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Collisional drift wave transport

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The turbulent transport produced by the standard collisional drift wave model is re-examined for the presence of nonlinear, self-focused vortex structures. The nonlinear instability of Scott (1993) is found and interpreted in terms of the coherent vortex structures which are well known to have weak shear damping rates compared to linear modes. The parametric scaling of the observed transport is examined with respect to the collisionality parameter and the gyroradius scaling. In the scrape-off layer (SOL) the long magnetic field line length and high plasma collisionality allow the possibility of Bohm-like transport. Comparisons with the large scale, coherent fluctuations observed the SOL plasma of ASDEX are made.

I. INTRODUCTION

The importance of plasma transport in the scrape-off layer plasma (SOL) and the other high collisionality edge plasmas such as in the Texas Experimental Tokamak Upgrade (TEXT-U) where high levels of fluctuations are ubiquitous leads one to re-examine the dynamics of the nonlinear collisional drift wave instability. This instability is also of fundamental importance in the subject of anomalous transport since it was the first drift wave instability to be identified in the laboratory using the quiescent Q-machine device. The fundamental studies of Hendel*et al.* (1967, 1968) and the associated nonlinear calculations of Stix (1969), Hinton and Horton (1971) and Simon and X (1973) have given one of the most detailed, quantitative comparisons between experiments and transport theory for magnetized, confined plasmas. In these experiments in a straight cylindrical geometry the low m-modes are unstable and there is no magnetic shear. The theoretical calculations were based on bifurcation analysis using a third order expansion from the state of marginal stability. The same method has been used in sheared magnetic fields for the resistive (Sugama and Wakatani, 1988, 1991; Hamaguchi, 1989) modes and the ion temperature gradient modes (Hamaguchi and Horton, 1990).

The collisional drift wave equations are well known from Hinton-Horton (1971), Varma-Horton (1973), Drake and Hassam), Terry and Diamond (1985), and Scott (1992). Here we follow the notation of Scott (1992), so as to be better able to compare with his extensive numerical simulations.

The collisional drift wave dynamics involves the interaction of the four fluctuations $\tilde{\varphi}, \tilde{n}, \tilde{u}_{\|}$, and $\widetilde{T}_e$ as governed by the collisional $\overline{k}_{\|}\, v_e \leq \nu_e$ transport equations. In this study we adopt the recent standard reductions of the drift wave problem to only include the $\mathbf{E}\times\mathbf{B}$ convective nonlinearities, dropping many of the nonlinearities considered important in the early Q-machine work. The extent to which this reduction is justified in the SOL plasma must be considered *a posteriori*.

In this work we compare the turbulence and the associated transport produced in two regimes: (i) the shearless geometry characteristic of the plasma outside the separatrix where the open field lines terminate on a limiter or diverter plate (Xu, 1993) and (ii) in the extreme edge of low temperature tokamak plasma where the magnetic shear is important. In both cases other physical processes will also need to be considered, such as the influence of plasma flows and the effect of radiation and recombination. These processes are neglected in the present study.

II. MODEL EQUATIONS

The collisional drift wave equations are well known from the early works on drift waves. In nonlinear studies the equations are reduced (Hinton and Horton, 1971; Hasegawa-Wakatani, 1983; Scott, 1992). Here we used the reduced form given by Scott (1992) in order to make direct comparisons with his finding. We point out that some terms included in the early study of

Hinton and Horton (1971) but dropped in the $\mathbf{E}\times\mathbf{B}$ nonlinearity reduction may be important in actual applications in the SOL plasma where the plasma gradients are strong.

The standard drift wave space-time units $(\rho_s, L_n/c_s)$ and amplitude (ρ_s/L_n) units are used to yield the dimensionless equations with only the $\mathbf{E}\times\mathbf{B}$-convective nonlinearities with unit coefficients.

