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Electron Physics and Ambipolarity in the Tokamak Scrape-off Layer

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Abstract

Models of the electron behavior in the scrape-off layer (SOL) of diverted and limited tokamak plasmas must retain the abrupt change in boundary conditions that occurs across the separatrix or last closed flux surface as well as the electron reflecting Debye sheath established at the limiter or divertor plates. The balance between ion radial diffusion and streaming to the plates sets the SOL width and the electrons must adjust the Debye sheath at the plates to maintain quasineutrality and ambipolarity in the SOL beyond the last closed flux surface. We consider the long mean-free-path limit where a bounce-averaged kinetic electron model results in a steady-state balance in the SOL between radial diffusive feed from the core and velocity space diffusive loss to the plates due to collisional detrapping. In this double diffusion model a velocity space boundary layer occurs about the trapped-passing boundary where strong velocity space gradients must balance the streaming of the newly de-trapped electrons to the plates. The behavior of the electron distribution function in the velocity space boundary layer provides the information needed to evaluate the Debye-sheath-dependent electron loss rate.

I Introduction

The scrape-off layer near the edge of a limiter tokamak, or near the separatrix of a divertor tokamak, results from the abrupt change in boundary conditions that occurs across the critical radius, a. However, the effect of this change is very different for ion and electrons. In the zero gyration radius and long mean-free-path limit, an ion outside a quickly streams to the limiter or divertor collector plate, so that the layer thickness, w, is set by balancing radial diffusion against parallel streaming:

$$w \sim \left(\frac{DL}{v_t}\right)^{1/2} ,$$

where D is the diffusion coefficient, L the connection length and v_t an ion thermal speed.[1] The behavior of electrons is more complicated. Because of the Debye sheath surrounding any collecting surface, most electrons outside a are electrostatically trapped, and merely bounce back and forth between plates. As in a magnetic mirror device, some collisional process is necessary to detrap electrons and allow their escape.[2] Hence, even in the small collisionality limit, collisions, as well as the sheath potential, play a crucial role in electron scrape-off physics.

Note that *two* diffusion processes must occur in order for a typical electron to be collected. It must first diffuse radially to reach the scrape-off region where it becomes electrostatically trapped. Then, and equally importantly, it must diffuse in velocity space in order to overcome the sheath potential barrier. The main objective of the present work is to analyze this double diffusion process.

The radial structure of the scrape-off is fixed by ion dynamics, essentially because ions are relatively insensitive to the sheath potential. Ion scrape-off physics has been studied in a number of previous papers.[1, 3, 4] Electron scrape-off physics is studied in order to obtain, from quasineutrality and ambipolarity conditions, a prediction for the sheath potential amplitude. A main result of this work, Eq. (41), specifies the ambipolar sheath potential as a function of the (radially local) ion loss rate.

Loss of particles from a tokamak is a distinctly two-dimensional process, involving both radial motion and motion along the magnetic field (in addition to its velocity space dependence). The present analysis, like our previous ion treatment,[1] emphasizes this two-dimensional feature. Other aspects of the scrape-off layer, such as more detailed geometrical effects, are ignored.[3,4] Our purpose is to bring out certain physical effects that seem to be crucial, rather than to present a detailed, realistic description of the tokamak scrape-off layer.

II Model Geometry

We use the two-dimensional model of Ref. 1. The x-direction is radial, with separatrix radius at $x = 0$: electrons with $x < 0$ move on closed flux surfaces, while those at larger radii can stream to the divertor (or limiter) collector plates. The poloidal direction is denoted by y, with collector plates at $y = \pm L$. For $x < 0$, all physical quantities must be periodic in y, while for $x > 0$ absorbing boundary conditions are applied at $y = \pm L$.

This model oversimplifies the geometry and omits several potentially important physical processes. It does however include geometrical and kinetic features that will play an essential role in any more elaborate model of the scrape-off layer.

