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IMPACT OF CLUMPS ON PLASMA STABILITY AND THE NATURE OF TURBULENCE IN A SATURATED STATE

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Abstract

Phase-space density granulations (clumps) are studied using the theory of two-point phase-space density correlation. A novel mechanism of extraction of expansion-free energy is described. This mechanism affects questions pertaining to nonlinear stability. Theories for two-point correlation for the universal mode in a slab geometry with shear and trapped electrons in toroidal geometry are discussed. Results are presented which show destabilization of the universal mode and enhancement of the trapped electron growth rate. An analytic formula for the width of the frequency spectrum is obtained. By specifying the collective resonance damping mechanism, the wavenumber spectrum is also calculated. A formula for energy flux illustrates the impact of clumps on transport and energy confinement.

I. INTRODUCTION

In recent years, it has become increasingly apparent that a description of plasma behavior only in terms of collective normal modes is incomplete, particularly in the turbulent state or near an instability in the presence of fluctuations.[1-4] The inadequacy of the normal mode description of plasma behavior, basically a legacy of linear theory, is underscored by experimental measurements of the fluctuation spectrum of low-frequency turbulence in tokamak plasmas.[5] These measurements of broad frequency spectra at fixed wavenumbers are inconsistent with the normal mode picture in which at saturation the dielectric is zero, the modes are marginally stable (nonlinearly), and hence the collective mode resonance width is very narrow. Absent from this picture are fluctuations which are not phase coherent with the potential and, hence, are not mode-like in nature. These incoherent fluctuations are produced by the mode coupling associated with the nonlinearity. Represented mathematically in Fourier transform space as a convolution with the potential, the nonlinearity thus provides a component of the distribution at wavenumber k which is proportional to the potential at some other wavenumber k'. When fluctuations resonate with particles, the incoherent fluctuations become particle-like in nature: they are a granulation in phase-space resulting from a mixing process in which the conservation of phase-space density along the particle orbit inhibits the intermixing of different densities. This can be understood by considering the correlation of neighboring particles in the presence of the turbulent mixing. Neighboring trajectories diverge due to the mixing process. However, particles closely separated in phase-space remain correlated for a longer time since the turbulent potential each particle sees is nearly the same. The particles in a sufficiently small volume in phase-space stay correlated with each other for a time exceeding the typical correlation time of the turbulence and are thus scattered turbulently as a macroparticle. The mixing process continues, however, and the granulation or clump eventually decays in time. The decay process is offset by the continuous creation of microscale structure caused by the turbulent rearrangement of the average distribution which has a gradient. The granulations are thus said to have a source. Systems having such a scale dependent mixing process and a gradient driven source are common in plasma physics. They include systems in which, for example, ion sound waves and drift instabilities might occur, as well as turbulent plasmas existing in conditions below the threshold of these instabilities, but which nevertheless have a free-energy source. Although historically the description of clumps and associated phenomena has been couched in a kinetic formalism, these considerations and processes are also germane to fluid descriptions, including both one-fluid (MHD)[6] and two-fluid models.[7]

The decay of the microscale correlation, formulated in terms of a relative diffusion process has been extensively studied; a number of investigations have dealt exclusively with relative diffusion.[8] Here, we seek to emphasize the crucial role played by the source. The source is proportional to the rate of relaxation of the average distribution and, hence, to the rate of extraction of expansion-free energy. Through the source, the incoherent fluctuations have access to the expansion-free energy. The incoherent fluctuations act as noise, exciting the collective modes. In the saturated state, the modes are necessarily damped in order to balance the noise excitation. This damping then constitutes the width of the collective resonance centered at the frequency $\omega(k)$ of the collective resonance (mode). The standard view of turbulence in the saturated state as consisting of a spectrum of waves is thus replaced by a description in terms of collective resonances broadened by incoherent noise emission.

The relationship of the source of microscale correlation to the extraction of expansion-free energy provides a new accessibility mechanism and resultant clump-induced instability. The width of the frequency spectrum, as the collective mode's damping response to the instability, is thus strongly related and may be considered a signature of the clump-induced instability. Considerations of free-energy extraction and relaxation of the average distribution also point to a strong effect on transport arising from clumps. Thus, a consideration of the source introduces the issues of plasma stability and transport into the study of clump-related phenomena as well as alters the classical view of steady-state turbulence.

II. EVOLUTION OF THE TWO-POINT CORRELATION

We have already described the action of the turbulent mixing on neighboring phase-space trajectories and the scale-dependent correlation function that results. This correlation is a two-point phase-space density correlation. We seek an equation which describes the evolution of two-point phase-space density correlation under the influence of the turbulent mixing and the source. An equation which correctly describes this evolution cannot be obtained from a standard one-point renormalization theory. A two-point equation constructed from a renormalized one-point equation incorrectly predicts that two points will diffuse independently even when their separation is very small. Following Dupree[1,2], we start with the evolution equation for two-point phase-space density correlation obtained from the Vlasov hierarchy and renormalize the triplet nonlinearity. It is then readily ascertainable that the relative diffusion indeed reflects correlation at small separation.

We have already alluded to measurements of the fluctuation spectrum of low-frequency turbulence in tokamaks[5] as motivation for considering incoherent fluctuations. We formulate the theory of two-point correlation for drift-wave fluctuations, widely considered responsible for anomalous transport processes in tokamaks. In contrast to Dupree, we treat clumps in the presence of collective modes -- the dielectric is zero and the collective resonance shields the clumps, as governed by Poisson's equation. Drift-wave fluctuations occur at frequencies approximately given by the electron diamagnetic drift frequency, ω_{*e}. Incoherent fluctuations, which propagate ballistically, <u>i.e.</u>, are resonant, are therefore generated in the electron species. Collisionless electron dynamics are described by the gyrokinetic equation,

$$\left(\frac{\partial}{\partial t} + \underset{\sim}{V}_d \cdot \underset{\sim}{\nabla}_\perp + v_\parallel \hat{n} \cdot \underset{\sim}{\nabla}\right) g - \frac{c}{B} \nabla\Phi \times \hat{n} \cdot \nabla_\perp g = -\frac{|e|}{T_e} \langle f \rangle \frac{\partial \Phi}{\partial t} - \frac{c}{B_0} \nabla_\perp \Phi \times \hat{n} \cdot \nabla_\perp \langle f \rangle \ , \tag{1}$$

where g is the nonadiabatic part of the fluctuating electron distribution, Φ is the potential, $\langle f \rangle$ is the average distribution, and $\hat{n}$ is the unit vector in the direction of the magnetic field. The velocity $\underset{\sim}{V}_d$ represents the ∇B and curvature drifts. The last term on the left-hand side is the $E \times B$ drift, which is the dominant nonlinearity in the problem. Simplifications of Eq. (1) are appropriate for the problems we shall consider herein. When g is taken to describe a distribution of trapped electrons, a bounce-average is performed. When the bounce frequency represents the fastest time-scale, the parallel gradient term averages to zero and only the ∇B and curvature-drift resonance remains. When g describes the electron distribution for the universal mode, the ∇B and curvature drifts are negigible and the parallel resonance remains. For concreteness, our description of the theory will focus on the later application. Details for the former case are presented in Ref. 4.