In the dimensionless variables the equations for the four dimensionless fluctuations $e\widetilde{\phi}/T_e$, $\widetilde{n}/n$, $\widetilde{u}_{\|i}/c_s$ and $\widetilde{T}_e/T_{e0}$ in the units of ρ_s/L_n are as follows:

$$\frac{d}{dt}\nabla_\perp^2\phi = C^{-1}\nabla_\|^2(n-\phi+(1+\alpha)T)+\mu_\perp\nabla_\perp^4\phi \tag{1}$$

$$\frac{d}{dt}n = -v_d\frac{\partial\phi}{\partial y}-\nabla_\| v + C^{-1}\nabla_\|^2(n-\phi+(1+\alpha)T)+\mu_\perp\nabla_\perp^2 n \tag{2}$$

$$\frac{d}{dt}v = -\nabla_\|(n+T)+\mu_\|\nabla_\|^2 v+\mu_\perp\nabla_\perp^2 v \tag{3}$$

$$\begin{aligned}\tfrac{3}{2}\frac{d}{dt}T = -\tfrac{3}{2}\eta_e v_d\frac{\partial\phi}{\partial y}-\nabla_\| v+(1+\alpha)C^{-1}\nabla_\|^2(n-\phi+(1+\alpha)T)\\ +1.07C^{-1}\nabla_\|^2 T+\mu_\perp\nabla_\perp^2 T\end{aligned} \tag{4}$$

where

$$\frac{d}{dt}=\frac{\partial}{\partial t}+\hat{\mathbf{z}}\cdot\nabla\phi\times\nabla\ . \tag{5}$$

For the parallel diffusion operator $\nabla_\|^2$ we use two models. For plasma inside the separatrix we use the standard sheared slab model with

$$\nabla_\| = Sx\frac{\partial}{\partial y}\to iSxk_y\ .$$

For plasma outside the separatrix we model the parallel variation of $\phi(x, y, s, t) = f(s)\ \phi(x, y, t)$ and take the projection of the equations (1)–(4) onto the fixed parallel wave function $f(s)$ by operating with $\int ds f(s)/B(s)$. Carrying out this integration along the field line yields end-point contributions from the fluctuation current $\delta j_{\|}(x, y, s = \pm L_z, t)$. Here we may follow Berk *et al.* (1993) introducing the sheath impedance (or admittance) function $\delta j_{\|}(x, y, s = \pm L_z/2) = \pm enc_s f(s = \pm L_z/2)(\Sigma_{11}\tilde{\phi} + \Sigma_{12}\tilde{T})$ to describe physics of the sheath plasma surrounding the limiter or diverter plate. Taking appropriate Σ's leads to the wall impedance driven instabilities of Berk *et al.* (1993).

In the example considered by Xu (1993) $\Sigma_{\|} = 1$ and $\Sigma_{12} = -\Lambda_s = -3$. The projection onto $f(s)$ reduces the system (1)–(4) to two dimensions with the $\nabla_{\|} v$ and $\nabla_{\|}(n + p) \to 0$ and the parallel diffusion terms replaced by constants

$$\nabla_{\|}^2 \to -\frac{\int (df/ds)^2 ds}{\int f^2 ds} \equiv -k_{\|}^2$$

where the effective $k_{\|}$ is a constant of order $\pi/\mathcal{L}$ where $\mathcal{L}$ is the characteristic length of the open field line. The divergence of the current $\nabla_{\|} j_{\|}$ where $j_{\|} = C^{-1}\nabla_{\|}(n - \phi + (1 + \alpha)T)$ introduces the wall impedance through $f(s) j_{\|}(s)\Big|_{s=-\mathcal{L}/2}^{s=\mathcal{L}/2}$. When the wall admittance is set equal to the zero the system reduces to a generalized form of the Hasegawa-Wakatani equations. The generalization is the inclusion of the electron temperature fluctuation and the associated electro-thermal effect α and the electron thermal diffusivity

$\widehat{\chi}_{\|}$. The ion acoustic flows can be dropped from the system to yield essentially the three-pole system $(\widetilde{\varphi}, \widetilde{n}, \widetilde{T})$ analyzed by Hinton and Horton (1971). Dropping the ion acoustic waves is valid provided $\omega \sim \omega_* \gg \pi c_s/\mathcal{L}$.

