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Neoclassical and anomalous transport in axisymmetric toroidal plasmas with electrostatic turbulence

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Abstract

Neoclassical and anomalous transport fluxes are determined for axisymmetric toroidal plasmas with weak electrostatic fluctuations. The neoclassical and anomalous fluxes are defined based on the ensemble-averaged kinetic equation with the statistically averaged nonlinear term. The anomalous forces derived from that quasilinear term induce the anomalous particle and heat fluxes. The neoclassical banana-plateau particle and heat fluxes and the bootstrap current are also affected by the fluctuations through the parallel anomalous forces and the modified parallel viscosities. The fluctuating part of the drift kinetic equation gives the response of the distribution function to the potential fluctuation, from which the quasilinear term, the anomalous forces, and the anomalous particle and heat fluxes are evaluated. The averaged drift kinetic equation with the quasilinear term is solved for the plateau regime to derive the parallel viscosities modified by the fluctuations. The entropy production rate due to the anomalous transport processes is formulated and used to identify conjugate pairs of the anomalous fluxes and forces, which are connected by the matrix with the Onsager symmetry. Estimates are given from the dispersion relation for the ion temperature gradient driven mode for the anomalous fluxes and viscosity.

I. INTRODUCTION

Extensive theoretical and experimental studies on transport processes of magnetically confined plasmas have been performed over many years since it is crucially important to understand the transport rates for realizing controlled nuclear fusion. The neoclassical transport theory[1-3] is based on Coulomb collisions of particles moving in toroidal magnetic configurations. Particle and energy transport fluxes observed in most fusion devices exceed the predictions of the neoclassical theory and thus are called anomalous transport.[4] The anomalous transport is considered to result from the turbulent fluctuations caused by various instabilities existing in confined plasmas. Most of the theoretical works on the anomalous (or turbulent) plasma transport have been done separately from the neoclassical transport. Shaing[5,6] and Balescu,[7] however, have attempted to unify the neoclassical and anomalous transport theories. The present work treats this same problem from a different approach.

Here we investigate the weakly turbulent regime as in the theories of Shaing and Balescu. Only electrostatic fluctuations in the axisymmetric toroidal system are considered for simplicity. The principal difference between the present theory and the formulation of Shaing and Balescu lies in the way of dividing the physical variables into the average and fluctuating parts. Two-scale separation of spatio-temporally varying quantities is essentially important in treating both the neoclassical and turbulent effects. Variables treated by the neoclassical theory are spatio-temporally smooth and regarded as the ensemble-averaged parts while the fluctuating parts are treated mainly by the anomalous or turbulent transport theory. For example, the averaged flow is incompressible to the lowest order in the neoclassical theory, which is crucially important in deriving the neoclassical banana-plateau transports fluxes. This incompressibility is derived from the continuity equation with the slow temporal variation of the ensemble-average density neglected. On the other hand, the fluctuating part of

the flow can be compressible, which causes the ion sound wave, the ion temperature gradient driven mode and other effects that influence the anomalous transport. Here we emphasize that it matters significantly how the separation of variables into the average and fluctuating parts is defined. In our treatment, strict separation into the ensemble-average and fluctuating parts is done at the level of the kinetic equation. We define the average part of fluid variables such as densities, flow velocities, temperatures and heat flux from the average kinetic distribution function. These definitions are different from those given by ensemble-average of random fluid variables. For example, the flow velocities and temperatures given from the average kinetic distribution function (which we call the 'kinetic definition') deviate from the average of the random flow velocities and temperatures given from the random kinetic distribution function (which we call the 'fluid definition'). In the works of Shaing and Balescu, clear definitions for the average and fluctuating parts are not written although they seem to obey the fluid definition. When the fluid definition is employed, the averaged fluid equations include many nonlinear terms with respect to the fluctuations of the fluid variables. On the other hand, by using the kinetic definition, each fluid equation contains only a single nonlinear term with respect to the fluctuations, and thus the complexities are reduced.

Furthermore, the kinetic definition makes clearer the division of the total transport into the neoclassical and anomalous parts. The averaged kinetic equation is a starting point of the neoclassical part of the theory, where the effects of the fluctuations are contained through the term including the statistically averaged quadratic nonlinearity. We define anomalous particle and heat fluxes from this term quite naturally according to the analogy to the definitions of the classical and neoclassical fluxes. This definition of the anomalous heat flux is different from that of Shaing[5,6] and Balescu.7 Thus, this statistically averaged nonlinear term plays a essential role in the unification of the neoclassical and anomalous transport theories, and it is calculated by the quasilinear technique in the weakly turbulent regime.

Evaluation of this quasilinear term requires the fluctuating part of the kinetic distribution function. In our formulation, owing to the kinetic definition, the fluctuating part of the kinetic equation coincides with the standard drift or gyrokinetic equation for the plasma turbulence, and the fluctuation of the kinetic distribution function takes a well-known form used for microinstabilities. In contrast Shaing[5,6] and Balescu[7] employ Shaing's ansatz[5–7] for the kinetic distribution and derive the kinetic response to the fluctuations from a drift or gyrokinetic equation including non-diamagnetic flow dependence which we treat by the average part of the drift kinetic equation.

In the present formulation the modified averaged parallel viscosities are obtained in the plateau regime from the solution of the averaged drift kinetic equation and the fluctuations affect the neoclassical banana-plateau transport fluxes. We find the new relation between the parallel viscosities and the average poloidal flows caused by the electrostatic fluctuations, which are not described by Shaing and Balescu since the quasilinear fluctuation effects on the average kinetic distribution are neglected by them. Physically, the parallel viscosity modification arises from the pressure anisotropy induced by the parallel velocity diffusion produced by the drift wave fluctuations. This anisotropy competes with the neoclassical anisotropy mechanism from the poloidal flow in producing the parallel viscosity.

We also analyze the Onsager relations and the entropy production functional in the anomalous transport system. Shaing argues that the Onsager relation holds for the anomalous transport coefficients while Balescu claims that it does not. Here, we emphasize that the Onsager relation is closely related to the entropy production. For example, in the classical process, the entropy production defined in terms of the collision operator is represented by the product of transport fluxes and thermodynamic forces and the Onsager symmetry is valid for the transport matrix which relates these fluxes and forces to each other.[3] Thus, conjugated pairs of the fluxes and forces are determined through the entropy production functional. According to the neoclassical and anomalous transport processes, there exist

two types of entropy production: one is derived from the collision operator as mentioned above, and the other is due to the transfer of energy and momentum through the anomalous processes. Since neither Shaing nor Balescu give clear expressions for the entropy production in the anomalous transport process from which the conjugate pairs of the anomalous transport fluxes and forces should be defined, their arguments on the Onsager relation seem to be incomplete. We define the anomalous entropy production in terms of the anomalous quasilinear term and thus give the conjugate flux-force pairs. The resulting expression of the anomalous entropy production coincides with that derived by Horton.[8] It is also shown that the Onsager symmetry is satisfied by the anomalous transport matrix connecting these anomalous fluxes and forces. However, this anomalous transport relation has the structure which is totally different from the classical or neoclassical one, since the anomalous transport matrix is also a highly nonlinear function of the forces such as the density and temperature gradients through the eigenfrequencies. Furthermore, in order to complete the transport relations, the spectrum of the potential fluctuations remains to be determined. The fluctuation spectrum is given by the nonlinear saturation mechanism although here we only treat the spectrum as given following the works of Shaing and Balescu. We estimate the anomalous transport and the parallel viscosities from the dispersion relation for the ion temperature gradient driven mode.

This work is organized as follows. In Sec. II, basic equations for the density, momentum, energy and energy fluxes are derived from the ensemble-averaged kinetic equation including the effects of the electrostatic fluctuations through the quasilinear term. In Sec. III, the neoclassical and anomalous transport fluxes are defined. In Sec. IV, the average and fluctuating parts of the drift kinetic equation are shown, which give the bases for the neoclassical and anomalous parts of our theory, respectively. In Sec. V, we derive the entropy production in the anomalous transport process and show the Onsager relation between the conjugate pairs of the anomalous transport fluxes and forces. In Sec. VI, the averaged drift kinetic equation

with the quasilinear term is solved for the plateau regime to give the average parallel viscosities. There, estimates are given for the anomalous transport and parallel viscosities from the dispersion relation for the ion temperature gradient driven mode. Finally, conclusions and discussion are given in Sec. VIII.

II. BASIC EQUATIONS

We start from an ensemble-averaged kinetic equation for species a:

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\cdot\nabla f_a + \frac{e_a}{m_a}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\frac{\partial f_a}{\partial \mathbf{v}} = C_a + \mathcal{D}_a \tag{1}$$

where C_a is a collision term and $\mathcal{D}_a$ is fluctuation-averaged nonlinear term defined by

$$\mathcal{D}_a = -\frac{e_a}{m_a}\left\langle \widehat{\mathbf{E}}\cdot\frac{\partial \widehat{f}_a}{\partial \mathbf{v}}\right\rangle_{ens} \tag{2}$$

$$\widehat{\mathbf{E}} = -\nabla\widehat{\phi} \tag{3}$$

Here $\langle\cdot\rangle_{ens}$ denotes the ensemble average and we divided the distribution function (the electric field) into the ensemble-averaged part f_a ($\mathbf{E}$) and the fluctuating part $\widehat{f}_a$ ($\widehat{\mathbf{E}}$). Taking the moments of the kinetic equation yields the following fluid equations. The continuity equation:

$$\frac{\partial n_a}{\partial t} + \nabla\cdot(n_a\mathbf{u}_a) = 0. \tag{4}$$

The momentum balance equation:

$$m_a n_a\left(\frac{\partial}{\partial t} + \mathbf{u}_a\cdot\nabla\right)\mathbf{u}_a = n_a e_a\left(\mathbf{E} + \frac{1}{c}\mathbf{u}_a\times\mathbf{B}\right) - \nabla p_a - \nabla\cdot\boldsymbol{\pi}_a + \mathbf{F}_{a1} + \mathbf{K}_{a1} \tag{5}$$

The energy balance equation:

$$\frac{3}{2}\frac{\partial p_a}{\partial t} = -\nabla\cdot\left(\mathbf{q}_a + \frac{5}{2}p_a\mathbf{u}_a + \boldsymbol{\pi}_a\cdot\mathbf{u}_a\right) + \mathbf{u}_a\cdot\nabla p_a + \mathbf{u}_a\cdot(\nabla\cdot\boldsymbol{\pi}_a) + Q_a + H_a. \tag{6}$$

The energy flux equation:

$$\begin{aligned} m_a n_a\frac{\partial \mathbf{Q}_a}{\partial t} = {} & \frac{e_a}{m_a}\left[\mathbf{E}\cdot\left(\frac{5}{2}p_a\mathsf{I} + \boldsymbol{\pi}_a + m_a n_a\mathbf{u}_a\mathbf{u}_a\right) + \frac{1}{c}\mathbf{Q}_a\times\mathbf{B}\right] \\ & + \frac{T_a}{m_a}\left[\frac{5}{2}(\mathbf{F}_{a1} + \mathbf{K}_{a1}) + (\mathbf{F}_{a2} + \mathbf{K}_{a2})\right] - \nabla\cdot\mathsf{r}_a \end{aligned} \tag{7}$$

