DE-FG05-80ET-53088-683 IFSR #683

Envelope Evolution of a Laser Pulse in an Active Medium

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November 1994

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Abstract

We show that the envelope velocity, v_{env}, of a short laser pulse can, via propagation in an active medium, be made less than, equal to, or even greater than c, the vacuum phase velocity of light. Simulation results, based on moving frame propagation equations coupling the laser pulse, active medium and plasma, are presented, as well as equations that determines the design value of super- and sub-luminous v_{env}. In this simulation the laser pulse evolves in time in a moving frame as opposed to our earlier work where the profile was fixed. The elimination of phase slippage and pump depletion effects in the laser wakefield accelerator is discussed as a particular application. Finally we discuss media properties necessary for an experimental realization of this technique.

PACS numbers: 42.65.Re, 52.75.Di, 32.80.-t

I. INTRODUCTION

It has been suggested[1] that an active laser medium may be used to control the envelope propagation velocity of a laser pulse in an active (active-plasma) medium to any design velocity v_{env}, including the speed of light or greater. Such a pulse is said to be superluminous. It is possible with appropriate construction of the laser pulse profile in the direction of propagation to "accelerate" the group velocity of that pulse. This is related to the process of self induced transparency[2] and to the triple soliton solutions already obtained for a (nonactive) plasma system.[3,4] The pre-excited active medium plays the role of a nonlinear amplifier, amplifying the front of the laser pulse and absorbing energy at the rear of the pulse in such a manner as to maintain the pulse shape but at the same time to increase the overall pulse speed. This apparent acceleration of the group velocity even beyond the speed of light does not violate the special theory of relativity as energy and information flow in fact does not exceed c. The leading edge of the pulse which is necessary for acceleration already contains information about the pulse and this information is extracted through the nonlinear amplification process. The energy to form the envelope comes from two sources; one is the normal energy flow of the existing laser pulse which flows at the normal group velocity ($v_{\mathrm{normal}} < c$) and the other contribution for the envelope acceleration is the energy already present in the active medium.

The process of effective acceleration of the photon group velocity and the recovery of laser energy loss by the active medium has applications in photonics and telecommunications as well as to wakefield accelerators.[5] Concerning application to wakefield acceleration, since the pulse can travel at the design speed with no change in its structure, the phase velocity of the accelerating field will also be at the design speed in an active-plasma medium. The increased phase velocity of the wakefield could help overcome slippage between the particle bunch and accelerating field as the particle bunch otherwise outruns the accelerating field due to the difference between the phase velocity of the field and the particle velocity $v \approx c$. Secondly, since the energy used to induce the wake comes from the active medium and not from the laser pulse, pump depletion[6] would be alleviated. Thirdly, using a properly shaped active medium channel in the transverse direction, we should be able to replenish energy loss due to diffraction, thereby optically guiding and effectively focusing the laser pulse without relying on self focusing[7,8] of the laser pulse.

In the previous paper[1] we considered only stationary structures and not its evolution and stability. Here we consider both stationary and nonstationary structures and their stability and evolution. This paper looks at some of questions regarding more realistic conditions and parameters for the laser pulse. First the relationship between the energy density of the medium and design velocity is considered in Sec. II. Then effects of evolving from a vacuum into the medium and the effect of finite length are investigate in Sec. III. Applications to wakefield acceleration and effects of a gaussian laser profile are also investigated in Sec. IV. In Sec. V we discuss possible parameters for and preparation of an active-plasma medium.

II. THEORY

We start with the electromagnetic wave equation in the Coulomb gauge

$$\left(\nabla^2 - \frac{\eta^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{A} + c\nabla\frac{\partial\phi}{\partial t} = -\frac{4\pi}{c}\mathbf{J} - \frac{4\pi}{c}\frac{\partial\mathbf{P}}{\partial t}\ , \tag{1}$$

where $\mathbf{A}$ is the vector potential of the laser pulse, ϕ is the electrostatic potential for the plasma electric field, and the transverse component of polarization vector $\mathbf{P}$ contains only the active material dependent (atomic) response. None of the plasma effects are included in $\mathbf{P}$; they enter through ϕ and J. The index of refraction η is for the nonactive portion of the material response (atomic). The form of the fields in one dimension for a laser of frequency

ω_0 and wavenumber k_0 are

$$P = [U(z,t) + iV(z,t)]e^{i[k_0 z - \omega_o t + \psi(z,t)]} \ , \tag{2}$$

$$A = a(z,t) i e^{i[k_0 z - \omega_o t + \psi(z,t)]} \ , \tag{3}$$

$$J = -\frac{e^2 n A}{\gamma m c} \ , \tag{4}$$

where $U(z,t)$ and $V(z,t)$ are the slowly varying real (electric dipole dispersion) and imaginary (absorption) components of the material polarization vector,[2] $\psi(z,t)$ and $a(z,t)$ are the slowly varying phase and envelope of the laser pulse. The transverse current J is obtained from the conservation of canonical momentum and that initially the plasma is stationary. With the assumption that the laser oscillation time and length scales are much shorter than those for $a(z,t)$, $\psi(z,t)$ and the plasma (ω_p), we multiply Eq. (1) by $\mathbf{A}^*$ and average over the fast laser scales.[2]

In our previous paper[1] we were interested in the exact stationary shape and transformed to a moving frame $\lambda = z - ct$ without looking at the evolution of the pulse. However, since we are now interested in the evolution of a laser pulse traveling at approximately $v_g \approx c$, we allow explicit time evolution of the envelope of the pulse and transform to the moving coordinates $\lambda = z - ct$, and $t = t$. In this moving frame the envelope is slowly varying and we assume $\frac{\partial^2}{\partial t^2} \ll \omega_0 \frac{\partial}{\partial t}$. Dropping the $\frac{\partial^2}{\partial t^2}$ terms and using the zeroth order dispersion relation $\omega_0^2 = k_0^2 c^2 + \frac{\omega_p^2}{\gamma}$, the transformation gives the evolution equations for the slowly varying quantities a and ψ.

