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Electron-Neutrino Separation Instability

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Deduced from Weinberg-Salam electroweak theory, a Boltzmann equation and subsequent fluid equations are derived for the primordial electron-positron-neutrino-photon plasma. A collective instability that separates the phases of electrons (and positrons) and neutrinos (and anti-neutrinos) is discussed. Cosmological implications are mentioned.

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The primordial plasma in the epoch approximately between 10^{-4} sec to 1 sec after the Big Bang is believed to be made up primarily of electrons, positrons, neutrinos and anti-neutrinos, and photons (with a small amount of baryonic matter).[1] During this epoch (particularly its early part) electrons and positrons not only coupled strongly with photons through electromagnetic interaction, but also coupled with neutrinos (and anti-neutrinos) through the weak interaction. After the temperature of the plasma dropped below the rest mass of $W^{\pm}$ and Z bosons,[2] by integrating out the boson propagators and using the Fierz transformation,[2] we have an effective Lagrangian for the $e-\nu$ interaction. Then, the effective Lagrangian for the $e-\nu$ parts is:

$$\mathcal{L}_{\text{int}}^{W} = -\frac{\sqrt{2}G}{c^2}\left(\overline{\nu}\gamma_{\mu}\frac{1+\gamma_5}{2}\nu\right)\left[\overline{e}\gamma_{\mu}\left(\frac{1+\gamma_5}{2}+2\sin^2\theta_w\right)e\right] , \tag{1}$$

and the electromagnetic interaction Lagrangian is $\mathcal{L}_{\text{int}}^{\text{EM}} = \frac{q}{c}(i\overline{e}\gamma_{\mu}e)A_{\mu}$, where G is the Fermi constant and θ_w is the Weinberg angle in the Weinberg-Salam theory ($\sin^2\theta_w \approx 0.25$). Our present approach is to start from Eq. (1) in a semiclassical way to systematically derive classical hydrodynamic Lagrangian of the electroweak plasma. We will then arrive at the Boltzmann equation and its associated moment hydrodynamical equations. These equations allow us to analyze collective modes of the electroweak plasma.

The classicalization is done by suppressing the handedness and by reducing $i\overline{e}_L\gamma_{\mu}e_L$ to $\frac{1}{2}j_{\mu}^{e} = \frac{1}{2}(n_e\mathbf{v}_e, icn_e)$ and $i\overline{\nu}_L\gamma_{\mu}\nu_L$ to $\frac{1}{2}j_{\mu}^{\nu_L} = \frac{1}{2}(n_{\nu}\mathbf{v}_{\nu}, icn_{\nu})$.

We then obtain the interaction Lagrangian as

$$L_{\rm int} = \int \mathcal{L}_{\rm int} d^3x = \frac{q}{c}\mathbf{v}_e \cdot \mathbf{A}(\mathbf{x}_e, t) - q\phi(\mathbf{x}_e, t)$$

$$+ \frac{\sqrt{2}G}{c^2}\left(n_\nu(\mathbf{x}_e, t)\mathbf{v}_e \cdot \mathbf{v}_\nu(\mathbf{x}_e, t) - c^2 n_\nu(\mathbf{x}_e, t)\right) , \tag{2}$$

for a single electron and a similar one for a single neutrino, where $n_\sigma(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{x}_\sigma)$ $(\sigma = e,\ \nu)$. The equations of motion for electrons and neutrinos are obtained by the Euler-Lagrange equation.

The Euler-Lagrange equation thus derived defines the characteristics of each species of particles and thus allow us to construct the Boltzmann equation: $\frac{\partial f_\sigma}{\partial t} + \mathbf{v}_\sigma \cdot \nabla_{\mathbf{x}} f_\sigma + \mathbf{F}_\sigma \cdot \nabla_{\mathbf{p}} f_\sigma = C_\sigma$, where C_σ is the appropriate collision operator including the annihilation and creation of particles. By making the velocity moments and summing over many particles, we finally arrive at the classical hydrodynamical equations

$$n_e \frac{d\mathbf{p}_e}{dt} = n_e q\left(\mathbf{E} + \frac{\mathbf{v}_e \times \mathbf{B}}{c}\right) - \nabla P_e - \nabla P_\gamma + \frac{\sqrt{2}Gn_e}{c^2}\Bigg[\nabla \cdot (n_\nu \mathbf{v}_\nu)\mathbf{v}_e$$