Here $\mathbf{F}_{a1}$, Q_a, and $\mathbf{F}_{a2}$ represents the collisional generation rates of momentum, heat, and heat flux, respectively, which are defined in Ref. 2. The density n_a, pressure p_a, flow velocity $\mathbf{u}_a$, heat flux $\mathbf{q}_a$, total energy flux $\mathbf{Q}_a$, viscosity tensor $\boldsymbol{\pi}_a$, and energy weighted stress tensor r_a are all defined from the ensemble-averaged distribution function f_a in the standard way as given in Ref. 2. It should be noted that these 'kinetic' definitions of the average fluid variables are different from the conventional 'fluid' definitions: for example, the both definitions give the same average density and pressure although the average flow velocity in the latter is defined by the average of the random flow velocity given from the total kinetic distribution and is different from that in the former as

$$\left\langle \frac{\int d^3v (f_a + \widehat{f}_a)\mathbf{v}}{\int d^3v (f_a + \widehat{f}_a)} \right\rangle_{ens} \neq \frac{\int d^3v f_a \mathbf{v}}{\int d^3v f_a} \equiv \mathbf{u}_a \tag{8}$$

and similarly the different average temperatures are given depending on the definitions. By our kinetic definitions, the above fluid equations contain the nonlinear terms with respect to the kinetic fluctuations, which are all derived from $\mathcal{D}_a$, while, when the fluid definitions were used, there are large numbers of nonlinear terms with respect to the fluctuations at each level of the averaged fluid equations. For example, the averaged momentum balance equation is written in the fluid definition as

$$\begin{aligned}
& m_a \frac{\partial}{\partial t} \left(n_a \mathbf{u}_a + \langle \widehat{n}_a \widehat{\mathbf{u}}_a \rangle_{ens} \right) \\
& + m_a \nabla \cdot \left(n_a \mathbf{u}_a \mathbf{u}_a + n_a \langle \widehat{\mathbf{u}}_a \widehat{\mathbf{u}}_a \rangle_{ens} + \langle \widehat{n}_a \widehat{\mathbf{u}}_a \rangle_{ens} \mathbf{u}_a + \mathbf{u}_a \langle \widehat{n}_a \widehat{\mathbf{u}}_a \rangle_{ens} + \langle \widehat{n}_a \widehat{\mathbf{u}}_a \widehat{\mathbf{u}}_a \rangle_{ens} \right) \\
& = n_a e_a \left(\mathbf{E} + \frac{1}{c} \mathbf{u}_a \times \mathbf{B} \right) - \nabla p_a - \nabla \cdot \pi_a + \mathbf{F}_{a1} + e_a \left\langle \widehat{n}_a \widehat{\mathbf{E}} \right\rangle_{ens} + \frac{e_a}{c} \langle \widehat{n}_a \widehat{\mathbf{u}}_a \rangle_{ens} \times \mathbf{B}
\end{aligned}$$

where all the random fluid variables are defined from the total distribution function $f_a + \widehat{f}_a$ and divided into the average $(\mathbf{u}_a, \cdots)$ and fluctuating parts $(\widehat{\mathbf{u}}_a, \cdots)$.

The heat generation rate and forces resulting from the fluctuation term $\mathcal{D}_a$ are given by

$$H_a = \int d^3v \mathcal{D}_a \frac{1}{2} m_a (\mathbf{v} - \mathbf{u}_a)^2 = e_a \left\langle (\widehat{\Gamma}_a - \widehat{n}_a \mathbf{u}_a) \cdot \widehat{\mathbf{E}} \right\rangle_{ens}$$

$$\mathbf{K}_{a1} = \int d^3v \mathcal{D}_a m_a \mathbf{v} = e_a \left\langle \widehat{n}_a \widehat{\mathbf{E}} \right\rangle_{ens}$$

$$\mathbf{K}_{a2} = \int d^3v \mathcal{D}_a m_a \mathbf{v} \left(\frac{m_a v^2}{2T_a} - \frac{5}{2} \right)$$

$$= \frac{e_a}{T_a} \left\langle \frac{5}{2} (\widehat{p}_a - \widehat{n}_a T_a) \widehat{\mathbf{E}} + \widehat{\boldsymbol{\pi}}_a \cdot \widehat{\mathbf{E}} \right\rangle_{ens} \tag{9}$$

where we defined the fluctuating density, particle flux and pressure (scalar and stress parts) as

$$\widehat{n}_a = \int d^3v \widehat{f}_a$$

$$\widehat{\boldsymbol{\Gamma}}_a = \int d^3v \widehat{f}_a \mathbf{v}$$

$$\widehat{p}_a = \frac{1}{3} \int d^3v \widehat{f}_a m_a v^2$$

$$\widehat{\boldsymbol{\pi}}_a = \int d^3v \widehat{f}_a m_a \left(\mathbf{vv} - \frac{1}{3} v^2 \mathsf{I} \right) \tag{10}$$

Let us define an ordering parameter Δ for the fluctuating variables as

$$\frac{\widehat{f}_a}{f_a} \sim \frac{e_a \widehat{\phi}}{T_a} \sim \frac{\widehat{n}_a}{n_a} \sim \frac{k_\parallel}{k_\perp} \sim \Delta \tag{11}$$

where $k_\parallel \sim L^{-1}$ (L: the scale length of the plasma equilibrium quantities) and $k_\perp$ denotes the parallel and perpendicular wavenumbers of fluctuations, respectively. Then we have

$$\mathbf{K}_{aj\perp} \sim \Delta$$

$$\mathbf{K}_{aj\parallel} \sim \Delta^2 \qquad (j = 1, 2). \tag{12}$$

Another ordering parameter is a drift-ordering parameter δ given by

$$\delta \sim \rho_a / L \tag{13}$$

where ρ_a is a thermal gyroradius. When $\Delta \sim \delta$, we should use the gyrokinetic equation to obtain the gyrophase dependence of $\widehat{f}_a$. If $\Delta \gg \delta$, the lowest-order part of $\widehat{f}_a$ has no

gyrophase dependence and is obtained from the drift-kinetic equation. In the latter case, $\widehat{\pi}_a$ has a CGL form:

$$\widehat{\pi}_a = (\widehat{p}_{\parallel a} - \widehat{p}_{\perp a})\left(\mathbf{nn} - \frac{1}{3}\mathsf{I}\right)$$
$$\widehat{p}_{\parallel a} = \int d^3v \widehat{f}_a m_a v_\parallel^2$$
$$\widehat{p}_{\perp a} = \frac{1}{2}\int d^3v \widehat{f}_a m_a v_\perp^2 \tag{14}$$

where $\mathbf{n} = \mathbf{B}/B$, $v_\parallel = \mathbf{v}\cdot\mathbf{n}$ and $\mathbf{v}_\perp = \mathbf{v} - v_\parallel\mathbf{n}$.

From the perpendicular components of Eqs. (5) and (7), we have the perpendicular classical and neoclassical fluxes of particles and heat of $\mathcal{O}(\delta^2)$ and the anomalous perpendicular fluxes driven by $\mathbf{K}_{aj\perp}$ of $\mathcal{O}(\delta\Delta)$. In the parallel components of Eqs. (5) and (7), forces such as $(\nabla\cdot\boldsymbol{\pi}_a)_\parallel$ and $(\nabla\cdot\mathsf{r}_a)_\parallel$ are $\mathcal{O}(\delta)$ while the parallel forces $\mathbf{K}_{aj\parallel}$ are $\mathcal{O}(\Delta^2)$. Thus, if we take $\Delta\sim\delta$, the order of the anomalous perpendicular fluxes is equal to that of classical and neoclassical perpendicular fluxes, and the fluctuation effects due to $\mathbf{K}_{aj\parallel}$ on the neoclassical banana-plateau fluxes and the bootstrap current are neglected. Hereafter, we employ the ordering $\Delta\sim\delta^{1/2}\gg\delta$. Then the anomalous perpendicular fluxes exceed the neoclassical ones and besides the banana-plateau fluxes and the bootstrap current are significantly affected by the fluctuations as shown in the following sections.

III. NEOCLASSICAL AND ANOMALOUS FLUXES

In axisymmetric systems such as tokamaks, the magnetic field is given by

$$\mathbf{B} = I\nabla\zeta + \frac{1}{2\pi}\nabla\zeta\times\nabla\chi \tag{15}$$

where ζ denotes the toroidal angle, χ the poloidal magnetic flux and I the covariant toroidal component of the magnetic field (here as in Ref. 2, we consider a general case in which I is not treated as a flux surface quantity without assuming no radial current). Hereafter, we

assume that the ensemble-averaged distribution function f_a is independent of the toroidal angle ζ and that all the average fluid variables defined from f_a and the average electric field $\mathbf{E}$ are axisymmetric.

The lowest-order parts of the continuity equation (4) and the energy balance equation (6) show that the divergence of the lowest-order flows $\mathbf{u}_a$ and $\mathbf{q}_a$ vanishes. Using this zero-divergence constraint and the lowest-order parallel components of Eqs. (5) and (7) with the friction-flow relations,[2,3] we have the relations:

$$\begin{bmatrix} -\frac{1}{en_e}\left\langle Bj_\parallel \right\rangle \\ -\frac{2}{5}p_e^{-1}\left\langle Bq_{\parallel e} \right\rangle \end{bmatrix} = \left\langle B^2 \right\rangle \begin{bmatrix} u_{e\theta} - u_{i\theta} \\ -\frac{2}{5}\frac{q_{e\theta}}{p_e} \end{bmatrix} + \begin{bmatrix} \langle B(V_{1e} - V_{1i}) \rangle \\ \langle BV_{2e} \rangle \end{bmatrix}$$

$$= -\frac{\tau_e}{n_e m_e} \begin{bmatrix} \widetilde{\sigma}_\parallel & -\sqrt{\frac{2}{5}}\widetilde{\alpha}_\parallel \\ -\sqrt{\frac{2}{5}}\widetilde{\alpha}_\parallel & \frac{2}{5}\widetilde{\kappa}_\parallel^e \end{bmatrix} \begin{bmatrix} n_e e \left\langle BE_\parallel^{(A)} \right\rangle - \langle \mathbf{B}\cdot\mathbf{K}_{e1} \rangle + \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_e \rangle \\ -\langle \mathbf{B}\cdot\mathbf{K}_{e2} \rangle + \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\Theta}_e \rangle \end{bmatrix} \tag{16}$$

and

$$\frac{2}{5}p_i^{-1}\left\langle Bq_{\parallel i} \right\rangle = \frac{2}{5}\frac{q_{i\theta}}{p_i}\left\langle B^2 \right\rangle + \langle BV_{2i} \rangle = -\frac{2}{5}\widetilde{\kappa}_\parallel^i \frac{\tau_i}{n_i m_i}\left(-\langle \mathbf{B}\cdot\mathbf{K}_{i2} \rangle + \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\Theta}_i \rangle\right). \tag{17}$$

where $\langle\cdot\rangle$ denotes the magnetic flux surface average, and $\boldsymbol{\Theta}_a = (m_a/T_a)[\mathsf{r}_a - \frac{1}{3}\mathsf{I}(\mathsf{I}:\mathsf{r}_a)] - \frac{5}{2}\boldsymbol{\pi}_a$. Here $u_{a\theta}$ and $q_{a\theta}$ is the flux functions[2] defined by

$$u_{a\theta}(\psi) \equiv \frac{\mathbf{u}_a\cdot\nabla\theta}{\mathbf{B}\cdot\nabla\theta} \qquad q_{a\theta}(\psi) \equiv \frac{\mathbf{q}_a\cdot\nabla\theta}{\mathbf{B}\cdot\nabla\theta} \tag{18}$$

with the poloidal angle θ and an arbitrary flux label ψ. The dimensionless coefficients $\widetilde{\sigma}_\parallel$, $\widetilde{\alpha}_\parallel$, $\widetilde{\kappa}_\parallel^e$ and $\widetilde{\kappa}_\parallel^i$ are given in Ref. 3. The diamagnetic flow contributions V_{1a} and V_{2a} are defined by