$$2a\frac{\eta^2}{c^2}\frac{\partial \psi}{\partial t}\left(\omega_0 + c\frac{\partial \psi}{\partial \lambda}\right) = (1-\eta^2)\frac{\partial^2 a}{\partial \lambda^2} - 2a\left(k_0 - \frac{\eta^2 \omega_0}{c}\right)\frac{\partial \psi}{\partial \lambda} - a(1-\eta^2)\left(\frac{\partial \psi}{\partial \lambda}\right)^2$$
$$+ \frac{2\eta^2}{c}\frac{\partial^2 a}{\partial \lambda \partial t} + \frac{\eta^2 a}{c^2}\left(\frac{\partial \psi}{\partial t}\right)^2 - \frac{4\pi}{c}\frac{e^2 a \delta n}{m\sqrt{1+\left(\frac{ea^2}{mc^2}\right)}} + \frac{4\pi}{c}\left(\frac{\partial V}{\partial t} - c\frac{\partial V}{\partial \lambda}\right) \ , \tag{5}$$

$$\frac{2\eta^2}{c^2}\left(\omega_0 + c\frac{\partial \psi}{\partial \lambda} - \frac{\partial \psi}{\partial t}\right)\frac{\partial a}{\partial t} = 2\left(\frac{\omega_0 \eta^2}{c} - k_0\right)\frac{\partial a}{\partial \lambda} - \frac{2\eta^2}{c}\frac{\partial a}{\partial \lambda}\frac{\partial \psi}{\partial t} - \frac{2\eta^2 a}{c}\frac{\partial^2 \psi}{\partial \lambda \partial t} +$$

$$\frac{4\pi}{c}\left[V\left(\omega_0+c\frac{\partial\psi}{\partial\lambda}-\frac{\partial\psi}{\partial t}\right)\right]-(1-\eta^2)a\frac{\partial^2\psi}{\partial\lambda^2}\ , \tag{6}$$

where δn is the electron density variation from the uniform equilibrium density n_0.

We consider the laser medium in an approximate fashion, taking into account only two levels of interest. This approximation is valid when the interacting electromagnetic wave frequency is at or near resonance and the natural widths of the levels are sufficiently narrow. Laser medium equations for a two state system with dipole transitions at exact resonance ($\mathcal{E}_{\rm res}=\hbar\omega_0$) without relaxation may be given by[2]

$$\frac{\partial V}{\partial t}=c\frac{\partial V}{\partial\lambda}+\frac{\kappa^2}{c}aW\ , \tag{7}$$

$$\frac{\partial W}{\partial t}=c\frac{\partial W}{\partial\lambda}-\frac{\omega_0^2}{c}aV\ , \tag{8}$$

where $\mathcal{E}_{\rm res}$ is the atomic transition energy between the two states under consideration, $e\hbar\kappa$ is the specific dipole moment for the active state and the time constants for incoherent damping effects and energy damping of the active system are assumed to be long compared to the interaction time. The energy density of the active medium W is defined with respect to the ground state $W_0=-N_a\frac{\hbar\omega_0}{2}$ where N_a is the density of possible states active.

The longitudinal laser-plasma interaction is modeled through the ponderomotive force in the electron momentum Eq. (10) and the ions are assumed immobile. The continuity equation takes the form

$$\frac{\partial n}{\partial t}=c\frac{\partial\left(n\left(1-\frac{v}{c}\right)\right)}{\partial\lambda}\ , \tag{9}$$

and the momentum equation is

$$\frac{\partial\gamma v}{\partial t}=c\left(1-\frac{v}{c}\right)\frac{\partial\gamma v}{\partial\lambda}-\frac{eE}{m}+\frac{e^2}{m^2}\frac{1}{2\gamma}\frac{\partial a^2}{\partial\lambda}\ . \tag{10}$$

Poisson's equation becomes

$$\frac{\partial E}{\partial\lambda}=4\pi e n_0\left(\frac{n}{n_0}-1\right)\ . \tag{11}$$

The last seven equations are numerically integrated with specific initial conditions and results are given in Sec. III.

Three conditions must be met to accelerate the pulse envelope to the design speed $v_{\rm env}$ and retain a stationary structure. First is the resonance condition: the laser photon energy should approximately match that of the transition energy in the active medium. Here we consider the case of exact resonance, but small detuning might be both possible and desirable to minimize unwanted nonlinear optical effects. Secondly the laser pulse duration must match the time scale of energy exchange between the active medium and laser pulse (reciprocal of the average Rabi frequency). Thirdly a certain energy density in the active medium is necessary for the acceleration of the pulse envelope.

The third condition states that there be enough energy Γ stored in the active medium prior to the pulse arrival to amplify the leading edge of the pulse to the peak value in the distance L_v ($L_v = l_{\rm pulse} v_{\rm env}/(v_{\rm env} - v_g)$) for the accelerated pulse envelope to outrun the otherwise nonaccelerated pulse over the pulse length $l_{\rm pulse}$. That is, there must be energy equal to that of the pulse stored in the length L_v of the active medium. For envelope deceleration (self-induced transparency) there must be enough empty states to absorb the leading edge to its peak value in the distance L_v. When the atomic effect dominates $\frac{1}{\eta} < \sqrt{1 - (\frac{\omega_p}{\omega_0})^2}$ with $\eta \approx 1$ and $\frac{\omega_0}{\omega_p} \gg 1$ (underdense plasma), this yields

$$W(-\infty) + \mid W_0 \mid = \Gamma_v \approx \frac{l_{\rm pulse}}{L_v}\frac{E^2}{4\pi} = \left(1 - \frac{c}{\eta v_{\rm env}}\right)\frac{E^2}{4\pi}\ , \tag{12}$$

where $W(-\infty)$ is the value of W prior to the arrival of the pulse, Γ_v energy density necessary to accelerate the stationary structure to the design velocity $v_{\rm env}$ and E is the electric field of the laser. The interaction between the laser pulse and active medium is characterized by two time scales, the average inverse Rabi frequency[9] Ω_R^{-1} and the laser pulse scale Ω_l^{-1}. The frequency of energy exchange between the laser and active medium (Ω_R) is derived using Eqs. (7) and (8) along with the approximate average laser intensity $\frac{a^2}{2}$ stationary in the moving frame to yield

$$\frac{\partial^2 V}{\partial \lambda^2} = -\frac{\kappa^2 \omega_0^2 a^2}{2c^4} V \,. \tag{13}$$

This gives $\Omega_R \approx \frac{\kappa\omega_0 a}{\sqrt{2}c^2}$. Additionally the laser time scale comes from Eq. (6), keeping only leading order terms yields

$$\frac{\partial^2 a}{\partial \lambda^2} = \frac{4\pi\kappa^2 W a}{(\eta^2 - 1)c^2} \,. \tag{14}$$

Using Eq. (12) ($v_{\text{env}} = c$) for the energy density gives $\Omega_l \approx \frac{\kappa\omega_0 a}{\sqrt{2}c^2}$. The second necessary condition

$$\Omega_R = \Omega_l \approx \frac{\kappa\omega_0 a}{\sqrt{2}c^2} \tag{15}$$

for a stationary superluminous structure is in fact demonstrated: these two time scales must match. This match is necessary since the energy must return to the active medium on the average Rabi time scale. If the Rabi time scale does not match the duration of the laser pulse, the energy return will be incomplete if the pulse is short and will again extract energy if the pulse is too long.

