t=60/ωpe)
(a)
γE βE
1.81
0
-0.94
PLASMA ELECTRONS
BEAM ELECTRONS
0 3 6 9 12 15
X (in cm)
ION PHASE SPACE (TIME t=60/ωpe)
(b)
γi βi
0.27
0
-0.07
IONS
0 3 6 9 12 15
X (in cm)
ELECTRIC FIELD VS. DISTANCE FROM ANODE (TIME t=60/ωpe)
(c)
E (x 100 KV/cm)
0.292
0.081
-0.129
-0.339
-0.550
-0.760
0 3 6 9 12 15
X (in cm)

FIG. 4