$$V_{1a} \equiv -\frac{2\pi I}{\chi'}\frac{cT_a}{e_a B}\left(\frac{p_a'}{p_a} + \frac{e_a\Phi'}{T_a}\right)$$

$$V_{2a} \equiv -\frac{2\pi I}{\chi'}\frac{cT_a'}{e_a B} \tag{19}$$

The species summation of the lowest-order parallel component of the momentum balance equation (5) gives

$$\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_e \rangle + \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_i \rangle = 0 \tag{20}$$

where we should note that the species summation of the anomalous forces $\sum_a \mathbf{K}_{a1}$ vanishes due to the quasineutrality condition $\sum_a e_a \widehat{n}_a = 0$. The parallel viscosities $\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_a\rangle$ and $\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\Theta}_a\rangle$ are calculated by solving the ensemble-averaged drift kinetic equation which contains the quasilinear fluctuation term as shown in the following sections. The source of the parallel viscosities consists of two parts : one is the same as in the conventional neoclassical theory[2,3] due to the poloidal flows and the other is the anomalous drive due to the quasilinear fluctuation term. Thus, the parallel viscosities are written as

$$\begin{aligned}\begin{bmatrix}\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_a\rangle \\ \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\Theta}_a\rangle\end{bmatrix} &= 3\left\langle(\mathbf{n}\cdot\nabla B)^2\right\rangle \begin{bmatrix}\mu_{a1} & \mu_{a2}\\ \mu_{a2} & \mu_{a3}\end{bmatrix}\begin{bmatrix}u_{a\theta}\\ \frac{2}{5}\frac{q_{a\theta}}{p_a}\end{bmatrix} + \begin{bmatrix}Y_{a1}\\ Y_{a2}\end{bmatrix} \\ &= 3\left\langle(\mathbf{n}\cdot\nabla B)^2\right\rangle \begin{bmatrix}\mu_{a1} & \mu_{a2}\\ \mu_{a2} & \mu_{a3}\end{bmatrix}\begin{bmatrix}u_{a\theta}-W_{a1}\\ \frac{2}{5}\frac{q_{a\theta}}{p_a}-W_{a2}\end{bmatrix}. \end{aligned} \tag{21}$$

Here the neoclassical viscosity coefficients μ_{aj} $(j=1,2,3)$ are given in Ref. 2 for the Pfirsch-Schlüter, plateau, and banana regimes, and written in Ref. 3 as

$$3\left\langle(\mathbf{n}\cdot\nabla B)^2\right\rangle \begin{bmatrix}\mu_{a1} & \mu_{a2}\\ \mu_{a2} & \mu_{a3}\end{bmatrix} = \frac{n_a m_a}{\tau_a}\left\langle B^2\right\rangle \varphi \begin{bmatrix}\mu^a_{11} & \sqrt{\frac{5}{2}}\mu^a_{13}\\ \sqrt{\frac{5}{2}}\mu^a_{31} & \frac{5}{2}\mu^a_{33}\end{bmatrix} \tag{22}$$

for the banana regime, where φ and μ^a_{jk} should be replaced with φ_{a*} and λ_{jk} for the plateau regime (see Ref. 3 for the definitions of these coefficients). In Eq. (21), Y_{aj} $(j=1,2)$ denote the anomalous parts of the parallel viscosities, W_{aj} $(j=1,2)$ represent the shifts of the poloidal flows due to the fluctuations, and they are calculated in the Sec. VI for the plateau regime.

Taking the flux surface averages of the toroidal components of Eqs. (5) and (7) yields the flux-surface-averaged radial particle and heat fluxes up to $\mathcal{O}(\delta^2)$ as

$$\begin{aligned}\langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle &= \langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{cl} + \langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{PS} + \langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{bp} \\ &+ \frac{2\pi}{\chi'} n_a c\left\langle \frac{E^{(A)}_{\parallel}}{B}\left(\langle I\rangle\frac{B^2}{\langle B^2\rangle} - I\right)\right\rangle + \left\langle \frac{n_a c}{B}\mathbf{E}^{(A)}\times\mathbf{n}\cdot\nabla\psi\right\rangle \\ &+ \langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle^{\text{anom}}_{PS} + \langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle^{\text{anom}}\end{aligned}$$

$$\langle \mathbf{q}_a \cdot \nabla\psi \rangle = \langle \mathbf{q}_a \cdot \nabla\psi \rangle_{cl} + \langle \mathbf{q}_a \cdot \nabla\psi \rangle_{PS} + \langle \mathbf{q}_a \cdot \nabla\psi \rangle_{bp}$$

$$+ \langle \mathbf{q}_a \cdot \nabla\psi \rangle_{PS}^{\text{anom}} + \langle \mathbf{q}_a \cdot \nabla\psi \rangle^{\text{anom}} \tag{23}$$

where the first three in the right-hand sides represent the classical, Pfirsch-Schlüter and banana-plateau fluxes while the last two are the anomalous contributions defined later. The classical and Pfirsch-Schlüter fluxes are given in terms of the pressure and temperature gradients in the same way as in the case of no fluctuations and their transport relations are written in Refs. 1–3. However, the fluctuation effects appear in the anomalous fluxes as well as in the banana-plateau transport relations as shown later.

Anomalous terms $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$ and W_{aj} (or Y_{aj}) are incorporated into the modified forces and poloidal flows by defining

$$\langle B(V_{1e} - V_{1i}) \rangle^{(m)} = \langle B(V_{1e} - V_{1i}) \rangle + \sqrt{\frac{2}{5}} \tilde{\alpha}_{\parallel} \frac{\tau_e}{n_e m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + \left\langle B^2 \right\rangle (W_{e1} - W_{i1})$$

$$\langle BV_{2e} \rangle^{(m)} = \langle BV_{2e} \rangle - \frac{2}{5} \tilde{\kappa}_{\parallel}^{e} \frac{\tau_e}{n_e m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + \left\langle B^2 \right\rangle W_{e2}$$

$$\langle BV_{2i} \rangle^{(m)} = \langle BV_{2i} \rangle - \frac{2}{5} \tilde{\kappa}_{\parallel}^{i} \frac{\tau_i}{n_i m_i} \langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle + \left\langle B^2 \right\rangle W_{i2}$$

$$\left\langle BE_{\parallel}^{(A)} \right\rangle^{(m)} = \left\langle BE_{\parallel}^{(A)} \right\rangle - (n_e e)^{-1} \langle \mathbf{B} \cdot \mathbf{K}_{e1} \rangle$$

$$u_{a\theta}^{(m)} = u_{a\theta} - W_{a1}$$

$$\frac{2}{5} \frac{q_{a\theta}^{(m)}}{p_a} = \frac{2}{5} \frac{q_{a\theta}}{p_a} - W_{a2} \tag{24}$$

If we use the above modified forces and poloidal flows, the neoclassical expressions for the parallel viscosities and accordingly for the banana-plateau particle and heat fluxes which are valid even in the presence of fluctuations. From Eq. (16), we can obtain the contributions of fluctuations to the parallel current. Then, from Eqs. (16), (17), (20), (21), and (22), we

obtain the banana-plateau transport equations as

$$\begin{bmatrix} \langle \mathbf{\Gamma}_e \cdot \nabla\psi \rangle_{bp} \\ \langle \mathbf{q}_e \cdot \nabla\psi \rangle_{bp} \\ \langle \mathbf{q}_i \cdot \nabla\psi \rangle_{bp} \\ \langle B^2 \rangle^{-1/2} \langle Bj_\parallel \rangle^{(m)} \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} -n_e^{-1}(dP/d\psi)^{(m)} \\ -T_e^{-1}(dT_e/d\psi)^{(m)} \\ -T_i^{-1}(dT_i/d\psi)^{(m)} \\ \langle B^2 \rangle^{-1/2} \langle BE_\parallel^{(A)} \rangle^{(m)} \end{bmatrix} \tag{25}$$

where the fluxes in the left-hand side are defined by

$$\langle \mathbf{\Gamma}_a \cdot \nabla\psi \rangle_{bp} = -\frac{2\pi I}{\chi'} \frac{c \langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_a \rangle}{e_a \langle B^2 \rangle}$$

$$T_a^{-1} \langle \mathbf{q}_a \cdot \nabla\psi \rangle_{bp} = -\frac{2\pi I}{\chi'} \frac{c \langle \mathbf{B} \cdot \nabla \cdot \mathbf{\Theta}_a \rangle}{e_a \langle B^2 \rangle}$$

$$\langle Bj_\parallel \rangle^{(m)} = \langle Bj_\parallel \rangle - \frac{n_e e^2 \tau_e}{m_e} \tilde{\sigma}_\parallel \langle BE_\parallel^{(A)} \rangle^{(m)} - \sqrt{\frac{2}{5}} \tilde{\alpha}_\parallel \frac{e\tau_e}{m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle \tag{26}$$

and the modified pressure and temperature gradients are given from Eq. (24) as

$$\left(\frac{dP}{d\psi}\right)^{(m)} = \frac{dP}{d\psi} + \frac{\chi'}{2\pi I} \frac{e}{c} \left(\sqrt{\frac{2}{5}} \tilde{\alpha}_\parallel \frac{\tau_e}{m_e} \langle \mathbf{B} \cdot \mathbf{K}_{e2} \rangle + n_e \langle B^2 \rangle (W_{e1} - W_{i1}) \right)$$

$$\left(\frac{dT_a}{d\psi}\right)^{(m)} = \frac{dT_a}{d\psi} + \frac{\chi'}{2\pi I} \frac{e_a}{c} \left(\frac{2}{5} \tilde{\kappa}_\parallel^a \frac{\tau_a}{n_a m_a} \langle \mathbf{B} \cdot \mathbf{K}_{a2} \rangle - \langle B^2 \rangle W_{a2} \right). \tag{27}$$

The transport matrix is given by

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} & L_{14} \\ L_{21} & L_{22} & L_{23} & L_{24} \\ L_{31} & L_{32} & L_{33} & L_{34} \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} = \frac{n_e \rho_e^2}{2 T_e \tau_e} \left(\frac{2\pi I}{\chi'}\right)^2 \varphi \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{5}{2}} T_e & 0 & 0 \\ 0 & 0 & \sqrt{\frac{5}{2}} \frac{T_i}{Z_i} & 0 \\ 0 & 0 & 0 & -e\frac{\chi'}{2\pi I} |\Omega_e| \tau_e \end{bmatrix}$$

$$\times \begin{bmatrix} l_{11}^{ee} & l_{13}^{ee} & l_{13}^{ei} & l_{1E}^{e} \\ l_{31}^{ee} & l_{33}^{ee} & l_{33}^{ei} & l_{3E}^{e} \\ l_{31}^{ie} & l_{33}^{ie} & A l_{33}^{ii} & l_{3E}^{i} \\ l_{E1}^{e} & l_{E3}^{e} & l_{E3}^{i} & l_{EE} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{\frac{5}{2}} T_e & 0 & 0 \\ 0 & 0 & \sqrt{\frac{5}{2}} \frac{T_i}{Z_i} & 0 \\ 0 & 0 & 0 & -e\frac{\chi'}{2\pi I} |\Omega_e| \tau_e \end{bmatrix} \tag{28}$$

where $\rho_a = v_{Ta}/|\Omega_a|$ and $\Omega_a = e_a \langle B^2 \rangle^{1/2} / m_a c$. The dimensionless coefficients $l_{11}^{ee}, \cdots$ and A are defined in Ref. 3. From Eqs. (17), (21), (22) and (24), the parallel ion viscosity is written

as

$$\langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_i\rangle = \frac{n_i m_i}{\tau_i}\left\langle B^2\right\rangle \varphi\left[\left(\mu_{11}^i - (\mu_{13}^i)^2\widetilde{\kappa}_\parallel^i \frac{\varphi}{1+\widetilde{\kappa}_\parallel^i\mu_{33}^i\varphi}\right)u_{i\theta}^{(m)}\right.$$

$$\left. - \sqrt{\frac{5}{2}}\frac{\mu_{13}^i}{1+\widetilde{\kappa}_\parallel^i\mu_{33}^i\varphi}\frac{\langle BV_{2i}\rangle^{(m)}}{\langle B^2\rangle}\right] \tag{29}$$