$$+ \nabla(n_\nu \mathbf{v}_\nu \cdot \mathbf{v}_e) - c^2 \nabla n_\nu - \frac{d}{dt}(n_\nu \mathbf{v}_\nu)\Bigg] - \eta_e n_e m_e \mathbf{v}_e , \tag{3}$$

and

$$n_\nu \frac{d\mathbf{p}_\nu}{dt} = \frac{\sqrt{2}Gn_\nu}{c^2}\left[\nabla \cdot (n_e \mathbf{v}_e)\mathbf{v}_\nu + \nabla(n_e \mathbf{v}_e \cdot \mathbf{v}_\nu) - c^2 \nabla n_e - \frac{d}{dt}(n_e \mathbf{v}_e)\right]$$

$$- \eta_\nu n_\nu m_\nu \mathbf{v}_\nu , \tag{4}$$

where $\frac{d}{dt}$ is the Lagrangian time derivative, n_σ is now the density, η_σ is the collisional frequency of fluid species σ with photons or other fluid components, and P_e and P_γ are electron and photon pressures. Other equations are the continuity equations for each species: $\frac{\partial n_\sigma}{\partial t} + \nabla \cdot (n_\sigma \mathbf{v}_\sigma) = 0$. We have neglected the cosmological background expansion, as the Hubble expansion time proves to be much greater than the growth time as we shall see.

Since the plasma is highly relativistic and opaque, we can safely assume $P_\gamma = P_e = n_e T_e = \frac{1}{\sigma_s^{1/3}} n_e^{4/3}$, where $\sigma_s = \frac{\pi^2}{45\hbar^3 c^3}$. We assume that the equilibrium of the plasma is uniform: $n_e = \overline{n}_e$ and $n_\nu = \overline{n}_\nu$ and no large scale EM fields and flows $\mathbf{E} = \mathbf{B} = \mathbf{v}_e = \mathbf{v}_\nu = 0$. We then solve Eqs. (3) and (4) by linearizing about the equilibrium and transforming these into Fourier space:

$$(-i\omega)m_e\gamma_e\overline{n}_e\delta\mathbf{v}_e = \overline{n}_e q_e \delta\mathbf{E} - \sqrt{2}G\overline{n}_e(i\mathbf{k})\delta n_\nu - \frac{\sqrt{2}G}{c^2}\overline{n}_e\overline{n}_\nu(-i\omega)\delta\mathbf{v}_\nu$$
$$- \eta_e\overline{n}_e m_e\delta\mathbf{v}_e - \frac{8}{3\sigma^{1/3}}\overline{n}_e^{1/3}(i\mathbf{k})\delta n_e \ , \qquad (5)$$

$$(-i\omega)m_\nu\gamma_\nu\overline{n}_\nu\delta\mathbf{v}_\nu = -\sqrt{2}G\overline{n}_\nu(i\mathbf{k})\delta n_e$$
$$- \frac{\sqrt{2}G}{c^2}\overline{n}_e\overline{n}_\nu(-i\omega)\delta\mathbf{v}_e - \eta_\nu\overline{n}_\nu m_\nu\delta\mathbf{v}_\nu \ , \qquad (6)$$

and similarly, for the continuity equations, where γ_σ is the relativistic factor for the species σ. These equations are coupled with Maxwell's equations for electric and magnetic fields. The determinant of the matrix of these equations results in an equation $f \cdot g^2 = 0$, which leads to two independent dispersion relations $f = 0$ or $g = 0$, where f and g are given by

$$f = (1-a)\omega^4 + i(\eta_e + \eta_\nu)\omega^3 - (bc^2k^2 - 2ac^2k^2 + \eta_e\eta_\nu + \omega_p^2)\omega^2$$

$$- i\eta_\nu(\omega_p^2 + bc^2k^2)\omega - ac^4k^4 = 0 \;, \tag{7}$$

or

$$g = A - \eta_e\eta_\nu\omega^2(\omega^2 - c^2k^2) + i(\eta_e + \eta_\nu)(\omega^5 - c^2k^2\omega^3)$$

$$+ i\eta_\nu(bc^4k^4\omega - bc^2k^2\omega^3 - \omega_p^2\omega^3) = 0 \;, \tag{8}$$

and

$$A = (1-a)\omega^6 + (3ac^2k^2 - bc^2k^2 - c^2k^2 - \omega_p^2)\omega^4 + (b-3a)c^4k^4\omega^2 + ac^6k^6 \;.$$

