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Effect of a NonUniform Resistive Wall on the Stability of Tokamak Plasmas*

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Abstract

The effect of a nonuniform resistive wall on the stability of plasma MHD modes is examined. For the case of a tokamak plasma interacting with a wall possessing toroidally nonuniform electrical resistance the kink mode dispersion relation is found to reduce to a surprisingly simple form, provided that the scale of variation of the resistance is sufficiently large. The influence of a wall with toroidal gaps on tokamak plasma stability is investigated in some detail. Under some circumstances kink modes are found to 'explode' through the gaps with ideal growth rates. A similar investigation is made for a modular wall constructed of alternate thick and thin sections.

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I Introduction

The interaction of magnetohydrodynamical (MHD) plasma instabilities with a resistive wall in toroidal pinches has been extensively studied in the literature.[1–22] It is generally accepted that eddy currents induced in the wall can moderate the growth of an otherwise ideally unstable kink mode, so that it evolves on some characteristic resistive time scale of the wall. Such modes are usually referred to as 'wall modes'. The interaction of a rotating tearing mode island with self-induced wall eddy currents is thought to generate a nonlinear slowing torque which effectively brakes the rotation once a critical island width is exceeded.[19,22] This effect is important because a nonrotating (or 'locked') tearing mode is generally more unstable than a rapidly rotating one, since the nonrotating mode is able to penetrate through the wall.[11]

This paper is concerned with the stability of MHD modes in tokamaks which possess close fitting walls with nonuniform electrical resistance. In fact, most modern tokamaks are of this type since their vacuum vessels are of modular construction, with thick low resistance sections (containing the diagnostic ports) separated by thin high resistance bellows sections.[23–25] It is clearly of interest to establish whether wall modes grow on the relatively slow resistive time scale of the thick sections, the much faster resistive time scale of the thin sections, or some appropriate average of the two. For obvious engineering reasons magnetic pick-up coils tend to be attached to the thick sections of the vacuum vessel. Distortions induced in the structure of MHD modes by the nonuniform eddy currents flowing in the vacuum vessel (e.g. 'ballooning' of modes through the thin sections of the vessel) need to be taken into account during the interpretation of pick-up coil data, otherwise spurious results may be obtained. Some tokamaks possess thick conducting walls with insulating toroidal breaks.[26,27] In such devices it may be possible for an ideally unstable kink mode to defeat the moderating effect of wall eddy currents by 'ballooning' through the insulating breaks where no eddy currents can flow.

In next-generation tokamaks, such as the International Tokamak Experimental Reactor (ITER),[28] the interaction of MHD instabilities with the wall is likely to be of particular significance because of the large dimensions envisaged for such devices. This follows since the critical island width for the 'locking' of rotating tearing modes to the wall is a rapidly decreasing function of machine dimensions, due to the comparatively feeble mode rotation found in large devices.[22] Thus, 'locked modes', which interact strongly with the wall, may be a common occurrence in next-generation tokamaks.

The above discussion highlights the importance of gaining as complete an understanding as possible of the interaction of MHD instabilities with *realistic* walls, including the effects of modularity, insulating breaks, gaps, diagnostic ports, etc. In Sec. II of this paper a general formalism is developed for analyzing the influence of a wall with nonuniform resistance on the stability of kink modes. In Sec. III the kink mode dispersion relation for conventional tokamaks possessing walls with toroidally varying resistance is found to reduce to a surprisingly simple form, provided that the scale of variation of the resistance is sufficiently large. Section IV investigates the effect of a wall with toroidal gaps on the stability of both kink modes and tearing modes. In Sec. V a similar investigation is made for a wall of modular construction. Finally, this paper is summarized in Sec. VI.