The electrostatic potential, $\Phi(x, y)$, varies in two dimensions. However we assume that its y-dependence is restricted to a narrow region near $y = \pm L$; in other words, Φ is constant along the magnetic field everywhere except in the vicinity of the Debye sheaths, of width λ_D, at each plate. The radial dependence of Φ is not restricted. We take the plates to be conducting, and set the plate potential at zero:

$$\Phi(x, \pm L) = 0 \ . \quad (1)$$

In velocity space, convenient variables are the total energy

$$E = \tfrac{1}{2} m v^2 - e\Phi \tag{2}$$

and the parallel energy,

$$U = \tfrac{1}{2} m v_\parallel^2 - e\Phi \ . \tag{3}$$

Here $v_\parallel$ is the velocity component in the direction of the magnetic field, and the minus signs reflect the sign of the electron charge, whose magnitude is $e > 0$. Note that in view of (1) we may assume

$$\Phi(x, y) > 0 \ , \tag{4}$$

since the potential acts to confine electrons. Because the magnetic field has constant magnitude in our model, the adiabatic invariance of the magnetic moment implies that both E and U are constants of the collisionless motion.

It is convenient to denote the parallel velocity by

$$v_\parallel = \sigma u \ ,$$

where $\sigma = \pm 1$ and $u = |v_\parallel|$. In terms of the basic variables,

$$u = \left\{ \left(\frac{2}{m}\right) [U + e\Phi(x, y)] \right\}^{1/2} \ . \tag{5}$$

Thus, in view of (4), the trapped and passing regions of velocity space are demarcated by U:

$$U > 0 \Rightarrow \text{ passing electrons ;}$$

$$U < 0 \Rightarrow \text{ trapped electrons .} \tag{6}$$

Finally we note that velocity space Jacobian is given by

$$\int d^3v = \left(\frac{2\pi}{m^2}\right) \sum_\sigma \int \frac{dU dE}{u} \ . \tag{7}$$

III Electron Kinetic Theory

We express the electron drift kinetic equation as

$$v_\parallel \nabla_\parallel f + \mathbf{v}_D \cdot \nabla f = C(f) \tag{8}$$

where f is the electron distribution, $\mathbf{v}_D$ is the guiding center drift velocity and C denotes the collision operator. The drift term could be replaced by an explicit diffusion term, as in Ref. 1, without changing the argument; in either case the integral of this term gives the divergence of the particle flux, Γ:

$$\int d^3v\, \mathbf{v}_D \cdot \nabla f = \nabla \cdot \Gamma\,. \tag{9}$$

The first term in (8) represents streaming of electrons along the magnetic field, and is measured by the transit frequency, ω_t:

$$v_\parallel \nabla_\parallel \sim \omega_t \equiv \frac{v_t}{L}\,. \tag{10}$$

Here

$$v_t \equiv \left(\frac{2T}{m}\right)^{1/2} \tag{11}$$

is the thermal speed, with T the electron temperature. The drift term represents the radially local electron loss rate (that is, the rate at which electrons are lost from a particular flux surface); it is consistent to assume this rate to be comparable to the electron collision frequency, ν:

$$\mathbf{v}_D \cdot \nabla f \sim C(f) \sim \nu f\,. \tag{12}$$

Thus (8) contains two frequencies, ω_t and ν. Our analysis assumes that the dominant physical process is electron streaming:

$$\omega_t \gg \nu\,. \tag{13}$$

This long mean-free-path ordering is consistent with that of Ref. 1; of course it is not pertinent to every tokamak scrape-off layer.

We expand the distribution function in powers of ν/ω_t,

$$f = f_0 + f_1 + \ldots, \qquad f_n \sim \left(\frac{\nu}{\omega_t}\right)^n ,$$

and thus decompose (8) into a sequence of equations for the f_n. Only two members of that sequence are needed:

$$\nabla_{\parallel} f_0 = 0 , \tag{14}$$

$$\sigma u \nabla_{\parallel} f_1 + \mathbf{v}_D \cdot \nabla f_0 = C(f_0) . \tag{15}$$

Because the plates are assumed to absorb incident electrons, (6) and (14) imply that

$$f_0 = 0 , \quad \text{for} \quad U > 0 . \tag{16}$$

Hence (15) is of interest only in the trapped region. We annihilate its first term by means of the orbital or "bounce" average,

$$\langle A \rangle \equiv \tfrac{1}{2} \sum_{\sigma} \frac{\oint \frac{dy}{u} A}{\oint \frac{dy}{u}} \tag{17}$$

where the loop integral is performed between bounce points. Since

$$\left\langle \sigma u \nabla_{\parallel} f_1 \right\rangle = 0 , \quad \text{for} \quad U < 0 , \tag{18}$$

we have

$$\langle \mathbf{v}_D \cdot \nabla f_0 \rangle = \langle C(f_0) \rangle , \quad \text{for} \quad U < 0 . \tag{19}$$

