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Abstract

A simplified two-dimensional nonlinear fluid model is developed to elucidate the basic mechanism of the formation of multifaceted asymmetric radiation from the edge of tokamak plasmas (MARFE). In the framework of a mixed Eulerian-Lagrangian description, the problem is reduced to a reaction-diffusion-type equation with nonlocality, which obeys the constraints of length constancy and mass conservation along the magnetic field. With a simple cooling function, this model predicts formation of stationary MARFE-like structures.

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The edge plasma plays an important role in the problem of plasma confinement in tokamaks. One of the major factors determining the global stability of a tokamak discharge is impurity radiation at the edge.[1, 2, 3, 4, 5] At the so called density limit (or Murakami limit) the total impurity radiation power (which for the optically thin plasma is proportional to the square of density) becomes equal to the total input power into the tokamak discharge. Above the density limit the discharge undergoes a thermal collapse.[4, 5, 6] MARFEs, toroidally symmetric, but poloidally asymmetric standing or moving radiative condensations, form at the edge of tokamak plasmas, and they were observed in almost every tokamak somewhat below the density limit.[7, 8, 9, 10, 11, 12, 13] MARFEs significantly increase the impurity radiation power. On the other hand, experiments with reproducible MARFEs provide a convenient means of the edge plasma diagnostics.[13] MARFEs are related to radiative condensation instability (RCI),[14, 15] discussed earlier in astrophysical contexts,[16] and they are indeed similar to a number of self-organizing radiative condensation phenomena in astrophysics (solar prominences,[17] interstellar clouds,[18] etc.).

Theoretical studies of MARFEs proceeded in the following three directions: (i) the linear analysis of the radiative condensation instability, (ii) search for possible MARFE-like steady states of a tokamak edge plasma, (iii) attempts to simulate the formation of MARFEs numerically. Drake[19] was the first to understand that an adequate theory of MARFEs must be at least two-dimensional (2d). Using fluid equations in the cylindrical geometry, he analyzed the linear stability of a simple (constant density, but radially non-uniform temperature) thermal equilibrium. He showed that the RCI starts when the plasma density exceeds a critical threshold that is somewhat lower than the density limit, in agreement with observations. Also, he found that the poloidally symmetric perturbations are normally damped, which agrees with the observed poloidal asymmetry of MARFEs. Subsequent extensions of Drake's linear theory addressed ionization effects, edge density gradient, different forms of the radiative cooling function and "detached plasma" regime.[20, 22, 21]

The search for possible MARFE-like steady states involves the analysis of a 2d-thermal balance equation, which includes a (nonlinear) heating-cooling function and anisotropic heat conduction. Krasheninnikov[23] and Kaw *et al.*[24] analyzed simplified versions of this equation analytically[23, 24] and numerically[24] for possible poloidally symmetric and asymmetric (MARFE-like) solutions and transition between them. Also, dependence of the transverse heat conduction on the poloidal angle was introduced,[25] to interpret the localization of steady-state MARFEs at the smaller major radius side of tokamaks.

The linear stability and steady-state analyses, though important steps, are not sufficient for elucidating the (still poorly understood) physical mechanism of pattern formation in this system. This problem requires a nonlinear time-dependent analysis. Neuhauser *et al.*[15] studied the nonlinear RCI dynamics numerically, employing a one-dimensional multi-fluid code with the ionization-recombination balance for the hydrogen and impurities. A number of interesting effects were found, such as relaxation oscillations during a plasma density rise, "condensation" front propagation and effects related to inhomogeneous power and impurity input. However, it became clear after the works of Drake,[19] Krasheninnikov[23] and Kaw *et al.*,[24] that 2d nonlinear dynamics should be addressed.