The Pfirsch-Schlüter type fluxes induced by the parallel fluctuation forces are given by

$$\langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle_{PS}^{\text{anom}} = -\frac{2\pi}{\chi'}\left\langle \frac{\mathbf{n}\cdot\mathbf{K}_{a1}}{m_a\Omega_a}\left(I - \langle I\rangle\frac{B^2}{\langle B^2\rangle}\right)\right\rangle$$

$$T_a^{-1}\langle \mathbf{q}_a\cdot\nabla\psi\rangle_{PS}^{\text{anom}} = -\frac{2\pi}{\chi'}\left\langle \frac{\mathbf{n}\cdot\mathbf{K}_{a2}}{m_a\Omega_a}\left(I - \langle I\rangle\frac{B^2}{\langle B^2\rangle}\right)\right\rangle. \tag{30}$$

The anomalous transport induced by the perpendicular fluctuation forces are given by

$$\langle \boldsymbol{\Gamma}_a\cdot\nabla\psi\rangle^{\text{anom}} = \left\langle \frac{\nabla\psi}{m_a\Omega_a}\cdot(\mathbf{K}_{a1}\times\mathbf{n})\right\rangle$$

$$T_a^{-1}\langle \mathbf{q}_a\cdot\nabla\psi\rangle^{\text{anom}} = \left\langle \frac{\nabla\psi}{m_a\Omega_a}\cdot(\mathbf{K}_{a2}\times\mathbf{n})\right\rangle. \tag{31}$$

The anomalous fluxes in Eq. (30) are negligibly smaller than those in Eq. (31) since $k_\parallel \ll k_\perp$ is assumed.

IV. DRIFT KINETIC EQUATION WITH ELECTROSTATIC FLUCTUATIONS

The drift kinetic equation[9,10] is given by

$$\frac{\partial \overline{f}_a}{\partial t} + (v_\parallel\mathbf{n} + \mathbf{v}_{da})\cdot\nabla\overline{f}_a + e_a\left[\frac{\partial\Phi}{\partial t} + (v_\parallel\mathbf{n}+\mathbf{v}_{da})\cdot\mathbf{E}^{(A)}\right]\frac{\partial\overline{f}_a}{\partial E} = C_a(\overline{f}_a) \tag{32}$$

where $\overline{f}_a$ is a gyroangle-averaged distribution function in the phase space of guiding center variables $(\mathbf{x}, E, \mu)$ and $\mathbf{v}_{da}$ is the guiding center drift velocity. The potential Φ consists of the time-independent ensemble average part $\Phi_0 = \langle\Phi\rangle_{ens}$ and the fluctuating part $\widehat{\phi}$:

$$\Phi = \Phi_0 + \widehat{\phi}, \qquad \frac{\partial\Phi_0}{\partial t} = 0. \tag{33}$$

Then the energy variable E is divided into the average and fluctuating parts:

$$E = E_0 + e_a\widehat{\phi}$$

$$E_0 = \frac{1}{2}m_a v^2 + e_a\Phi_0. \tag{34}$$

Hereafter let us use $(\mathbf{x}, E_0, \mu)$ as independent guiding center variables instead of $(\mathbf{x}, E, \mu)$. Then the drift kinetic equation is rewritten for $\overline{f}_a(\mathbf{x}, E_0, \mu)$ as

$$\frac{\partial \overline{f}_a}{\partial t} + (v_\parallel \mathbf{n} + \mathbf{v}_{da0} + \widehat{\mathbf{v}}_E)\cdot\nabla\overline{f}_a + e_a(v_\parallel \mathbf{n} + \mathbf{v}_{da0} + \widehat{\mathbf{v}}_E)\cdot(\widehat{\mathbf{E}} + \mathbf{E}^{(A)})\frac{\partial \overline{f}_a}{\partial E_0} = C_a(\overline{f}_a) \tag{35}$$

where

$$\widehat{\mathbf{E}} = -\nabla\widehat{\phi}, \qquad \widehat{\mathbf{v}}_E = \frac{c}{B}\widehat{\mathbf{E}}\times\mathbf{n}.$$

$$\mathbf{v}_{da0} = \frac{cT_a}{e_a B}\mathbf{n}\times\left(\frac{e_a}{T_a}\nabla\Phi_0 + x_{\perp a}^2\nabla\ln B + 2x_{\parallel a}^2\mathbf{n}\cdot\nabla\mathbf{n}\right) \tag{36}$$

with $x_\perp = v_\perp/v_{Ta}$, $x_\parallel = v_\parallel/v_{Ta}$ and $v_{Ta} = (2T_a/m_a)^{1/2}$. Due to the electrostatic fluctuation terms, the solution $\overline{f}_a$ of the drift kinetic equation also includes average and fluctuating parts:

$$\overline{f}_a = \left\langle \overline{f}_a \right\rangle_{ens} + \widehat{\overline{f}}_a \tag{37}$$

Taking an ensemble average of the drift kinetic equation and retaining the terms up to $\mathcal{O}(\delta)$, we obtain

$$(v_\parallel \mathbf{n} + \mathbf{v}_{da0})\cdot\nabla\left\langle \overline{f}_a \right\rangle_{ens} + e_a v_\parallel E_\parallel^{(A)}\frac{\partial\left\langle \overline{f}_a \right\rangle_{ens}}{\partial E_0} = \left\langle C_a(\overline{f}_a)\right\rangle_{ens} + \overline{\mathcal{D}}_a \tag{38}$$

where

$$\overline{\mathcal{D}}_a = -e_a v_\parallel\left\langle \widehat{E}_\parallel\frac{\partial\widehat{\overline{f}}_a}{\partial E_0}\right\rangle_{ens} \tag{39}$$

To the lowest order, we have

$$v_\parallel\mathbf{n}\cdot\nabla\left\langle \overline{f}_{a0}\right\rangle_{ens} = C_a(\left\langle \overline{f}_{a0}\right\rangle_{ens}). \tag{40}$$

As a solution of the lowest order equation, we use the Maxwellian distribution function

$$\overline{f}_{a0} = \left\langle \overline{f}_{a0} \right\rangle_{ens} = f_{aM} = \pi^{-3/2} n_a v_{Ta}^{-3} \mathrm{e}^{-x_a^2} = n_a \left(\frac{m_a}{2\pi T_a} \right)^{3/2} \exp\left(\frac{e_a \Phi_0 - E_0}{T_a} \right) \tag{41}$$

where $x_a = v/v_{Ta}$. We should note that the lowest order distribution function $\overline{f}_{a0}$ does not include a fluctuating part, i.e., $\widehat{\overline{f}}_{a0} = 0$. To $\mathcal{O}(\delta)$, we have

$$v_\parallel \mathbf{n} \cdot \nabla \left\langle \overline{f}_{a1} \right\rangle_{ens} + \mathbf{v}_{da0} \cdot \nabla \overline{f}_{a0} + e_a v_\parallel E_\parallel^{(A)} \frac{\partial \overline{f}_{a0}}{\partial E_0} = C_a(\left\langle \overline{f}_{a1} \right\rangle_{ens}) + \overline{\mathcal{D}}_a. \tag{42}$$

In the right-hand side, we use the linearized collision operator and

$$\overline{\mathcal{D}}_a = -e_a v_\parallel \left\langle \widehat{E}_\parallel \frac{\partial \widehat{\overline{f}}_{a1}}{\partial E_0} \right\rangle_{ens}. \tag{43}$$

The linearized drift kinetic equation for the fluctuating part $\widehat{\overline{f}}_{a1}$ is written as

$$\frac{\partial \widehat{\overline{f}}_{a1}}{\partial t} + (v_\parallel \mathbf{n} + \mathbf{v}_{da0}) \cdot \nabla \widehat{\overline{f}}_{a1} + \widehat{\mathbf{v}}_E \cdot \nabla \overline{f}_{a0} + e_a (v_\parallel \mathbf{n} + \mathbf{v}_{da0}) \cdot \widehat{\mathbf{E}} \frac{\partial \overline{f}_{a0}}{\partial E_0} = C_a(\widehat{\overline{f}}_{a1}) \tag{44}$$

where higher order terms than $\mathcal{O}(\delta)$ are neglected. Defining the nonadiabatic part $\widehat{h}_a$ by

$$\widehat{\overline{f}}_{a1} = -\frac{e_a \widehat{\phi}}{T_a} f_{aM} + \widehat{h}_a \tag{45}$$

we obtain

$$\left[\frac{\partial}{\partial t} + (v_\parallel \mathbf{n} + \mathbf{v}_{da0}) \cdot \nabla + C_a \right] \widehat{h}_a = \left(\frac{\partial}{\partial t} + \frac{cT_a}{e_a B} \mathbf{n} \times \nabla \ln f_{aM} \cdot \nabla \right) \frac{e_a \widehat{\phi}}{T_a} f_{aM} \tag{46}$$

which has the well known form of the gyrokinetic equation[9,11,12] in the zero gyroradius limit. Using the Fourier representation for the rapid spatio-temporal variation of the fluctuating quantities as

$$\begin{bmatrix} \widehat{\phi} \\ \widehat{h}_a \end{bmatrix} = \sum_{\mathbf{k}} \begin{bmatrix} \widehat{\phi}_{\mathbf{k}} \\ \widehat{h}_{a\mathbf{k}} \end{bmatrix} \exp[i\mathbf{k} \cdot \mathbf{x} - i\omega_{\mathbf{k}} t] \tag{47}$$

and neglecting the bounce motion of the trapped ions that is justified in the plateau regime, we have

$$\widehat{h}_{a\mathbf{k}} = \frac{\omega - \omega_E - \omega_{*a}^T}{\omega - \omega_E - \omega_{Da} - k_\parallel v_\parallel + i\nu_a} \frac{e_a \widehat{\phi}_{\mathbf{k}}}{T_a} f_{aM} \tag{48}$$

where $C_a(\widehat{h}_{a\mathbf{k}})$ is replaced with $\nu_a \widehat{h}_{a\mathbf{k}}$ and

$$\omega_E = \mathbf{k} \cdot \frac{c}{B} \mathbf{n} \times \nabla \Phi_0$$

$$\omega_{Da} = \mathbf{k} \cdot (\mathbf{v}_{\nabla B} + \mathbf{v}_{\mathrm{curv}}) = \mathbf{k} \cdot \frac{cT_a}{e_a B} \mathbf{n} \times (x_{\perp a}^2 \nabla \ln B + 2 x_{\parallel a}^2 \mathbf{n} \cdot \nabla \mathbf{n})$$

$$\omega_{*a}^T = \omega_{*a} \left[1 + \eta_a \left(x_a^2 - \frac{3}{2} \right) \right]$$

$$\omega_{*a} = \mathbf{k} \cdot \frac{cT_a}{e_a B} \mathbf{n} \times \nabla \ln n_a$$

$$\eta_a = d \ln T_a / d \ln n_a \tag{49}$$

are used. Using Eqs. (43), (45), (47), and (48) and assuming that $\nu_a \ll |\omega|$, we have

$$\overline{\mathcal{D}}_a = -e_a v_\parallel \sum_{\mathbf{k}} \mathrm{Real} \left\langle \widehat{E}_{\parallel \mathbf{k}}^* \frac{\partial \widehat{h}_{a\mathbf{k}}}{\partial E_0} \right\rangle_{ens}$$

$$= -\pi \sum_{\mathbf{k}} k_\parallel v_\parallel \frac{e_a^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_a} \frac{\partial}{\partial E_0} \left[(\omega - \omega_E - \omega_{*a}^T) \delta(\omega - \omega_E - \omega_{Da} - k_\parallel v_\parallel) f_{aM} \right]. \tag{50}$$

