REDUCED MAGNETOHYDRODYNAMICS AND THE HASEGAWA-MIMA EQUATION

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Abstract

Reduced magnetohydrodynamics consists of a set of simplified fluid equations which has become a principal tool in the interpretation of plasma fluid motions in tokamak experiments. The Hasegawa-Mima equation is applied to the study of electrostatic fluctuations in turbulent plasmas. The relation between these two nonlinear models is elucidated. It is shown that both models can be obtained from appropriate limits of a third, inclusive, nonlinear system. The inclusive system is remarkably simple.

I. INTRODUCTION

This note examines the relation between two intensively studied fluid models of nonlinear plasma dynamics.

The first model, reduced magnetohydrodynamics (RMHD), originated in the work of Rosenbluth, Strauss, Monticello, and White[1], and was importantly generalized and developed by Strauss.[2,3] Its resistive, low-beta version, which we consider, describes the nonlinear dynamics of kink and tearing instabilities in large aspect-ratio tokamak geometry. Numerical solution of the RMHD equations reveals an evolution of magnetic flux surfaces and plasma flows in remarkably close agreement with important observed phenomena, including tokamak plasma disruption.[4]

The second model was constructed by Hasegawa and Mima[5] to describe electrostatic plasma turbulence in slab geometry. We refer to it as "CHM" because the same equation was derived earlier, in an entirely different context, by Charney.[6] Various versions of CHM -- we consider only the simplest -- have been applied to analytical and numerical investigations of drift-wave turbulence and solitary wave phenomena.[5,7]

The two models differ sharply in their intentions, in their assumed orderings, and in the fact that one is inherently electromagnetic, the other strictly electrostatic. However, the derivations of the two models, which are outlined in Sec. II, can be made sufficiently parallel as to suggest that CHM might be the electrostatic limit of RMHD. In fact, the relation between RMHD and CHM is not quite this simple: the "smallest" nonlinear system which includes both RMHD and CHM as limiting cases is necessarily "larger" -- in the sense of involving an additional, independent field variable -- than RMHD.

Section III is devoted to a derivation of the inclusive nonlinear model. Its conclusion, which is discussed further in Sec. IV, may be summarized as follows. RMHD involves two fields, the normalized magnetic flux, ψ , and the normalized electrostatic (or velocity flow) potential, ϕ . Of course, CHM involves only ϕ . The inclusive system requires three fields: ψ , ϕ , and χ , where χ measures the plasma density perturbation. The new field appears with a coupling coefficient,

$$\alpha = \frac{\rho_s^2}{a^2} . \qquad (1)$$

Here, a represents the presumed scale length for variation perpendicular to the magnetic field,

$$\nabla_\perp \sim a^{-1} \qquad (2)$$

and $\rho_s^2 = (T_e/m_i)\Omega_T^{-2}$, where T_e is the electron temperature, m_i is the ion mass, and Ω_T is the ion gyrofrequency in the toroidal magnetic field. The normalization of ϕ and χ are such that the special case

$$\phi = \alpha\chi \qquad (3)$$

corresponds to adiabatic electrons, i.e., $\tilde{n}/n_c = e\Phi/T_e$, where $\tilde{n}$ (n_c) is the perturbed (equilibrium) electron density, e is the electron charge and Φ is the unnormalized electostatic potential.

Consider the class of fluid disturbances which vary over length scales much larger than ρ_s [and for which Eq. (3) does not hold, so that ϕ and χ are independent]. For this class, Eq. (1) instructs us to neglect α in the inclusive system; the result is RMHD. On the other hand, for $\alpha \sim 1$, the inclusive system describes electromagnetic micro-fluctuations and reduces to CHM in the adiabatic case of Eq. (3). [More precisely, Eq. (3) decouples the inclusive system into CHM and ordinary resistive diffusion.]

We can restate this conclusion by characterizing RMHD as the small ρ_s/a limit of an electromagnetic (nonadiabatic) generalization of CHM.