In these expressions, we include the viscosity effect μ by lumping $\eta + \mu k^2$ in a simple η, where we can approximately evaluate as $\eta = c\overline{n}_e\sigma_{\rm KN}$ and $\mu = \frac{c}{3\overline{n}\sigma_{\rm KN}}$, with the Klein-Nishina cross section $\sigma_{\rm KN} = \frac{\pi e^4}{m_e c^2 \hbar\omega}$. A dimensionless coupling coefficient a between electrons and neutrinos is defined as

$$a = \frac{2G^2\overline{n}_e\overline{n}_\nu}{c^4 m_e m_\nu} \;,$$

and a dimensionless coupling coefficient b between electrons and photons is

$$b = \frac{8}{3}\left(\frac{\overline{n}_e}{\sigma_s}\right)^{1/3}\frac{1}{m_e c^2} \;.$$

The dispersion relation $f = 0$ gives rise from the longitudinal mode (i.e. the electric polarization $\mathbf{E}$ is parallel to $\mathbf{k}$), while the dispersion relation $g^2 = 0$ from the transverse ($\mathbf{E} \perp \mathbf{k}$) ($g^2 = 0$ since two polarizations). Each relation reduces to the familiar form of $\omega^2 = \omega_p^2 + bc^2k^2$ (plasmons) and $\omega^2 = \omega_p^2 + k^2c^2$ (polaritons) respectively, when the electron-neutrino coupling $a \to 0$ and collisions $\eta \to 0$. Note that the coefficient a is mass (m_ν) dependent (we took the form for $m_\nu \neq 0$; for $m_\nu = 0$ case appropriate modifications to nonsingularize the present equations can be carried out).

We seek analytical solutions for Eqs. (7) and (8). Let $\Gamma = -i\frac{\omega}{ck}$ (normalized growth rate). The dispersion relation becomes respectively from (7)

$$
\begin{aligned}
f(\Gamma) = (1-a)\Gamma^4 + \frac{\eta_e + \eta_\nu}{ck}\Gamma^3 + \left(b - 2a + \frac{\eta_e\eta_\nu}{c^2k^2} + \left(\frac{\omega_p}{ck}\right)^2\right)\Gamma^2 \\
+ \frac{\eta_\nu}{ck}\left(\frac{\omega_p}{ck}^2 + b\right)\Gamma - a = 0\ ,
\end{aligned}
\tag{9}
$$

and from (8)

$$
\begin{aligned}
g(\Gamma) = (1-a)\Gamma^6 + \left(b + 1 + \left(\frac{\omega_p}{ck}\right)^2 - 3a\right)\Gamma^4 + (b-3a)\Gamma^2 - a + \frac{\eta_e\eta_\nu}{c^2k^2}\Gamma^2(\Gamma^2+1) \\
+ \frac{\eta_e + \eta_\nu}{ck}\Gamma^3(\Gamma^2+1) + \frac{\eta_\nu}{ck}\Gamma\left(b + b\Gamma^2 + \left(\frac{\omega_p}{ck}\right)^2\Gamma^2\right) = 0\ .
\end{aligned}
\tag{10}
$$

In our plasma, the coupling coefficient a is a small parameter($a \ll 1$) and the parameter $b \gg a$. We already know the high frequency behavior of these equilibrium plasmons and polaritons. We thus focus on low frequency behaviors of collective modes which are influenced by the presence of electroweak coupling: $|\Gamma| \ll 1$.