From Eq. (1) we construct the Vlasov hierarchy equation for two-point phase-space correlation:

$$\left(\frac{\partial}{\partial t}+\frac{v_{\parallel 1}}{Rq(r_1)}\frac{\partial}{\partial \eta_1}+\frac{v_{\parallel 2}}{Rq(r_2)}\frac{\partial}{\partial \eta_2}\right)\left\langle \hat{g}(\eta_1,\phi_1)\ \hat{g}(\eta_2,\phi_2)\right\rangle$$

$$+\sum_{n}\sum_{n'}\sum_{n''}\Bigg\langle \exp(in\phi_1)\ \exp(in'\phi_1)\ \exp(in''\phi_2)\ \frac{c}{B_0}\sum_{m'} k_\theta k_\theta' \hat{s}(2\pi m')$$

$$\times\ \exp(-2\pi in'qm')\ \hat{\Phi}_{\substack{n'\\ \omega'}}\left(\eta_1+2\pi m'\right)\ \hat{g}_{\substack{n\\ \omega}}(\eta_1)\ \hat{g}_{\substack{n''\\ \omega''}}(\eta_2)\Bigg\rangle$$

$$=\sum_{\substack{n'\\ \omega'}}\frac{i|e|}{T_e}\exp[in'(\phi_1-\phi_2)]\ \langle f(1)\rangle(\omega'-\omega_{*e}')\left\langle \hat{\Phi}_{\substack{-n'\\ -\omega'}}(\eta_1)\ \hat{g}_{\substack{n'\\ \omega'}}(\eta_2)\right\rangle\ . \tag{2}$$

where $k_\theta = rq/r$. We have employed the ballooning representation[9], given by the transformation

$$\left\{\begin{matrix}\Phi\\ g\end{matrix}\right\}=\sum_{n}\exp(in\phi)\sum_{m}\exp(-im\theta)\int d\eta\ \exp[i(m-nq)\eta]\ \left\{\begin{matrix}\hat{\Phi}_n(\eta)\\ \hat{g}_n(\eta)\end{matrix}\right\}, \tag{3}$$

where ϕ is the toroidal angle and η is the variation in the direction of the magnetic field. This eikonal representation extracts the radial variation in $(m - nq)$ as rapid variation; the remaining explicit radial dependence is slow. Thus, for lowest-order theory, the radial variation is contained in η ; $k_r = -k_\theta \hat{s}\eta$, and r enters as a parameter. A compact form for the $E\times B$ nonlinearity is so obtained[10], in which the interaction is between n and n' at r along η .

It is necessary to close the two-point equation, since its nonlinearity contains a three-point correlation. This is accomplished by renormalizing the nonlinearity using the direct interaction approximation. We anticipate the separation of variables into average motion $\phi_+ = \phi_1 + \phi_2$ and relative motion $\phi_- = \phi_1 - \phi_2$. The averaging is performed by integrating over ϕ_+ to yield $\delta(n + n' + n'')$. The nonlinear term T_{12} thus becomes

$$T_{12} = \sum_n \sum_{n'} \Bigg[\exp(in\phi_-) \sum_{m'} k_\theta k'_\theta \hat{s}(2\pi m') \exp(2\pi i n' q m') \left\langle \hat{\Phi}_{-n'} (\eta_1 + 2\pi m') \, \hat{g}_{n+n'}(1) \, \hat{g}_{-n}(2) \right\rangle - \exp(in\phi_- + in'\phi_-) \sum_m k_\theta k'_\theta \times \hat{s}(2\pi m') \exp(2\pi i n' q m') \left\langle \hat{\Phi}_{-n'}(\eta_1 + 2\pi m') \, \hat{g}_{-n'}(\eta_1) \, \hat{g}_{n+n'}(\eta_2) \right\rangle \Bigg] + (1 \rightarrow 2) , \qquad (4)$$

where shielding effects of a third term, $\left\langle \hat{\Phi}_{n+n'} \, \hat{g}_{-n'} \, \hat{g}_n \right\rangle$ are neglected. Wave-particle resonances dominate and the clumps themselves are resonant so that $\hat{g}_{\substack{n+n' \\ \omega+\omega'}} \cong \hat{g}^{(2)}_{\substack{n+n' \\ \omega+\omega'}}$ where

$$\hat{g}^{(2)}_{\substack{n+n' \\ \omega+\omega'}} = L_{\substack{n+n' \\ \omega+\omega'}} \frac{c}{B_0} \sum_{n''} \sum_{m'} (-k''_\theta)(k_\theta + k'_\theta + k''_\theta) \hat{s} \times \exp(2\pi i n'' q m') \left[\hat{\Phi}_{-n''}(\eta_1 + 2\pi m) \, \hat{g}_{n+n'+n''} \right] \qquad (5)$$

and

$$L_{n+n' \atop \omega+\omega'} = \left[-i(\omega + \omega') + i(k_\parallel v_\parallel + k_\parallel' v_\parallel) \right] .$$