II. REDUCED FLUID EQUATIONS

A. Normalization

We briefly review, with slight modification, the RMHD derivation of Strauss.[2] (Note that our sign conventions differ from those of Strauss.) Let (R,ζ,Z) be cylindrical coordinates centered on the tokamak symmetry axis; thus, R is the major radius, ζ is the toroidal angle, and Z varies along the symmetric axis. Also, let a be a pertinent length scale such that

$$\varepsilon \equiv \frac{a}{R_0} \ll 1 ,$$

where R_0 is the major radius of the magnetic axis. We introduce dimensionless coordinates (x,y,z), where

$$x = \frac{R - R_0}{a} , \quad y = \frac{Z}{a} , \quad z = -\zeta . \tag{4}$$

Hence, the normalized gradient of any scalar, S , assumes the form

$$a\nabla S = \nabla_\perp S + \hat{z} \frac{\varepsilon}{1 + \varepsilon x} \frac{\partial S}{\partial z} \tag{5}$$

with

$$\nabla_\perp \equiv \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} .$$

Similarly one finds, for an arbitrary vector,

$$\underset{\sim}{V} = \hat{z} V_z + \underset{\sim}{V}_\perp , \qquad V_z = \hat{z} \cdot \underset{\sim}{V} ,$$

the normalized divergence,

$$a\nabla \cdot \underset{\sim}{V} = \nabla_\perp \cdot \underset{\sim}{V}_\perp + \frac{\varepsilon}{1 + \varepsilon x} \left(\frac{\partial V_z}{\partial z} + V_x \right) \tag{6}$$

and the normalized curl,

$$a\nabla \times \underset{\sim}{V} = -\hat{z} \times \nabla_\perp V_z + \hat{z}\hat{z} \cdot \nabla_\perp \times \underset{\sim}{V}_\perp + \frac{\varepsilon}{1 + \varepsilon x} \left(\hat{z} \times \frac{\partial \underset{\sim}{V}_\perp}{\partial z} - \hat{y} V_z \right) . \tag{7}$$

In these expressions, the curvature terms have been made explicit, so that the unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ can be treated as constant, Cartesian unit vectors (for example, $\nabla_\perp \cdot \underset{\sim}{V}_\perp = \partial V_x / \partial x + \partial V_y / \partial y$). Notice that both curvature terms and z-derivative terms appeared weighted by ε .

The magnetic field is written as

$$\underset{\sim}{B} = \frac{B_T \hat{z}}{1 + \varepsilon x} + \underset{\sim}{\nabla} \times \underset{\sim}{A} \tag{8}$$

where B_T is constant and $\underset{\sim}{A}$ is a vector potential. The first term in Eq. (8) is evidently the vacuum field due to toroidal coils; because this field component dominates in tokamak configurations, we write

$$\underset{\sim}{A} = \varepsilon B_T a \hat{\underset{\sim}{A}} \quad . \tag{9}$$

Then Eq. (7) shows that

$$\frac{\underset{\sim}{B}}{B_T} = \hat{z}(1 + \varepsilon x)^{-1} + \varepsilon \hat{z}\hat{z}\cdot\underset{\sim}{\nabla}_\perp \times \hat{\underset{\sim}{A}}_\perp - \varepsilon \hat{z}\times\underset{\sim}{\nabla}_\perp \psi + O(\varepsilon^2) \quad . \tag{10}$$

Here, we introduced the abbreviation

$$\psi(\underset{\sim}{x},t) = \hat{A}_z(\underset{\sim}{x},t) = A_z(\underset{\sim}{x},t)(\varepsilon B_T a)^{-1} \quad . \tag{11}$$

We similarly introduce a dimensionless electrostatic potential,

$$\phi(\underset{\sim}{x},t) = \left(\frac{c}{\varepsilon B_T a v_A} \right) \Phi(\underset{\sim}{x},t) \tag{12}$$

where v_A is the Alfvén speed

$$v_A^2 = \frac{B_T^2}{(4\pi n_c m_i)} \qquad (13)$$

with m_i the ion mass and n_c a constant measure of the plasma density (for example, the average density).

Finally, we introduce a dimensionless time variable τ , such that

$$t = \frac{\tau_A \tau}{\varepsilon} \qquad (14)$$

where

$$\tau_A \equiv \frac{a}{v_A} \qquad (15)$$

is the Alfvên time. The normalization (τ_A/ε) is appropriate to the relatively slow, shear-Alfvên motions of interest.