The vorticity equation (6) is the statement that the divergence of the current vanishes. The balance of the divergence of the polarization current with electron parallel current requires that potential evolve on the fast-time scale $1/\nu_\phi$ given by

$$\nu_\phi = \frac{C^{-1}\left\langle k_\|^2 \right\rangle}{\langle k_\perp^2 \rho_s^2 \rangle}$$

to the state with $\delta j_\| = C^{-1}\nabla_\| \ (n - \phi + (1+\alpha)T) \simeq 0$. The presence of the density gradient v_d and temperature gradient η_e will drive the system away from this $\delta j_\| = 0$ state producing a net ohmic dissipation $P_c = C^{-1}\left\langle \left[\nabla_\| (n - \phi + (1+\alpha)\widetilde{T})\right]^2 \right\rangle$. The density equation (2) describes the usual electron drift wave response which for small coupling to the ion-acoustic mode is $\widetilde{n} = \left[(\nu_\| - i\omega_*)/(\nu_\| - i\omega)\right] \phi$ where $\nu_\| = k_\|^2 v_e^2/\nu_e \ [c_s/L_n]$.

The parallel ion acceleration in Eq. (3) is written in the approximation $\nabla p_i \rightarrow 0$ and that $E_\| = \eta j_\| - \nabla_\| p_e/n_e - (\alpha/e)\nabla_\| T_e$ from the massless electron parallel force balance equation. The electron thermal balance equation describes the driving of $\widetilde{T}$ from the ambient electron temperature gradient $\eta_e v_d = -\frac{c}{eB}\frac{dT_e}{dr}$ and the parallel electron compression $\nabla_\| u_{\|e}$ using $j_\|$ and $u_{\|i}$ to calculate $u_{\|e}$.

It is useful to recall that in the local approximation the truncated n-ϕ

dynamics of the Hasegawa-Wakatani system gives the dispersion relation

$$\omega(1+k_\perp^2\rho_s^2) - k_y v_d = \frac{i\omega^2 k_\perp^2\rho_s^2}{\nu_\parallel}$$

where $\nu_\parallel = k_\parallel^2 v_e^2/\nu_e$. For small $\nu_\parallel/|\omega_*|$ the two branches are $\omega = \pm\sqrt{\frac{i\nu_\parallel|\omega_*|}{k_\perp^2\rho_s^2}} \leq |\omega_*|$ and for large $\nu_\parallel/|\omega_*|$ the drift wave mode is unstable with

$$\omega = \frac{k_y v_d}{1+k_\perp^2\rho_s^2} + i\frac{k_\perp^2\rho_s^2\omega_*^2}{\nu_\parallel(1+k_\perp^2\rho_s^2)^3}$$

and the second branch is $\omega \simeq -\nu_\phi = -i\nu_\parallel(1+k_\perp^2\rho_s^2)/(k_\perp^2\rho_s^2)$.

The reference parameters are given by

$$\begin{aligned} &\alpha = 0.71, \quad \eta_e = 1, \quad S = 0.1 \ , \\ &\mu_\perp = 0.01 \ , \mu_\parallel = 0.01, C^{-1} = 0.1 \ , \\ &L_x = 80, \quad N_x = 200, k_{y\min} = 0.1 \ , \\ &N_y = 64 \ . \end{aligned} \tag{6}$$

It should be noted that Fourier transforms are in terms of the following normalization

$$\phi(x,y) = 2\,\mathrm{Re}\sum_{m=1}^{N_y}\phi_m(x)\exp(imy) \tag{7}$$

and $\phi_{-m} = \phi_m^*$; $k_m = m\,k_{y\min}$.

We define the volume average $\langle \quad \rangle$ by

$$\langle A(x,y)\rangle \equiv \frac{1}{L_x}\int_{-L_x/2}^{L_x/2} dx\; 2\,\mathrm{Re}\sum_{k=1}^{N_y} A_k(x) \tag{8}$$

where $A(x,y) = 2\,\mathrm{Re}\sum_{k=1}^{N_y} A_k(x)\exp(iky)$. The four energy components W_2 are given by

$$
\begin{aligned}
W_1 &= \left\langle \tfrac{1}{2}\left(\left|\frac{\partial \phi_k}{\partial x}\right|^2 + k_y^2|\phi_k|^2\right)\right\rangle \\
W_2 &= \left\langle \tfrac{1}{2}|n_k|^2\right\rangle \\
W_3 &= \left\langle \tfrac{1}{2}\,|v_k|^2\right\rangle \\
W_4 &= \left\langle \tfrac{3}{4}|T_k|^2\right\rangle
\end{aligned}
\tag{9}
$$