It is important to appreciate that (19) pertains only to electrons trapped by the Debye sheath, and does not hold in the entire velocity space. The point is that its right-hand side conserves particles at each radius, so that extending the equation to all U would lead quickly to contradiction. (An underlying circumstance is that the streaming term has no general annihilator in the presence of passing particles hitting an absorbing wall.) Indeed, there is a stronger restriction on (19): it fails, not only for $U > 0$, but also in the close vicinity of $U = 0$, where a narrow velocity-space boundary layer develops, invalidating the ordering. We consider the boundary layer in detail below, but note here that (19) holds only in a region R given by

$$R = \left\{ -e\Phi_0 < E < \infty\,,\ -e\Phi_0 < U < -\Delta U \text{ for } E \geq 0 \text{ and } -e\Phi_0 < U < E \text{ for } E < 0 \right\} \tag{20}$$

where Φ_0 is the maximum value of the potential along a field line (that is, the potential far from the Debye sheath) and ΔU, which satisfies

$$\Delta U \ll T\,, \tag{21}$$

is the width of the boundary layer.

Our next step is to integrate both sides of (19) over the region R. For the left-hand side, we note that the lowest order distribution is localized to the trapped region, so that the integral can be extended over all U without changing its value. This approximation mistreats only the contribution from boundary layer electrons; it does not require $e\Phi_0 \gg T$. Hence Eq. (9) is applicable; since we have already integrated over y, the result is

$$\int_R d^3v\, \langle \mathbf{v}_D \cdot \nabla f_0 \rangle = \frac{d\Gamma}{dx} \tag{22}$$

the (surface-averaged) radially local loss rate.

To compute the same integral of the right-hand side of (19), we need an explicit expression for C. The electron collision process is represented by a Lorentz gas operator,

$$C(f) = \nu \left(\frac{v_t}{v}\right)^3 \left(\frac{\partial}{\partial \xi}\right)(1-\xi^2)\left(\frac{\partial}{\partial \xi}\right)\,, \tag{23}$$

where $\xi = v_\parallel / v$. This form describes collisions accurately when the scrape-off region contains a significant fraction of impurities, and gives qualitatively correct scrape-off physics in general. In terms of the variables U and E we find that

$$C = \nu T^{3/2}(U + e\Phi)^{1/2}\left(\frac{\partial}{\partial U}\right)(U + e\Phi)^{1/2}\frac{(E-U)}{(E+e\Phi)^{3/2}}\left(\frac{\partial}{\partial U}\right)\,. \tag{24}$$

Here $\Phi(x,y)$ can be replaced by $\Phi_0(x)$, simply by restricting our attention to the y-region inside the two Debye sheaths. The bounce average then becomes trivial, and, with (7) and (24), it is a simple matter to find that

$$\int_R d^3v \, \langle C(f_0) \rangle = \pi v_t^3 \nu (e\Phi_0)^{1/2} \int_0^\infty \frac{dE\, E}{(E+e\Phi_0)^{3/2}} \frac{\partial f_0}{\partial U}\bigg|_{U=-\Delta U} . \tag{25}$$

The evaluation of the derivative at the edge of the boundary layer might seem problematic, in that boundary layers do not have well defined edges. However we find in the next section that $\partial f_0/\partial U$ approaches a constant near the layer, so that (25) is unambiguous. A similar procedure can be used to evaluate the energy loss rate.