B. RMHD Equations

When the plasma pressure is sufficiently small, RMHD can be derived using only two of the equations of resistive magnetohydrodynamics: the equation of motion,

$$mn \left(\frac{\partial \underset{\sim}{V}}{\partial t} + \underset{\sim}{V} \cdot \underset{\sim}{\nabla} \underset{\sim}{V}\right) + \underset{\sim}{\nabla} p = \frac{1}{c} \underset{\sim}{J} \times \underset{\sim}{B} \qquad (16)$$

and the Ohm's law,

$$-\frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} - \underset{\sim}{\nabla}\Phi + \frac{1}{c} \underset{\sim}{V} \times \underset{\sim}{B} = \eta \underset{\sim}{J} \quad , \qquad (17)$$

where $\underset{\sim}{J} = (c/4\pi)\underset{\sim}{\nabla}\times\underset{\sim}{B}$ is the plasma current, p is the pressure, $\underset{\sim}{V}$ is the flow velocity and η is the resistivity, which is assumed to be small. Equations (16) and (17) suffice, because at this point one begins to approximate for small ε. Thus, one finds from Eqs. (12) and (17) that

$$\underset{\sim}{V} = \varepsilon v_A \hat{z} \times \underset{\sim}{\nabla}_\perp \phi + O(\varepsilon^2) + O(\eta) \quad . \tag{18}$$

Here the toroidal flow has been presumed small, as can be shown to be consistent with Eq. (16).[2,3] The parallel component of Eq. (17), when written in terms of normalized variables, takes the form

$$\frac{\partial\psi}{\partial\tau} + \frac{\partial\phi}{\partial z} + \hat{z}\cdot\underset{\sim}{\nabla}_\perp\phi\times\underset{\sim}{\nabla}_\perp\psi = \hat{\eta}\underset{\sim}{\nabla}_\perp^2\psi \tag{19}$$

where

$$\hat{\eta} \equiv \frac{c^2\eta}{4\pi a^2}\frac{\tau_A}{\varepsilon} \tag{20}$$

is the dimensionless resistivity, and all $O(\varepsilon)$ corrections are omitted.

Consider next Eq. (16). Its largest terms are $O(\varepsilon)$ and correspond to compressional Alfvén wave equilibration: $c\underset{\sim}{\nabla}p \simeq \underset{\sim}{J}\times\underset{\sim}{B}$, or

$$\underset{\sim}{\nabla}_\perp\left(\frac{4\pi p}{B_T^2} + \varepsilon\hat{z}\cdot\underset{\sim}{\nabla}_\perp\times\hat{\underset{\sim}{A}}_\perp\right) = O(\varepsilon^2) \quad . \tag{21}$$

We adopt a low-beta tokamak ordering,

$$\nabla\left(\frac{8\pi p}{B_T^2}\right) \sim \varepsilon^2 ,$$

which makes the longitudinal field perturbation, $\hat{z}\cdot\nabla_\perp\times\hat{A}_\perp$, nearly constant and irrelevant. The most important information, describing shear-Alfvén motion, is then extracted by computing the $\hat{z}$-component of the curl of Eq. (16). This yields

$$\frac{\partial}{\partial\tau}\nabla_\perp^2\phi + \hat{z}\cdot\nabla_\perp\phi\times\nabla_\perp(\nabla_\perp^2\phi) + \frac{\partial}{\partial z}\nabla_\perp^2\psi - \hat{z}\cdot\nabla_\perp\psi\times\nabla_\perp(\nabla_\perp^2\psi) = 0 , \quad (22)$$

when $O(\varepsilon^3)$ terms are neglected.