II General Analysis

A The wall flux

In the following, the standard cylindrical tokamak limit is adopted, and the usual right-handed cylindrical polar coordinates r, θ, z are employed. The perturbed magnetic field $\delta\mathbf{B}$ is written in terms of the perturbed poloidal flux ψ, so that $\delta\mathbf{B} = \nabla \wedge (\psi\hat{\mathbf{z}}) \equiv \nabla\psi \wedge \hat{\mathbf{z}}$. The perturbed flux in the wall is written $\Psi_{\rm w}(\theta, \phi) \equiv \psi(r_{\rm w}, \theta, \phi)$, where $r_{\rm w}$ is the wall minor radius, $\phi = z/R_0$, and R_0 is the simulated major radius. The wall is assumed to lie in the 'thin-shell' limit, where the skin depth is much larger than the actual wall thickness, so that the flux is approximately constant across the wall.[10]

B The wall eddy currents

The radially integrated eddy currents induced in the wall are written in terms of a stream function, so that $\delta\mathbf{I}_w = \nabla \wedge (J_w \hat{\mathbf{r}}) \equiv \nabla J_w \wedge \hat{\mathbf{r}}$, where $J_w \equiv J_w(\theta, \phi)$. Here, it is assumed that the eddy currents have negligible radial components, so that the current pattern in the wall is essentially two dimensional. This is a reasonable assumption in the 'thin shell' limit.

Ohm's law (integrated across the wall) takes the form $\delta\mathbf{I}_w = \sigma_w \delta_w \, \delta\mathbf{E}$, where $\delta\mathbf{E}$ is the perturbed electric field, $\sigma_w(\theta, \phi)$ the wall conductivity, and $\delta_w(\theta, \phi)$ the wall thickness. It follows from the radial component of Faraday's law that

$$\frac{1}{\sigma_w \delta_w} \nabla^2 J_w + \nabla \left(\frac{1}{\sigma_w \delta_w} \right) \cdot \nabla J_w = \frac{\gamma}{r_w} \frac{\partial \Psi_w}{\partial \theta} \,, \tag{1}$$

where $\gamma \equiv d \ln \Psi_w / dt$ is the growth rate.

C The 'jump' conditions at the wall

The 'jump' in radial derivative of the wall flux induced by the eddy currents is written[22]

$$\Delta\Psi_w(\theta, \phi) = \left[r \frac{\partial \psi(r, \theta, \phi)}{\partial r} \right]_{r_w-}^{r_w+} . \tag{2}$$

It follows from the z-component of Ampère's law that

$$\Delta\Psi_w = \mu_0 \frac{\partial J_w}{\partial \theta} \,. \tag{3}$$

Let

$$\Psi_w(\theta, \phi) = \sum_{m,n} \Psi_w^{m/n} \exp[\mathrm{i}\,(m\theta - n\phi)] \,,$$

$$\Delta\Psi_w(\theta, \phi) = \sum_{m,n} \Delta\Psi_w^{m/n} \exp[\mathrm{i}\,(m\theta - n\phi)] \,,$$

$$J_{\rm w}(\theta,\,\phi) = \sum_{m,n} J_{\rm w}^{m/n}\,\exp[{\rm i}\,(m\theta - n\phi)]\ , \tag{4}$$

then Eqs. (3) and (1) transform to

$$\Delta\Psi_{\rm w}^{m/n} = {\rm i}\,m\mu_0\,J_{\rm w}^{m/n}, \tag{5}$$

and

$$\Psi_{\rm w}^{m/n} = \sum_{j,\,k}\left[1 + \frac{n(n+k)}{m(m+j)}\,\epsilon_{\rm w}^2\right]\oint\oint \frac{\exp[{\rm i}\,(j\theta - k\phi)]}{\gamma\tau_{\rm w}(\theta,\,\phi)}\,\frac{d\theta}{2\pi}\,\frac{d\phi}{2\pi}\ \ \Delta\Psi_{\rm w}^{m+j/n+k}\ , \tag{6}$$

respectively. Here, $\epsilon_{\rm w} = r_{\rm w}/R_0$ is the inverse aspect ratio of the wall, and

$$\tau_{\rm w}(\theta,\,\phi) = \mu_0 r_{\rm w}\sigma_{\rm w}\delta_{\rm w} \tag{7}$$

is the poloidally and toroidally varying wall time constant.

