Theoretical Interpretation of Alfvén Cascades in Tokamaks with Non-monotonic q-profiles

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Abstract

Alfvén spectra in a reversed-shear tokamak plasma with a population of energetic ions exhibit a quasiperiodic pattern of primarily upward frequency sweeping (Alfvén Cascade). Presented here is an explanation for such asymmetric sweeping behavior which involves finding a new energetic particle mode localized around the point of zero magnetic shear.

The presence of energetic particles in a plasma can alter its behavior from that predicted by conventional magneto-hydrodynamics (MHD) theory in two ways. First these particles can perturbatively destabilize a basic MHD mode. Alternatively, a sufficient number of these particles can non-perturbatively alter the very structure of the MHD modes. This latter behavior is relevant to certain shear Alfvénic perturbations often called energetic particle modes (EPM) [1–3]. In addition, in recent years there has been a great deal of interest in plasmas with reversed magnetic shear profiles, where transport and MHD stability properties have been shown to improve [4,5]. It is important for fusion experiments in shear reversed fields to understand the collective properties associated with energetic particles. Experiments in JT-60U [6] and JET [7] have investigated reversed shear regimes and have produced energetic particles with ion cyclotron heating (ICRH) [8]. Alfvén modes emerge in these experiments but their spectrum is often puzzling. This paper presents an example of how a purely MHD description is incompatible with the data while a description which accounts for the non-perturbative energetic particle response explains a large part of the data. The interpretation suggests a sensitive method to experimentally determine q_{min} (the minimum safety factor) in reversed magnetic shear tokamaks.

The JET experiments exhibit upward frequency sweeping phenomena, named Alfvén wave cascades (ACs) [9] (see Fig. 1a). Each cascade consists of several modes with different toroidal mode numbers and different frequencies. The toroidal mode numbers vary from $n = 1$ to $n = 6$. The frequency starts from $20 - 40\,\mathrm{kHz}$ and increases up to $100 - 120\,\mathrm{kHz}$ which is the TAE gap frequency. Similar data were obtained some time ago on JT-60U [6]. In both the JET and JT-60U data, the modes with higher toroidal mode numbers exhibit a more rapid frequency sweeping, and the higher n-modes re-occur more often than the lower n-modes. It is striking that downward frequency sweeping either does not appear, or appears only rarely. In both JET and JT-60U experiments, the minimum value of q decreases in time and a population of energetic ions is created by ICRH heating.

ACs resemble the global Alfvén eigenmode [10,11], whose frequency is close to the local value of the Alfvén wave frequency at the zero shear point in minor radius, $r = r_0$, i.e.

$2\pi f_{\rm AC} \approx \omega_A(r_0) \equiv |k_{\|}(r_0)|V_A(r_0)$, where V_A is Alfvén velocity and $k_{\|}$ is the wave-vector component along the equilibrium magnetic field B_0. To avoid strong damping, the frequency $f_{\rm AC}$ needs to be somewhat larger than $\omega_A(r_0)$ if $\omega_A(r)$ has a maximum at $r = r_0$ and smaller than $\omega_A(r_0)$ if $\omega_A(r)$ has a minimum there. Otherwise, continuum resonance inhibits mode excitation by a moderate population of energetic ions.

In the standard theory of the global Alfvén Eigenmode, the mode is associated with a minimum of the local Alfvén frequency $\omega_A(r)$ [10,11]. However from both analytic considerations, and numerical calculations with the use of ideal MHD CSCAS code [12] we infer that a maximum of $\omega_A(r)$ is needed to explain the data. This conclusion follows from the local shear Alfvén wave dispersion relation which is $\omega_A/V_A = |k_{\|}(r)| = |n - m/q(r)|/R$, where m is the poloidal mode number. We impose the convention that ω_A and n are positive, and positive m is required to allow the mode frequency to be smaller than the TAE frequency, $f_{\rm TAE} = V_A/(4\pi qR)$. It is readily established that $\omega_A(r_0)$ is a maximum at $q = q_{\rm min}$ when $k_{\|}(r_0) < 0$ and a minimum when $k_{\|}(r_0) > 0$. We now assume that q_{min} decreases in time as it does in the JET and JT-60U experiments.