When the plasma effect dominates over the atomic effect $\frac{1}{\eta} \gg \left(1 - \left(\frac{\omega_p}{\omega_0}\right)^2\right)$, the stored energy for acceleration to the design velocity becomes

$$W(-\infty) + \mid W_0 \mid = \Gamma_v \approx \left(1 - \frac{c}{v_{\text{env}}}\left(1 - \frac{\omega_p^2}{2\omega_0^2}\right)\right)\frac{E^2}{4\pi} \,. \tag{16}$$

Setting the velocity of the laser pulse envelope $v_{env} = c$ yields

$$\Gamma_c = W(-\infty) + \mid W_0 \mid \approx \frac{\omega_p^2}{2\omega_0^2}\frac{E^2}{4\pi} \,. \tag{17}$$

The energy density $W(-\infty)$ for the stationary pulse structure must supply the energy for the wakefield. That is, to be stationary, the pulse must lose no energy. For a laser pulse $l_{\text{pulse}} \approx \lambda_p$ we have the energy density necessary to overcome pump depletion

$$W_{\text{pump}} \approx \frac{E_p^2}{4\pi} \,. \tag{18}$$

With Eq. (17) gives

$$W_{\text{pump}}(0) \approx \left(\frac{\omega_p}{\omega}\right)^2 \left(\frac{eE}{m\omega_p c}\right)^2 \frac{\omega_p^2}{2\omega^2}\frac{E^2}{4\pi} = \left(\frac{\omega_p}{\omega}\right)^2 \left(\frac{eE}{m\omega_p c}\right)^2 \Gamma_c \,. \tag{19}$$

In all regimes for acceleration $(\frac{\omega_p}{\omega})^2 \left(\frac{eE}{m\omega_p c}\right)^2 \ll 1$. Therefore, the energy density necessary for acceleration to c is much larger than that to overcome pump depletion.

It is foreseeable that very short laser pulses may be produced such that the pulse length is short compared to the ionization time for that material ($\tau_i \gg \tau_{\text{pulse}}$). We note the recent progress of ultrashort laser pulse technology[10,11] in particular. This could allow $\frac{N_a}{n_0} \gg 1$ with a very short plasma wavelength and therefore very large accelerating gradients with a phase velocity equal to c. Also with a properly prepared short pulse it may be possible to induce and absorb the plasma oscillation within the laser pulse producing no wake and therefore not destroying dense material.

III. SIMULATION AND RESULTS

Equations (5)–(11) were numerically integrated with initial conditions. The initial condition include material dependent constants, the dipole moment κ, the initial imaginary part of the polarization V_0, energy density W_0, $\frac{\omega_0}{\omega_p}$ and the index of refraction η for the inactive portion of the response of the material. In the simulations all the active states are considered to be initially full or depleted. This implies that the imaginary part of the polarization $V_0 = 0$ and the initial energy density $= \pm W_0$ where for all states initially active we have $+W_0$ and for all states initially depleted $-W_0$. The initial condition of a partially full energy density corresponds to the initial imaginary part of the polarization not equal to zero ($V_0 \neq 0$). To simulate realistic conditions, we have the laser pulse starting outside of the active medium (active-plasma medium) traveling into the medium.

In Ref. 1 we demonstrated that there is a stable structure ($\approx$ sech). However, to simulate most experiments, it is necessary to consider the evolution of the laser pulse under realistic conditions. First of these properties is the evolution of the laser pulse from vacuum into the active medium (active-plasma medium). In all the simulations the laser pulse is initialized in a vacuum and evolves into the active (active-plasma) medium which has a gaussian edge profile. Since the leading edge of the laser pulse enters the active medium ahead of the trailing edge of the pulse, it is amplified while the tail is not absorbed, increasing its width. Due to this initialization of the width of the pulse should be to a value less than the predicted value due to this broadening of the laser pulse as it enters the active medium. The higher the gain of the medium, the larger the initialized reduction in width. In all the simulations this had no noticeable effect on the long term stability of the laser pulse, whether the pulse was a sech or gaussian. The first parameter we checked is the relationship between the energy density (W_0) and the envelope velocity of the laser pulse. In table 1 we show the results of different simulation runs with an energy density W_0 given by Eq. (12) for a sech laser pulse with different $\frac{v_{\text{env}}}{c}$. For these cases $\frac{\omega_0}{\omega_p} = 100$ so that the plasma will have little effect on the velocity ($v_g = 0.999995c$). The other parameters were $a = 0.1\frac{mc^2}{e}$, $\kappa = 0.19\frac{e}{mc}$, $\eta = 1$. From this table we see that the results compare very well with the theory. For "deceleration" of the envelope velocity we start with the initial conditions of a negative energy density and the imaginary part of the polarization vector equal to zero. An energy density $W_0 < 0$ (with $V_0 = 0$) corresponds to an active medium with no states active. In this case as the laser pulse enters an inactive medium, the process of self-induced transparency takes place,[2] reducing the envelope velocity to less than c. In all the cases only the energy density is varied to produce the acceleration and all other parameters were left unchanged. The width of the laser pulse was about $4\frac{c}{\omega_p}$ approximately equal to that predicted by Eq. (15).

Figure 1(a) shows the results for $v_{\text{env}} = 1.01c$ ($W_0 = 0.5n_0$ mc^2) with all parameters as above for the initial laser pulse and the pulse at $\omega_p t = 50$. Note the advancement of the envelope in this moving frame, indicating an envelope velocity greater than c. Figure 1(b) shows the energy density of the active medium at $\omega_p t = 25$ and $\omega_p t = 50$ and Fig. 1(c) shows the wakefield at time $\omega_p t = 25$ and $\omega_p t = 50$. Note the accelerated phase velocity observed by an advancement of the wakefield in this moving frame also indicating that its phase velocity is greater than c ($v_{\text{phase}} = 1.01c$). Also note the complete return of the energy density to its value at infinity as is necessary for a stationary structure.

The second condition is that of a finite laser pulse, that is a sech laser pulse with a sharp leading edge. Simulations show that the velocity of the peak portion of the laser pulse travels with the design velocity (v_{des}) if the peak is far from the sharp edge. This edge travels with the normal group velocity as the peak portion travels up towards the sharp edge. As the peak approaches the leading edge, its velocity is reduced from the design velocity and approaches the normal group velocity. When the peak has caught up with the leading edge, its velocity is now equal to that of the edge, the normal group velocity. Figure 2 shows a sech laser pulse with a finite leading edge ($a = 0$ at $\lambda \approx 18\frac{c}{\omega_p}$) and compares its evolution to that of the same sech with an infinite leading edge. The simulation shows that the peak of the finite pulse travels up to the leading edge and the velocity of the sharp edge is equal to the normal group velocity, which in this case is $v_g = c$, since there is no plasma and $\eta = 1.0$. Initially the peak starts at the design velocity ($v_{\text{env}} = 1.01c$) but slows down, as it approaches the leading edge of the laser pulse. Upon reaching the leading edge the peak travels with the normal group velocity and is amplified. The same figure also shows where the same laser pulse would be if it had an infinite leading edge. Note that the infinite pulse has traveled past the finite pulse with the envelope velocity $v_{\text{env}} = 1.01c$.