Equations (19) and (22) define (low beta, resistive) RMHD. They are conveniently rewritten in terms of the normalized vorticity,

$$U \equiv \nabla_\perp^2\phi , \quad (23)$$

and the normalized toroidal current,

$$J \equiv \nabla_\perp^2\psi . \quad (24)$$

We also use a conventional bracket notation,

$$[f,g] \equiv \hat{z}\cdot\nabla_\perp f\times\nabla_\perp g , \quad (25)$$

for any functions f and g. From Eq. (10) we observe that, in lowest order, $B\cdot\nabla f \propto \partial f/\partial z - [\psi,f]$. Hence, we introduce the nonlinear parallel gradient, defined by

$$\hat{\nabla}_{\parallel} f = \frac{\partial f}{\partial z} - [\psi, f] \quad , \tag{26}$$

in order to write Eqs. (19) and (22) in the form

$$\frac{\partial U}{\partial \tau} + [\phi, U] + \hat{\nabla}_{\parallel} J = 0 \tag{27}$$

$$\frac{\partial \psi}{\partial \tau} + \hat{\nabla}_{\parallel} \phi = \hat{\eta} J \quad . \tag{28}$$

C. CHM Equation

The CHM equation is derived by Hasegawa and Mima[5] by making simplifying assumptions in the particle conservation law,

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = 0 \quad , \tag{29}$$

and in the equation of motion for (cold) ions,

$$nm_i\left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V}\right) = en\left(\mathbf{E} + \frac{1}{c}\,\mathbf{V}\times\mathbf{B}\right) \quad . \tag{30}$$

The electric field is presumed to be electrostatic,

$$\mathbf{E} = -\nabla\Phi \quad , \tag{31}$$

and, most importantly, the density is assumed to obey a linearized Maxwell-Boltzmann law,

$$n = n_c\left(1 + \frac{e\Phi}{T_e}\right) \quad , \qquad \frac{e\Phi}{T_e} \ll 1 \quad , \tag{32}$$

where T_e is presumed to be constant. The linearization is convenient, although not strictly necessary in electrostatic theory.[8] The point is that CHM treats the convective nonlinearity in Eq. (30) carefully, while neglecting nonlinear effects involving n. In Eq. (32), the coefficient n_c is constant [as in Eq. (13)]; spatial variation of n_c can be admitted, within the linearized context, with some sacrifice of simplicity.[5,7] The main physical content of Eq. (32) is electron parallel mobility, or electron adiabaticity, and quasi-neutrality.

CHM will be derived in Sec. III as a special case of the inclusive system. Here, we give only the result,

$$\frac{\partial U}{\partial \tau} + [\phi, U] = \frac{\partial \phi}{\partial \tau} \tag{33}$$

in RMHD notation. The term on the right-hand side of Eq. (33) results essentially from Eq. (32). Hasegawa and Mima observe that this term is absent in the analogous Navier-Stokes equation. For the same reason, this term does not appear in a strictly electrostatic ($\psi = J = 0$) version of RMHD.

We remark here that no fluid model is likely to be accurate over the range of plasma conditions relevant to controlled fusion. While RMHD and CHM may seem rather crude (for example, both models ignore electron temperature gradients and finite ion gyroradius effects[9]), their simplicity yields important benefits with regard to analytical and numerical tractability.

III. INCLUSIVE NONLINEAR SYSTEM

Our objective in this section is to derive a system of fluid equations which contains the physics of both RMHD and CHM, with as little additional physics as possible. Thus, we continue to assume

$$T_e = \text{const.} \quad ,$$

and we again treat the density as only mildly perturbed from a constant, n_c :

$$n(\underset{\sim}{x},t) = n_c + \tilde{n}(\underset{\sim}{x},t) \quad , \qquad \tilde{n} \ll n_c \quad .$$

We also neglect the ion parallel flow, so that the parallel current is

$$J_{\parallel} = -enV_{\parallel e} \quad . \tag{34}$$

Our procedure is based on the incorporation of electron parallel mobility into the RMHD framework. This will allow (without requiring) the electrons to respond adiabatically to electrostatic fields, as in Eq. (32). The fluid manifestation of electron parallel flow is the parallel pressure gradient term in a generalized Ohm's law, i.e., the Hall term. Thus, we replace the parallel component of Eq. (17) by

$$-\underset{\sim}{B}\cdot \left(\frac{1}{c}\frac{\partial \underset{\sim}{A}}{\partial t} + \underset{\sim}{\nabla}\Phi\right) = \eta \underset{\sim}{B}\cdot \underset{\sim}{J} - \frac{T_e}{en}\underset{\sim}{B}\cdot\underset{\sim}{\nabla} n \quad . \tag{35}$$