If the modes in the experiment just trace the Alfvén dispersion relation at $q = q_{\rm min}$, and $q_{\rm min}$ decreases, the frequency would increase in time when $k_{\|}(r_0) < 0$ and decrease in time when $k_{\|}(r_0) > 0$. This pattern is shown in Fig. 1b obtained from the CSCAS code that is applied to a series of JET experimental equilibria. In these CSCAS runs only the toroidal n - number is a precise quantum number, while the dominant poloidal mode number m changes in steps as $q_{\rm min}$ decreases in time. This code automatically transfers the dominance of the m-th poloidal harmonic to $m-1$ as $q_{\rm min}$ passes through $q_{\rm TAE} = (m - 1/2)/n$ to keep the mode frequency below the gap associated with the TAE frequency. It is also clear that the Alfvén continuum modes in Fig. 1b form bunches when $q_{\rm min}$ takes on integer values like 3, 4 and 5. In the experiment the emerging modes also appear in bunches.

It is important to note that the transition from m to $m-1$ changes the sign of $k_{\|}(r_0)$, which reverses the direction of frequency sweeping. However, the modes with downward sweeping seem to be strongly suppressed in the experiment. We therefore need a mechanism

that gives preference to the waves with negative $k_{\|}(r_0)$. The only way we have found to explain the asymmetry is to describe the fast particles response in a non-perturbative manner. This means that the fast particle contribution to the MHD equations affects the very existence of the mode rather than just the mode growth rate. We have also examined other candidate mechanisms, which will be briefly mention later in this paper. They all give preference to waves with positive $k_{\|}(r_0)$ which is inconsistent with the experiment.

Technically, the following feature of the deeply-reversed shear discharges in JET is essential for our interpretation: the fast particle ∇B drift rate across the mode structure is faster than the bounce frequency or the mode frequency ω. As a result of fast drift, the hot particle response is found to be spatially local, which simplifies our analysis considerably. In this aspect, our theory is substantially different from past theories for the EPM which deal with the non-local hot particle response [1,2].

For a typical energy of fast ions $\sim 500\,\mathrm{keV}$, and with other plasma parameters chosen to be compatible with the relevant equilibrium, the fast particle orbits are found to be nonstandard. Indeed, our numerical calculations with the particle-following code HAGIS [13] and the CASTOR-K code [14] show that the toroidal drift frequency exceeds the poloidal bounce frequency and the orbit width is a substantial fraction of the plasma radius. These features make it easy to satisfy the condition that the ∇B drift frequency exceeds the eigenmode frequency for high n-values, and marginally for $n = 1$.

Our formal derivation of the relevant energetic particle mode is based on the reduced MHD description of shear Alfvén perturbations and the drift kinetic description of energetic particles. We consider a low-beta plasma in a large-aspect-ratio torus, for which the perturbed vector potential, $\delta\boldsymbol{A}$ and the perturbed fields $\delta\boldsymbol{E}$ and $\delta\boldsymbol{B}$ for the shear Alfvén wave can be represented by a single scalar function in the following forms: $\delta\boldsymbol{A} = \nabla\delta\Phi - \frac{\boldsymbol{B}}{B^2}(\boldsymbol{B}\cdot\nabla\delta\Phi)$, $\delta\boldsymbol{E} = -\frac{1}{c}\frac{\partial\delta\boldsymbol{A}}{\partial t}$ and $\delta\boldsymbol{B} = \nabla\times\delta\boldsymbol{A}$, where $\boldsymbol{B}$ is the equilibrium magnetic field. The equation for $\delta\Phi$ follows from a derivation procedure presented in

Ref. [15], from which we find,

$$\nabla \cdot \frac{1}{V_A^2 B^2}\left[\boldsymbol{B} \times \left[\nabla \delta \ddot{\Phi} \times \boldsymbol{B}\right]\right] = (\boldsymbol{B} \cdot \nabla)\frac{1}{B^2}\nabla \cdot \left[\boldsymbol{B} \times \left[\nabla\left(\frac{1}{B^2}(\boldsymbol{B} \cdot \nabla \delta \Phi)\right) \times \boldsymbol{B}\right]\right]$$
$$- \left(\nabla\left(\frac{1}{B^2}(\boldsymbol{B} \cdot \nabla \delta \Phi)\right) \cdot \Delta \boldsymbol{B}\right) - \nabla \cdot \frac{4\pi}{B^2}[\boldsymbol{B} \times \delta \boldsymbol{F}], \tag{1}$$

where

$$\delta F_\alpha \equiv -\delta \frac{\partial}{\partial x_\beta}\left[P_\perp\left(\delta_{\alpha\beta} - \frac{B_\alpha B_\beta}{B^2}\right) + P_\parallel \frac{B_\alpha B_\beta}{B^2}\right] \tag{2}$$