IV. APPLICATIONS TO WAKEFIELD ACCELERATION

One of the major problems with the LWFA is that the laser pulse and therefore the phase velocity of the wakefield does not travel at the speed of light c in a nonactive plasma. This causes the ultra-relativistic particles to dephase with the accelerating field as the accelerated particles outrun the accelerating field.[6] For application of wakefield acceleration we show that the laser envelope and wakefield are "accelerated" to the speed of light c in an active-plasma medium ($v_{\text{env}} = c$) in accordance with Eq. (17) from its original velocity of $v_g = \sqrt{1 - \frac{\omega_p}{\omega_0}} c < c$. Figure 3(a)–(b) shows an initial sech laser pulse and the same pulse at time $\omega_p t = 50$ with $\frac{\omega_0}{\omega_p} = 10$ and no active medium. The simulation shows that the pulse traveled with the normal group velocity of $v_{\text{normal}} = 0.995c$, as expected (this corresponds to a velocity of $v_{\text{normal}} = -0.005$ in the frame moving with the speed of light). The laser pulse is retarded in this moving frame as is the wakefield. This reduction in pulse velocity from the speed of light causes a reduction in the phase velocity of the wakefield ($v_{\text{phase}} = 0.995c$), thereby causing phase slippage between the ultrarelativistic particles and the accelerating field. In Fig. 3(c)–(d) we show the same laser pulse as above except in an active-plasma medium. The plasma parameters are as above and the energy density of the active medium is given according to Eq. (17) ($W_0 = 0.0025 n_0 mc^2$) to accelerate the envelope velocity to the speed of light. The simulation recorded a velocity of $v_g = 1.000c$. Also note that the phase velocity of the accelerating field is now the speed of light and there is no slippage between the fields in this moving frame. This will eliminate the problem of phase slippage between the accelerating field and the particles to be accelerated.

The last parameter we looked at is the evolution of a gaussian laser pulse. Since it is not the stationary form, the pulse shape will evolve in time. The important parameters here are the way the form of the pulse evolves and the time scale for that evolution. The importance of the way the pulse shape changes depends upon the application. Figure 4(a,b) shows a gaussian laser pulse in a non-active medium shown with $\frac{\omega_0}{\omega_p} = 10$, having a group velocity in the simulation of $v_g = 0.995c$. Again we see the phase velocity of the wakefield less than c ($v_p = 0.995c$). Since a gaussian shaped laser pulse is not the shape for a stationary structure, we want to study the stability of the pulse in an active medium. In Fig. 4(c)–(d) we see the evolution of a gaussian laser pulse in an active medium with an energy density of $W_0 = 0.0025 n_0\,\mathrm{mc}^2$, the same value as in Fig. 3(c)–(d) to accelerate the sech laser pulse to the speed of light c. Note that the peak portion of the laser pulse travels at the design velocity c. However, the front and tail of the laser pulse do not travel at c. Also the amplitude of the peak increases to evolve towards the stationary form of a sech. For application to acceleration it is the peak portion of the laser pulse which induces the wake and therefore controls the wakefield amplitude and phase velocity. This can be seen as there is almost no dephasing in the moving frame between the wakefield at the end and at the earlier time as there is in the nonaccelerated case. Since the gaussian pulse is not the stationary structure, a higher energy density is necessary than that given by Eq. (17). We see that the velocity of the peak portion of the gaussian laser pulse is stable for a long period of time and may be used to accelerate the phase velocity of the wakefield overcoming the problem on phase slippage between particles and the accelerating field.

V. ACTIVE-PLASMA MEDIUM

A number of conditions must be met for a material to be used as the active medium-plasma source. The basic concept we suggest here is to use the outer shell electrons of atoms to form a plasma and inner shell electron resonant transitions as the active source, as in the basic scheme used in X-ray lasers.[12] The inner shell transition energy of the active material must match that of the laser frequency (resonance). For applications using high intensity laser pulses, the problems of the competing processes of ionization, power broadening of the resonance and locating resonances in an intense electric field must be addressed.

For a resonance to be sharp enough to allow the lasing transitions, the maximum electric field of the laser must be small when compared to the atomic ion field. Using the simple Bohr model of the atom ionized to a charge of $Z-1$, the electric field at the outer most electron (resonance electron) is approximately

$$E_{\rm atom} \approx \frac{Z^3 e}{n^{*4} a_0^2} , \tag{20}$$

where n^* is the effective principle quantum number and a_0 is the Bohr radius. In this model we assume for simplicity a single outer electron with a core of the remaining un-ionized electrons and the nucleus. If this outer electron is in the closest shell to the core, then $n^* \approx 1$. For the atomic electric field to be 10 (100) times that of the peak laser field, the critical intensity is

$$I_{c10(100)} = \frac{1}{10(100)} \frac{Z^6 e^2 c}{n^{*8} a_0^2} . \tag{21}$$

Considering a laser pulse with peak intensity of 10^{18} W/cm^2, we find that the atom must be ionized to 4 (8) for the atomic field to be 10 (100) times that of the peak laser field according to Eq. (21) with $n^* = 1$. This seems realizable.

In addition ionization takes place when the Coulomb barrier is suppressed by the laser field at a critical strength, and the atomic electron can freely escape. According to the Coulomb Barrier Suppression (CBS) theory[13] this corresponds to a laser intensity threshold of

$$I_{th} \left(\frac{\mathrm{W}}{\mathrm{cm}}^2 \right) = 4.00 \times 10^9 \frac{E^4(\mathrm{eV})}{Z^2} , \tag{22}$$

where $E = \frac{Z^2}{2n^{*2}}$ or as above when the atomic and laser fields are about equal. Numerical simulations based on the CBS model (see Fig. 5) show that for a 100 fs laser pulse at an intensity of 10^{18} W/cm^2 the degree to which ionization will take place from this pulse is 8(Ne) to 19(Xe). Atoms ionized to above these levels will have their outermost surviving electrons (un-ionized) which may be used for the laser resonance.

Power broadening of the energy levels due to the Stark shift[14] by the laser electric field is proportional to the laser power E^2. This gives the Stark shift $\Delta\mathcal{E}_{\mathrm{Stark}}$ of the level with energy $\mathcal{E}$

$$\Delta\mathcal{E}_{\mathrm{Stark}} \approx \mathcal{E} \left(\frac{E_{\mathrm{atom}}}{E} \right)^2 . \tag{23}$$

The requirement that $\Delta\mathcal{E}_{\mathrm{Stark}} \leq \frac{\mathcal{E}_{\mathrm{res}}}{10}$ gives another condition for the laser intensity I_{Stark}

$$I_{\mathrm{Stark}} \leq \frac{1}{10} \frac{\Delta\mathcal{E}_{\mathrm{res}}}{\mathcal{E}} \frac{E_{\mathrm{atom}}^2}{4\pi} . \tag{24}$$

Given $\frac{E_{\text{atom}}}{E} \approx 10(100)$, the shift in the energy will be approximately 10^{-2} (10^{-4}) and for an energy level of approximately 1000 eV the corresponding energy shift is approximately 10 (0.1) eV. With the requirement that the shift be no more than 1/10 the energy of the laser for a 1-10 eV photon resonance the $\frac{E_{\text{atom}}}{E}$ has to be approximately 100, giving an ionization depth of about 10 electrons.