In this Letter, we develop the first 2d nonlinear time-dependent fluid model, aimed at elucidating the basic mechanism of MARFEs formation. Similar to a number of previous studies of MARFEs,[18, 25] we shall assume that the tokamak magnetic field completely suppresses the plasma motions across the field, but not the transverse heat conduction. We start with the simple fluid equations, including the thermal balance equation for an optically thin plasma:

$$\frac{dn}{dt} + n\mathbf{b} \cdot \nabla v_{\|} = 0 \ , \tag{1}$$

$$m_i n \frac{dv_\parallel}{dt} = -\mathbf{b}\cdot\nabla p \;, \tag{2}$$

$$\frac{3}{2}\frac{dp}{dt} + \frac{5}{2} p\mathbf{b}\cdot\nabla v_\parallel - \mathbf{b}\cdot\nabla\kappa_\parallel \mathbf{b}\cdot\nabla T - \nabla_\perp\cdot\kappa_\perp\nabla_\perp T + L - H = \;, \tag{3}$$

where n, T and $p = nT$ are the plasma concentration, temperature and pressure, respectively; $\mathbf{b} = \mathbf{B}/B$ is a unit vector along the magnetic field $\mathbf{B}$; $v_\parallel$ is the longitudinal (along $\mathbf{B}$) velocity; m_i is the ion mass; $\kappa_\parallel$ and $\kappa_\perp$ are the coefficients of longitudinal and transverse heat conductions; $L = n^2 F(T)$ is the radiative cooling function; H is a heating function. The total time derivative in Eqs. (1)–(3) is $d/dt = \partial/\partial t + v_\parallel \mathbf{b}\cdot\nabla$. Now, let us explicitly use the fact that the typical longitudinal acoustic time scale is normally much shorter than the other relevant time scales (the radiative cooling time scale and longitudinal and transverse heat conduction time scales). In this case, the plasma pressure rapidly becomes uniform along the magnetic field lines, so that the Euler equation (2) can be replaced by the simple relation $\mathbf{b}\cdot\nabla p = 0$.

MARFEs form in a thin region at the plasma edge. Therefore, the problem can be considered in slab geometry (x, z), if we supplement it by periodic boundary conditions with respect to the magnetic field direction z. Let the left boundary of the plasma, $x = 0$, be kept at a constant temperature $T = T_0$ (this condition simulates the hot plasma inside the discharge), while the right boundary, $x = a$, be kept at a (significantly lower) constant temperature T_a, which simulates the discharge periphery. The boundary conditions along z are periodic: $n(x, 0) = n(x, l)$; $T(x, 0) = T(x, l)$ and $v_\parallel(x, 0) = v_|(x, l)$, where $l = 2\pi R$, and R can be identified with the major radius of the tokamak. From this periodicity immediately follows mass conservation along z:

$$M(x,t) = \int_0^l n(x,z,t)dz = \mathrm{const}(t) \;. \tag{4}$$

Introduce the specific volume of the fluid, $u(x, z, t) = n^{-1}(x, z, t)$, and eliminate the temperature, using the equation of state. We arrive at the following two equations for the three variables $u(x,z,t)$, $v_{\|}(x,z,t)$ and $p(x,t)$:

$$\frac{du}{dt} - u\frac{\partial v_{\|}}{\partial z} = 0 \ , \tag{5}$$

$$\frac{3}{2}\frac{\partial p}{\partial t} + \frac{5}{2}p\frac{\partial v_{\partial|}}{\partial z} - p\frac{\partial}{\partial z}\left(\kappa_{\|}\frac{\partial u}{\partial z}\right) - \frac{\partial}{\partial x}\left(\kappa_{\perp}\frac{\partial(pu)}{\partial x}\right) + L - H = 0 \ , \tag{6}$$

where it is assumed that $\kappa_{\|}, \kappa_{\perp}, L$ and H are expressed as functions of u and p. Integrating Eq. (6) with respect to z over the period l, we obtain a nonlocal evolution equation for the pressure:

$$\frac{\partial p}{\partial t} = -\frac{2}{3l}\int_0^l (L-H)dz + \frac{2}{3l}\int_0^l \frac{\partial}{\partial x}\left(\kappa_{\perp}\frac{\partial(pu)}{\partial x}\right) dz \ . \tag{7}$$

The equations (5)-(7) are closed, and they represent the Eulerian version of our reduced model. They can be further simplified by introducing a Lagrangian mass coordinate

$$m(x,z,t) = \int_0^z u^{-1}(x,z',t)dz' \ . \tag{8}$$

Transforming to the coordinates x and m, we are left with only two governing equations:

$$\frac{\partial u}{\partial t} = \frac{2}{5}\frac{\partial}{\partial m}\left(\frac{\kappa_{\|}}{u}\frac{\partial u}{\partial m}\right) + \frac{2u}{5p}\frac{\partial}{\partial x}\left(\kappa_{\perp}\frac{\partial(pu)}{\partial x}\right) - \frac{2u}{5p}(L-H) - \frac{3u}{5p}\frac{\partial p}{\partial t} \ , \tag{9}$$