and the rate of change of each energy component is given by

$$
\begin{aligned}
\frac{dW_1}{dt} &= \left\langle C^{-1}(Sk_yx)^2\phi_k^*(n_k - \phi_k + (1+\alpha)T_k)\right\rangle - \left\langle \mu_\perp\left|\frac{\partial^2\phi_k}{\partial x^2} - k_y^2\phi_k\right|^2\right\rangle \\
\frac{dW_2}{dt} &= v_d\,\Gamma_n - \langle iSk_yxn_k^*v_k\rangle - \left\langle \mu_\perp\left(\left|\frac{\partial n}{\partial x}\right|^2 + k_y^2|n_k|^2\right)\right\rangle \\
&\quad - \left\langle C^{-1}(Sk_yx)^2 n_k^*(n_k - \phi_k + (1+\alpha)T_k)\right\rangle \\
\frac{dW_3}{dt} &= -\left\langle iSk_yxv_k^*(n_k + T)\right\rangle - \left\langle \mu_\parallel (Sk_yx)^2\,|v_k|^2\right\rangle - \left\langle \mu_\perp\left(\left|\frac{\partial v_k}{\partial x}\right|^2 + k_y^2|v_k|^2\right\rangle\right\rangle \\
\frac{dW_4}{dt} &= \tfrac{3}{2}\,\eta_e\Gamma_T - \langle iSk_yxT_k^*v_k\rangle - \left\langle (1+\alpha)C^{-1}(Sk_yx)^2T_k^*(n_k - \phi_k + (1+\alpha)T_k)\right\rangle \\
&\quad - \left\langle \tfrac{3}{2}\times 1.07C^{-1}S^2k_y^2x^2\,|T_k|^2\right\rangle - \left\langle \tfrac{3}{2}\mu_\perp\left(\left|\frac{\partial T}{\partial x}\right|^2 + k_y^2|T_k|^2\right)\right\rangle .
\end{aligned}
\tag{10}
$$

The rate of change of the sum of the four energy components is given by

$$\frac{d}{dt}(W_1+W_2+W_3+W_4) = v_d\Gamma_n+\tfrac{3}{2}\eta_e v_d\Gamma_T-\Gamma_c-\Gamma_k-\Gamma_s-\Gamma_1-\Gamma_2-\Gamma_3-\Gamma_4 \quad (11)$$

which is driven by the products of the particle flux Γ_n with density gradient v_d,

$$\Gamma_n = -\langle ik_y\phi_k n_k^*\rangle = -D\frac{dn}{dx},$$

the thermal flux Γ_T with the electron temperature gradient $\eta_e v_d$

$$\Gamma_T = -\langle ik_y\phi_k T_k^*\rangle = -n\chi\frac{dT}{dx},$$

and the absorption of fluctuation energy by the seven positive definite collisional dissipations

$$\Gamma_c = \left\langle C^{-1}(Sk_yx)^2\left|n_k-\phi_k+(1+\alpha)T_k\right|^2\right\rangle$$

$$\Gamma_k = \left\langle \tfrac{3}{2}\times 1.07C^{-1}S^2k_y^2x^2\left|T_k\right|^2\right\rangle$$

$$\Gamma_s = \left\langle \mu_\parallel(Sk_yx)^2\left|v_k\right|^2\right\rangle$$

$$\Gamma_1 = \left\langle \mu_\perp\left|\frac{\partial^2\phi_k}{\partial x^2}-k_y^2\phi_k\right|^2\right\rangle$$

$$\Gamma_2 = \left\langle \mu_\perp\left(\left|\frac{\partial n_k}{\partial x}\right|^2+k_y^2|n_k|^2\right)\right\rangle$$

$$\Gamma_3 = \left\langle \mu_\perp\left(\left|\frac{\partial v_k}{\partial x}\right|^2+k_y^2|v_k|^2\right)\right\rangle$$

$$\Gamma_4 = \left\langle \tfrac{3}{2}\mu_\perp \left(\left| \frac{\partial T_k}{\partial x} \right|^2 + k_y^2 |T_k|^2 \right) \right\rangle .$$

In the steady state, the fluctuation power balance is

$$\Gamma_n + \tfrac{3}{2}\eta_e \Gamma_T = \Gamma_c + \Gamma_k + \Gamma_s + \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 . \tag{12}$$

The dominant terms in the balance Eq. (12) are $3\eta_e\Gamma_T/2 \simeq \Gamma_c$ for typical parameters.