From Eqs. (9), (10), (45), (47), and (48), we can calculate the anomalous heat generation rate and forces as

$$H_a = \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens} \int d^3 v f_{aM} \delta(\omega - \omega_E - \omega_{Da} - k_\parallel v_\parallel)(\omega - \omega_E - \omega_{*a}^T)$$

$$\times (k_\parallel v_\parallel - \omega_E - \omega_{*a}[1 + \eta_a])$$

$$\mathbf{K}_{a1} = \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens} \int d^3 v f_{aM} \delta(\omega - \omega_E - \omega_{Da} - k_\parallel v_\parallel)(\omega - \omega_E - \omega_{*a}^T) \mathbf{k}$$

$$\mathbf{K}_{a2} = \pi \frac{e_a^2}{T_a} \sum_{\mathbf{k}} \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens} \int d^3 v f_{aM} \delta(\omega - \omega_E - \omega_{Da} - k_\parallel v_\parallel)(\omega - \omega_E - \omega_{*a}^T)$$

$$\times \left[\left(3x_{\parallel a}^2 + x_{\perp a}^2 - \frac{5}{2} \right) k_\parallel \mathbf{n} + \left(x_{\parallel a}^2 + 2x_{\perp a}^2 - \frac{5}{2} \right) \mathbf{k}_\perp \right] \tag{51}$$

When the fluctuation spectrum $\left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}$ and the dispersion relation $\omega = \omega_{\mathbf{k}}$ are given, we can obtain from the above equations the parallel anomalous forces, which are necessary for

the determination of the banana-plateau transport in Eq. (25) and the anomalous Pfirsch-Schlüter fluxes in Eq. (30). We can also obtain the anomalous fluxes from the perpendicular anomalous forces as

$$\Gamma_a^{\rm anom} = \frac{c}{e_a B}\mathbf{K}_{a1}\times\mathbf{n}$$

$$\frac{1}{T_a}\mathbf{q}_a^{\rm anom} = \frac{c}{e_a B}\mathbf{K}_{a2}\times\mathbf{n}. \tag{52}$$

The flux surface averages of the radial components of Eq. (52) were already found in Eq. (31).

V. ENTROPY PRODUCTION AND CONJUGATE PAIRS OF FLUXES AND FORCES IN ANOMALOUS TRANSPORT PROCESSES

We define the microscopic entropy S_m and macroscopic entropy S_M per unit volume by

$$S_m = -\int d^3v(\langle f\rangle + \widehat{f})\ln(\langle f\rangle + \widehat{f}) \tag{53}$$

$$S_M = -\int d^3v\,\langle f\rangle \ln\langle f\rangle\,. \tag{54}$$

In this section, subscripts for species and ensemble average are suppressed. We have the relation between these entropies by retaining terms up to $\mathcal{O}(\Delta^2)$ as

$$S_M = \langle S_m\rangle + \frac{1}{2}\int d^3v\frac{\langle\widehat{f}^2\rangle}{\langle f\rangle}. \tag{55}$$

Without collisions, the total microscopic entropy, i.e., the species summation and the spatial integration of S_m is conserved although the total macroscopic entropy can be increased by the turbulent or anomalous transport process. In this section, we are concerned with the entropy production by the turbulent process and neglect the collisional effect by assuming that the time scale of the turbulent fluctuations is much shorter than the collision time. Then we see from the average kinetic (Vlasov) equation that the conservation of S_M is broken by

$\mathcal{D}$ and the entropy production rate due to the anomalous or turbulent process is defined by

$$\sigma_A = -\int d^3v \mathcal{D} \ln \langle f \rangle . \tag{56}$$

From the average and fluctuating parts of the Vlasov equation, we also obtain up to $\mathcal{O}(\Delta^2)$

$$\sigma_A = \frac{1}{2}\int d^3v \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)\frac{\langle \widehat{f}^2 \rangle}{\langle f \rangle}. \tag{57}$$

Here we can see that σ is due to the spatio-temporal variation of the second term in the right-hand side of Eq. (55). Using $\langle f \rangle \simeq f_M$ and the linearized drift kinetic equation (44) for $\widehat{f}$ in the collisionless limit and assuming that the temporal variation of f_M is much slower than that of $\langle \widehat{f}^2 \rangle$, we have

$$\sigma_A = \frac{1}{2}\int d^3v \frac{1}{f_M}\left(\frac{\partial}{\partial t} + v_\parallel \mathbf{n}\cdot\nabla\right)\langle \widehat{f}^2 \rangle = \int d^3v \left\langle \widehat{f}\left[-\widehat{\mathbf{v}}_E\cdot\nabla \ln f_M + \frac{e}{T}(v_\parallel \mathbf{n} + \mathbf{v}_{d0})\cdot\widehat{\mathbf{E}}\right]\right\rangle$$

$$= \mathbf{J}_1\cdot\mathbf{X}_1 + \mathbf{J}_2\cdot\mathbf{X}_2 + J_3 X_3 \tag{58}$$

where we defined conjugate pairs of forces

$$\mathbf{X}_1 = -\nabla \ln n, \qquad \mathbf{X}_2 = -\nabla \ln T, \qquad X_3 = 1/T \tag{59}$$

and fluxes

$$\mathbf{J}_1 = \mathbf{\Gamma}_A = \int d^3v \langle \widehat{f}\widehat{\mathbf{v}}_E \rangle = \pi \sum_{\mathbf{k}} \frac{e^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle}{T^2} \int d^3v f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E - \omega_*^T)\frac{cT}{eB}\mathbf{k}\times\mathbf{n}$$

$$\mathbf{J}_2 = \frac{1}{T}\mathbf{q}_A = \int d^3v \left(x^2 - \frac{3}{2}\right)\langle \widehat{f}\widehat{\mathbf{v}}_E \rangle$$

$$= \pi \sum_{\mathbf{k}} \frac{e^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle}{T^2} \int d^3v \left(x^2 - \frac{3}{2}\right) f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E - \omega_*^T)\frac{cT}{eB}\mathbf{k}\times\mathbf{n}$$

$$J_3 = e\int d^3v \langle \widehat{f}\widehat{\mathbf{E}} \rangle \cdot (v_\parallel \mathbf{n} + \mathbf{v}_{\nabla B} + \mathbf{v}_{\mathrm{curv}})$$

$$= \pi \sum_{\mathbf{k}} \frac{e^2 \langle |\widehat{\phi}_{\mathbf{k}}|^2 \rangle}{T} \int d^3v f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E - \omega_*^T)(\omega - \omega_E). \tag{60}$$

Here $\mathbf{J}_1 = \mathbf{\Gamma}_A$ is the particle flux conjugate to the density gradient force $\mathbf{X}_1 = -\nabla \ln n$ and is the same as the anomalous particle flux $\mathbf{\Gamma}^{\rm anom}$ defined in Eq. (52). $\mathbf{J}_2 = \mathbf{q}_A/T$ is the heat flux divided by the temperature conjugate to the temperature gradient force $\mathbf{X}_2 = -\nabla \ln T$ although it is different from the anomalous flux $\mathbf{q}^{\rm anom}/T$ defined in Eq. (52). The flux J_3 is conjugate to the force $X_3 = 1/T$ and represents the rate of the energy transfer from the electrostatic fluctuations to the particles moving along the guiding center orbits.

The positive definiteness of σ_A is shown as

$$\sigma_A = \pi \sum_{\mathbf{k}} \frac{e^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle}{T^2} \int d^3v f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E - \omega_*^T)^2 \geq 0. \tag{61}$$

The relation between the fluxes and forces are given by

$$\begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} \mathsf{L}_{11} & \mathsf{L}_{12} & \mathbf{L}_{13} \\ \mathsf{L}_{12} & \mathsf{L}_{22} & \mathbf{L}_{23} \\ \mathbf{L}_{13} & \mathbf{L}_{23} & L_{33} \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ X_3 \end{bmatrix} \tag{62}$$

where

$$\mathsf{L}_{lm} = \pi \sum_{\mathbf{k}} \frac{e^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle}{T^2} \int d^3v \left(x^2 - \frac{3}{2}\right)^{l+m-2} f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel) \left(\frac{cT}{eB}\right)^2 (\mathbf{k} \times \mathbf{n})(\mathbf{k} \times \mathbf{n})$$

$$\mathbf{L}_{l3} = \pi \sum_{\mathbf{k}} \frac{e^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle}{T} \int d^3v \left(x^2 - \frac{3}{2}\right)^{l-1} f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E) \frac{cT}{eB} (\mathbf{k} \times \mathbf{n})$$

$$L_{33} = \pi \sum_{\mathbf{k}} e^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle \int d^3v f_M \delta(\omega - \omega_E - \omega_D - k_\parallel v_\parallel)(\omega - \omega_E)^2 \qquad (l, m = 1, 2). \tag{63}$$

Thus the matrix relating the fluxes to the forces satisfies the Onsager symmetry and the positive definiteness although it is qualitatively different from the classical and neoclassical transport matrices in that the former matrix depends also on the forces through the eigenfrequencies, which are determined from the dispersion relation, and through the spectrum of the fluctuation amplitudes, which is given by the nonlinear saturation.

The effects of the finite gyroradius can be derived from using the gyro-kinetic equation and the results are easily obtained by including $[J_0(k_\perp v_\perp/\Omega)]^2$ into the integrands in the matrix coefficients given by Eq. (63).

VI. ANOMALOUS EFFECTS ON THE PARALLEL VISCOSITIES

Now, let us find the solution of the averaged drift kinetic equation (42). Here, we consider a large aspect ratio tokamak in order to derive the approximate solution for the plateau regime, and, as shown in Appendix A, the solution is written as

$$\left\langle \overline{f}_{a1} \right\rangle_{ens} = \overline{F}_{a1} + \overline{g}_a^{(l=1)} - (\nu_a \mathcal{L}_a)^{-1} \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle + \overline{h}_a \tag{64}$$

where we denoted the contribution of the thermodynamic forces by given by

$$\overline{F}_{a1} = -\frac{2\pi I}{\chi'} \frac{v_\parallel}{\Omega_a} \frac{\partial \overline{f}_{a0}}{\partial \psi} = \frac{2v_\parallel}{v_{Ta}^2} \left[V_{1a} + \left(x_a^2 - \frac{5}{2} \right) V_{2a} \right] \overline{f}_{a0}, \tag{65}$$

the poloidal flow part by

$$\overline{g}_a^{(l=1)} = \frac{2v_\parallel}{v_{Ta}^2} B \left[u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left(x_a^2 - \frac{5}{2} \right) \right] \overline{f}_{a0}, \tag{66}$$

the part contributing to the parallel viscosity by

$$\overline{h}_a = \epsilon \overline{\nu}_a^{-1/3} \left\{ \frac{x_a}{v_{Ta}} B \left[u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left(x_a^2 - \frac{5}{2} \right) \right] \overline{f}_{a0} + \frac{1}{\nu_a} \int_0^1 \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle d\xi \right\}$$
$$\times \int_0^\infty d\tau \sin(\theta - \overline{\nu}_a^{-1/3} \xi \tau) \mathrm{e}^{-\tau^3/6} \tag{67}$$

and the definition of $(\nu_a \mathcal{L}_a)^{-1} \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle$ is described in Appendix A. Here, $\xi \equiv v_\parallel / v$ denotes the cosine of the pitch angle. Fluctuation effects on the solution are explicitly included through $\left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle$ in the third and fourth terms in Eq. (64). The third term $-(\nu_a \mathcal{L}_a)^{-1} \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle$ results from the balance between the collisional pitch angle scattering and the quasilinear anisotropic deformation of the distribution in the velocity space. The fourth term $\overline{h}_a$ represents the distribution of the resonant particles ($|\xi| \ll 1$). The anisotropic distribution (or ξ-dependence) in the velocity space caused by the second term (poloidal flow) and that by the third term (quasilinear effect) give the sources of the resonant particles that are responsible for the neoclassical and anomalous parallel viscosities, which are shown in Eq. (67).