Proximity to resonance permits the Markovian approximation, $L_{n+n'} \to L_{n'}$. Selecting from the sum over n'' , the directly-interacting triplet $(n'' = -n')$, we arrive at the renormalized kinetic equation for the two-point phase-space density correlation,

$$\left[\frac{\partial}{\partial t} + \left(\frac{v_{\parallel -}}{Rq(r)} + \frac{v_{\parallel +} r_-}{Rqr(r)} \right) \frac{\partial}{\partial \eta_-} - D_- \frac{\partial^2}{\partial y_-^2} \right] \left\langle \hat{g}(1)\hat{g}(2) \right\rangle$$

$$= \sum_{k' \atop \omega'} \frac{i|e|}{T_e} \exp(ik'y_-)(\omega' - \omega'_{*e}) \langle f(1) \rangle \left\langle \hat{g}(2)\hat{\Phi}(1) \right\rangle_{n' \atop \omega'} , \qquad (6)$$

where $y = r\phi/q$, $D = 2D - D^{(2,1)} - D^{(1,2)}$,

$$D = \frac{c^2}{B_0^2} \sum_{k' \atop \omega'} k_\theta'^2 \hat{s}^2 \left(\mathrm{Re} L_{k' \atop \omega'} \right) \sum_m (2\pi m)^2 \left\langle \hat{\Phi}(\eta + 2\pi m)^2 \right\rangle_{k' \atop \omega'} \qquad (7)$$

and

$$D^{(1,2)} = \frac{c^2}{B_0^2} \sum_{k' \atop \omega'} \exp(ik'y_-)\, k_\theta'^2 \hat{s}^2 \left(\mathrm{Re} L_{n' \atop \omega'} \right) \sum_m (2\pi m)^2$$

$$\times \exp(2\pi i m r_- k' \hat{s}) \left\langle \hat{\Phi}(\eta_1 + 2\pi m)\hat{\Phi}(\eta_2 + 2\pi m) \right\rangle . \qquad (8)$$

The diffusion coefficient D represents independent diffusion which is derived from a one-point renormalization, and hence characterizes coherent mode coupling. The coefficients $D^{(1,2)}$ and $D^{(2,1)}$

represent the correlated diffusion caused by incoherent mode coupling. Correlated diffusion vanishes for large relative separation and approaches 2D as the separation goes to zero. The relative diffusion D_- thus tends to zero as the separation of neighboring orbits approaches zero. The scales over which $D^{1,2} \neq 0$ as determined from Eq. (8) define the scale of the clump. For phase-space separations small compared to the clump scale, the relative diffusion is quadratic in the relative phase-space variables,

$$D_- = k_0^2 \left(y_-^2 + \hat{s}^2 r_-^2 + \frac{\eta_-^2}{k_0^2 \Delta\eta^2} \right) \qquad (9)$$

where k_0^{-1}, $k_0^{-1}\hat{s}^{-1}$, and $\Delta\eta$ are the clump scales for the toroidal, radial, and parallel variations, respectively.

The evolution of the separation of neighboring trajectories permits a determination of the lifetime of microscale correlations. The time required for the separation to reach clump scales from an initially smaller separation defines the clump lifetime, $\tau_{c\ell}$;

$$\tau_{c\ell} = \frac{\tau_c}{2} \ln \left\{ k_0^{-2} \left[y_-^2 + \hat{s}^2 r_-^2 + \frac{\eta_-^2}{k_0^2 \Delta\eta^2} + \frac{\tau_c}{k_0^2 (\Delta\eta)^2} \eta_- \left(v_{\parallel -} + \frac{v_\parallel r_-}{r} \right) \frac{1}{Rq(r)} + \frac{\tau_c^2}{2 k_0^2 \Delta\eta^2} \frac{1}{[Rq(r)]^2} \left(v_{\parallel -} + \frac{v_\parallel r_-}{r} \right)^2 \right]^{-1} \right\} \qquad (10)$$

where τ_c is the correlation time of the coherent fluctuations for scales comparable to the clump scale $\left(\tau_c = 1/k_0^2 D\right)$. The clump lifetime exhibits the logarithmic peaking in the phase-space variables which is characteristic of the exponential separation of trajectories occuring over most of the clump scale. An earlier assertion that particles in a sufficiently small volume stay correlated for a time $(\tau_{c\ell})$ exceeding the correlation time (τ_c) of the turbulence is now obvious from Eq. (5).

III. THE SOURCE

The clump lifetime $\tau_{c\ell}$ is the decay time of the microscale correlation, which characterizes the evolution of the two-point equation, Eq. (2). In the steady state, the solution of the two-point equation is given approximately by $\langle g(1)g(2)\rangle = \tau_{c\ell} S_{12}$. The decay is counteracted by the source which drives the microscale correlation,

$$S_{12}(\kappa_1,\kappa_2,v_\parallel) = \frac{i|e|}{T_e} \sum_{\substack{k' \\ \omega'}} \exp(ik'y_-)\left(\omega' - \omega'_{*e}\right)\langle f(1)\rangle \left\langle \hat{g}(\kappa_2)\hat{\Phi}(\kappa_1)\right\rangle_{\substack{k' \\ \omega'}} . \tag{11}$$

For convenience, we work in the space of κ_1 and κ_2 obtained from Fourier transforming,

$$S_{12}(\kappa_1,\kappa_2) = \int d\eta_1 \exp\left(i\kappa_1\eta_1\right) \int d\eta_2 \exp\left(i\kappa_2\eta_2\right) S_{12}(\eta_1,\eta_2) .$$

The source is proportional to the rate of relaxation of the average distribution $S = -\langle f\rangle\, \partial\langle f\rangle/\partial t$. Consequently, associated with the driving of the microscale correlation is the extraction of the expansion-free energy stored in the density gradient of the average distribution. A detailed picture giving insight into the mechanism by which the expansion-free energy is made accessible through the source is provided by an analogy with the physics of discreteness as described in the Balescu-Lenard equation.[1,2] This analogy is suggested when the source is separated into coherent and incoherent components. We recall that

$$\hat{g}_k(\kappa) = \hat{g}_k^{(c)}(\kappa) + \tilde{g}_k(\kappa) \tag{12}$$

where

$$\hat{g}_k^{(c)}(\kappa) = \frac{-(\omega - \omega_{*e})}{[\omega - (\kappa v_{\parallel}/Rq)]} \frac{|e|}{T_e} \langle f \rangle \hat{\Phi}_k(\kappa) \tag{13}$$

is the coherent response and $\tilde{g}_k(\kappa)$ is the incoherent fluctuation. Substituting Eqs. (12) and (13) into Eq. (11), the source is written as

$$S_{12} = \sum_{\substack{k' \\ \omega'}} \left[(\omega' - \omega'_{*e})^2 \frac{|e|^2}{T_e^2} \langle f(1) \rangle \langle f(2) \rangle \pi \delta\left(\omega' - \frac{\kappa v_{\parallel}}{Rq}\right) \left\langle \hat{\Phi}(1) \hat{\Phi}(2) \right\rangle_{k', \omega'} \right.$$

$$\left. + \frac{i|e|}{T_e} (\omega' - \omega'_{*e}) \langle f(1) \rangle \left\langle \tilde{g}(2) \hat{\Phi}(1) \right\rangle_{k', \omega'} \right] . \tag{14}$$

The first term is the coherent component and the second is the incoherent component. In writing the first term, we anticipate that we will require the real part of the source; hence, we retain only the residue contribution $i\pi\delta(\omega - \kappa v_{\parallel}/Rq)$ from the pole $(\omega - \kappa v_{\parallel}/Rq)^{-1}$.