D The stability of wall modes

Consider the stability of wall modes, which behave ideally (i.e. with no reconnection) at all rational surfaces within the plasma. The perturbed poloidal flux can be written

$$\psi(r,\theta,\phi) = \sum_{m,n} \Psi^{m/n}(r)\,\exp[{\rm i}\,(m\theta - n\phi)]\ , \tag{8}$$

where $\Psi^{m/n}(r)$ satisfies the m/n cylindrical tearing mode equation[22] and the physical boundary conditions at $r = 0$ and $r \to \infty$. In addition, $\Psi^{m/n}(r)$ is zero at the m/n rational surface, provided it lies within the plasma. There is a real wall stability index associated with each harmonic of the perturbed poloidal flux:

$$E_{\rm ww}^{m/n} \equiv \left[r\,\frac{d\Psi^{m/n}}{dr}\right]_{r_{\rm w-}}^{r_{\rm w+}} \Big/\,\Psi^{m/n}(r_{\rm w})\ . \tag{9}$$

Asymptotic matching across the wall yields

$$\Delta\Psi_{\rm w}^{m/n} = E_{\rm ww}^{m/n}\,\Psi_{\rm w}^{m/n} \tag{10}$$

for each harmonic.

III Analysis for a Toroidally Nonuniform Wall

A Introduction

Consider a wall whose resistance only varies in the toroidal direction, so that $\tau_{\rm w} \equiv \tau_{\rm w}(\phi)$. It follows from Eq. (6) that

$$\Psi_{\rm w}^{m/n} = \sum_k \left[1 + \frac{n(n+k)}{m^2}\,\epsilon_{\rm w}^2\right] \oint \frac{\exp(-{\rm i}\,k\phi)}{\gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi}\,\Delta\Psi_{\rm w}^{m/n+k}\,. \qquad (11)$$

Suppose that the coupled toroidal harmonics lie in the range $|n| < n_{\rm max}$, where $n_{\rm max} \ll m/\epsilon_{\rm w}$. This is equivalent to a limit on the scale of variation of the wall resistance:

$$L_\phi \sim R_0 \Big/ \left(\frac{d\ln\tau_{\rm w}}{d\phi}\right)_{\rm min} \gg 2\pi\,\frac{r_{\rm w}}{m}\,. \qquad (12)$$

In fact, $2\pi\,r_{\rm w}/m$ is the poloidal spacing between eddy current cells, so (12) implies that the scale of variation of the wall resistance is much greater than the poloidal spacing of current eddies. If this is the case then Eqs. (10) and (11) yield

$$\Delta\Psi_{\rm w}^{m/n} \Big/ E_{\rm ww}^{m/n} = \sum_k \oint \frac{\exp(-{\rm i}\,k\phi)}{\gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi}\,\Delta\Psi_{\rm w}^{m/n+k}\,, \qquad (13)$$

which can be inverted to give

$$E_{\rm ww}^{m/n}\,\Psi_{\rm w}^{m/n} = \sum_k \oint \gamma\tau_{\rm w}(\phi)\exp(-{\rm i}\,k\phi)\,\frac{d\phi}{2\pi}\,\Psi_{\rm w}^{m/n+k}\,. \qquad (14)$$

Finally, let

$$\Psi_{\rm w}^{m/n} = \int_{-\infty}^{\infty} \overline{\Psi}_{\rm w}(\eta)\exp({\rm i}\,n\eta)\,\frac{d\eta}{2\pi}\,, \qquad (15)$$

so that Eq. (14) yields

$$E_{\rm ww}^{m/n}\,\Psi_{\rm w}^{m/n} = \int_{-\infty}^{\infty} \gamma\tau_{\rm w}(\eta)\overline{\Psi}_{\rm w}(\eta)\exp({\rm i}\,n\eta)\,\frac{d\eta}{2\pi}\,. \qquad (16)$$

It is easily demonstrated that for large-n (i.e. $1 \ll |n| \ll m/\epsilon_{\rm w}$) the $\Psi^{m/n}(r)$ (see Sec. II.D) become vacuum-like (i.e. they are essentially unaffected by the equilibrium plasma current) and $E_{\rm ww}^{m/n} \simeq -2m$. Note that the standard tokamak orderings (and the cylindrical tearing mode equation) break down at very high-n (i.e. $|n| \sim m/\epsilon_{\rm w}$), so $E_{\rm ww}^{m/n} \neq -2m$ in this limit. However, the constraint (12) ensures that such high-n modes can be neglected.