There are other conditions which, although not necessary, would help to optimize the application for a wakefield accelerator. Since the interaction of the laser with the plasma (ponderomotive force) is proportional to $(\frac{\omega_0}{\omega_p})^2$, for the strongest wakefield we prefer $\frac{\omega_0}{\omega_p} \geq 1$. The optimal pulse length for wakefield production is $l_{\text{pulse}} \approx \frac{c}{\omega_p}$, although our latest work[15] shows that l_{pulse} can be substantially longer than this in a sufficiently strong laser intensity regime. This translates to a present day short pulse of approximately 100 fs. Near future lengths of 10 fs makes the preferred transition and therefore laser photon energy in the visible to ultraviolet region. For these low energy resonance transitions in highly ionized atoms we want transitions corresponding only to Δl or Δm (as in the Argon ion laser which uses the $4p$ to $4s$ line) but not those with change in principle quantum number n. Transitions corresponding to Δn are in the X-ray regime and require future development of X-ray lasers to be employed for this application.

The synthesis of the material for use (ionization, activation etc.) might involve several stages of preparation. Initial formation of the plasma and the ion species could be accomplished by a number of methods such as a long laser pulse or strong static field. Next the material could be activated (if not already accomplished by the first step) through another laser pulse at the resonance frequency, by cascading of electrons down to fill the inversion level or some physical mechanism such as collisions.

A hypothetical example of a possible candidate for a material is Argon. Coulomb Barrier Suppression simulations show that at 10^{18}W/cm^2 Argon will not be ionized past its outer shell 8 electrons for a 100 fs laser pulse. A long ionization prepulse could ionize all the outer shell electrons (8) and one of the next inner shell electrons. We would then look for possible inversions in the $2p - 2s$ transition. Alternately we could ionize the outer shell of Argon and active one of the lower level electrons to the now empty 3 shell looking for an inversion in this shell. This would take a Δn transition to activate and use a Δl in the third shell as the active transition. The Δn transition may possibly be accomplished through collisions, multiphoton absorption of a laser pulse or a high energy photon laser (X-ray). These laser pulses (for activation) would not necessarily need to be short or very high power.

Using our simple model, it is possible to construct a laser pulse in an active medium without pump depletion whose group velocity is adjustable to the speed of light c. In the limit that the plasma density goes to zero ($\phi \rightarrow 0$) this stationary structure has the form $a^2 \propto \mathrm{sech}^2$.[2] Since the energy transferred to the plasma is a small portion of the energy of the pulse, we expect the stationary structure with a plasma to be similar to the sech^2. Simulation results shows this holds for our examples.[1] Since the plasma does not couple directly to the active medium, the energy to form the wakefield originates in the active medium, flows to the laser pulse and then finally to the wakefield.

An alternate approach would be to use an alternating series of accelerating regions (plasma) each followed by an active medium region (nonplasma) used to shape the laser pulse.[6] The laser intensity in the active region could be reduced using optics to expand the laser pulse and then focus down after the pulse has left the active medium region. This would eliminate the problems associated with the high intensity laser pulse. In this case the phase velocity of the wakefield would be that of a normal plasma (less than c) and therefore the accelerating length would have to be less than the dephasing length. This would still be able to overcome the problems of pump depletion and focusing.

VI. CONCLUSION

In conclusion we find that with an appropriately constructed laser pulse in an active medium we are able to control the envelope velocity to any desired velocity including greater than the speed of light c. We show that it is possible to use this to overcome the dephasing of accelerated particles in a LWFA scheme by increasing the phase velocity of the accelerating field by controlling the envelope velocity of the laser pulse. It is shown that even a nonstationary shape (gaussian) may be used to accelerate the phase velocity of the wakefield. Finally, the present solution of accelerated group velocity and the amplification of the pulse may be applicable to other situations such as photonics and telecommunications.[16] The present model of the two level atoms as well as other idealizations is a first step toward understanding the resonant interaction of the laser with the medium to create a superluminous pulse. Other effects such as multilevel atomic states, ionization,[17] and more precise identification of appropriate materials have to be addressed experimentally and by more detailed and perhaps less transparent theories in future investigations.

Acknowledgments

This work was supported by the U.S. Dept. of Energy contract no. DE-FG05-80ET-53088.

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TABLE 1

Design β	W_0	Simulation β
0.980	-1.0204	0.980
0.990	-0.505	0.990
1.000	0.00	1.000
1.010	0.50	1.010
1.020	0.98	1.020
1.030	1.456	1.029

Table 1 shows the theoretical initial energy density (W_0) for different $\beta = \frac{v_{env}}{c}$ from Eq. (12) compared to the simulation β for that initial energy density. Initial conditions were $\eta = 1$, $a = 0.1\frac{mc^2}{e}$, $\frac{\omega_0}{\omega_p} = 100$ and $\kappa = 0.19\frac{e}{mc}$ for a sech laser pulse.

FIGURE CAPTIONS

Fig. 1. (a) Initial sech laser pulse (solid), and that laser pulse at $\omega_p t = 50$ (dashed) for the initial conditions of $W_0 = 0.5 n_0 mc^2$ (theoretical value for $\beta = 1.01$). 1(b,c) the energy density of the active medium (wakefield) at $\omega_p t = 25$ (solid) and $\omega_p t = 50$ (dashed). Initial conditions as used in Table 1.

Fig. 2. Initial finite sech pulse (solid) with a sharp leading edge (edge at $\approx 18\frac{c}{\omega_p}$). Also shown that finite pulse at time $\omega_p t = 220$ (dashed)and the position of the same initial sech pulse if it had be infinite also at time $\omega_p t = 220$ (long dashed). Note that the leading edge of the finite pulse travels at $v_g = c$ since there is no plasma and $\eta = 1.0$. The initial energy density of the active medium is $W_0 = 0.5 n_0 mc^2$, $\frac{\omega_0}{\omega_p} = 100$, $\eta = 1$, and $\kappa = 0.19\frac{e}{mc}$.

Fig. 3. (a) Initial laser pulse (solid) and at $\omega_p t = 50$ (dashed) for a sech laser pulse with $\frac{\omega_0}{\omega_p} = 10$ showing reduction from the vacuum velocity due to the plasma with no active medium. 3(b) shows the reduction in the phase velocity of the wakefield at time $\omega_p t = 25$ (solid) and final time $\omega_p t = 50$ (dashed). 3(c) Initial laser pulse (solid) and that pulse at $\omega_p t = 50$ (dashed) as above, however, now in an active-plasma medium with energy density $W_0 = 0.0025 n_0 \mathrm{mc}^2$ the amount required according to Eq. (17) to accelerate the envelope to $v_g = c$. 3(d) shows the wakefield at time $\omega_p t = 25$ (solid) and at time $\omega_p t = 50$ (dashed) showing a phase velocity of c.