Since $S_{12} = -\langle f \rangle (\partial \langle f \rangle / \partial t)$, we associate with the right-hand side of Eq. (14) the processes that drive the evolution of $\langle f \rangle$. The equation $\partial \langle f \rangle / \partial t = S_{12} / \langle f \rangle$ is analogous to the Balescu-Lenard equation and allows us to identify with the coherent and incoherent components of the source the processes of diffusion and drag, respectively. The diffusion is quasi-linear diffusion, except that here the coherent response, through Poisson's equation, shields the clump. The drag term represents the friction experienced by the clump as it emits Cherenkov-like radiation while moving through the plasma.

The analogy with particle collisions is useful in understanding an important cancellation which occurs in the source. To see the cancellation, it is necessary to express the clump-shielded potential $\hat{\Phi}$ in terms of the incoherent potential $\tilde{\Phi}_k = 4\pi q \int dv_\parallel \tilde{g}_k(\kappa, v_\parallel)$. The relationship between $\hat{\Phi}$ and $\tilde{\Phi}$ is obtained from Poisson's equation $L_k(\kappa)\hat{\Phi}_k(\kappa) = \tilde{\Phi}_k$ where $L_k(\kappa)$ is the eigenmode operator. We further recall that clumps are resonant, moving at the ballistic velocity $u = \omega / k_\parallel = \omega R q / \kappa$. Thus,

$$\left\langle \tilde{g}(2)\hat{\Phi}(1) \right\rangle_{k',\omega'} = 2\pi\delta(\omega - k_\parallel' v_\parallel) L_k^{-1}(\kappa_1) \left\langle \tilde{g}(2)\hat{\Phi}(1) \right\rangle_{k'} ,$$

$$\left\langle \hat{\Phi}(1)\hat{\Phi}(2) \right\rangle_{k',\omega'} = L_{-k'}^{-1}(\kappa_1) L_{k'}^{-1}(\kappa_2) \int dv_\parallel \left\langle \tilde{g}(2)\tilde{\Phi}(1) \right\rangle_{k'} 2\pi\delta(\omega' - k_\parallel' v_\parallel)$$

$$= \frac{2\pi}{|k_\parallel|} L_{-k'}^{-1}(\kappa_1) L_{k'}^{-1}(\kappa_2) \left\langle \tilde{g}(u')\tilde{\Phi} \right\rangle_{k'} \qquad (15)$$

and

$$S_{12} = \sum_{\substack{k' \\ \omega'}} \left\{ \left[\frac{|e|}{T_e} \langle f(u) \rangle (\omega' - \omega'_{*e}) 2\pi \delta(\omega' - k_\parallel u) L^{-1}_{-k'}(\kappa_1) \, L^{-1}_{k'}(\kappa_2) \right] \right.$$

$$\left. \times \left[\frac{|e|}{T_e} (\omega' - \omega'_{*e}) \frac{\pi}{|k_\parallel|} \langle f(u) \rangle \left\langle \tilde{g}(u') \tilde{\Phi} \right\rangle_{k'} + i L_{k'}(\kappa_2) \left\langle \tilde{g}(u) \tilde{\Phi} \right\rangle_{k'} \right] \right\} . \tag{16}$$

From the imaginary part of $L_{k'}(\kappa)$ comes the drag term's contribution to the source. This consists of the electron and ion dissipation involved in the mode. The electron dissipation, representing the electron-electron drag, is that of inverse Landau damping; the ion dissipation, representing the electron-ion drag, consists of the shear damping (linear) and the nonlinear damping (here, ion compton scattering) which satrurates finite amplitude turbulence. Since the diffusion term is proportional to the electron dissipation, the diffusion cancels with the electron-electron drag. This cancellation is generic to clump phenomena and reflects the shielding of clumps by collective modes. For quasi-neutral fluctuations, the shielding of a single clump species, say electrons, is expressed by a shielded quasi-neutrality condition, $n_e = n_e^{(c)} + \tilde{n}_e = -n_i^{(c)}$. Rewriting the right-hand side as an ion response function times the potential, the imaginary part of this condition yields a relation between electron density and ion dissipation. This velocity-averaged shielding relation is extended to phase-space densities because ballistic, particle-like clumps select from the velocity continuum a single velocity, projecting onto the configuration space. This leaves only the drag with the ions, so that

$$S_{12} = \sum_{\substack{k' \\ \omega'}} \left\{ \left[\langle f(u) \rangle \left(\omega' - \omega'_{*e}\right) 2\pi\delta\left(\omega' - k_{\parallel} u\right) \; i\varepsilon_{IM}^{ION} \left(k', \omega', \kappa_2\right) \right] \right.$$

$$\left. \times \; L^{-1}_{-k'}(\kappa_2) \; L^{-1}_{k'}(\kappa_2) \left\langle \tilde{g}(u')\tilde{n} \right\rangle_{k'} \right\} \quad , \qquad (17)$$

where $i\varepsilon_{IM}^{ION}$ is the ion dissipation. The scaling of the source with ion dissipation reflects emission by an electron clump due to its drag on the ions.