Equation (35) can be rewritten in terms of RMHD-normalized variables, and reduced by the neglect of $O(\varepsilon^3)$ terms. The result is

$$\frac{\partial\psi}{\partial\tau} + \hat{\nabla}_{\parallel}\phi = \hat{\eta}J + \frac{\Delta}{2\varepsilon}\hat{\nabla}_{\parallel}\left(\frac{\tilde{n}}{n_c}\right) \quad . \tag{36}$$

Here,

$$\Delta \equiv \frac{\beta ec}{\Omega_T \tau_A} , \tag{37}$$

where βec measures the electron beta,

$$\beta ec = \frac{8\pi n_c T_e}{B_T^2}$$

and $\Omega_T = eB_T/m_i c$ is the ion gyrofrequency. The appropriate ordering of the last term on the right-hand side of Eq. (36) will be considered presently.

As usual, an equation for $\tilde{n}/n_c$ is obtained from the particle conservation law. A short cut (which is appropriate for quasi-neutral plasmas) is to consider <u>electron</u> conservation; for low beta, the transverse electron velocity coincides with the ion velocity of Eq. (18), while the electron flow along $\underset{\sim}{B}$ is given by Eq. (34). Thus, Eq. (29) can be reduced to

$$\frac{\partial}{\partial\tau}\frac{\tilde{n}}{n_c} + \left[\phi, \frac{\tilde{n}}{n_c}\right] + \frac{2\varepsilon\alpha}{\Delta}\hat{\nabla}_{\parallel}J = O(\varepsilon) \quad . \tag{38}$$

The parameter $\alpha = \rho_s^2/a^2$ was introduced in Eq. (1).

There is more than one way to order the last term on the left-hand side of Eq. (38). In a maximal ordering, it is considered comparable to the $O(\varepsilon)$ correction terms on the right-hand side, involving toroidal curvature (Pfirsch-Schlüter flows), which must therefore be included.[10,11] However, our present objective is to derive the simplest inclusive system. This is obtained by assuming $\Delta \sim \varepsilon$, so that the left-hand side of Eq. (38) contains all the lowest order terms. An appropriate normalization for $\tilde{n}$ is then clearly indicated:

$$\frac{\tilde{n}}{n_c} = 2\,\frac{\varepsilon\alpha}{\Delta}\,\chi \quad , \tag{39}$$

where $\chi(\underset{\sim}{x},t)$ satisfies

$$\frac{\partial \chi}{\partial \tau} + [\phi,\chi] + \hat{\nabla}_{\parallel} J = 0 \quad .$$

Notice that Eq. (39) also simplifies the last term on the right-hand side of Eq. (36).

Our final equation is obtained from the equation of motion, Eq. (16), by the RMHD procedure which was outlined in subsection II.B. The result, Eq. (27), is not modified by present considerations.

In summary: We have deduced, using conventionally simplified moment equations and an internally consistent ordering scheme, the following nonlinear fluid system:

$$\frac{\partial U}{\partial \tau} + [\phi,U] + \hat{\nabla}_{\parallel} J = 0 \tag{40}$$

$$\frac{\partial\psi}{\partial\tau} + \hat{\nabla}_{\parallel}\phi = \hat{\eta}J + \alpha\hat{\nabla}_{\parallel}\chi \tag{41}$$

$$\frac{\partial\chi}{\partial\tau} + [\phi,\chi] + \hat{\nabla}_{\parallel}J = 0 \tag{42}$$

where

$$J = \nabla_{\perp}^{2}\psi \quad , \qquad U = \nabla_{\perp}^{2}\phi \quad ,$$

$$[f,g] = \hat{z}\cdot\underset{\sim}{\nabla}_{\perp}f \times \underset{\sim}{\nabla}_{\perp}g \quad ,$$

and

$$\hat{\nabla}_{\parallel}f = \frac{\partial f}{\partial z} - [\psi,f] \quad .$$

The normalized fields, ψ , ϕ , and χ , are defined, respectively, by Eqs. (11), (12), and (39). The parameters α and $\hat{\eta}$ are defined, respectively, by Eqs. (1) and (20).