Since the quasilinear term $\left\langle \overline{\mathcal{D}}_a \right\rangle$ contains a delta function of ξ, all the Legendre function components with $l = 2, 3, 4, \cdots$ homogeneously contribute to $\left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle$ although its even parts with $l = 2, 4, 6, \cdots$ vanish in the integral $\int_0^1 \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle d\xi$. Then, noting that only $\overline{h}_a$ contributes to the flux averaged parallel viscosities, we obtain

$$\begin{aligned}
\begin{bmatrix} \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_a \rangle \\ \langle \mathbf{B}\cdot\nabla\cdot\boldsymbol{\Theta}_a \rangle \end{bmatrix} &= \begin{bmatrix} \left\langle \int d^3v m_a v_\parallel^2 \mathbf{B}\cdot\nabla\overline{h}_a \right\rangle \\ \left\langle \int d^3v m_a v_\parallel^2 \left(x_a^2 - \frac{5}{2}\right) \mathbf{B}\cdot\nabla\overline{h}_a \right\rangle \end{bmatrix} \\
&= \frac{\sqrt{\pi}}{2}\epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} \\ \frac{2}{5}\frac{q_{a\theta}}{p_a} \end{bmatrix} + \begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix} \\
&= \frac{\sqrt{\pi}}{2}\epsilon^2 n_a m_a \omega_{Ta} B_0^2 \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{13}{4} \end{bmatrix} \begin{bmatrix} u_{a\theta} - W_{a1} \\ \frac{2}{5}\frac{q_{a\theta}}{p_a} - W_{a2} \end{bmatrix}
\end{aligned} \tag{68}$$

where the anomalous contributions from the quasilinear fluctuation term are given by

$$\begin{bmatrix} Y_{a1} \\ Y_{a2} \end{bmatrix} = \frac{\sqrt{\pi}}{2}\epsilon^2 n_a m_a \omega_{Ta} v_{Ta} \tau_{aa} B_0 \int_0^\infty dx_a \mathrm{e}^{-x_a^2} \frac{x_a^4}{\tau_{aa}\nu_a(x_a)} \begin{bmatrix} 1 \\ \left(x_a^2 - \frac{5}{2}\right) \end{bmatrix} \int_0^1 \frac{\left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle}{f_{aM}} d\xi$$

$$\begin{bmatrix} W_{a1} \\ W_{a2} \end{bmatrix} = \frac{v_{Ta}\tau_{aa}}{B_0} \int_0^\infty dx_a \mathrm{e}^{-x_a^2} \frac{x_a^4}{\tau_{aa}\nu_a(x_a)} \begin{bmatrix} \left(-\frac{3}{2} + \frac{1}{6}x_a^2\right) \\ \left(1 - \frac{1}{3}x_a^2\right) \end{bmatrix} \int_0^1 \frac{\left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle}{f_{aM}} d\xi \tag{69}$$

where the quasilinear term $\overline{\mathcal{D}}_a^{(l\geq 2)}$ is given by

$$\overline{\mathcal{D}}_a^{(l\geq 2)} = \overline{\mathcal{D}}_a - \frac{1}{2}\int_{-1}^1 \overline{\mathcal{D}}_a d\xi - \frac{3\xi}{2}\int_{-1}^1 \xi \overline{\mathcal{D}}_a d\xi$$

$$\begin{aligned}
\overline{\mathcal{D}}_a = \pi f_{aM} \sum_{\mathbf{k}} & \frac{e_a^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_a^2} \frac{k_\parallel}{|k_\parallel|} \\
& \times \left[(\omega - \omega_E - \omega_{*a}^{T'})\alpha\delta(\xi - \alpha) - \frac{1}{2x_a^2}(\omega - \omega_E - \omega_{*a}^T)\left(1 + \frac{2}{x_a\xi}\frac{\omega_{\mathrm{curv}}}{k_\parallel v_{Ta}}\right)\delta'(\xi - \alpha) \right]
\end{aligned} \tag{70}$$

with

$$\omega_{*a}^{T'} = \omega_{*a}\left[1 + \eta_a\left(x_a^2 - \frac{5}{2}\right)\right], \qquad \omega_{\mathrm{curv}} = \mathbf{k}\cdot\mathbf{v}_{\mathrm{curv}}, \qquad \alpha = \frac{\omega - \omega_E - \omega_{Da}}{k_\parallel v}.$$

Here, if the anomalous contributions Y_{aj} or W_{aj} vanish, Eq. (68) reduces to the conventional plateau parallel viscosities.

As seen from Eqs. (51), (69) and (70), the anomalous effects are negligibly small when the phase velocities of the fluctuations in the reference frame moving with the guiding center particles are much larger than the thermal velocity, i.e., $|(\omega - \omega_E - \omega_{Da})/k_\parallel| \gg v_{Ta}$. Thus we now investigate the unstable modes with $|(\omega - \omega_E - \omega_{Da})/k_\parallel| \sim v_{Ta}$ to give the detailed expressions of the anomalous transport and the parallel viscosities. In the case of the electron drift wave driven by the density gradient $dn_a/d\psi < 0$ with $\eta_a \simeq 0$ and $\omega_{Da} \simeq 0$, the anomalous effects are small since $v_{Ti} \ll |(\omega - \omega_E)/k_\parallel| \ll v_{Te}$ is required for the unstable modes and the anomalous contributions $\mathbf{K}_{ej}$, Y_{ej} and W_{ej} $(j = 1, 2)$ are all proportional to $|(\omega - \omega_E - \omega_{*e})/\omega_{*e}| \simeq \frac{1}{2}k_\perp^2 \rho_i^2 (1 + Z_i T_e/T_i) \ll 1$, where Z_i is the ion charge number.

The most relevant fluctuations that resonantly exchange energy-momentum between the ions and the fluctuation is the ion-temperature gradient driven turbulence. Numerous studies of the stability and quasilinear fluxes from this form of drift wave turbulence are available. For the small perpendicular wavenumbers satisfying $k_\perp \rho_i \ll 1$ or $\omega_{Di} \ll k_\parallel v_{Ti}$, we obtain the slab ion temperature gradient (ITG) driven modes[13,14] where the instabilities with $|(\omega - \omega_E)/k_\parallel| \sim v_{Ti}$ are found as shown in Appendix B. The relationship of the small ω_{Di} limit to the strong toroidal regime of $\omega_{Di} \sim k_\parallel v_{Ti}$ is developed in Kim and Horton[15] and Kim *et al.*[16] When $Z_i T_e/T_i = 1$ and $dT_i/d\psi < 0$, we can write the anomalous effects for ions from Appendix B as

$$T_i^{-1} \langle \mathbf{q}_i \cdot \nabla\psi \rangle^{\text{anom}} = 2.61\, n_i \frac{cT_i}{e_i B_0} \sum_{\mathbf{k}'} |\mathbf{k} \times \mathbf{n} \cdot \nabla\psi| \frac{e_i^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_i^2}$$

$$\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle = -1.30\, p_i B_0 \sum_{\mathbf{k}'} \sigma_{*i} k_\parallel \frac{e_i^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_i^2}$$

$$Y_{ij} = C_{yj}\epsilon^2 p_i \omega_{Ti}\tau_{ii} B_0 \sum_{\mathbf{k}'} \sigma_{*i} k_\parallel \frac{e_i^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_i^2} \quad (j = 1, 2)$$

$$W_{ij} = C_{wj} \frac{v_{Ti}^2 \tau_{ii}}{B_0} \sum_{\mathbf{k}'} \sigma_{*i} k_\parallel \frac{e_i^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_i^2} \quad (j = 1, 2) \tag{71}$$

where $\sum'$ represents the summation over the wavenumber region where $|\omega_{*e}\eta_i / k_\parallel v_{Ti}| \sim 2$ and σ_{*i} is the sign of the ion diamagnetic drift frequency. The dimensionless constants in Eq. (71) are given by

$$C_{y1} = -3.05, \quad C_{y2} = 0.85, \quad C_{w1} = 1.94, \quad C_{w2} = -0.45. \tag{72}$$

We should remark on the symmetry properties of the eigenfrequency and the fluctuation spectrum with respect to the parallel wavenumber $k_\parallel$. It is found that the eigenfrequency given is an even function of $k_\parallel$, which results from our use of the Maxwellian distribution with no flow velocity as an equilibrium. If we assume that the spectrum $\left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}$ is also even in $k_\parallel$, the parallel anomalous forces $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$ and the anomalous effects on the parallel viscosities Y_{aj} and W_{aj} vanish. These are confirmed by noting that the wavenumber spectra of $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$, Y_{aj} and W_{aj} are odd functions of $k_\parallel$. Thus, in this case, the banana-plateau transport is not modified by the fluctuations. The $k_\parallel$-symmetry of the dispersion relation is broken, for example, if we take account of sheared flows in the equilibrium distribution function, although they are neglected here by the δ-ordering.

Using $m_e/m_i \ll 1$ and Eq. (20), we have $\langle \mathbf{B} \cdot \nabla \cdot \boldsymbol{\pi}_i \rangle \simeq 0$, from which we obtain the poloidal flow velocity $u_P \equiv B_P u_{i\theta}$ as

$$u_P = -\frac{1}{2}\frac{c}{e_i B_T}\frac{dT_i}{dr} + 1.95\,\epsilon\, q^{-1} v_{Ti}^2 \tau_{ii} \sum_{\mathbf{k}'} \sigma_{*i} k_\parallel \frac{e_i^2 \left\langle |\widehat{\phi}_{\mathbf{k}}|^2 \right\rangle_{ens}}{T_i^2} \tag{73}$$

where Eqs. (17), (19), (21), (68), and (71) are used. In the right-hand side, the first term represents the ion temperature gradient driven poloidal flow in the plateau regime given by the conventional neoclassical theory, the second is due to $\langle \mathbf{B} \cdot \mathbf{K}_{i2} \rangle$ and Y_{ij} (or W_{ij}). As

previously mentioned, the second term vanishes if the spectrum $\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}$ is even in $k_\parallel$. We can see that first term contributes to the direction of the electron diamagnetic rotation with $dn_e/dr < 0$ and $dT_i/dr < 0$ assumed, while the sign of the second term depends on the wavenumber spectrum. The ratio of the anomalous poloidal flow to the ion temperature gradient driven flow has the same order of magnitude as the ratio of the anomalous parallel current to the pressure gradient driven bootstrap current and that is estimated for the plateau regime from Eq. (73) as

$$\frac{u_P^{\rm anom}}{u_P^{\rm neo}} \sim \frac{j_\parallel^{\rm anom}}{j_\parallel^{\rm neo}} \sim \epsilon(\omega_{Ti}\tau_{ii})\left(\frac{L_{Ti}}{\rho_i}\right)\sum_{\mathbf{k}}{}'\sigma_{*i}k_\parallel R\frac{e_i^2\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}}{T_i^2} \tag{74}$$

where a dimensionless numerical constant is omitted and $L_{Ti} = |d\ln T_i/dr|^{-1}$ is used. When we write the parallel wavenumber as that $k_\parallel = (m - nq)/Rq$ with the poloidal and toroidal mode numbers (m, n), Eq. (74) implies that the anomalous rotation and the anomalous parallel currents have opposite signs on different sides of the mode rational surfaces as shown by Shaing.[6] As shown by Dong *et al.*,[17] in the presence of the parallel shear flow which breaks the radial symmetry, the peak of the fluctuating potential shifts radially. In that case, the anomalous forces on the both sides of the mode rational surface do not cancel out and they generate the net rotation and current. If we use $\sigma_{*i}k_\parallel R \sim 1$ and $\sum_{\mathbf{k}}' e_i^2\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}/T_i^2 \sim \Delta^2 \sim \delta \sim \rho_i/L_{Ti}$, the ratio in Eq. (74) reduces to $\epsilon(\omega_{Ti}\tau_{ii})$ and we have $\epsilon \ll \epsilon(\omega_{Ti}\tau_{ii}) \ll \epsilon^{-1/2}$ for the plateau regime. Thus, the anomalous contribution to the transport along the magnetic flux surface is expected to become dominant in the weak collisional plateau regime. For comparison, let us consider the anomalous contribution to the anomalous heat flux across the magnetic flux surface. Using Eq. (71), the ratio of the anomalous to the banana-plateau heat flux is estimated for the plateau regime as

$$\frac{q_i^{\rm anom}}{q_i^{\rm neo}} \sim \frac{\chi_i^{\rm anom}}{\chi_i^{\rm neo}} \sim \frac{1}{q}\left(\frac{L_{Ti}}{\rho_i}\right)\sum_{\mathbf{k}}{}'k_\perp R\frac{e^2\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}}{T_i^2}. \tag{75}$$