$$\frac{\partial p}{\partial t} = -\frac{2}{3l}\int_0^{M(x)} u(L-H)dm + \frac{2}{3l}\int_0^{M(x)} u\frac{\partial}{\partial x}\left(\kappa_{\perp}\frac{\partial(pu)}{\partial x}\right) dm \ , \tag{10}$$

where $M(x) = \int_0^l u^{-1}(x,z,t)dz$ is the (x-dependent) total mass content of each magnetic field line. Because of mass conservation along z, the quantity $M(x)$ is independent of time and determined solely by the initial density profile. Note that the constancy of the system length l in the z-direction, while trivial in the Eulerian coordinates, appears as a conservation law in the Lagrangian description:

$$\int_0^{M(x)} u(x,m,t)dm = l \ . \tag{11}$$

The Eulerian-Lagrangian form of the reduced model (9) and (10) has similarity to some known equations. Indeed, Eq. (9) for $u(x, m, t)$, which should be solved subject to the periodic boundary condition with respect to m, $u[x, m = M(x), t] = u(x, m = 0, t)$, resembles a well-known reaction-diffusion (RD) equation for an active medium (see, e.g.[26]) However, in our problem it is coupled to the nonlocal evolution equation (10). It is important that this nonlocality arises due to the constraints of the time-independent mass $M(x)$ and length l of the system along the magnetic field. Qualitatively similar nonlocalities have arisen in a number of one-dimensional *bistable* RD systems, where they introduce "global negative feedback" and contribute to the formation of persistent stationary structures, normally impossible in non-constrained one-component RD equations.[26, 27, 28, 29] The mechanism of stationary pattern formation in bistable systems with global negative feedback consists, essentially, in the arrest of motion of "phase transition" fronts, which otherwise would make the system uniform. The arguments developed in[26, 27, 28, 29] can be extended to the more complicated (in particular, non-bistable) 2d model (9) and (10). Therefore, we can interpret the development of MARFEs in terms of the formation of stationary patterns in a RD system with global negative feedback.

To verify this idea and follow the MARFE formation, we solved the reduced equations (9) and (10) numerically. As neither the linear theory of the RCI,[19, 21] nor the MARFE-like steady states[23, 24] are very sensitive to the precise form of the cooling function, we chose a rather crude model of the line radiation from the tokamak edge: $L = u^{-2}L_0$, for the radiatively unstable temperature interval $T_a \leq pu \leq T_L$, and $L = 0$ outside this interval. Also, T_L was taken much smaller than T_0, so that the radiative cooling was localized near the edge. For simplicity, we assumed the following model forms of the heat conduction coefficients: $\kappa_{\|} = T = pu$, $\kappa_{\perp} = 1$. (Renormalizing properly the coordinates x and m and rescaling the dimensions a and l, we can always scale down the numerical coefficients in the heat conduction coefficients). The simplest case of a zero heating, $H = 0$, was chosen.

Figures 1 and 2 show a typical example of our model edge plasma dynamics leading to the formation of a steady-state MARFE-like pattern. The scaled "edge plasma" dimensions were $a = 15$ and $l = 120$, while the parameters were the following: $T_0 = 45, T_a = 1, T_L = 3$, and $L_0 = 3$. In this run, we started with a small 2d "seed" plasma condensation (with a maximum of $5 \cdot 10^{-2}$ on the background of a constant (and equal to 1) plasma density. The "seed" condensation was exponentially localized, and we put it close to the middle of the "cold wall" of the system. For this initial density profile, the mass content of each magnetic field line is almost independent of x and equal to $M = 120$ (with an accuracy better than 10^{-3}). Recall that M serves as the Lagrangian "length" of the system in the z-direction. The initial pressure profile is one-dimensional and linear in x: $p(x, m, t = 0) = T_0 - (T_0 - T_a)(x/a)$. We checked the constancy of l (Eq. (11)) to monitor the accuracy of computations, and the maximum relative error in l was found to be 1 percent.