Fig. 4. (a) Initial gaussian laser pulse (solid) and that pulse at $\omega_p t = 50$ (dashed) with $\frac{\omega_0}{\omega_p} = 10$ showing reduction from the vacuum velocity due to the plasma with no active medium. 4(b) shows the reduction in the phase velocity of the wakefield with a time of $\omega_p t = 25$ (solid) and final time $\omega_p t = 50$ (dashed). 4(c) Initial laser pulse (solid) and that pulse at $\omega_p t = 50$ (dashed) as above but in an active medium (as

in 3c,d). 4(d) shows the wakefield at time $\omega_p t = 25$ (solid) and final time $\omega_p t = 50$ (dashed) showing a phase velocity of c.

Fig. 5. Degree of ionization for noble gases according to CBS theory for a particular intensity $(\frac{W}{\mathrm{cm}^2})$.

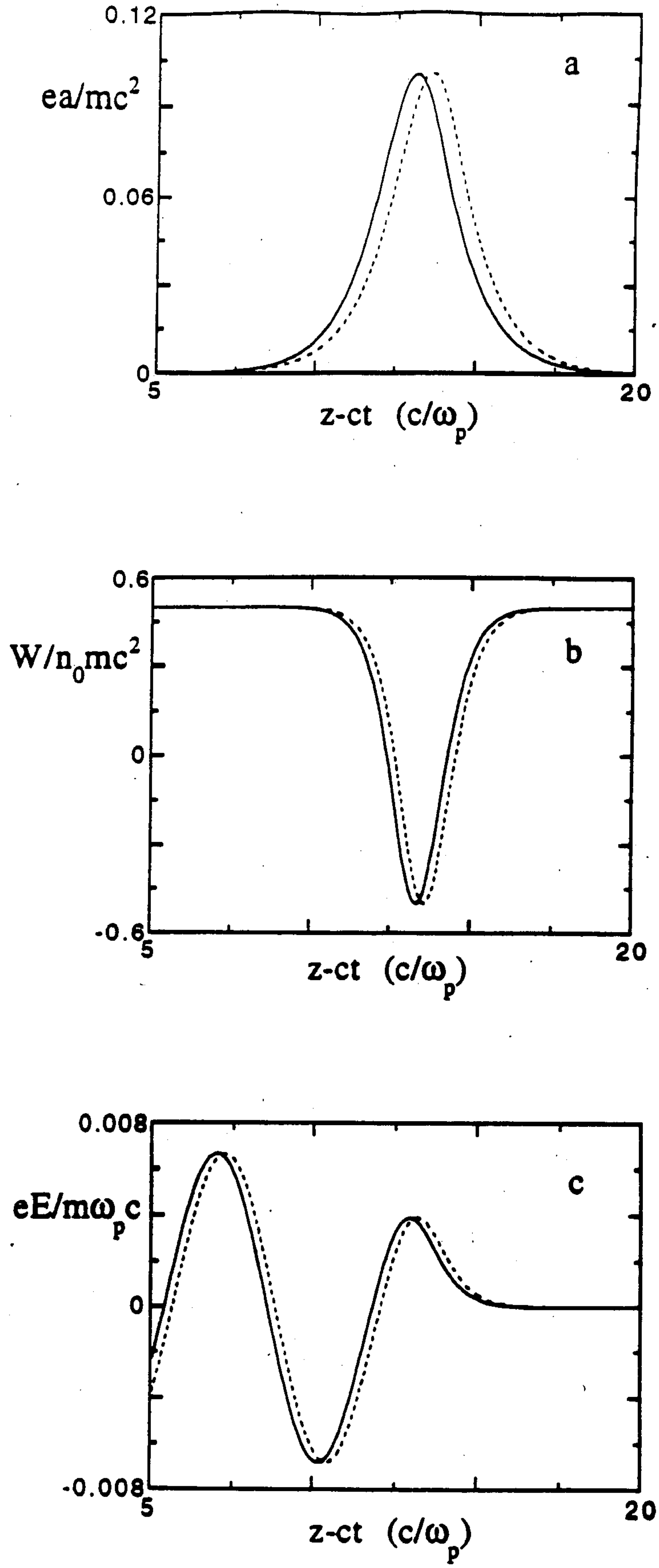

0.12
0.06
0
ea/mc^2
a
5
20
z-ct (c/ω$_p$)
0.6
0
-0.6
W/n$_0$mc^2
b
5
20
z-ct (c/ω$_p$)
0.008
0
-0.008
eE/mω$_p$c
c
5
20
z-ct (c/ω$_p$)