IV Boundary Layer

Since f_0 has a finite slope for $U < 0$ but uniformly vanishes for $U > 0$, we expect a boundary layer to appear near $U = 0$. In this layer, the sharp U-curvature of f causes collisional effects to be magnified, without any enhancement of radial motion. The dominant terms in the kinetic equation are therefore

$$\sigma u \frac{\partial f}{\partial y} = C(f) , \qquad \text{for} \qquad U \approx 0 . \tag{26}$$

This form corresponds to a tokamak with $L = \pi q R$, where R is the major radius and q the safety factor. It is convenient to introduce the normalized variables

$$s = \frac{y}{L} , \qquad \varphi = \frac{e\Phi_0}{T} , \qquad \psi = \frac{E}{T} . \tag{27}$$

and

$$\eta = \frac{U}{\Delta U} . \tag{28}$$

where

$$\Delta U = \left(\frac{m}{2}\right) (\nu\, v_t^3)^{1/2} L^{1/2} \psi^{1/2} \varphi^{1/4} / (\psi + \varphi)^{3/4} . \tag{29}$$

Equation (26) then becomes

$$\sigma \frac{\partial f}{\partial s} = \frac{\partial^2 f}{\partial \eta^2} \,. \tag{30}$$

Here f must be periodic in s, for $\eta < 0$, while satisfying absorbing wall conditions at $s = \pm 1$, for $\eta > 0$. These conditions, together with (30), define a familiar Wiener-Hopf problem whose solution is well known.[5] (The same solution, with a different interpretation of the variable η, was used in Ref. 1 to analyze the ion scrape-off layer. The point is that an ion outside the separatrix is subject to the same loss process as an electron in the untrapped velocity-space region.) Without repeating the analysis that solves (3), we note that it yields the asymptotic behavior [cf. Eq. (15) of Ref. 1]

$$\frac{\partial f}{\partial \eta} \rightarrow \frac{\sqrt{\pi}}{\zeta(1/2)}\, f_M \,, \quad \eta \rightarrow -\infty \tag{31}$$

where ζ is the Riemann ζ-function, satisfying $\zeta(1/2) \approx -1.46$, and f_M is a Maxwellian distribution:

$$f_M = \frac{n_0}{\pi^{3/2}} v_t^{-3}\, e^{-(E+e\Phi_0)/T} \,. \tag{32}$$

Here the density n_0 measures the amplitude of the Maxwellian distribution describing the deeply trapped electrons; because only trapped electrons contribute, the actual electron density is

$$n_e = \int_R f_M d^3v = n_0 \,\mathrm{erf}(\sqrt{\varphi}) \,. \tag{33}$$

Equation (31) is equivalently expressed as

$$\frac{\partial f_0}{\partial U} = \frac{\sqrt{\pi}}{\zeta(1/2)} \frac{f_M}{\Delta U} \,. \tag{34}$$

Note that this quantity is independent of U, as anticipated: f_0 has asymptotically constant slope.

We substitute (34) into (25) and recall (29) to obtain

$$\int_R d^3v \, \langle C(f_0) \rangle = -(\nu\omega_t)^{1/2} n_0 \, F(\varphi) \tag{35}$$

where ω_t is the transit frequency of (10) and

$$F(\varphi) \equiv |1/\zeta(1/2)|\varphi^{1/4} e^{-\varphi} \int_0^\infty d\psi\, \psi^{1/2}(\psi + \varphi)^{-3/4} e^{-\psi} . \tag{36}$$

This is proportional to a confluent hypergeometric function (Kummer's function).[6] The asymptotic form

$$F(\varphi) \sim 0.61\varphi^{-1/2} e^{-\varphi} , \quad \varphi \gg 1 , \tag{37}$$

is easily deduced from (36).

V Scrape-off Ambipolarity

We now return to Eq. (19), which can be made explicit using (22), (33), and (35):

$$n_e \frac{F(\varphi)}{\mathrm{erf}(\sqrt{\varphi})} = -(\nu\omega_t)^{-1/2} \frac{d\Gamma}{dx} , \tag{38}$$

where $\nu/\omega_t \propto n/T$ depends on x. Equation (38), whose right-hand side measures the electron loss rate, determines the sheath potential $\varphi(x)$. Notice that, since φ is presumed to vanish on the plate and to be constant along $\mathbf{B}$, it effectively measures the radial *plasma potential* in the $x > 0$ part of the scrape-off. Our model does not contain enough physics to determine the plasma potential inside the separatrix ($x < 0$), where there is no parallel loss mechanism. The calculation of the core plasma potential profile, apparently involving the effects of finite Larmor radius, velocity shear, or other phenomena which modify the Maxwell-Boltzmann response, is beyond the scope of the present study.