From the ordering $k_\perp/k_\parallel \sim \Delta^{-1} \gg 1$, this ratio is much larger than the ratio given by Eq. (74) and reduces to Δ^{-1}.

VII. CONCLUSIONS AND DISCUSSION

In this work, we have investigated the neoclassical and anomalous transport in axisymmetric toroidal systems with electrostatic fluctuations. The total transport is clearly separated into the neoclassical and anomalous parts by using the kinetic definitions which give the average fluid variables from the average kinetic distribution function. The neoclassical banana-plateau transport fluxes modified by the fluctuations were shown in Eq. (25), where the parallel fluctuation-induced forces $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$, $(a = 1, 2; j = i, e)$ and the corrections Y_{aj} (or W_{aj}) to the parallel viscosities due to the fluctuations appear in the definitions of the modified thermodynamic forces and the parallel current. The anomalous transport fluxes were defined by Eq. (31) or Eq. (52) in terms of the perpendicular components of the fluctuation-induced forces $\mathbf{K}_{aj}$ in the similar way to the definition of the classical transport fluxes by the collisional friction forces $\mathbf{F}_{aj}$. Thus, the anomalous fluxes are defined compactly in terms of $\mathbf{K}_{aj}$ in our treatment, which give expressions different from the anomalous heat fluxes defined by Shaing and by Balescu. The parallel components of $\mathbf{K}_{aj}$ produce the Pfirsch-Schlüter like anomalous fluxes as given by Eq. (30) although they are negligibly smaller than those in Eq. (31) for the fluctuations with $k_\parallel \ll k_\perp$.

The fluctuation-induced forces $\mathbf{K}_{aj}$ were defined by Eq. (9) in terms of the statistically nonlinear term $\mathcal{D}_a$ which appears in the ensemble-averaged kinetic equation and they were calculated from the solution of the fluctuating part of the linear drift kinetic equation as given in Eq. (45). The anisotropic distribution in the velocity space caused by the quasilinear fluctuation source gives the corrections to the parallel viscosities Y_{aj} (or W_{aj}), which were obtained for the plateau regime in Eq. (69) from the solution of the ensemble-averaged drift kinetic equation. Thus from $\mathbf{K}_{aj}$ and Y_{aj} (or W_{aj}), we can evaluate the neoclassical and anomalous transport fluxes when the fluctuation spectrum $\langle |\phi_{\mathbf{k}}|^2 \rangle_{ens}$ and the frequency $\omega_{\mathbf{k}}$ are specified. The results using the slab ITG mode dispersion relation for the fluctuations

with the small perpendicular wavenumbers were shown in Sec. VI.

Neither Shaing nor Balescu described the anomalous contributions Y_{aj} or W_{aj} to the parallel viscosities since they did not take account of the ensemble-averaged drift kinetic equation with the quasilinear fluctuation term. The anomalous effects on the banana-plateau fluxes appear through $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$ and Y_{aj} (or W_{aj}). If $\langle |\phi_{\mathrm{k}}|^2 \rangle_{ens}$ is even in $k_\parallel$, $\langle \mathbf{B} \cdot \mathbf{K}_{aj} \rangle$ and Y_{aj} (or W_{aj}) vanish. The spectra of $\mathbf{K}_{aj\perp}$ are larger than those of $\mathbf{K}_{aj\parallel}$ by an order of $k_\perp/k_\parallel (\gg 1)$ so that the anomalous effects on the perpendicular transport is much larger than those on the parallel transport.

The entropy production in the anomalous transport process was given in Eqs. (56)–(58) and its positive definiteness was shown. Then, we identified conjugated pairs of the anomalous fluxes and forces and found the Onsager symmetry satisfied by the transport matrix connecting them. This matrix is a highly nonlinear function of the forces such as the density and temperature gradients through the eigenfrequencies and the fluctuation spectrum.

The magnetic flux surface average of Eqs. (4) and (6) yields the basic equations used for the particle and energy transport analyses. The results of our work suggest that the modified neoclassical fluxes as well as the anomalous fluxes should be included in the total transport fluxes and that the anomalous heat generation terms H_a should be added into the energy transport equations. The anomalous particle fluxes in Eqs. (30) and (31) are intrinsically ambipolar due to the quasineutrality condition $\sum_a e_a \widehat{n}_a = 0$ which is used for the dispersion relation. Thus, in the axisymmetric systems even with the electrostatic fluctuations, the ambipolarity gives no constraint to determine the average radial electric field. In the conventional neoclassical transport theory for the axisymmetric systems, the radial electric field does not affect the particle and heat fluxes and it is not required for the transport analyses. In our case, the radial electric field affects the anomalous energy exchange between the electrons and the ions through the dependence of H_a on the $\mathbf{E} \times \mathbf{B}$ drift frequency ω_E.

However, the anomalous fluxes and the dispersion relations contain ω_E only in the form of $(\omega - \omega_E)$ as seen in Eqs. (51) and (B1), which implies the Doppler shift and no explicit dependence of the anomalous fluxes on ω_E. Further investigations for the determination of the radial electric field and the fluctuation spectrum $\langle|\phi_{\mathbf{k}}|^2\rangle_{ens}$ are required. We are also considering the direct extension of this work to the case of the nonaxisymmetric system with the magnetic fluctuations, which will be reported elsewhere.

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APPENDIX A: SOLUTION OF QUASILINEAR EQUATION IN THE PLATEAU REGIME

Here, we find the solution of the averaged drift kinetic equation (42). Putting

$$\left\langle \overline{f}_{a1} \right\rangle_{ens} = \overline{F}_{a1} + \overline{g}_a \tag{A1}$$

where

$$\overline{F}_{a1} = -\frac{2\pi I}{\chi'} \frac{v_\parallel}{\Omega_a} \frac{\partial \overline{f}_{a0}}{\partial \psi} = \frac{2v_\parallel}{v_{Ta}^2} \left[V_{1a} + \left(x_a^2 - \frac{5}{2} \right) V_{2a} \right] \overline{f}_{a0}. \tag{A2}$$

Equation (42) is rewritten as

$$v_\parallel \mathbf{n} \cdot \nabla \overline{g}_a - C_a^T(\overline{g}_a) = -e_a v_\parallel E_\parallel^{(A)} \frac{\partial \overline{f}_{a0}}{\partial E_0} + C_a^T(\overline{F}_a) + C_a^F + \overline{\mathcal{D}}_a \tag{A3}$$

where C_a^T and C_a^F are the test and field particle parts of the linear collision operator, respectively. The $l = 1$ part in the Legendre polynomial expansion of $\left\langle \overline{f}_{a1} \right\rangle_{ens}$ as a function of ξ is written in the 13M approximation[3] as

$$\left\langle \overline{f}_{a1} \right\rangle_{ens}^{(l=1)} = \frac{2v_\parallel}{v_{Ta}^2} \left[u_{\parallel a} + \frac{2}{5} \frac{q_{\parallel a}}{p_a} \left(x_a^2 - \frac{5}{2} \right) \right] \overline{f}_{a0} \tag{A4}$$

and that of $\overline{g}_a$ is given as

$$\overline{g}_{a1}^{(l=1)} = \frac{2v_\parallel}{v_{Ta}^2} B \left[u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left(x_a^2 - \frac{5}{2} \right) \right] \overline{f}_{a0}. \tag{A5}$$

We find

$$(v_\parallel \mathbf{n} \cdot \nabla \overline{g}_a)^{(l=0)} \equiv \frac{1}{2} \int_{-1}^{1} d\xi v_\parallel \overline{n} \cdot \nabla \overline{g}_a = 0. \tag{A6}$$

Then the $l = 0$ part of Eq. (A3) is given by

$$-C_a^T(\overline{g}_a^{(l=0)}) = \overline{\mathcal{D}}_a^{(l=0)} + C_a^{F(l=0)}. \tag{A7}$$

Subtracting Eq. (A7) from Eq. (A3) yields

$$v_\parallel \mathbf{n} \cdot \nabla \overline{g}_a - \nu_a \mathcal{L}_a \overline{g}_a = \xi F(v, \psi, \theta) + C_a'^{(l \geq 2)} + \overline{\mathcal{D}}_a^{(l \geq 2)} \tag{A8}$$

where $\nu_a \mathcal{L}_a$ denotes the pitch angle scattering part of the collision operator and

$$C'_a = C_a^T(\overline{g}_a) + C_a^F - \nu_a \mathcal{L}_a \overline{g}_a. \tag{A9}$$

The superscript $(l \geq 2)$ represents the sum of the Legendre polynomial components with $l \geq 2$. Here $F(v, \psi, \theta)$ is an isotropic function in the velocity space and its functional form will not affect the results of the following analysis.