is the force density due to the perturbed anisotropic pressure that can be calculated with the use of the kinetic guiding center theory [16]. The quantities $P_\perp$ and $P_\parallel$ are the standard perpendicular and parallel pressure obtained form appropriate moments of the particle distribution function f. It was shown in Ref. [15] that the following relation holds for the pressure term in Eq. (1) in the limit of low beta and large aspect ratio

$$-\nabla \cdot \frac{1}{B^2}[\boldsymbol{B} \times \delta \boldsymbol{F}] = \frac{e}{c}\int d^3v\,(\boldsymbol{v}_D \cdot \nabla \delta f)\,. \tag{3}$$

Here e is the energetic particle charge, $\boldsymbol{v}_D$ is the magnetic field gradient and curvature drift velocity, and the gradient ∇ operates on the perturbed distribution function δf with energy w and magnetic moment μ held fixed. Equation (3) generally involves summation over all species, but we will only keep the response from the ICRF heated energetic ions since one can show that the contribution from the background plasma pressure is relatively small in our ultimate eigenmode equation.

In order to evaluate the right-hand side of Eq. (3) we use the linearized drift kinetic equation neglecting equilibrium electric fields,

$$\frac{\partial \delta f}{\partial t} + v_\parallel \boldsymbol{b} \cdot \nabla \delta f + \boldsymbol{v}_D \cdot \nabla \delta f + \left[\delta(v_\parallel \boldsymbol{b})\right] \cdot \nabla f + (\delta \boldsymbol{v}_D) \cdot \nabla f$$
$$+ \boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}} \cdot \nabla f + \frac{v_\parallel}{\Omega}\left[\boldsymbol{b} \times \frac{\partial \delta \boldsymbol{b}}{\partial t}\right] \cdot \nabla f + \left[\mu \frac{\partial \delta B}{\partial t} + e\left(v_\parallel \boldsymbol{b} \cdot \delta \boldsymbol{E} + \boldsymbol{v}_D \cdot \delta \boldsymbol{E}\right)\right]\frac{\partial f}{\partial w} = 0 \tag{4}$$

with

$$w \equiv \frac{m v_\parallel^2}{2} + \mu B; \quad \boldsymbol{v}_D \equiv \frac{1}{\Omega}\boldsymbol{b} \times \left(\frac{\mu}{m}\nabla B + v_\parallel^2 (\boldsymbol{b} \cdot \nabla)\boldsymbol{b}\right); \quad \boldsymbol{v}_{\boldsymbol{E}\times\boldsymbol{B}} \equiv \frac{c}{B^2}\delta \boldsymbol{E} \times \boldsymbol{B} \tag{5}$$

where Ω is the fast particle gyrofrequency, and the symbol δ denotes a perturbation of a quantity. Note that the integrand in Eq. (3) is exactly the third term on the left-hand side of Eq. (4). We will limit our consideration to the case of sufficiently fast drift velocity $\boldsymbol{v}_D$, as discussed in the introduction. Then the third term in Eq. (4) is the only term involving δf that need be retained. This simplification will lead to a differential equation rather than an integral equation for determining $\delta\Phi$. For the JET experiment under consideration this approximation is marginally good at $n = 1$ and improves for larger values of n. Further simplifications occur when we use $\boldsymbol{b}\cdot\delta\boldsymbol{B} = 0$ and $\boldsymbol{b}\cdot\delta\boldsymbol{E} = 0$ for shear Alfvén perturbations. We also take into account that $\frac{v_\parallel}{\Omega B}\left[\boldsymbol{b}\times\frac{\partial\delta\boldsymbol{B}}{\partial t}\right]\cdot\nabla f \ll \frac{v_\parallel}{B}\delta\boldsymbol{B}\cdot\nabla f$ since the mode frequency is much smaller than the gyro-frequency. In addition, we neglect $\delta\boldsymbol{v}_D\cdot\nabla f$. Then, with the elimination of some other small terms Eqs. (3) and (4) yield,