We assume that in equilibrium the total loss is locally ambipolar,

$$\frac{d\Gamma}{dx} = \frac{d\Gamma_i}{dx} . \tag{39}$$

Then, since

$$n_e = n_i , \tag{40}$$

we can write (38) in the form

$$G(\varphi) = (\nu\omega_t)^{-1/2}\, S_i\ , \tag{41}$$

where

$$G(\varphi) \equiv \frac{F(\varphi)}{\mathrm{erf}(\sqrt{\varphi})}\ , \tag{42}$$

and

$$S_i \equiv n_i^{-1}\left|\frac{d\Gamma_i}{dx}\right| \tag{43}$$

is the density-normalized ion loss rate. It is evident that for large φ, G has the same asymptotic behavior as F, given by (37). At very small φ, $G \sim \varphi^{-1/4}$ diverges. The singular behavior can be traced back to a breakdown of the thin trapped-passing boundary layer assumption. As $\varphi \to 0$ the boundary layer response becomes comparable to the adiabatic response of the deeply trapped electrons. However, this singularity affects our results only for $\varphi \lesssim (L\nu/v_t)^{2/3} \ll 1$, a domain that is not observed experimentally, so we are content to use (36). A plot of $G(\varphi)$ is shown in Fig. 1.

Equation (41) determines the ambipolar potential in terms of quantities characterizing the ion scrape-off. Its content can be compared to that of conventional, one-dimensional theories of the Debye sheath. Note that such theories are consistent only if some source is provided to maintain the assumed steady-state. In the present two-dimensional theory, this source is made explicit in terms of radial diffusion across the separatrix. On the other hand we do not study the parallel structure of the Debye sheath, on the scale length λ_D, predicting only the potential barrier height.

The quantity S_i is the loss rate for a single particle; from particle conservation we find that

$$S_i = \frac{\Gamma_\parallel(y=L) + |\Gamma_\parallel(y=-L)|}{2Ln_i} \tag{44}$$

where $\Gamma_\parallel$ is the ion parallel flux satisfying $\partial\Gamma/\partial x + \partial\Gamma_\parallel/\partial y = 0$. Since ion streaming to the plates is unimpeded by the potential, this rate is essentially determined by the radial motion that brings ions to the magnetic surface at x. In the simplest diffusion model,[1] for example,

$$\Gamma_i = -D\,\frac{dn_i}{dx}$$

we would have

$$S_i = D\,\frac{|d^2n_i/dx^2|}{n_i}\,. \tag{45}$$

One does not expect this function to vary sharply with x. Furthermore, within the context of the present idealizations (ignoring charge exchange, for example), S_i seems generally unlikely to show sharp variation. In other words (41) predicts a nearly radially uniform φ:

$$\frac{e\Phi_0}{T_e} \approx \text{ constant}\,. \tag{46}$$

The value of the constant can be estimated using (45) and the conventional estimate,[1]

$$D\left|\frac{d^2n_i}{dx^2}\right| \approx \frac{n_i c_s}{L} = n_i\left(\frac{m_e}{m_i}\right)^{1/2}\omega_t$$

where c_s is the sound speed. Hence

$$G \approx \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\omega_t}{\nu}\right)^{1/2} \tag{47}$$

For concreteness we consider a typical TEXT discharge, in which $\omega_t/\nu \approx 6$ in the edge region. Then $G \approx 0.06$ and (41) yields

$$\frac{e\Phi_0}{T_e} \approx 1.7\,. \tag{48}$$

The excellent agreement between (48) and TEXT experimental measurements[7] is not conclusive, because of the idealizations present in our model. Furthermore it must be noted that more conventional, one-dimensional sheath theories lead to similar results. Yet the intrinsically two-dimensional nature of the tokamak scrape-off problem is worth emphasizing: radial diffusion provides the necessary source for the collector plate sheath. Equation (48) shows that a self-consistent two-dimensional treatment, involving both radial diffusion and velocity-space scattering, gives experimentally reasonable predictions.