From $f(0) = -a < 0$, $f(\infty) > 0$ and $g(0) = -a < 0$, $g(\infty) > 0$ we know that there exists at least a solution with $\Gamma > 0$. Recalling the definition of

Γ, we immediately realize that this is an unstable (exponentially temporary growing) mode. We can solve for small Γ by neglecting higher order terms like Γ^6, Γ^5 and Γ^4. For the transverse modes $g(\Gamma)$ we have

$$\left(b + \frac{\eta_e \eta_\nu}{c^2 k^2}\right)\Gamma^2 + \frac{\eta_\nu}{ck} b\Gamma - a \approx 0 \ . \tag{11}$$

Solve Γ as $\Gamma(k)$. Find the wavenumber value for maximum growth $k_{mg} \approx \sqrt{\frac{\eta}{\mu}}$ such that $\frac{d\Gamma}{dk}(k_{mg}) = 0$, where we now express η and μ explicitly. And the maximum Γ value $\Gamma_{\max} = \Gamma(k_{mg}) \approx \frac{ac}{b\sqrt{\eta\mu}}$. Then the maximum growth rate is given by

$$\gamma_{\max} = ck_{mg}\Gamma_{\max} = \frac{ac^2}{b\mu} \ . \tag{12}$$

For the longitudinal modes $f(\Gamma)$ we have similar results.

From Eqs. (5), (6), and (11), along with the continuity equations, it is evident that the Fourier components of density perturbations δn_e and δn_ν are 180° out of phase (optical phonon-like). For example, the negative gradient of the neutrino fluid exerts a weak force to reinforce the positive density gradient of the electron fluid and vice versa. This mutual reinforcing force leads to the instability. The reason why both the transverse and longitudinal dispersion relations $f = 0$ and $g = 0$ produce the same instability is now clear. That is, the origin of this instability is not related to photon coupling but to the weak coupling (or weak collisions). The latter is blind to the optical polarization and comes in equally in transverse and longitudinal equations. On the other hand, the mode that moves the neutrino and electron fluids together (acoustic phonon-like) is stable, as expected due to the photon viscosity. The instability we found tends to separate the neutrinos (and anti-neutrinos) from the electrons (and positrons) and each species into small "bubbles." The bubble size is approximately determined by the lower threshold value of k as a result of the nonlinear evolution of this instability. Various characteristics of this instability at representative cosmological epochs are listed in Table 1. The growth rate as a function of the wavenumber is shown in Fig. 1. Note that the present fluid theory fails to apply to quantum mechanical wavenumber regime $k > n_e^{-1/3}$.[3] The growth rate of the instability far exceeds the Hubble expansion rate H for $t < 1\,\text{sec}$, which justifies the neglect of this effect in Eqs. (3) and (4).

A new area of investigation opens up as to how this instability grows in its nonlinear evolution stages. It should be noted that the modes with wavelengths shorter than the neutrino mean free path will not grow nonlinearly because of neutrino penetration. See Fig. 1. It is anticipated, as is the case for this kind of hydrodynamic instabilities, that the mode with the minimum wavenumber $k_{\min}$ of the unstable spectrum eventually dominates in its nonlinear evolution. It remains to be seen whether the bubble formation and its surface tension help form coalescence of bubbles[4] and, if so, how fast.

What are possible observational relics of this instability? As the present instability vanishes or the growth rate of the instability γ becomes on the same order of magnitude of the Hubble expansion rate H, most structures embedded in the density of electrons and positrons also vanish. This is because the weak interaction had sustained the phase separation, but the weak interaction was fading at this epoch ($t \approx 1\,\mathrm{sec}$). Thus it is unlikely that these bubbles influence primordial nucleosynthesis.[5] Do neutrino fluctuations due to bubbles show up in the neutrino flux "observed" at the present epoch through a hypothetical neutrino telescope? This is similar to the question if the photon fluctuations (Cosmic Microwave Background Radiation) should be observable reflecting the electron-photon plasma fluctuations at the time of recombination. The characteristic fluctuation wavelength at the present epoch that corresponds to the detached neutrino bubble scale length is $1 - 10\,\mathrm{m}$, an awfully difficult scales even though low energy neutrinos could be detected. The magnetic fluctuations associated with the plasma fluctuations such as current have been studied for thermally stable primordial plasmas.[3,6] It was found that the level of magnetic fluctuations is rather enormous, though the scale length is minuscule. It remains to be seen such magnetic fields can lead to structure formation.[7,8] In the present investigation, however, we have found that weak interaction can induce fluctuations through exponentially growing unstable modes. This will certainly enhance the magnetic fluctuation level found in thermally stable situations,[3] as the former (unstable) case allows far beyond the thermal critical opalescence level.[9] Such enhanced magnetic fields and associated fluctuations, albeit with small scales, are worth investigating their consequences in later epochs.[10,11] It is also possible for the present theory to impact on magnetic or other fluctuations generated prior to the epoch.[12] Finally a comment is due on the role of this instability in supernova explosion: in spite of the parameter a being near the cosmological plasma at $t = 10^{-2}$ sec, the mean free path of neutrinos is greater than the typical bubble size and thus cannot contribute to the explosion of the outer shell of dying stars.