$$\nabla\cdot(\frac{1}{B^2}\boldsymbol{B}\times\delta\boldsymbol{F}) = \frac{e}{B}\delta\boldsymbol{E}\times\boldsymbol{B}\cdot\nabla\frac{\int f d^3v}{B} + \frac{e}{c}\delta\boldsymbol{B}\cdot\nabla\frac{\int v_\parallel f d^3v}{B}$$
$$-\frac{e}{B^2}[\delta\boldsymbol{E}\times\boldsymbol{B}\cdot(\boldsymbol{b}\cdot\nabla)\boldsymbol{b}]\int f d^3v, \tag{6}$$

where we have transformed independent variables in the distribution function from $\boldsymbol{r}$, μ, and w to $\boldsymbol{r}$, $v_\parallel$ and $v_\perp$. Equations (1) and (6), together with $\delta\boldsymbol{E}$, and $\delta\boldsymbol{B}$ lead to a single equation of a form, $\widehat{L}\delta\Phi = 0$, where $\widehat{L}$ is a linear differential operator. In a torus, this operator is a periodic function of poloidal angle θ. Therefore, poloidal Fourier components of $\delta\Phi$ are generally coupled in the solution of this equation. However, in the case of non-monotonic q-profile in the presence of energetic particles the construction of essentially "cylindrical" modes is allowable if their frequencies are not too close to the TAE gap frequency. Formally, this means that we average all coefficients in $\widehat{L}$ over θ. We then seek a "cylindrical" solution of the form, $\delta\Phi = \psi(r)\exp(-i\omega t + in\varphi - im\theta)$, where φ is the toroidal angle and $\psi(r)$ is the radial eigenfunction. A straightforward averaging procedure with the added assumptions of $m \gg 1$ and large aspect ratio equilibrium with circular flux surfaces, gives the following equation for $\psi(r)$:

$$\frac{m^2}{r^2}\left(\frac{\omega^2}{V_A^2} - k_\parallel^2\right)\psi - \frac{\partial}{\partial r}\left(\frac{\omega^2}{V_A^2} - k_\parallel^2\right)\frac{\partial\psi}{\partial r} = -\frac{4\pi e}{cB}\frac{m}{r}\psi\frac{\partial}{\partial r}\left[\omega\left\langle n_h\right\rangle - k_\parallel\left\langle\frac{1}{e}j_{\parallel h}\right\rangle\right]. \tag{7}$$

where the subscript "h" denotes fast particles and the angular brackets denote flux surface averaging. The parallel wave number, $k_{\|} = \frac{1}{R}\left(n - \frac{m}{q(r)}\right)$, can be expanded about the point $r = r_0$ where $q = q_{\min}$ (the point of zero shear). In the vicinity of r_0, we have

$$k_{\|}^2 = \frac{1}{R^2}\left(n - \frac{m}{q_{\min}}\right)^2 + \frac{1}{R^2}\frac{mq''_{\min}}{q^2_{\min}}(r - r_0)^2\left(n - \frac{m}{q_{\min}}\right). \tag{8}$$

We assume that this expression for $k_{\|}$ is accurate over a region Δr where the mode is localized. Thus we require $(\Delta r)^2 < |nq_{\min} - m|\frac{q_{\min}}{mq''_{\min}}$. We can then replace ω and $k_{\|}$ on the right-hand side of Eq. (7) by the lowest order expressions, $\omega = \omega_A \equiv \frac{V_A}{R}\left|n - \frac{m}{q_{\min}}\right|$ and $k_{\|} = \frac{1}{R}\left(n - \frac{m}{q_{\min}}\right)$, respectively. Then defining the dimensionless radial independent variable $x \equiv m(r - r_0)/r_0$ and a new dependent variable $\Psi(x) = \psi(x)(S + x^2)^{1/2}$, we obtain,

$$\Psi - \frac{\partial}{\partial x}\frac{\partial \Psi}{\partial x} = \Psi\,\frac{Q}{(S + x^2)} - \Psi\,\frac{S}{(S + x^2)^2}. \tag{9}$$

with

$$S \equiv \frac{\omega^2 - \omega_A^2}{\omega_A^2}\,m\frac{q_{\min}}{q''_{\min}r_0^2}(m - nq_{\min}); \quad Q \equiv -\frac{4\pi e R q^2_{\min}}{cBr_0 q''_{\min}}\frac{\partial}{\partial r}\left[\frac{V_A|m - nq_{\min}|}{(m - nq_{\min})}\langle n_h\rangle + \left\langle \frac{1}{e}\,j_{\|h}\right\rangle\right]_{r=r_0}.$$

It should be noted that the eigenvalue S has to be positive to avoid singularity in $\Psi(x)$ that would induce strong continuum damping. However, with Q neglected, a positive S does not give a radially localized eigenmode. To obtain one, a positive Q is required that exceeds a certain critical value $Q_{\rm cr}$. Indeed, if Q is negative or zero, a "Schrödinger potential well" does not exist in Eq. (9).