A initial condition used to test the nonlinear self-focusing of the system is the dipole vortex of radius r_0 and amplitude ϕ_0 as given by

$$\phi_k^{\rm int} = \phi_0 \frac{1}{\sqrt{2\pi}} \frac{x \exp}{r_0} \left(-\frac{x^2}{2r_0^2} \right) \exp \left(-\frac{k_y^2 r_0^2}{2} \right) \tag{13}$$

where $r_0 = 6$. The control parameter A_0, as defined by Scott (1992), is related to ϕ_0 by

$$\phi_0 = \sqrt{\frac{A_0}{\langle |\phi_k^{\rm int}|^2 \rangle}} . \tag{14}$$

We choose $A_0 = 0.5, 1, 2$ and so on. Another initial condition is a spectrum of low amplitude, randomly phased waves.

III. MIXING LENGTH TRANSPORT SCALING

From the local scaling laws of the linear fluctuation derived from Eqs. (1)–(4) we obtain two regimes: a low collisionality regime in which $D \sim \chi \sim$

$\gamma/k_y^2 \sim \nu_{ei}$ and a high collisionality regime in which $D \sim \chi \sim \gamma/k_y^2 \sim 1/\nu_{ei}^{1/2}$. The scaling laws in these two regimes and the transition region are as follows:

(I) Sheared Slab

(i) low collisionality $\nu_e < \nu_e^*(k_y)$

$$\chi \cong D = \left(\frac{\nu_e L_s}{v_e}\right)\left(\frac{v_d^2 L_s \rho_s^2}{v_e \Delta x^2}\right) \tag{15}$$

(ii) high collisionality $\nu_e > \nu_e^*(k_y)$

$$\chi \cong D = \left(\frac{v_e}{\nu_e L_s}\right)^{1/2} \frac{\Delta x (v_e v_d)^{1/2}}{|k_y|^{3/2} L_s^{1/2} \rho_s} \tag{16}$$

where the maximum diffusivity occurs for $\nu_e = \nu_e^*(k_y)$ given by the condition

$$\frac{\nu_e^* L_s}{v_e} = \frac{v_e \Delta x^2}{v_d L_s \rho_s^2 |k_y|} \tag{17}$$

where Δx is the mode width.

Now condition (17) may be viewed as determined the most effective $k_y = k_y^*$ as a function of collisionality $\nu_e L_s/v_e$ and it must be noted that in general there is a (weak) k_y-dependence of the mode width Δx. At the maximum of the transport coefficient given by condition (17) we find that (either Eq. (1) or Eq. (2)). The diffusivity is given by the simple law

$$\chi_{\max} = D_{\max} = \frac{v_d}{|k_y|} \, . \tag{18}$$

Now if the $|k_y|$ must be interpreted not as the maximum of the linear growth rate, but rather the result of the dominant mode in the nonlinear turbulence or vortex structure. Depending on the scaling of $|k_y|$, we have the following results.

For large scale vortices will $k_y \to m_*/r$ then we have Bohm transport and for small scale structures tied to the ion inertial scale ρ_s we have gyro-Bohm scaling:

$$D_{\max} = \begin{cases} \dfrac{1}{m_*} \dfrac{r}{L_n} \dfrac{cT_e}{eB} & \text{for} \quad k_y = \dfrac{m_*}{r} \\[2ex] \dfrac{\rho_s}{L_n} \dfrac{cT_e}{eB} \left(\dfrac{L_s}{L_n}\right)^{p_1} & \text{for} \quad k_y \rho_s \lesssim 1 \; . \end{cases} \tag{19}$$

Thus an important goal in the numerical simulation is to determine the laws governing the dominant k_y appropriate for Eq. (18). In particular, we search for the scaling exponent p_1 for magnetic shear variation in the small scale turbulence and collisionality scaling of m_* in the large scale turbulence.