Of course if φ is presumed known, then (41) predicts the parallel flux, $\Gamma_{\|}$, on the plates. An explicit formula is obtained from (41) and (44):

$$\left[\Gamma_{\|}(y=L)+|\Gamma_{\|}(y=-L)|\right]2L = n_i(\nu\omega_t)^{1/2}G(\varphi)\ . \tag{49}$$

This relation should bear on experiments that artificially adjust the plasma potential. That its right-hand side monotonically decreases with increasing φ is physically obvious: reducing the plate sheath potential enhances the ambipolar plate flux. However the dependence on collision frequency, and, especially, the specific functional form of $G(\varphi)$, are not obvious. Both features reflect the critical role of pitch-angle scattering, in allowing electrostatically trapped electrons to escape the scrape-off.

Equation (49) assumes *local* ambipolarity. However, non-ambipolar radial transport can occur beyond the separatrix causing the radial derivatives of the electrons and ion fluxes to differ. In such cases current flows in the SOL[8] and $\nabla\cdot\mathbf{J}=0$ may be used to obtain the relation

$$\frac{d\Gamma}{dx}=\frac{d\Gamma_i}{dx}+e^{-1}\left[J_{\|}(x,y=L)-J_{\|}(x,y=-L)\right]$$

which can be used in (38) to generalize (49). Since equal numbers of ions and electrons must be lost from the plasma the constraint

$$\int_0^\infty dx\, J_{\|}(x,y=\pm L)=0$$

must be satisfied. Employing the preceding in Eq. (38) leads to the *global* ambipolarity constraint

$$\int_0^\infty dx\int_R d^3v\,\langle C\rangle=\int_0^\infty dx n_i(\nu\omega_t)^{1/2}G(\varphi)=|\Gamma(x=0)|=|\Gamma_i(x=0)|\ ,$$

which must hold in the *local* ambipolarity limit as well. This constraint determines how rapid the radial variation of φ must be for a specified collisionality and flux at $x=0$.

VI Summary

The main result of this work is Eq. (41), relating the sheath potential, near a limiter or divertor collector plate, to the ion loss rate. An equivalent expression is given by (49). This prediction stems from a two-dimensional (in configuration space) kinetic model of the scrape-off region, accounting for both radial particle flux into the scrape-off, parallel streaming to the boundary wall, and collisional scattering across the loss boundary in velocity space.

An obvious feature of the result is that the loss rate indeed decreases with increasing sheath potential. More interesting is the detailed functional form, with departs from a simple Maxwell-Boltzmann factor, and the relevant electron time scale, which emerges as the geometric mean of the electron collision and transit times. The predicted value of the sheath potential agrees well with experimental measurements in TEXT.

The plasma potential can be influenced by several factors, including non-ambipolar radial transport, space charge and plasma rotation. Most of these processes depend on finite gyro-radius (or finite orbit-width) physics, which this work omits.[9] They may be especially important inside the scrape-off, where Eq. (41) does not apply. Nonetheless the processes which we have described are both fundamental and intrinsic to tokamak operation. It seems likely that any finite gyro-radius theory of the scrape-off would be influenced by double diffusion process studied here.

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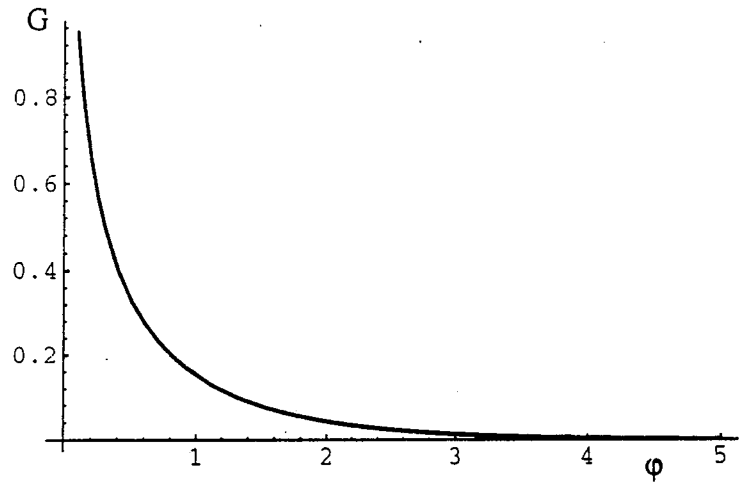

Figure 1. The function G(φ)