Figure 1

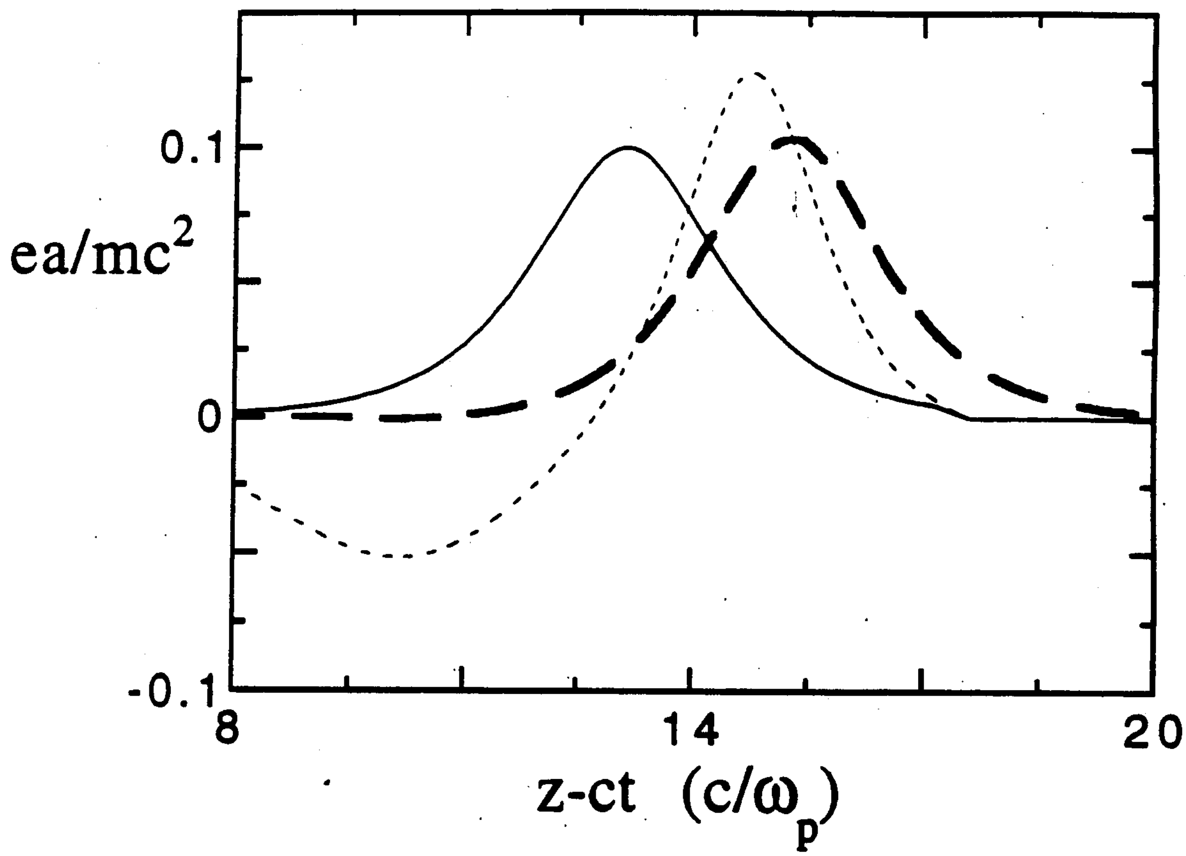
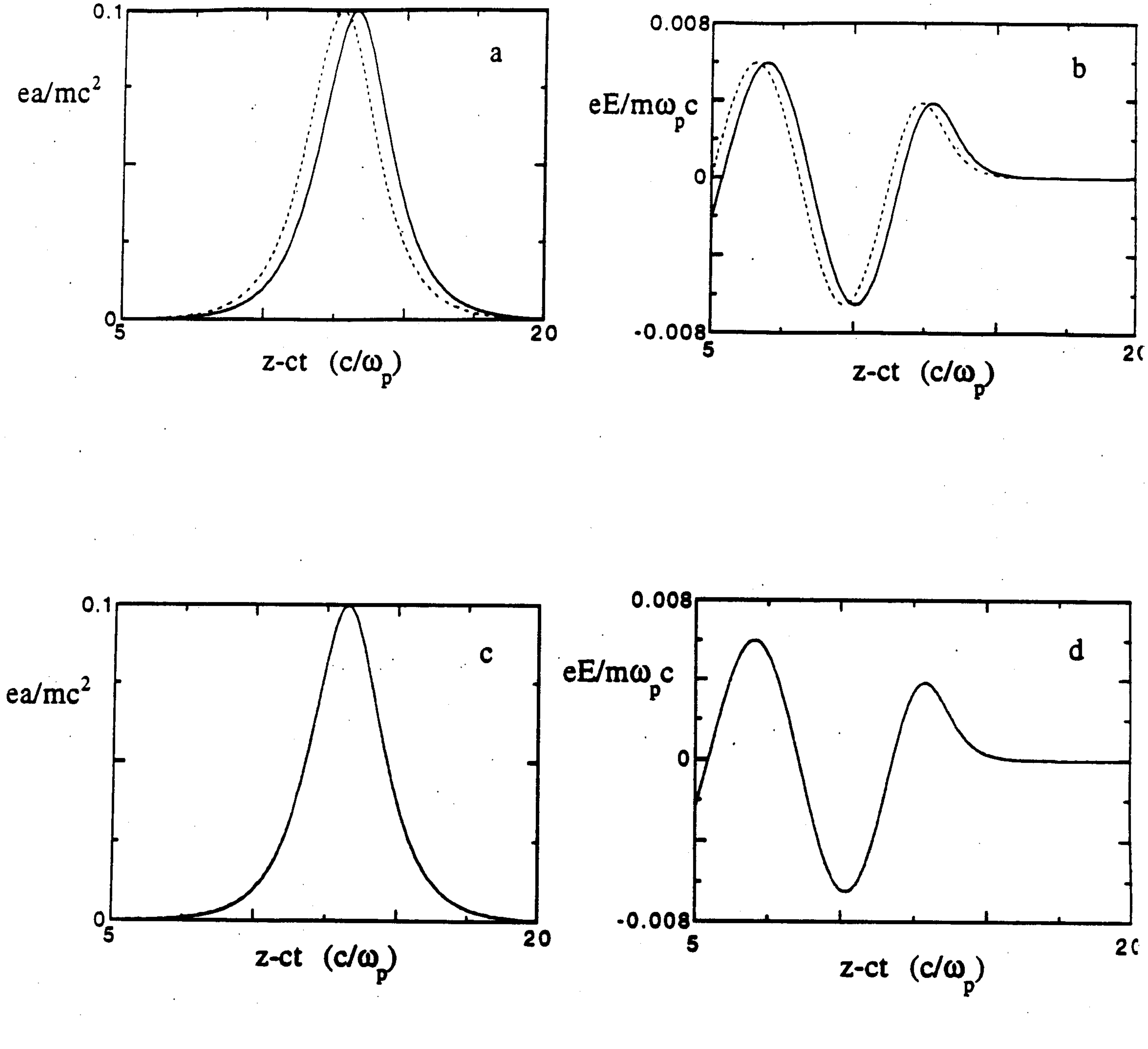


Figure 2



0.1
ea/mc²
a
0
5
20
z-ct (c/ωp)
0.008
eE/mωpc
b
0
-0.008
5
z-ct (c/ωp)
0.1
ea/mc²
c
0
5
20
z-ct (c/ωp)
0.008
eE/mωpc
d
0
-0.008
5
z-ct (c/ωp)