Let us put

$$\overline{g}_a = \overline{g}_a^{(l=1)} - (\nu_a \mathcal{L}_a)^{-1} \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle + \overline{h}_a \tag{A10}$$

where $(\nu_a \mathcal{L}_a)^{-1}$ represents the inverse of the pitch angle scattering operator. Here $\overline{g}_a^{(l=1)}$ and $(\nu_a \mathcal{L}_a)^{-1} \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle$ do not contribute to the parallel viscosities. Neglecting $C_a'^{(l\geq 2)}$ compared to the pitch angle scattering term in Eq. (A8), we have

$$\begin{aligned}
&\left[\xi \frac{\partial}{\partial \theta} - \frac{1}{2}\frac{\partial \ln B}{\partial \theta}(1-\xi^2)\frac{\partial}{\partial \xi} - \frac{\overline{\nu}_a}{2}\frac{\partial}{\partial \xi}(1-\xi^2)\frac{\partial}{\partial \xi}\right] \overline{h}_a \\
&= \frac{\partial \ln B}{\partial \theta}\left\{\frac{1-3\xi^2}{2}\frac{2x_a}{v_{Ta}} B \left[u_{a\theta} + \frac{2}{5}\frac{q_{a\theta}}{p_a}\left(x_a^2 - \frac{5}{2}\right)\right] \overline{f}_{a0} - \frac{1}{\nu_a}\int_{-1}^{\xi} \overline{\mathcal{D}}_a^{(l\geq 2)} d\xi\right\} \\
&\quad + (\omega_{Ta} x_a)^{-1}\left\{\overline{\mathcal{D}}_a^{(l\geq 2)} - \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle\right\} \\
&\quad + \xi\left\{(\omega_{Ta}x_a)^{-1} F - \frac{2\overline{\nu}_a}{v_{Ta}} x_a B \left[u_{a\theta} + \frac{2}{5}\frac{q_{a\theta}}{p_a}\left(x_a^2 - \frac{5}{2}\right)\right] \overline{f}_{a0}\right\}
\end{aligned} \tag{A11}$$

where

$$\begin{aligned}
\overline{\nu}_a &= (\omega_{Ta}\tau_{aa})^{-1}\frac{\tau_{aa}\nu_a(x_a)}{x_a} \\
\omega_{Ta} &= \frac{\chi'}{2\pi B\sqrt{g}} v_{Ta} \simeq \frac{v_{Ta}}{Rq}.
\end{aligned} \tag{A12}$$

Here the collision frequency τ_{aa} and the energy dependent collision frequency $\nu_a(x_a)$ are defined in Ref. 2, and we have used the Jacobian $\sqrt{g} = (\nabla\psi \times \nabla\theta \cdot \nabla\zeta)^{-1}$, the major radius $R = |\nabla\zeta|^{-1}$ and the safety factor q. In the large aspect ratio system, we have $\partial \ln B/\partial\theta \simeq \epsilon \sin\theta$

with the inverse aspect ratio ϵ and we also assume that $\left\{\overline{\mathcal{D}}_a^{(l\geq 2)} - \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle\right\} / \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle \sim \epsilon$. For the plateau regime $\epsilon^{3/2} \ll (\omega_{Ta}\tau_{aa})^{-1} \ll 1$, the ordering $\xi \sim \overline{\nu}_a^{1/3} \ll 1$ and the perturbation expansion with respect to $\epsilon/\overline{\nu}_a^{2/3}$ are used[3] to solve Eq. (A11) and the lowest order solution is given by

$$\overline{h}_a = \epsilon \overline{\nu}_a^{-1/3} \left\{ \frac{x_a}{v_{Ta}} B \left[u_{a\theta} + \frac{2}{5} \frac{q_{a\theta}}{p_a} \left(x_a^2 - \frac{5}{2} \right) \right] \overline{f}_{a0} + \frac{1}{\nu_a} \int_0^1 \left\langle \overline{\mathcal{D}}_a^{(l\geq 2)} \right\rangle d\xi \right\}$$
$$\times \int_0^\infty d\tau \sin(\theta - \overline{\nu}_a^{-1/3} \xi \tau) \mathrm{e}^{-\tau^3/6}. \tag{A13}$$

APPENDIX B: ANOMALOUS EFFECTS BY THE ITG MODES WITH $(\omega - \omega_E)/k_\parallel v_{Ti} = O(1)$

The dispersion relation for ion-temperature gradient (ITG) modes given by Eq. (48) is given in Kim *et al.*[15,16] here we simply analyze the limit of small $\omega_D/k_\parallel v_{Ti}$. Assuming that the response of the electrons to the electrostatic fluctuations is approximately adiabatic, the drift kinetic equation and the quasineutrality condition yield the following dispersion relation

$$1 + \frac{\omega_{*e}}{k_\parallel v_{Ti}} Z(\zeta) + \frac{Z_i T_e}{T_i}[1 + \zeta Z(\zeta)] + \frac{\omega_{*e}\eta_i}{k_\parallel v_{Ti}}\left[\zeta + \left(\zeta^2 - \frac{1}{2}\right) Z(\zeta)\right] = 0 \tag{B1}$$

where $Z(\zeta)$ is a plasma dispersion function and $\zeta = (\omega - \omega_E)/k_\parallel v_{Ti}$. Furthermore we assume that the density gradient is small $\omega_{*a} \simeq 0$. From the above dispersion relation, we find that instabilities occur when

$$\left|\frac{\omega_{*e}\eta_i}{k_\parallel v_{Ti}}\right| > \left[2\left(1 + \frac{Z_i T_e}{T_i}\right)\right]^{1/2}. \tag{B2}$$

The adiabatic response of the electrons, which is assumed in the dispersion relation in Eq. (B1), yields no anomalous contributions to the heat generation rate, the particle and heat transport fluxes and the parallel viscosities for electrons. Then the ion particle flux also vanishes according to the ambipolarity resulting from the quasineutrality. Thus, we treat the anomalous contributions to the heat generation rate, the heat flux and the parallel viscosities for ions only.

For $|\omega_{*e}\eta_i/k_\parallel v_{Ti}| \gg 1$, we obtain the typical slab ITG mode instability with

$$\omega - \omega_E \simeq |k_\parallel^2 c_s^2 \omega_{*i}\eta_i|^{1/3}(\sigma_{*i} + \sqrt{3}i)/2. \tag{B3}$$

Here $c_s \equiv (Z_i T_e/m_i)^{1/2}$ denotes the ion sound velocity and

$$\sigma_{*i} \equiv \frac{\omega_{*i}\eta_i}{|\omega_{*i}\eta_i|} = \frac{\mathbf{k}\times\mathbf{n}\cdot\nabla\psi}{|\mathbf{k}\times\mathbf{n}\cdot\nabla\psi|}\frac{\partial T_i/\partial\psi}{|\partial T_i/\partial\psi|} \tag{B4}$$

the sign of the ion diamagnetic frequency. Then, we have $|(\omega-\omega_E)/k_\parallel v_{Ti}| \gg 1$ and therefore the fluctuation-induced forces $\mathbf{K}_{ij}$ and parallel viscosities Y_{ij} $(j=1,2)$ for ions are small. Now, let us consider the unstable modes with $|(\omega-\omega_E)/k_\parallel v_{Ti}| \simeq O(1)$. These modes exist near the marginal point $|\omega_{*e}\eta_i/k_\parallel v_{Ti}| \sim [2(1+Z_iT_e/T_i)]^{1/2}$ and their real frequencies are given by

$$\frac{\omega-\omega_E}{|k_\parallel| v_{Ti}} \sim \sigma_{*i}\left[\frac{1}{2}\left(1+\frac{Z_iT_e}{T_i}\right)\right]^{1/2} \equiv \sigma_{*i}\zeta_0. \tag{B5}$$

Then, from Eqs. (51) and (69), we have $\langle H_i\rangle \simeq 0$ and

$$T_i^{-1}\langle \mathbf{q}_i\cdot\nabla\psi\rangle^{\rm anom} = -4\pi^{1/2}\frac{n_e e c}{Z_iT_e}\,\zeta_0\,\exp\left(-\zeta_0^2\right) \times {\sum_{\mathbf{k}}}'\left\langle \frac{1}{B}|\mathbf{k}\times\mathbf{n}\cdot\nabla\psi|\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}\right\rangle \frac{\partial T_i/\partial\psi}{|\partial T_i/\partial\psi|} \tag{B6}$$

$$\langle \mathbf{B}\cdot\mathbf{K}_{i2}\rangle = -2\pi^{1/2}\frac{n_e e^2}{T_e}\,\zeta_0\,\exp\left(-\zeta_0^2\right){\sum_{\mathbf{k}}}'\sigma_{*i}\left\langle k_\parallel B\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}\right\rangle \tag{B7}$$

$$\begin{bmatrix} Y_{i1}\\ Y_{i2}\end{bmatrix} = \pi^{3/2}\epsilon^2 p_i\omega_{Ti}\tau_{ii}B_0\,\zeta_0\int_0^\infty dx_i \mathrm{e}^{-x_i^2}\frac{x_i^4}{\tau_{ii}\nu_i(x_i)}\begin{bmatrix} 1\\ \left(x_i^2-\frac{5}{2}\right)\end{bmatrix} \times {\sum_{\mathbf{k}}}'\sigma_{*i}k_\parallel\left[y_1\left(x_i,\zeta_0\right)+\frac{2T_i}{Z_iT_e}y_2\left(x_i,\zeta_0\right)\right]\frac{e_i^2\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}}{T_i^2} \tag{B8}$$

$$\begin{bmatrix} W_{i1}\\ W_{i2}\end{bmatrix} = \pi\frac{v_{Ti}^2\tau_{ii}}{B_0}\,\zeta_0\int_0^\infty dx_i \mathrm{e}^{-x_i^2}\frac{x_i^4}{\tau_{ii}\nu_i(x_i)}\begin{bmatrix} \left(-\frac{3}{2}+\frac{1}{6}x_i^2\right)\\ \left(1-\frac{1}{3}x_i^2\right)\end{bmatrix} \times {\sum_{\mathbf{k}}}'\sigma_{*i}k_\parallel\left[y_1\left(x_i,\zeta_0\right)+\frac{2T_i}{Z_iT_e}y_2\left(x_i,\zeta_0\right)\right]\frac{e_i^2\left\langle|\widehat{\phi}_{\mathbf{k}}|^2\right\rangle_{ens}}{T_i^2} \tag{B9}$$

where $\sum'$ represents the summation over the wavenumber region where $|\omega_{*e}\eta_i/k_\parallel v_{Ti}| \sim [2(1+Z_iT_e/T_i)]^{1/2}$ and the functions y_1 and y_2 are defined by

$$y_1(x,\gamma) = \frac{1}{8\gamma}\delta(x-\gamma)+\frac{1}{8x^2}(4\gamma x-6\gamma^2-3)H(x-\gamma)$$

$$y_2(x,\gamma) = \frac{1}{8\gamma}\left(\gamma^2 - \frac{3}{2}\right)\delta(x-\gamma)$$

$$+ \frac{1}{16x^2}\{\gamma(8x^3 - 20x) - (12\gamma^2 + 6)x^2 + 30\gamma^2 + 9\}H(x-\gamma). \qquad \text{(B10)}$$

When $Z_i T_e/T_i = 1$, we obtain Eq. (71) from Eqs. (B6)–(B9).

REFERENCES

1. F.L. Hinton and R.D. Hazeltine, Rev. Mod. Phys. **42**, 239 (1976).

2. S.P. Hirshman and D.J. Sigmar, Nucl. Fusion **21**, 1079 (1981).

3. R. Balescu, *Transport Processes in Plasmas* (2 vols.) (North-Holland, Amsterdam, 1988).

4. P.C. Liewer, Nucl. Fusion **25**, 543 (1985).

5. K.C. Shaing, Phys. Fluids **31**, 8 (1988).

6. K.C. Shaing, Phys. Fluids **31**, 2249 (1988).

7. R. Balescu, Phys. Fluids B **2**, 2100 (1990).

8. W. Horton, Plasma Phys. **22**, 345 (1980).

9. R.D. Hazeltine and J.D. Meiss, *Plasma Confinement* (Addison-Wesley, Redwood City, California, 1992).

10. R.D. Hazeltine, Plasma Phys. **15**, 77 (1973).

11. P.H. Rutherford and E.A. Frieman, Phys. Fluids B **11**, 569 (1968).

12. J.B. Taylor and R.J. Hastie, Plasma Phys. **10**, 479 (1968).

13. W. Horton and R.K. Varma, Phys. Fluids **15**, 620 (1972).

14. W. Horton, Phys. Rep. **192**, 1 (1990).

15. J.Y. Kim and W. Horton, Phys. Fluids B **3**, 1167 (1991).

16. J.Y. Kim, Y. Kishimoto, W. Horton, and T. Tajima, Phys. Plasmas **1**, 927 (1994).

17. J.Q. Dong, W. Horton, R.D. Bengtson and G.X. Li, Phys. Plasmas **1**, 3250 (1994).