As the initial conditions are not in thermal equilibrium even without the small perturbation (they would be if it were not for the radiative cooling), the system starts to evolve.[30] The region with the temperature less than T_L is cooling down, as the radiative cooling operates here, while the heat conduction supplies heat from the core plasma and tends to smooth the temperature profile. Simultaneously, however, the initially small 2d-perturbation grows, and a pronounced 2d condensation develops. See Fig. 1(a) and (b) for the specific volume dynamics in the coordinates x and m. Finally, a localized MARFE-like structure is formed, as seen in Fig. 1(c) for the specific volume and in Fig. 2 for the temperature. Simulations with other initial pressure profiles gave similar results and, therefore, we conclude that the MARFE phenomenon has an extensive basin of attraction in the space of initial conditions.

In summary, we employed a natural hierarchy of time scales to derive a simplified model for the MARFE formation at the edge tokamak plasma. Using a mixed Eulerian-Lagrangian formalism, we were able to describe the MARFEs development in terms of formation of stable equilibrium patterns in a 2d active RD system with nonlocal constraints, leading to global negative feedback. We demonstrated numerically that persistent, MARFE-like patterns can develop in this system from a variety of initial conditions.

Acknowledgments

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[30]It would be, perhaps, more traditional to start from a small 2d-perturbation of a one-dimensional *thermal equilibrium*. However, we wanted to verify that the pattern formation in this system is insensitive to details of the initial conditions.

Figure Captions

1. Formation of a MARFE-like 2d pattern. Shown is the specific volume of the plasma as a function of x and Lagrangian coordinate m at successive time moments $t = 18$ (a), 36 (b), and 60 (c), when a steady state was achieved. The system lengths in the x- and z-directions are 15 and 120, respectively.

2. Steady-state plasma temperature as a function of x and Lagrangian coordinate m at time $t = 60$. The system lengths in the x- and z-directions are 15 and 120, respectively.

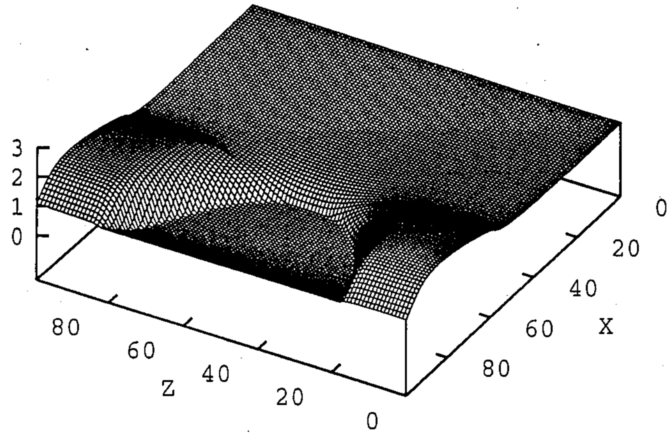

Fig. 1(a)

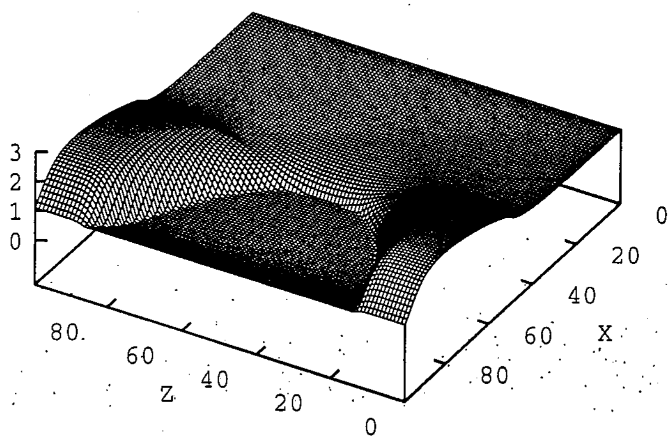

Fig. 1(b)

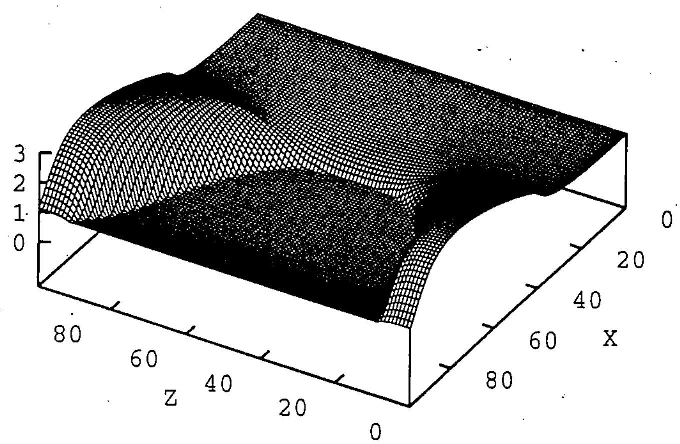

Fig. 1(c)