An analysis of Eq. (9) establishes that $Q_{\rm cr} = 1/4$ and that there is an infinite number of modes for $Q > Q_{\rm cr}$. Approximate analytic solutions can be obtained for $Q - Q_{\rm cr} \ll 1$ and $Q \gg 1$ and in between Eq. (9) has been solved numerically. The detailed analysis and results will be presented in a later publication. Here we note that if $Q - Q_{\rm cr} \ll 1$, the value of S is given by $S = \exp\left[-2l\pi/(Q - 1/4)^{1/2}\right]$ where l is a positive integer, while if $Q \gg 1$, we find $S = Q - (2l + 1)Q^{1/2}$ (assuming the second term much less than the first term). For $Q = 1$, the numerically evaluated eigenvalue is $S = .1003$ for the longest wavelength mode. Note that S is a relatively small even for $Q = 1$. The scale length of the longest

wavelength mode is $\Delta r \approx S^{1/2} r_0/\mathrm{m}$ for $S \ll 1$ and $\Delta r \approx Q^{1/4} r_0/\mathrm{m}$ for $Q \gg 1$. We expect that dissipative processes will suppress the short wavelength modes with $Q - Q_{\mathrm{cr}} \ll Q_{\mathrm{cr}}$, which may require $Q \gtrsim 1$ to allow the energetic particle drive to excite these modes in the experiment.

We now make additional remarks about experimental implications of our calculations. We have concluded that in order for the frequency to sweep upward we need $m > nq_{\mathrm{min}}$ and that to have the mode we need $Q > 1/4$ (though in practice a larger Q-value is required). There is strong bias in the expression for Q that favors $\partial \langle n_h \rangle / \partial r < 0$ near $r = r_0$ in order to meet the above two requirements. Then the frequencies of the allowed modes ($m > nq_{\mathrm{min}}$) increase as q_{min} decreases in time, whereas the condition for frequency decrease ($m < nq_{\mathrm{min}}$) is incompatible with mode existence. Further, we note that Q_{cr} is independent of m and n, a very satisfying result as many modes are characteristic for Alfvén cascades.

Once the existence of a mode is established, the mode growth rate, associated with resonant energetic particles, can be calculated with the use of straightforward perturbation theory. Also, the weakly nonlinear regime of mode saturation can be straightforwardly analyzed. This analysis goes beyond the scope of the present paper and will be described elsewhere.

We note that our eigenmode equation can be generalized to account for toroidal effects, if the mode frequency is not too close to the TAE frequency gap. Then the nearest sidebands of the principal poloidal mode are perturbative corrections to the cylindrical mode. Our calculations show that the corrections are second order in aspect ratio and background plasma beta and by themselves would give the opposite frequency sweeping asymmetry than is observed in experiment. The implication is that this effect is small and when the system is sufficiently above our predicted threshold these corrections only reduce the value of Q, so that a somewhat larger gradient of the fast particle density is needed to satisfy the condition for mode existence.

Similarly, we have found that kinetic contribution responsible for kinetic Alfvén waves

(KAW) is incompatible with upward frequency sweeping. The reason follows from the WKB dispersion relation for KAW, which is [17,18] $\alpha k_\perp^2 \rho_i^2 \omega^2 = \omega^2 - k_\parallel^2 V_A^2$, where ρ_i is the ion Larmor radius and $\alpha = \left(\frac{T_e}{T_i} + \frac{3}{4}\right)$. In order to obtain a localized mode, we require $k_r^2 < 0$ far enough from $r = r_0$ in either direction. Away from the mode center in either direction we have from Eq. (8), $\alpha k_r^2 \rho_i^2 \omega^2 = -\frac{V_A^2}{R^2} \frac{m q''_{\min}}{q^2_{\min}} (r - r_0)^2 \left(n - \frac{m}{q_{\min}}\right)$. Thus, as $q''_{\min} > 0$, mode localization requires $n > m/q_{\min}$, which is incompatible with experiment.