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References

[1]S. Weinberg, *Gravity and Cosmology*, (Wiley, New York, 1972).

[2]C. Itzykson, *Quantum Field Theory*, (McGraw-Hill, Singapore, 1987); S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).

[3]S.B. Cable and T. Tajima, Phys. Rev. A **46**, 3413 (1992). Also see A.G. Sitenko, *Electromagnetic Fluctuations in Plasma*, (Academic Press, New York, 1967).

[4]M. Yamada, *et al.*, Phys. Rev. Lett. **65**, 721 (1990); T. Tajima, S.B. Cable, and R.M. Kulsrud, Phys. Fluids B **4**, 2338 (1992).

[5]for examples C. Alcock, G.M. Fuller and G.J. Mathews, Ap.J. **320**, 439 (1987); H. Kurki-Suonio and R. Matzner, Phys. Rev. D **39**, 1046 (1989).

[6]P. Coles, Comments Astrophys. **16**, 45 (1992).

[7]J.C. McWilliams, J. Fluid Mech. **219**, 361 (1990) and reference therein.

[8]R.M. Kinney *et al.*, Phys. Plasmas **1**, 260 (1994).

[9]S. Ichimaru, D. Pines, and N. Rostoker, Phys. Rev. Lett. **8**, 231 (1962).

[10]A. Berera and L.Z Fang, Phys. Rev. Lett. **72**, 458 (1994).

[11]P.P. Kronberg, Rep. Prog. Phys. **57**, 325 (1994).

[12]B. Ratra, Phys. Rev. D **45**, 1913 (1992).

Time (s) Temperature (eV) Density (cm^{-3})	$t = 10^{-4}$ $T = 10^8$ $n = 6 \times 10^{37}$	$t = 10^{-2}$ $T = 10^7$ $n = 5 \times 10^{34}$	$t = 1$ $T = 10^6$ $n = 4 \times 10^{31}$
a	1.132×10^{-5}	7.859×10^{-12}	5.03×10^{-18}
b	668.45	62.9	5.839
$\sigma_{KN}(\text{cm}^2)$	1.275×10^{-27}	1.275×10^{-26}	1.275×10^{-25}
$\mu(\text{cm}^2/s)$	0.131	15.682	1960.21
$\eta(\text{s}^{-1})$	2.292×10^{21}	1.91×10^{19}	1.53×10^{17}
$\omega_p(\text{s}^{-1})$	6.183×10^{23}	1.785×10^{22}	5.049×10^{20}
$f(\Gamma) = 0, \gamma_{\text{max}}(\text{s}^{-1})$	1.164×10^{14}	7.16×10^{6}	0.3949
$f(\Gamma) = 0, k_{mg}(\text{cm}^{-1})$	6.872×10^{11}	8.59×10^{10}	5.369×10^{9}
$g(\Gamma) = 0, \gamma_{\text{max}}(\text{s}^{-1})$	1.164×10^{14}	7.16×10^{6}	0.3949
$g(\Gamma) = 0, k_{mg}(\text{cm}^{-1})$	1.323×10^{11}	1.104×10^{9}	8.835×10^{6}