Now in the SOL model where the modes are three dimensional and $k_\| \cong \pi/\mathcal{L}$ is a free parameter, the collisionality condition (17) is replaced by $k_\| = k_\|^*$

$$k_\|^* = \frac{|k_y|\rho_s}{v_e} \sqrt{\omega_{*e}\, \nu_e} \leq \frac{\nu_e}{v_e} \; . \tag{20}$$

The same behavior is found for the low and high collisionality limits and again the critical condition (20). We find the simple formula (18) again which reinforces the need to determine the law for the parametric dependence of the dominant $|k_y|$ for transport in the fluctuation spectrum.

IV. NUMERICAL ALGORITHM

Rewriting $ik_y \rightarrow k_y$ and $ik_y Sx \rightarrow k_\|$ and using the predictor-corrector scheme, we obtain

$$\left(1 - \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2\right)U^* = \left(1 + \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2\right)U^0 + \frac{\Delta t}{2}\left(-[\phi, \nabla_\perp^2\phi] + C^{-1}k_\|^2(n - \phi + (1+\alpha)T)\right)^0 \tag{21}$$

$$\nabla_\perp^2\phi^* = U^* \tag{22}$$

$$\left(1 - \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2 - \frac{\Delta t}{2}\frac{C^{-1}k_\|^2}{2}\right)n^* = \left(1 + \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2 + \frac{\Delta t}{2}\frac{C^{-1}k_\|^2}{2}\right)n^0 + \frac{\Delta t}{2}\left(-[\phi, n] - k_y\phi - k_\|v + C^{-1}k_\|^2(-\phi + (1+\alpha)T)\right)^0 \tag{23}$$

$$\left(1 - \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2 - \frac{\Delta t}{2}\frac{\mu_\| k_\|^2}{2}\right)v^* = \left(1 + \frac{\Delta t}{2}\frac{\mu_\perp}{2}\nabla_\perp^2 + \frac{\Delta t}{2}\frac{\mu_\| k_\|^2}{2}\right)v^0 + \frac{\Delta t}{2}\left(-[\phi, v] - k_\|(n+T)\right)^0 \tag{24}$$

$$\left(1 - \frac{\Delta t}{2}\frac{(2/3)\mu_\perp}{2}\nabla_\perp^2 - \frac{\Delta t}{2}\frac{(2/3)(1+\alpha)^2C^{-1}k_\|^2}{2} - \frac{\Delta t}{2}\frac{(2/3)1.07C^{-1}k_\|^2}{2}\right)T^* =$$

$$\left(1+\frac{\Delta t}{2}\frac{(2/3)\mu_\perp}{2}\nabla_\perp^2+\frac{\Delta t}{2}\frac{(2/3)(1+\alpha)^2C^{-1}k_\parallel^2}{2}+\frac{\Delta t}{2}\frac{(2/3)1.07C^{-1}k_\parallel^2}{2}\right)T^0$$

$$+\frac{\Delta t}{2}\left(-[\phi,T]-\eta_e\frac{\partial\phi}{\partial y}-\frac{2}{3}\nabla_\parallel v+\tfrac{2}{3}(1+\alpha)C^{-1}\nabla_\parallel^2(n-\phi)\right)^0 \tag{25}$$

$$\left(1-\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2\right)U^p=\left(1+\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2\right)U^0$$

$$+\Delta t\left(-[\phi,\nabla_\perp^2\phi]+C^{-1}k_\parallel^2(n-\phi+(1+\alpha)T)\right)^* \tag{26}$$

$$\nabla_\perp^2\phi^p=U^p \tag{27}$$

$$\left(1-\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2-\Delta t\frac{C^{-1}k_\parallel^2}{2}\right)n^p=\left(1+\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2+\frac{\Delta t}{2}\frac{C^{-1}k_\parallel^2}{2}\right)n^0$$

$$+\Delta t\left(-[\phi,n]-k_y\phi-k_\parallel v+C^{-1}k_\parallel^2(-\phi+(1+\alpha)T)\right)^* \tag{28}$$

$$\left(1-\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2-\Delta t\frac{\mu_\parallel k_\parallel^2}{2}\right)v^p=\left(1+\Delta t\frac{\mu_\perp}{2}\nabla_\perp^2+\Delta t\frac{\mu_\parallel k_\parallel^2}{2}\right)v^0$$