A final form for the source is obtained by expressing the inverse eigenfunction operators L^{-1} in terms of the wave function $\Psi_{\substack{k \\ \omega}}(\kappa)$ and dielectric response $\varepsilon(k,\omega)$,

$$L^{-1}_{\substack{k' \\ \omega'}} = \frac{\Psi_{\substack{k' \\ \omega'}}(\kappa_1)}{N_{\substack{k' \\ \omega'}} \varepsilon(k', \omega')} \int d\kappa' \Psi_{\substack{k' \\ \omega'}}(\kappa') \quad ,$$

where $N_{\substack{k' \\ \omega'}}$ is the normalization constant. Thus,

$$S_{12} = \sum_{\substack{k' \\ \omega'}} 2\pi \frac{\langle f(u) \rangle \left(\omega' - \omega'_{*e}\right) \varepsilon_{IM}^{ION}(k', \omega') \left| \Psi_{\substack{k' \\ \omega'}}(\kappa) \right|^2}{|\varepsilon(k', \omega')|^2} \left\langle \overline{\tilde{g}(u)\tilde{n}} \right\rangle_{k'} \qquad (18)$$

where

$$\left\langle \overline{\tilde{g}(u)\tilde{n}} \right\rangle_{k'} = \frac{1}{\left| N_{\substack{k' \\ \omega'}} \right|^2} \int d\kappa_1 \int d\kappa_2 \; \Psi_{\substack{k' \\ \omega'}}(\kappa_1) \; \Psi_{\substack{-k' \\ -\omega'}}(\kappa_2) \left\langle \tilde{g}(u)\tilde{n} \right\rangle_{k'} \; .$$

For electron phase-space granulations in low-frequency (ω_{*e})

turbulence, the consequences of Eq. (18) and its scaling with ion dissipation are important and account for the significant impact of clumps on the collective modes. As mentioned, the electron dissipation is destabilizing $\left(\varepsilon_{IM}^{ELEC} > 0\right)$ and the ions are stabilizing $\left(\varepsilon_{IM}^{ION} < 0\right)$. The ion dissipation includes the linear stabilization mechanisms (shear damping) and amplitude dependent damping mechanisms such as ion Compton scattering and is large, as it must balance the linear instability and its enhancement by the incoherent excitation. With $\omega < \omega_{*e}$, a large, positive source is thus provided for the electron microscale correlation.

IV. THE SPECTRUM BALANCE

The steady-state solution of the two-point phase-space density equation is given approximately by $\langle gg \rangle = \tau_{c\ell} S$. The incoherent part of the correlation is obtained from the total correlation by extracting the coherent part, and the $\langle g^{(c)}\tilde{g} \rangle$ cross correlation. Hence:

$$\langle \tilde{g}\tilde{g} \rangle = (\tau_{c\ell} - \tau_c) S \quad . \tag{19}$$

In the last section the source was expressed in terms of $\langle \overline{\tilde{g}(u)\tilde{n}} \rangle_{k'}$, the projection on the eigenfunctions of the incoherent part of velocity-integrated correlation. The quantity $\tilde{g}(u)$ results from the velocity integration of the incoherent density for the ballistic clump with velocity $v = u = \omega/k_\parallel$. By performing the velocity integrations, Fourier transforms in η and y, and eigenfunction projection on both sides of Eq. (19), it is possible to express Eq. (19) entirely in terms of $\langle\overline{\tilde{g}(u)\tilde{n}}\rangle_k$. The relationship between $\langle\tilde{g}(u)\tilde{n}\rangle_k$ and $\langle\tilde{g}\tilde{g}\rangle_k$ is expressed by

$$\left\langle\tilde{n}\tilde{n}\right\rangle_{\substack{k\\\omega}} =$$

$$\int dv_{\parallel +}\int dv_{\parallel -}\ 2\pi\delta\left(\omega - k_\parallel v_{\parallel +}\right)\int dy_-\ \exp(-iky_-)\int d\eta_-\ \exp(-i\kappa\_\eta_-)\left\langle\tilde{g}\tilde{g}\right\rangle_k$$

and

$$\int dv_{\parallel +}\ 2\pi\delta\left(\omega - k_\parallel v_{\parallel +}\right)\left\langle\tilde{g}(u)\tilde{n}\right\rangle_k = \frac{2\pi}{|k_\parallel|}\left\langle\tilde{g}\left(\frac{\omega}{k_\parallel}\right)\tilde{n}\right\rangle_k = \left\langle\tilde{n}\tilde{n}\right\rangle_{\substack{k\\\omega}}\ ,$$

where the two time correlation $\langle\tilde{n}\tilde{n}\rangle_{\substack{k\\\omega}}$ is obtained approximately from the one time correlation by operating with the propagator $2\pi\delta(\omega - k_\parallel v_\parallel)$ which represents ballistic propagation. Using these equations and the eigenfunction projections, we obtain the spectrum balance equation,

$$\left\langle\overline{\tilde{g}(u)\tilde{n}}\right\rangle_k = \int d\kappa_1\int d\kappa_2\ \frac{\Psi_k(\kappa_1)\Psi_{-k}(\kappa_2)}{|N_k|^2}\int dv_{\parallel -}$$
$$\times\int dy_-\ \exp(-iky_-)\int d\eta_-\ \exp(-i\kappa\eta_-)\ (\tau_{c\ell} - \tau_c)S_{12}\ ,\quad (20)$$

where $\tau_{c\ell}$ and S_{12} are given by Eqs. (10) and (18). The integration over $v_{\parallel -}$ has been given previously.[1] The Fourier transforms are performed with the aid of a polar transformation to variables ℓ and θ : $\ell\cos\theta = e\eta_-/\Delta\eta\sqrt{2}$; $\ell\sin\theta = k_0 e y_-$. A Bessel function expansion of $\exp(-i\kappa\eta_- - iky_-)$ then facilitates the θ integration to yield

$$\int dv_{\parallel -} \int d\eta_- \exp(i\kappa_-\eta_-) \int dy_- \exp(iky_-) \left(\tau_{c\ell} - \tau_c\right) =$$

$$\frac{16\pi\Delta\eta^2 Rq}{e^3 k_0^2} \cdot \frac{1}{k_\perp} \int_0^1 d\ell \cos^{-1}\ell \; J_1(k_\perp \ell)\ell \equiv a(k_\perp) \quad , \qquad (21)$$

where $k_\perp = \sqrt{(2\kappa_-^2\Delta\eta^2/e^2) + k^2/(k_0^2 e^2)}$. Combining this result with Eq. (20) gives

$$\left\langle \overline{\tilde{g}(u)\tilde{n}} \right\rangle_k = 2\pi \int d\kappa_1 \int d\kappa_2 \frac{\Psi_k(\kappa_1)\Psi_{-k}(\kappa_2)}{|N_k|^2} a(k_\perp) \langle f(u) \rangle$$

$$\int \sum_{\omega'} dk'(\omega' - \omega_{*e}') \Psi_{k'}(\kappa_1)\Psi_{-k'}(\kappa_2) \frac{\varepsilon_{IM}^{ION}(k',\omega')}{|\varepsilon(k',\omega')|^2} \left\langle \overline{\tilde{g}(u)\tilde{n}} \right\rangle_{k'} \quad . \qquad (22)$$