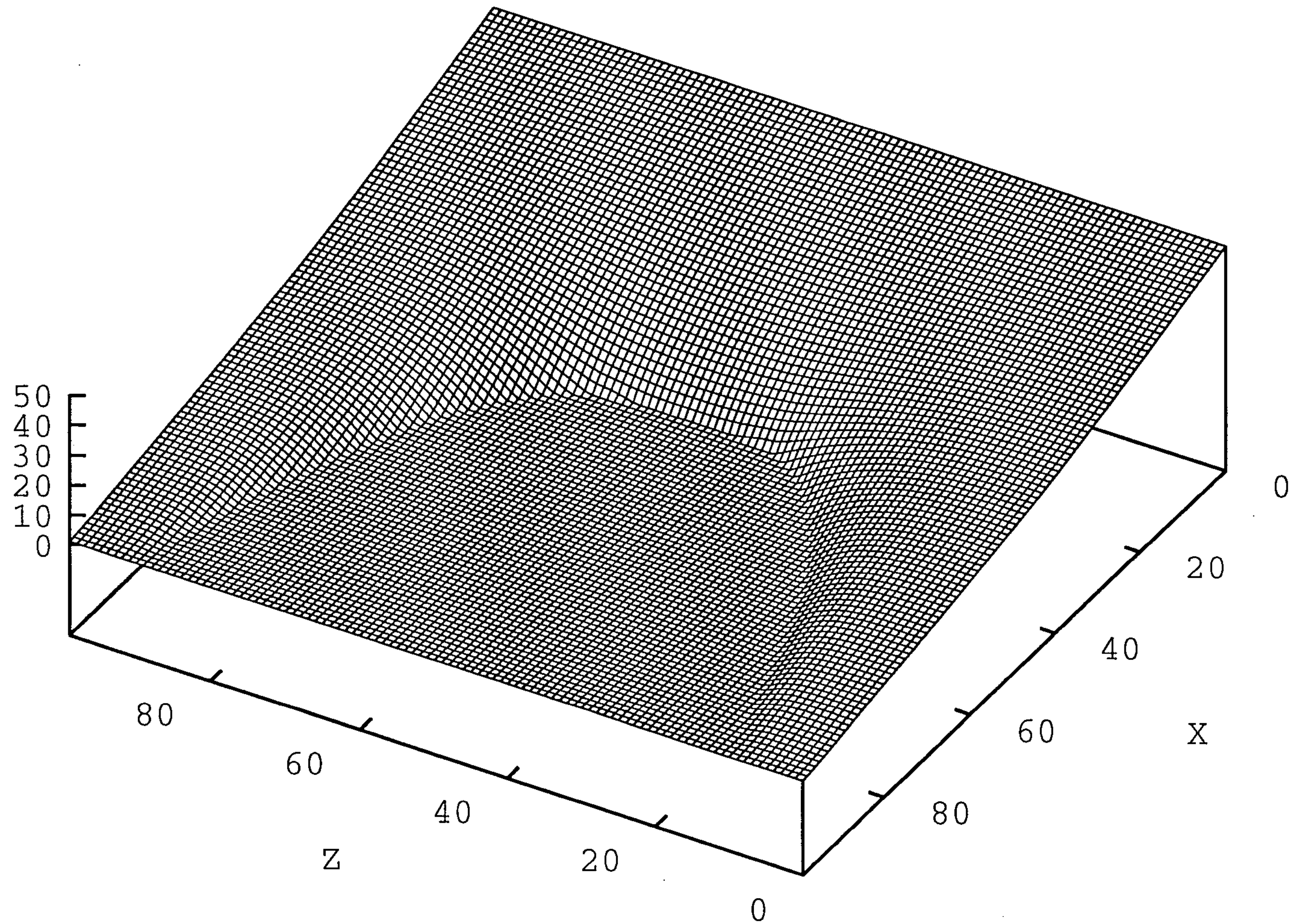

Fig. 2