IV. DISCUSSION

When the parameter $\alpha = \rho_s^2/a^2$ is negligibly small, χ is decoupled from ϕ and ψ . In this case, Eqs. (40) and (41) show that ϕ and ψ evolve according to RMHD dynamics. Thus, RMHD is obtained from the inclusive system when the fields are assumed to vary on scale lengths large compared to ρ_s .

On the other hand, for short scale-length variation, $\alpha \sim 1$ and Eqs. (40) - (42) can be interpreted as an electromagnetic, resistive generalization of CHM. The electromagnetic terms decouple in the special case of Eq. (3), $\phi = \alpha\chi$; Eq. (42) then implies $\hat{\nabla}_{\parallel}J = -\partial\chi/\partial\tau$, which, when substituted into Eq. (40) with $\alpha = 1$, yields the CHM equation. Notice that in this case Eq. (41) describes ordinary resistive diffusion.

From Eqs. (12), (37) and (39), one finds that $\phi = \alpha\chi$ is equivalent to Eq. (32), describing an adiabatic electron response. Hence, the reduced electromagnetic coupling results from nonadiabatic electrons.

It is instructive to consider the energy conservation law associated with the inclusive system. All quadratic conservation laws for systems with bracket nonlinearities are obtained from the identities

$$\int d\underset{\sim}{x}\, f[g,h] = \int d\underset{\sim}{x}\, g[h,f] = \int d\underset{\sim}{x}\, h[f,g] \qquad (43)$$

where f , g and h are arbitrary functions and surface contributions (which rarely matter) have been neglected. Equations (43) can be straightforwardly derived from Gauss's theorem, after it is noticed that

$$[f,g] = \hat{z}\cdot\underset{\sim}{\nabla}_\perp f\times\nabla_\perp g = a\underset{\sim}{\nabla}\cdot(g\hat{z}\times\underset{\sim}{\nabla}_\perp f) \quad .$$

We multiply Eq. (40) by ϕ , Eq. (41) by J , and Eq. (42) by $(-\alpha\chi)$. The results are added and integrated over $\underset{\sim}{x}$ to obtain, after use of Eqs. (43), the relation

$$\frac{\partial}{\partial\tau}\int d\underset{\sim}{x}\,\frac{1}{2}\left[(\nabla_\perp\phi)^2 + (\nabla_\perp\psi)^2 + \alpha\chi^2\right] + \int d\underset{\sim}{x}\,\hat{\eta}J^2 = 0 \quad . \qquad (44)$$

Thus, the total of fluid kinetic energy, magnetic field energy and thermal energy, changes at a rate prescribed by Ohmic dissipation.

Equation (44) reduces to the RMHD energy conservation law[2] when $\alpha = 0$, and to the CHM law[4] when $\nabla_\perp\psi \to 0$, $\phi = \alpha\chi$, $\hat{\eta} = 0$ and $\alpha = 1$.

Both RMHD and CHM possess additional quadratic invariants. Thus, CHM conserves enstrophy, while RMHD conserves the cross-helicity, $\int d\underset{\sim}{x}\underset{\sim}{\nabla}_\perp\phi\cdot\underset{\sim}{\nabla}_\perp\psi$. It can be shown that neither of these conservation laws survive in the inclusive system $(\alpha \neq 0, \phi \neq \alpha\chi)$.

In summary, we have derived a simple, internally consistent nonlinear system, Eqs. (40) - (42), which includes and generalizes the physics of both RMHD and CHM. The relation between RMHD and CHM is then easily understood in terms of scale-length differences and electron adiabaticity.

We remark that the inclusive system also possesses intrinsic interest. For example, its linearized version -- which has been considered previously[11] -- reproduces crucial features of the correct electron response at long mean-free-path, $\hat{\eta}J \ll \alpha\hat{\nabla}_\parallel\chi$. The long mean-free-path regime is experimentally relevant but inaccurately described by RMHD. With regard to nonlinear applications of the inclusive system, we mention that it yields an interesting electromagnetic generalization of the solitary-wave solutions to CHM.[12]

Acknowledgments

I wish to thank J. D. Meiss, R. E. Aamodt and R. E. Waltz for helpful exchanges. This work was supported by United States Department of Energy contract no. DE-FG05-80ET-53088 with the Institute for Fusion Studies.

Footnotes

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