B The wall mode dispersion relation

Suppose that

$$E_{\rm ww}^{m/n} \neq -2m \qquad \text{for } n_1 \leq n \leq n_2 \; ,$$

$$E_{\rm ww}^{m/n} = -2m \qquad \text{otherwise} \; , \tag{17}$$

which according to the above discussion is a reasonable assumption for tokamaks, but not for reversed field pinches (RFPs) where kink modes typically have $n \sim m/\epsilon_{\rm w}$.[16] Let

$$\overline{\Psi}_{\rm w}(\eta) = \sum_{k=n_1}^{n_2} \left\{ \overline{f}_k(\eta) + \alpha_k \,\frac{2\sin\eta}{\eta}\,\exp(-{\rm i}\,k\eta) \right\} \; , \tag{18}$$

so that

$$\Psi_{\rm w}^{m/n} = \sum_{k=n_1}^{n_2} \int_{-\infty}^{\infty} \overline{f}_k(\eta)\,\exp({\rm i}\,n\eta)\,\frac{d\eta}{2\pi} + \alpha_n \; . \tag{19}$$

Here, use has been made of the identity

$$\int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta}\,\exp[{\rm i}\,(n-n')\eta]\,\frac{d\eta}{2\pi} = \delta_{nn'} \; , \tag{20}$$

and the α_n are arbitrary complex constants satisfying $\alpha_n = 0$ for all n not in the range $n_1 \rightarrow n_2$. It follows from Eq. (16) that for all toroidal mode numbers not in this range

$$\sum_{k=n_1}^{n_2} \int_{-\infty}^{\infty} \left[(2m+\gamma\tau_{\rm w})\overline{f}_k + \alpha_k\,\gamma\tau_{\rm w}\,\frac{2\sin\eta}{\eta}\,\exp(-{\rm i}\,k\eta) \right] \exp({\rm i}\,n\eta)\,\frac{d\eta}{2\pi} = 0 \; . \tag{21}$$

The most general solution is

$$\overline{f}_k(\eta) = -\alpha_k\,\frac{2\sin\eta}{\eta}\,\frac{\beta_k + \gamma\tau_{\rm w}(\eta)}{2m+\gamma\tau_{\rm w}(\eta)}\,\exp(-{\rm i}\,k\eta) \; , \tag{22}$$

where the β_n are arbitrary complex constants satisfying $\beta_n = 0$ for n all not in the range $n_1 \rightarrow n_2$. Equations (18) and (22) give

$$\overline{\Psi}_{\rm w}(\eta) = \sum_{k=n_1}^{n_2} \alpha_k\,\frac{2\sin\eta}{\eta}\,\frac{2m-\beta_k}{2m+\gamma\tau_{\rm w}(\eta)}\,\exp(-{\rm i}\,k\eta) \; . \tag{23}$$

For each toroidal mode number in the range n_1 to n_2 Eq. (16) yields

$$\alpha_n\,E_{\rm ww}^{m/n} = \sum_{k=n_1}^{n_2} \alpha_k \times \tag{24}$$

$$\int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta}\,\frac{(E_{\rm ww}^{m/n}+2m-\beta_k)\gamma\tau_{\rm w}(\eta)+E_{\rm ww}^{m/n}\beta_k}{2m+\gamma\tau_{\rm w}(\eta)}\,\exp[{\rm i}\,(n-k)\eta]\,\frac{d\eta}{2\pi}\,,$$

which can be rearranged to give

$$\sum_{k=n_1}^{n_2} \widehat{\alpha}_k \int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta}\,\frac{2m+E_{\rm ww}^{m/n}}{2m+\gamma\tau_{\rm w}(\eta)}\,\exp[{\rm i}\,(n-k)\eta]\,\frac{d\eta}{2\pi} = \widehat{\alpha}_n\,, \tag{25}$$

where $\widehat{\alpha}_n = \alpha_n \times (2m-\beta_n)$.