This equation describes the detailed steady-state balance in the spectrum between linear destabilization, the enhancement by incoherent emission and the linear and nonlinear damping mechanisms. It plays an analogous role in the two-point theory to the wave-kinetic equation of weak turbulence theory. One important difference should be noted. Because $\int dv_\parallel \tau_{c\ell}$ is amplitude independent as Eq. (21) indicates, and because the source is proportional to $\langle \tilde{g}(u)\tilde{n} \rangle_{k'}$, the incoherent fluctuation amplitude appears to scale out of the balance. The incoherent emission process is ostensibly independent of fluctuation amplitude above some nominal level and the spectrum is determined only up to an unspecified function $N(k)$. However, because ε_{IM} details the balance between destabilization mechanisms and the linear and nonlinear amplitude-dependent damping, the spectrum balance equation does, in fact, depend on the collective mode amplitude. In the steady state it is possible, by supplying the details of the nonlinear damping process to determine N and completely specify the spectrum. Such a calculation will be outlined in the next section.

For this resonant system, we may expand the dielectric $|\varepsilon(k',\omega')|^2$ in the denominator of Eq. (22) about the eigenfrequency, assuming that the spectrum broadening, or damping, at saturation is not too large,

$$\varepsilon(k',\omega') = \left[k' - k_r'(\omega')\right] \frac{\partial\varepsilon}{\partial k'} + i\varepsilon_{IM}\left[k_r'(\omega),\omega'\right]$$

where $\varepsilon_r\left[k_r(\omega),\omega\right] = 0$. We perform the k' integration, evaluating the residue at the pole corresponding to the eigenmode. The correlation then cancels out of the spectrum balance leaving an equation expressing the relation between the total dissipation $\varepsilon_{IM}(k,\omega_k)$ and the dissipation in the ions, $\varepsilon_{IM}^{ION}(k,\omega_k)$,

$$|\varepsilon_{IM}(k,\omega_k)| = C(k,\omega_k)\ |\varepsilon_{IM}^{ION}(k,\omega_k)| \tag{23}$$

where

$$C(k,\omega_k) = 2\pi\int d\kappa_1 \int d\kappa_2\ \frac{|\Psi_k(\kappa_1)|^2\ |\Psi_{-k}(\kappa_2)|^2}{|N(k)|^2}\ a(k_\perp)\ \langle f(u)\rangle$$
$$\times\ \left(\omega_k - \omega_{*e}\right) \left|\frac{\partial\varepsilon}{\partial k}\right|_{k=k(\omega)} . \tag{24}$$

The integrations over κ_1 , and κ_2 are transformed to κ_+ and κ_- , and the integrations then allow the shielding response structure functions to sample the dependence of the clump on the two parallel scales. The evolution or clump decay enters into the fast scale (κ_-) sampling. The slow scale (κ_+) sampling reflects the degree to which the mode structure in η shields the clumps.

Since the total dissipation is composed of electron and ion contributions, $\varepsilon_{IM} = \varepsilon_{IM}^{ELEC} - |\varepsilon_{IM}^{ION}|$ and $\varepsilon_{IM} < 0$, as collective resonances must be damped to balance noise emission, we may rewrite Eq. (23) to obtain a saturation condition:

$$\varepsilon_{IM}^{ION}(k,\omega_k) = \frac{\varepsilon_{IM}^{ELEC}(k,\omega_k)}{[1 - C(k,\omega_k)]} \quad . \tag{25}$$

This expresses the balance in the steady state between the linear electron destabilization of inverse Landau damping enhanced by the incoherent noise emission $[1 - C(k,\omega_k)]^{-1}$ and the linear and nonlinear damping in the ions. A related expression,

$$\varepsilon_{IM}(k,\omega_k) = \frac{C(k,\omega_k)}{[1 - C(k,\omega_k)]}\,\varepsilon_{IM}^{ELEC}(k,\omega_k) \tag{26}$$

gives the total dissipative mode response to the incoherent emission process in terms of a numerical enhancement of the electron dissipation. The width of the frequency spectrum at fixed k is just $\varepsilon_{IM}/(\partial\varepsilon/\partial\omega)$ and is given by

$$\Delta\omega_k = \gamma_k^{ELEC}\frac{C(k,\omega_k)}{[1 - C(k,\omega_k)]} \quad . \tag{27}$$

V. ILLUSTRATIONS

We first report the results obtained for stability of the universal mode in the presence of incoherent emission. For comparison, we then consider a different problem, that of trapped electrons in toroidal geometry, and obtain formulas for the frequency and wavenumber spectrum as well as energy flux for the energy transport problem.

The universal mode is a density gradient-driven drift mode in a slab geometry with a sheared magnetic field $\underset{\sim}{B} = B_0(\hat{z} + x/L_s\hat{y})$. The linear properties of this mode have been extensively investigated. Destabilization is provided by electrons which resonate with the wave. Ion inertia is stabilizing, and is increasingly effective for stronger shear. It is now well established that the universal mode is linearly stable for all values of wavenumber and shear.[11,12] The level of thermal fluctuations in an ohmically-discharged plasma is sufficient to trigger incoherent fluctuations which tap the expansion-free energy as already described. The level of incoherent fluctuations grows, exciting the collective modes which reach finite amplitude. Nonlinear ion Compton scattering provides saturation and the overall damping necessary to maintain the steady state.

In previous sections, we have outlined the calculation of the incoherent spectrum relevant to the univeral mode. Using the ballooning representation, the radial eigenvalue problem is re-expressed as an eigenvalue problem in η . The x and y variations of the slab geometry pass over into the η and $y = \phi r/q$ variations of the ballooning representation. Hence, the κ_+ and κ_- integrations of Eq. (24) for the clump-enhanced growth factor $C(k,\omega_k)$ provide a sampling of radial structure by the radial eigenmode shielding response.