Thus the energetic particle mechanism described here is the only viable option we find to explain the observed Alfvén cascades. We conclude that the emerging bunches at times $t = 2.2$ s, 2.8 s and 3.7 s arise when $q_{\min}$ is 5, 4 and 3 respectively. One can see the $n = 1$ mode emerging at these times together with higher n modes. The $n = 2$ mode has an extra appearance between the $n = 1$ bursts and the $n = 3$ mode has two additional appearances between the $n = 1$ bursts, etc. Our identification of $q_{\min}$ correlates with the time behavior of the upper cascade frequency, which is close to the TAE frequency $V_A/4\pi R q_{\min}$.

ACKNOWLEDGMENTS

This work was partly performed under the European Fusion Development Agreement and was partly funded by Euratom and the UK Department of Trade and Industry, and the work of HLB and BNB was partly supported by the US Dept. of Energy Contract No. DE-FG03-96ER-54346. The authors are grateful to Claude Gormezano, Clive Challis, Joelle Mailloux, Nick Hawkes and the Task Force C for conducting the shear reversal experiments under the framework of JET Joint Undertaking. We thank Tim Hender for discussions, Ambrogio Fasoli, Robert Heeter and Duccio Testa for developing the Alfvén mode diagnostics and Mikhail Pekker for numerical calculations.

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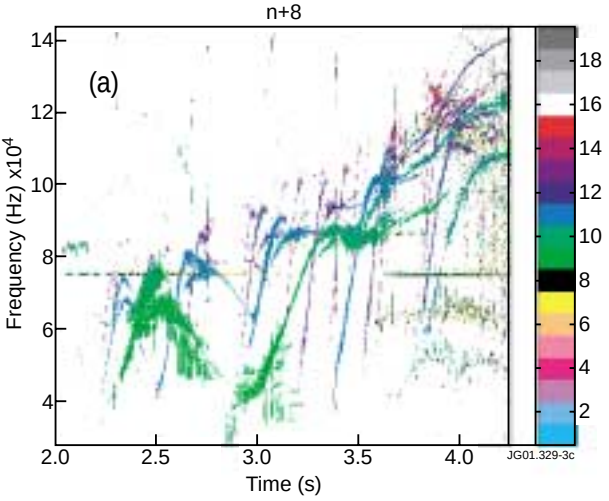
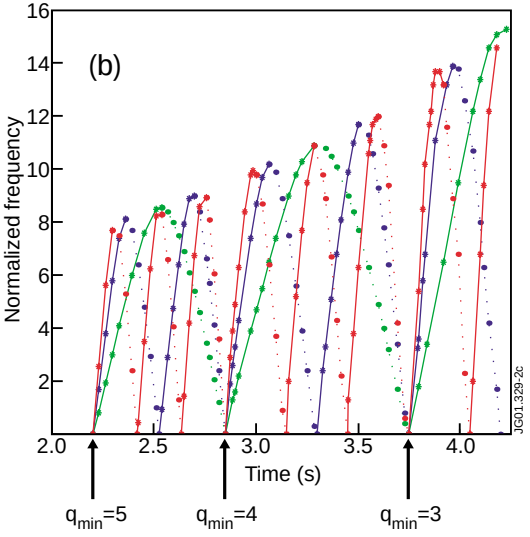
FIGURE CAPTION

Fig. 1a) Spectrogram of the magnetic perturbations, measured by the external Mirnov coils in JET plasma with non-monotonic $q(r)$(pulse #49382). Alfvén cascades of toroidal mode numbers from $n = 1$ through $n = 6$ are observed at frequencies below TAE frequency range, $f_{\mathrm{AC}} \cong$ 30-100 kHz$< f_{\mathrm{TAE}}$. The vertical legend color codes the quantity n + 8.

b) The CSCAS analysis of temporal evolution of the normalized frequency $\omega_A(r_0)R/V_A$ at $q = q_{\mathrm{min}}$ as $q_{\mathrm{min}}(t)$ varies. Mode numbers plotted are: $n = 1$ (green), $n = 2$ (blue) and $n = 3$ (red). Solid curves indicate local maxima of the Alfvén continuum, broken curves indicate local minima of the Alfvén continuum.


n+8
(a)
Frequency (Hz) x10^4
14
12
10
8
6
4
2.0
2.5
3.0
3.5
4.0
Time (s)
18
16
14
12
10
8
6
4
2
JG01.329-3c


(b)
Normalized frequency
16
14
12
10
8
6
4
2
2.0
2.5
3.0
3.5
4.0
Time (s)
q_min=5
q_min=4
q_min=3
JG01.329-2c