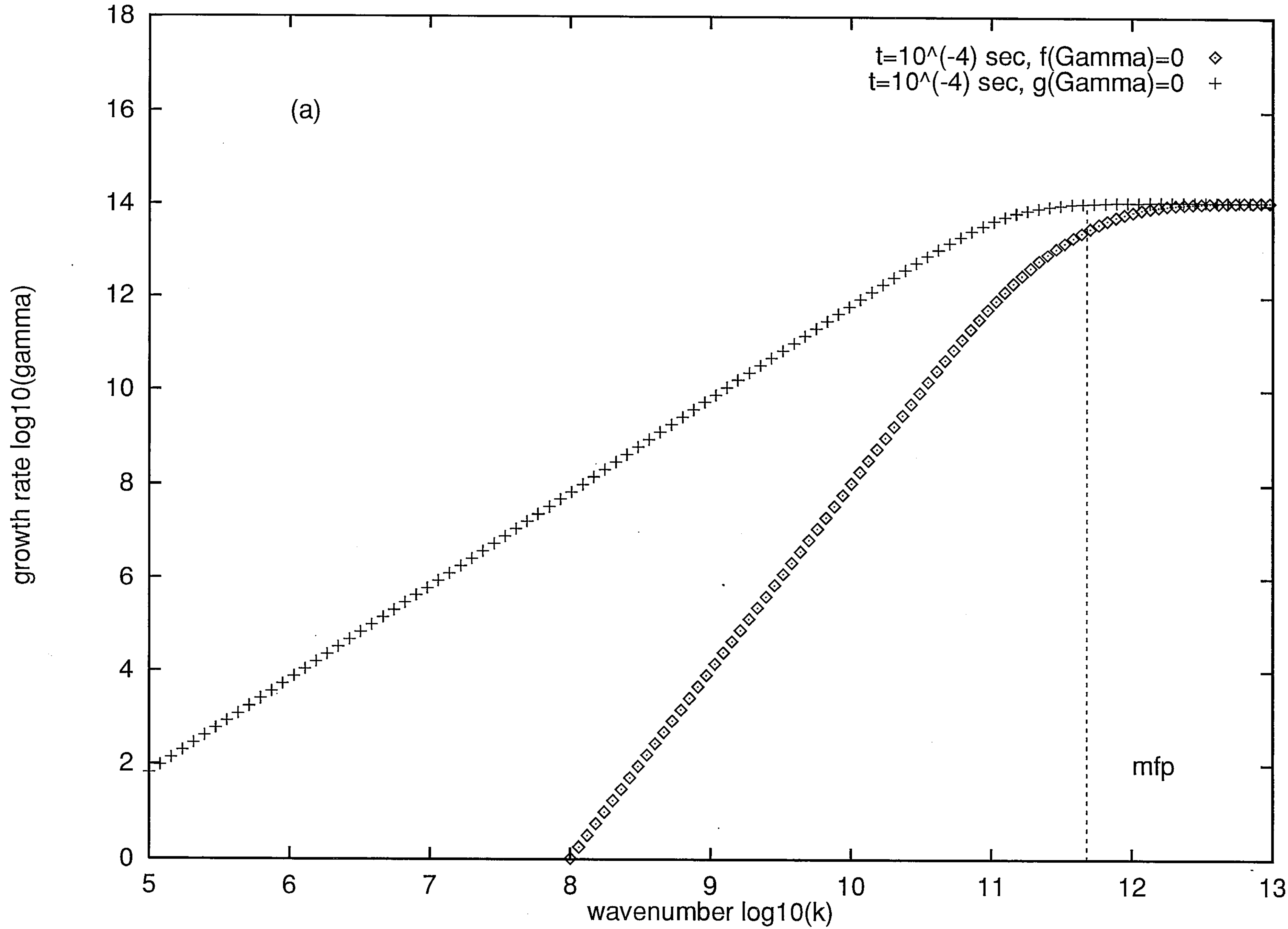
Table 1. Growth Rates of the Instability in a Primordial Plasma.