$$+\Delta t\left(-[\phi,v]-k_\parallel(n+T)\right)^* \tag{29}$$

$$\left(1-\Delta t\frac{(2/3)\mu_\perp}{2}\nabla_\perp^2-\Delta t\frac{(2/3)(1+\alpha)^2C^{-1}k_\parallel^2}{2}-\Delta t\frac{(2/3)1.07C^{-1}k_\parallel^2}{2}\right)T^*=$$

$$\left(1+\Delta t\frac{(2/3)\mu_\perp}{2}\nabla_\perp^2+\Delta t\frac{(2/3)(1+\alpha)^2C^{-1}k_\parallel^2}{2}+\Delta t\frac{(2/3)1.07C^{-1}k_\parallel^2}{2}\right)T^0$$

$$+\Delta t\left(-[\phi,T]-\eta_e\frac{\partial\phi}{\partial y}-\tfrac{2}{3}\nabla_\parallel v+\tfrac{2}{3}(1+\alpha)C^{-1}\nabla_\parallel^2(n-\phi)\right)^* . \qquad (30)$$

V. TEST CASES

The reference simulation parameters are taken as $\alpha = 0.71$, $\eta_e = 1$, $S = 0.1$, $\mu_\perp = 0.01$, $\mu_\parallel = 0.01$, $C^{-1} = 0.1$, with the discretization parameters $L_x = 160$, $N_x = 200$, $k_{y\min} = 0.1$, $N_y = 64$. First, we only change the parameter A_0 between $A_0 = 0.1$ and $A_0 = 1$. For these parameters, the drift wave is linearly unstable.

Figure 1 shows linear growth rate versus k_y. We choose $\delta t = 5\times10^{-4}$, and the number of time iterations is 2×10^4 which means $t = 10$. The maximum growth rate is about $\gamma = 0.08$ $[c_s/L_n]$ at $k_y\rho_s \sim 1$. This point is different from Scott's results. Because he did not observe the linear instability for these parameters.

Figure 2 shows the time evolution of energy defined by Eq. (10). For $A_0 = 0.1$, we can see firstly the energy is growing (linearly) then at $T \simeq 50$, the energy decreases.

Figure 3 shows the time evolution of Γ. Γ_n is negative value at $t \simeq 55$.

Figure 4 shows the power spectrum of the energy at $t = 90$.

Figure 5 shows the power spectrum of Γ at $t = 90$. In this figure we see Γ_t and Γ_n are not smooth function. Where Γ is x averaged. $\Gamma(k_y) = \frac{1}{L_x}\int \Gamma(x, k_y)dx$.

Figure 6 shows the spatial structure of Γ. Where $\Gamma(x) = 2\,\mathrm{Re}\sum_k \Gamma(x, k_y)$.

Figure 7 shows the contour plots of ϕ, n, v, T. Where the relation $\phi(x, y) = 2\,\mathrm{Re}\sum_k \phi_k(x)\,\exp(iky)$ is used.

Next we have changed the case $A_0 = 1$. Figure 8 shows energy versus time. In this case, at $t \sim 100$ the numerical instability is occurred and the calculation is stopped.

Figure 9 shows Γ versus time. Γ also increased.

Figure 10 shows the mode structure at $t = 90$. Three cases with $k_y = 0.1, 1.0, 2.0$ are plotted for ϕ, $v_\parallel$, p. From this figure we can see the longest wave number mode with $k_y = 0.1$ changed to be global and this cause the numerical instability. Therefore in this case the numerical instability is occurred in x space.

Figure 11 shows the energy spectrum at $t = 90$. In this case, energy spectrum in k_y space is decreasing function of k_y. Next we checked in the strong shear $S = 1.0$ with $A_0 = 80$ and $A_0 = 100$.

Figure 12 shows the time evolution of energy in case of $A_0 = 80$. In this case the energy is just dumping.

Figure 14 shows the mode structure at $t = 80$.

Figure 15 shows the power spectrum of energy at $t = 80$.

Figure 17 shows the time evolution of energy in case $A_0 = 100$. In this case firstly energy is decreasing then at $T = 150$, it is increasing and stopped due to overflow.

Figure 19 shows the mode structure at $t = 200$. In this case, the mode is broaden but is not completely global.

Figure 20 shows the power spectrum of the energy at $t = 200$. From this figure, we understand the numerical instability is driven in k_y space.

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