Figure 3

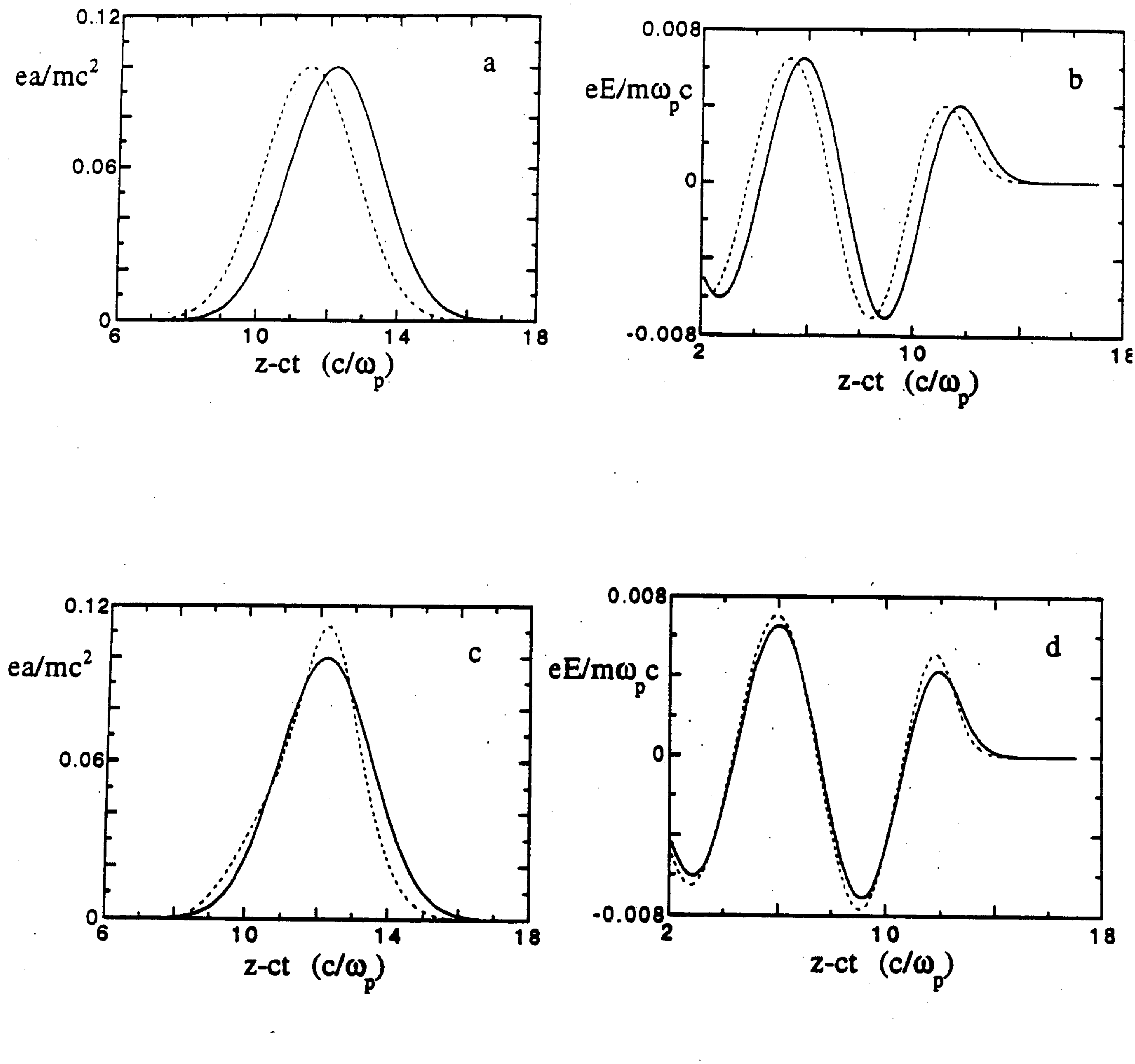
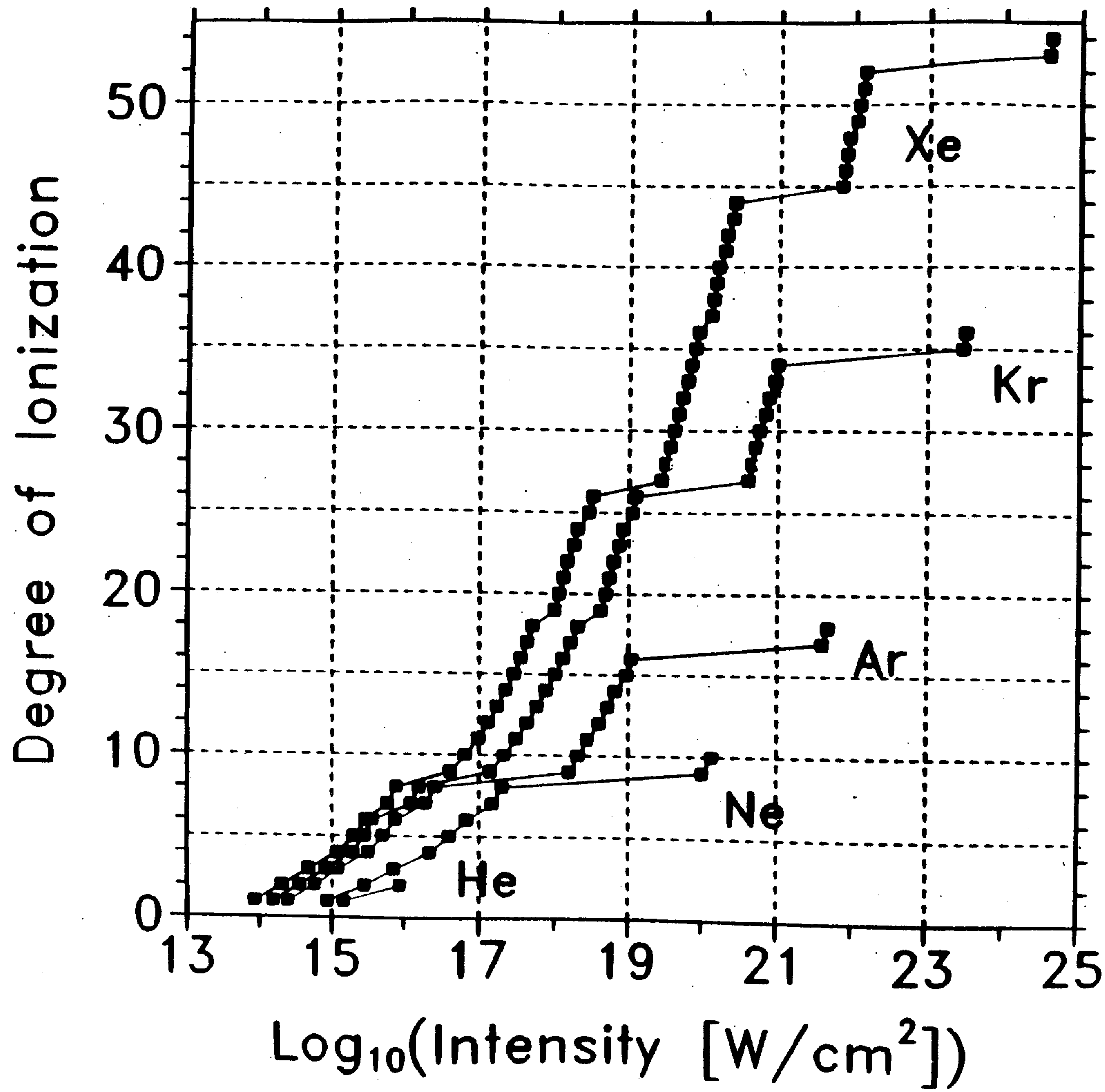

0.12
0.06
0
ea/mc^2
a
6
10
14
18
z-ct (c/ω$_p$)
0.008
0
-0.008
eE/mω$_p$c
b
2
10
18
z-ct (c/ω$_p$)
0.12
0.06
0
ea/mc^2
c
6
10
14
18
z-ct (c/ω$_p$)
0.008
0
-0.008
eE/mω$_p$c
d
2
10
18
z-ct (c/ω$_p$)

Figure 4



Degree of Ionization
50
40
30
20
10
0
13
15
17
19
21
23
25
Xe
Kr
Ar
Ne
He
Log10(Intensity [W/cm2])

Figure 5