Figure captions

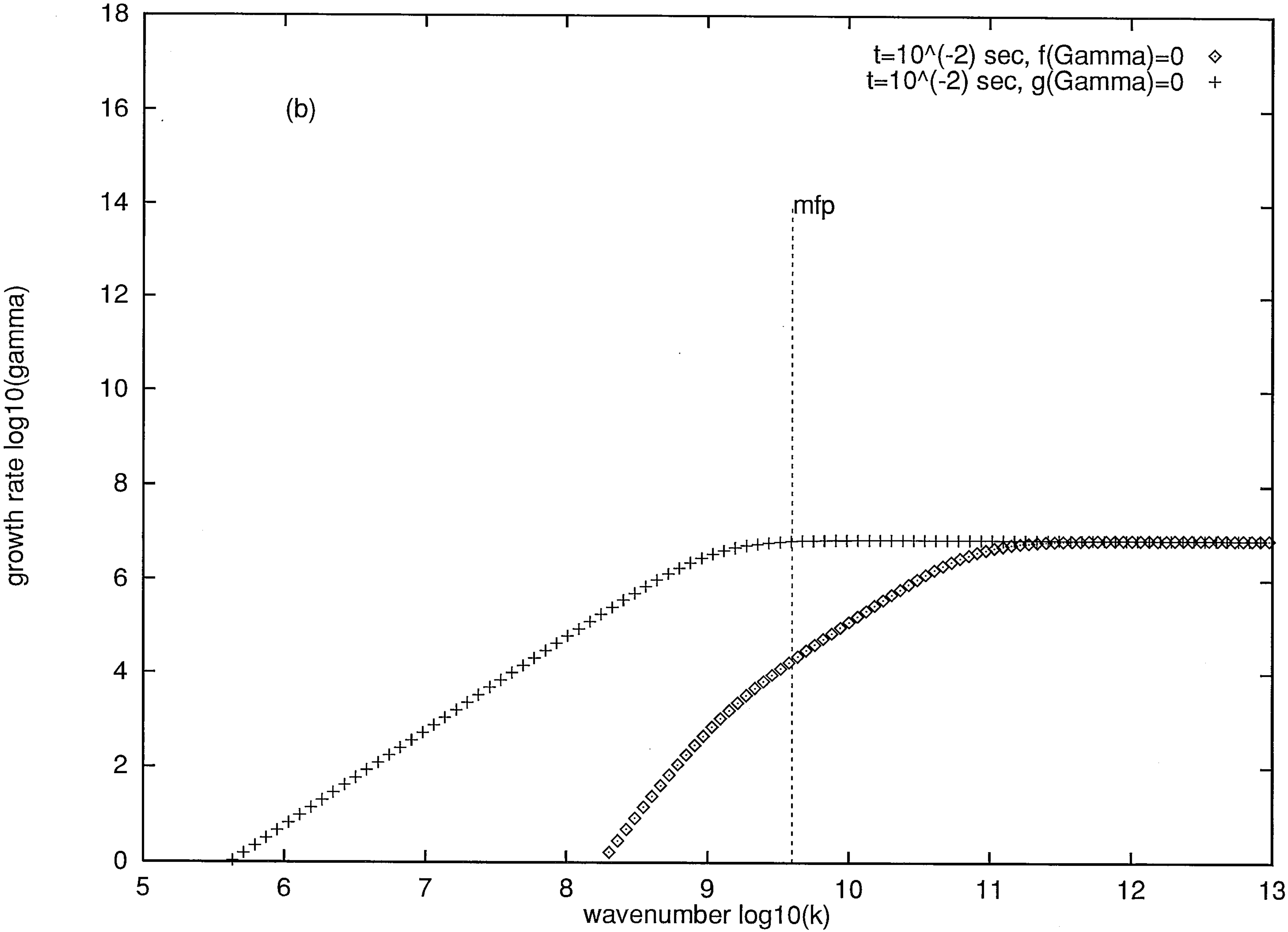
1. Growth rate of the phase separation at early cosmological epoch of electron-neutrino plasma.

   (a) a log-log plot of growth rate $\gamma(\mathrm{sec}^{-1})$ at $t = 10^{-4}\,\mathrm{sec}$ after the Big Bang as a function of wavenumber $k(\mathrm{cm}^{-1})$.

   (b) a log-log plot of growth rate $\gamma(\mathrm{sec}^{-1})$ at $t = 10^{-2}\,\mathrm{sec}$ after the Big Bang as a function of wavenumber $k(\mathrm{cm}^{-1})$. The labels "mfp" mark the wavenumber $k_{\mathrm{mfp}} = \frac{2\pi}{\lambda_{\mathrm{mfp}}}$, where λ_{mfp} is the neutrino mean free path.



t=10^(-4) sec, f(Gamma)=0
t=10^(-4) sec, g(Gamma)=0
(a)
mfp
growth rate log10(gamma)
wavenumber log10(k)
0
2
4
6
8
10
12
14
16
18
5
6
7
8
9
10
11
12
13


t=10^(-2) sec, f(Gamma)=0
t=10^(-2) sec, g(Gamma)=0
(b)
mfp
growth rate log10(gamma)
wavenumber log10(k)
0
2
4
6
8
10
12
14
16
18
5
6
7
8
9
10
11
12
13