Assuming Pearlstein-Berk mode structure by writing the shielding response as a Gaussian, $\Psi_k(\kappa) = \exp(-\alpha\kappa^2)$ (where α is, in general, complex) the κ_+ and κ_- integrations may be performed using saddle-point contour methods. With normalized wavefunctions, the final answer is relatively insensitive to the detailed value chosen for α, beyond the usual scaling of L_s/L_n, the ratio of shear length to density scale length which reflects the mode width of Pearlstein-Berk structure. The final result for $C(k,\omega_k)$ is

$$C(k,\omega_k) \cong 100(\Delta\eta)^2 \left(\frac{Rq}{L_n}\right) \frac{k\rho_s}{(1 + k^2\rho_s^2)} \cdot \frac{k_0}{k} \left(\frac{m_e}{2m_i}\right)^{1/2} . \tag{28}$$

The mass ratio dependence arises from a phase-velocity scaling $(\omega/k_\parallel v_{te})$ which reflects the emission process of ballistic electron clumps into modes at frequency ω_k. We note that the incoherent emission process occurs over the entire η extent of the mode, in contrast to another theory of nonlinear destabilization of the universal mode[13] which relies on effects within a small layer around the rational surface.

Evaluating Eq. (28) for parameter values consistent with the universal mode, we obtain a quantitative measure of the impact of clumps on the universal mode stability. We rewrite the saturation condition, Eq. (25), separating the nonlinear and linear damping in the ion term and writing the nonlinear damping rate at saturation as a function of linear dissipation:

$$\gamma_{n\ell}^{ION} = \gamma\big|_{C.I.} = \frac{\varepsilon_{IM}^{ELEC}(k,\omega_k)/(\partial\varepsilon/\partial\omega_k)}{[1 - C(k,\omega_k)]} - \frac{\varepsilon_{IM,\ell}^{ION}(k,\omega_k)}{(\partial\varepsilon/\partial\omega_k)} \quad . \qquad (29)$$

The nonlinear destabilization is the clump-induced growth rate. When positive, it indicates that incoherent emission has driven the modes to finite amplitude, despite shear damping, and has effectively destabilized the mode. The clump-induced growth rate is expressed in terms of the linear dispersion function. This reflects the fact that it is the response function with its associated dissipative processes which shields the clumps. Furthermore, we assume that the response structure in the nonlinear regime is well appoximated by the structure of the linear response. The validity of this assumption is verifiable *a posteriori* in terms of the magnitude of the ratio, $\gamma|_{C.I.}/\omega_{*e}$, and is already evident in Eq. (28) where the small parameter (u/v_{te}) plays a central role.

The evaluation of Eq. (29) is displayed graphically in Fig. 1, where $\gamma|_{C.I.}$ is plotted as a function of $k\rho_s$ and compared with $IM\,\omega(k)$ from linear theory. It is seen that clumps effectively destabilize an otherwise stable mode. A similar effect has been predicted and observed in simulations of a plasma below the threshold of the ion-acoustic instability.[14] In contrast to our calculation of the steady-state, clump-induced destabilization rate of a finite amplitude mode, Berman *et al.*[14] determine the nonlinear growth of the incoherent fluctuation $\langle\tilde{g}\tilde{g}\rangle$ [they solve for $\gamma = d/dt$ in Eq. (6)] away from conditions satisfying the linear dispersion. In the universal mode, the destabilization is not particularly large; for trapped electrons, however, the effect is more significant.

We consider electrons trapped in the magnetic mirrors created by the spiraling of the magnetic field on the toroidal magnetic flux surfaces of tokamak geometry. Such trapped electrons have banana-shaped orbits in a poloidal plane. The collisionless trapped electrons cause an unstable mode. In treating the dynamical equations, considerable simplification is obtained by performing the bounce average, which is possible when the trapped electron bounce frequency is associated with the fastest time scale in the problem. Further details are found in Ref. 4. The formula for the width of the frequency spectrum is given by Eq. (27), for trapped electrons,

$$C(k,\omega_k) = 2\sqrt{\pi}\, A(k\rho_s) \frac{\mathscr{S}_k}{\varepsilon_T} \exp\left(\frac{-\omega_k}{\bar{\omega}_d}\right)\left(\frac{\omega_k}{\bar{\omega}_d}\right)^{1/2}\left(1 - \frac{\omega_k}{\omega_{*e}}\right) \tag{30}$$

where $A(k\rho_s) = (k^2\rho_s^2)^{-1}[1 - J_0(\sqrt{2}\, k/k_0)]$, $\bar{\omega}_d = \varepsilon_T\omega_{*e}$ is the ballistic frequency of the clump corresponding to the phase velocity $u = \omega/\bar{\omega}_d$, and $\varepsilon_T = L_n/R$. As with the universal mode, phase-velocity dependence enters the relation, underscoring the emission process of ballistic clumps into modes at ω_k . The dependence on the shielding-response wavefunction $\mathscr{S}_k$ indicates that shielding-response structures which experience greater overlap with regions of clump activity allow for more efficient emission. For parameters consistent with the toroidicity-induced mode structure[15], the frequency broadening is computed to be $\Delta\omega_k/\omega_k \sim 1.1$. This broad line width is indicative of a strong enhancement of the linear growth due to the incoherent emission process.

As mentioned in the previous section, the spectrum may be completely specified from the saturation condition, Eq. (10), by supplying the details of the nonlinear damping process. We consider ion Compton scattering as the process of turbulent energy transfer. The nonlinear ion damping rate is obtained from perturbation theory, as it is in weak turbulence theory, however, the integral over frequency is performed using the broadened frequency spectrum $\Delta\omega_k/[(\omega - \omega_k)^2 + \Delta\omega_k^2]$ obtained from the two-point theory. The frequency broadening is responsible for broadening the beat resonance, inducing dissipation in the background fluctuations as well as altering the spectrum spatial structure. The net impact of these effects is an enhancement of the nonlinear wave-ion interaction. This may be interpreted as a spreading of the ion-Landau resonance point due to the uncertainity in ω represented by $\Delta\omega_k$. From these considerations, the wavenumber spectrum is determined and found to scale as $N(k_\perp\rho_s) \sim (k_\perp\rho_s)^{-3/2}$ asymptotically.

The fact that incoherent fluctuations are driven by the relaxation of the average distribution according to the drag-induced enhancement of free-energy accessibility implies that the quasi-linear prescription of transport is no longer valid. A direct calculation of the flux Γ of energy transport $(\partial E/\partial t + \Gamma\nabla\cdot E = S)$ shows that the flux is proportional to the clump-enhanced growth rate

$$\Gamma \propto \frac{\varepsilon_{IM}^{ELEC}}{[1 - C(k,\omega_k)]} , \qquad (31)$$

rather than γ_L, as in conventional quasi-linear theory. The enhancement of energy flux is obviously strong where significant broadening of the frequency spectrum occurs.