Now, $2m+\gamma\tau_{\rm w}(\eta)$ is a periodic function of η with period 2π. It follows that

$$\frac{1}{2m+\gamma\tau_{\rm w}(\eta)} = \sum_k \oint \frac{\exp({\rm i}\,k\phi)}{2m+\gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi}\,\exp(-{\rm i}\,k\eta)\,, \tag{26}$$

so using Eq. (20)

$$\int_{-\infty}^{\infty} \frac{2\sin\eta}{\eta}\,\frac{\exp({\rm i}\,n\eta)}{2m+\gamma\tau_{\rm w}(\eta)}\,\frac{d\eta}{2\pi} \equiv \oint \frac{\exp({\rm i}\,n\phi)}{2m+\gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi}\,. \tag{27}$$

Thus, the wall mode dispersion relation can be written

$$\sum_k \widehat{\alpha}_k \oint \frac{2m+E_{\rm ww}^{m/n}}{2m+\gamma\tau_{\rm w}(\phi)}\,\exp[{\rm i}\,(n-k)\phi]\,\frac{d\phi}{2\pi} = \widehat{\alpha}_n \tag{28}$$

for all n. Note that $\widehat{\alpha}_n = 0$ if $E_{\rm ww}^{m/n} = -2m$, as was initially assumed in Eq. (18).

The wall mode dispersion relation takes the form of a matrix equation:

$$\mathbf{A}\cdot\boldsymbol{\alpha} = \mathbf{0}\,, \tag{29}$$

where $\boldsymbol{\alpha}$ is the complex vector of the $\widehat{\alpha}_n$ values, and

$$A_{ab} = \delta_{ab} - \oint \frac{2m+E_{\rm ww}^{m/a}}{2m+\gamma\tau_{\rm w}(\phi)}\,\exp[{\rm i}\,(a-b)\phi]\,\frac{d\phi}{2\pi}\,. \tag{30}$$

Here, A is a complex $N\times N$ matrix, where N is the number of toroidal harmonics for which $E_{\rm ww}^{m/n} \neq -2m$. It is easily demonstrated that

$$\Psi_{\rm w}^{m/n} = \sum_k \widehat{\alpha}_k \oint \frac{\exp[{\rm i}\,(n-k)\phi]}{2m+\gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi}\,, \tag{31}$$

and

$$\Psi_{\rm w}(\theta,\,\phi) = \sum_n \hat{\alpha}_n\,\frac{\exp[{\rm i}\,(m\theta - n\phi)]}{2m + \gamma\tau_{\rm w}(\phi)}\,. \tag{32}$$

Finally, a comparison of Eqs. (13), (14), (31), and (32) yields

$$\left({\rm i}\,\frac{\mu_0}{2}\right) J_{\rm w}^{m/n} = \sum_k \hat{\alpha}_k \oint \frac{\gamma\tau_{\rm w}(\phi)}{2m + \gamma\tau_{\rm w}(\phi)}\,\exp[{\rm i}\,(n-k)\phi]\,\frac{d\phi}{2\pi}\,, \tag{33}$$

and

$$\left({\rm i}\,\frac{\mu_0}{2}\right) J_{\rm w}(\theta,\,\phi) = \sum_n \hat{\alpha}_n\,\frac{\gamma\tau_{\rm w}(\phi)}{2m + \gamma\tau_{\rm w}(\phi)}\,\exp[{\rm i}\,(m\theta - n\phi)]\,, \tag{34}$$

where use has been made of Eq. (5).

C Discussion

After some analysis, the wall mode dispersion relation is found to take the surprisingly simple form (29), in which only those coupled toroidal harmonics whose stability indices differ appreciably [i.e. by $\mathcal{O}(1)$] from the vacuum value $-2m$ are *explicitly* included. The remaining toroidal harmonics are, in fact, *implicitly* included in the calculation without any approximation. This great simplification is possible because in tokamaks with toroidally nonuniform walls satisfying the constraint (12) most of the coupled toroidal harmonics possess the same stability index, $-2m$. No corresponding simplification occurs for RFPs, or for tokamaks with poloidally nonuniform walls, because in both cases the coupled harmonics generally have widely different stability indices.

IV The Effect of a Wall with Toroidal Gaps

A The stability of wall modes

1 The single-mode approximation

Consider a tokamak possessing a wall with toroidal gaps. The metal sections of the wall are assumed to have uniform resistance. The constraint (12) is satisfied provided the toroidal angular extent of the individual metal and gap sections of the wall are all much greater than $\epsilon_{\rm w}/m$.

Suppose that only a single harmonic has a wall stability index which differs significantly from the vacuum value $-2m$. This is not an unusual situation, especially if a low mode number rational surface lies just outside the edge of the plasma current channel. Table 1 shows values of $E_{\rm ww}^{m/n}$ (for $m = 3$ poloidal harmonics) calculated for a Wesson-like equilibrium current profile $j_z(r) \propto (1 - r^2/a^2)^\nu$, with $\nu = 1.46$, $q(0) = 1.2$, $q(a) = 2.95$, and $r_{\rm w}/a = 1.0$. Here, $q(r)$ is the conventional tokamak safety factor profile.[22] Table 1 indicates that only the 3/1 harmonic has a wall stability index which differs appreciably from the vacuum value -6. In this situation, the wall mode dispersion relation (29) takes the particularly simple form

$$\oint \frac{2m + E_{\rm ww}^{m/n}}{2m + \gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi} = 1\,, \tag{35}$$

where $E_{\rm ww}^{m/n} \neq -2m$ is the 'special' stability index.

n	$E_{\rm ww}^{3/n}$
-4	-5.831
-3	-5.790
-2	-5.723
-1	-5.595
0	-5.244
1	+2.167
2	-7.077
3	-6.479
4	-6.309
5	-6.228
6	-6.181

Table I: Values of the resistive wall mode stability index for $m = 3$ modes calculated for various different toroidal mode numbers n. The equilibrium current profile is $j_z(r) \propto (1 - r^2/a^2)^\nu$, with $\nu = 1.46$, $q(0) = 1.2$, $q(a) = 2.95$, and $r_{\rm w}/a = 1.0$.

Suppose that the wall is made of metal with an intrinsic time constant $\tau_{\rm w}$, but that one or more toroidal sections of total fractional angular extent f are missing (so $f = 0$ corresponds to no gaps, and $f = 1$ to no metal). According to Eq. (35), the wall mode dispersion relation is written

$$\gamma\tau_{\rm w} = \frac{E_{\rm ww}^{m/n}}{1 - f\left(1 + E_{\rm ww}^{m/n}/2m\right)}\,, \tag{36}$$

and using Eq. (32),

$$\left|\frac{\Psi_{\rm w\,gap}}{\Psi_{\rm w\,mtl}}\right| = 1 + \frac{\gamma\tau_{\rm w}}{2m}\,. \tag{37}$$

Here, $\Psi_{\rm w\,gap}$ is the perturbed poloidal flux in the gaps, and $\Psi_{\rm w\,mtl}$ is the flux in the metal.

Equations (36) and (37) imply that as $E_{\rm ww}^{m/n}$ approaches the vacuum limit $-2m$, the flux at the wall radius is mostly concentrated in the metal sections (i.e. $\Psi_{\rm w\,gap} \to 0$), and the wall mode decays on the characteristic time constant of the metal (i.e. $\gamma \simeq E_{\rm ww}^{m/n}/\tau_{\rm w}$). As the mode approaches marginal stability (i.e. $E_{\rm ww}^{m/n} \to 0$), the fluxes in the gap and metal sections of the wall gradually even out, and the flux becomes uniform (i.e. $|\Psi_{\rm w\,gap}/\Psi_{\rm w\,mtl}| = 1$) at the marginal stability point $E_{\rm ww}^{m/n} = 0$. For weakly unstable/stable modes (i.e. $|E_{\rm ww}^{m/n}| \ll 1$), the typical growth/decay time scale is the average time constant of the metal and gap sections of the wall [i.e. $\gamma \simeq E_{\rm ww}^{m/n}/\oint \tau_{\rm w}(\phi)\,d\phi/2\pi = E_{\rm ww}^{m/n}/\tau_{\rm w}(1-f)$]. As the mode becomes significantly unstable [i.e. $E_{\rm ww}^{m/n} \sim \mathcal{O}(1)$], the flux at the wall radius starts to concentrate in the gap sections of the wall (i.e. $|\Psi_{\rm w\,gap}/\Psi_{\rm w\,mtl}| > 1$), and the characteristic growth time decreases. Eventually, at a critical wall mode stability index,