VI. CONCLUSIONS

We have considered herein the incoherent part of the fluctuating density, a constituent of turbulence usually neglected in theoretical treatments. These fluctuations are identified with clumps, granulations in the phase-space density resulting from the properties of the mixing process. We have used the theory of two-point phase-space density correlation as the natural vehicle for treating incoherent fluctuations. The source term has been emphasized and discussed in detail. In particular, we have shown that the driving of the microscale correlation by the source is proportional to the relaxation of the average distribution. Hence, we identify a novel mechanism for the extraction of expansion-free energy. The effect of incoherent fluctuations on the collective modes has been examined. The effect has been described as an emission process which excites the modes. We have considered the steady state in which the modes are damped to balance this emission. We obtain from the net damping in the steady state an analytic formula for the frequency spectrum at fixed k. The theory, then, effectively links the issue of the width of the frequency spectrum with nonlinear stability associated with incoherent fluctuations. Indeed, the width of the frequency spectrum provides a measure of the strength of the nonlinear instability.

We consider two drift-type modes in realistic geometries to obtain a quantitative measure of the impact of clumps. For the universal mode, we find that the mode is destabilized and reaches finite amplitude due to incoherent emission. The destabilization, however, is a small effect with $\gamma|_{C.I.}/\omega_{*e} \sim 0.02$ for $L_s/L_n = 32$. For trapped electrons, the impact of incoherent emission can be significant. In this case, with toroidicity-induced mode structure, the frequency spectrum is broad $(\Delta\omega_k/\omega_{*e} \sim 1.1)$. Considering the details of ion Compton scattering necessary to achieve saturation, we are able to completely specify the spectrum and show the wavenumber dependence. Finally, the effect on transport is assessed with the result that incoherent emission enhances the transport of energy increasing the flux by a numerical factor $(1 - C)^{-1}$.

Acknowledgment

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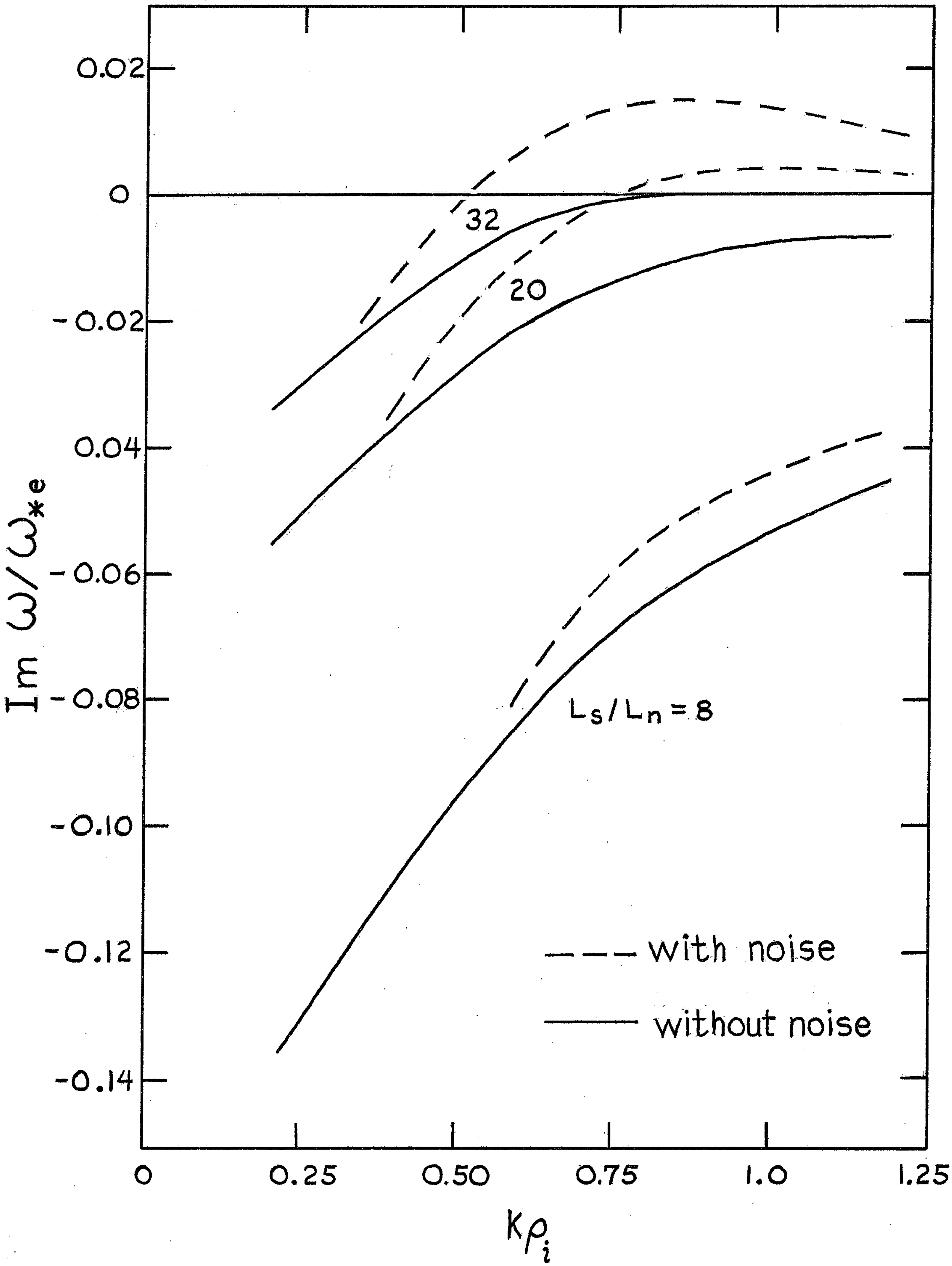
Figure Caption

The clump-induced growth rate normalized by ω_{*e} as a function of $k_{\perp}\rho_s$ for three values of the shear, compared to the linear growth rate in which incoherent fluctuations (noise) are neglected.


0.02
0
-0.02
0.04
-0.06
-0.08
-0.10
-0.12
-0.14
Im ω/ω*e
32
20
Ls/Ln = 8
with noise
without noise
0
0.25
0.50
0.75
1.0
1.25
kρi