$$(E_{\rm ww}^{m/n})_{\rm crit} = 2m\left(\frac{1}{f} - 1\right)\,, \tag{38}$$

the flux is entirely concentrated in the gap sections of the wall (i.e. $|\Psi_{\rm w\,mtl}| = 0$), and the mode becomes ideal in nature (i.e. the resistive growth rate tends to infinity). For $E_{\rm ww}^{m/n} > (E_{\rm ww}^{m/n})_{\rm crit}$, the wall eddy currents are insufficient to moderate the growth rate, and the mode 'explodes' through the gaps on a typical ideal external kink time scale (i.e. the growth rate is moderated by plasma inertia).

Equation (37) implies that the poloidal flux at the wall radius changes discontinuously at the metal/gap boundaries, corresponding to the situation shown schematically in Fig. 1. It can be seen that the eddy current vortices in the metal sections of the wall have 'square' ends, giving rise to infinite poloidal return currents flowing along the metal/gap boundaries [see Eq. (34)]. Such behaviour is clearly unphysical, and is a consequence of the approximation made in Sec. III.A. If the neglected term [i.e. the term in square brackets in Eq. (11)] is reinstated, the poloidal return currents are spread over a region of toroidal extent $2\pi\, r_{\rm w}/m$. This gives rise to round-ended eddy current vortices of the form shown schematically in Fig. 2. The constraint (12) ensures that the return current regions have negligible effect on mode stability.

2 The coupling of toroidal harmonics

Suppose that two toroidal harmonics (n_1 and n_2, say) have wall stability indices which differ significantly from the vacuum value $-2m$. The analysis of Sec. III.B yields:

$$\Psi_{\rm w\,gap}(\theta,\phi) \propto \widehat{\alpha}_1 \exp[{\rm i}\,(m\theta - n_1\phi)] + \widehat{\alpha}_2 \exp[{\rm i}\,(m\theta - n_2\phi)]\ , \tag{39a}$$

$$\left|\frac{\Psi_{\rm w\,gap}}{\Psi_{\rm w\,mtl}}\right| = 1 + \frac{\gamma\tau_{\rm w}}{2m}\ , \tag{39b}$$

$$\frac{\widehat{\alpha}_2}{\widehat{\alpha}_1} = \frac{\gamma\tau_{\rm w}(1+E_{\rm ww}^{m/n_2}/2m)c_{12}^*}{\gamma\tau_{\rm w}[1-f(1+E_{\rm ww}^{m/n_2}/2m)] - E_{\rm ww}^{m/n_2}}$$

$$= \frac{\gamma\tau_{\rm w}[1-f(1+E_{\rm ww}^{m/n_1}/2m)] - E_{\rm ww}^{m/n_1}}{\gamma\tau_{\rm w}(1+E_{\rm ww}^{m/n_1}/2m)c_{12}}\ , \tag{39c}$$

where

$$c_{12} = \int_{\rm gaps} \exp[{\rm i}\,(n_1-n_2)\phi]\,\frac{d\phi}{2\pi}\ . \tag{40}$$

According to Eq. (39c), the mode growth rate is given by

$$\gamma\tau_{\rm w}\times 2a = E_{\rm ww}^{m/n_1}[1-f(1+E_{\rm ww}^{m/n_2}/2m)] + E_{\rm ww}^{m/n_2}[1-f(1+E_{\rm ww}^{m/n_1}/2m)] \tag{41}$$

$$\pm\sqrt{(E_{\rm ww}^{m/n_1}-E_{\rm ww}^{m/n_2})^2(1-f)^2 + 4|c_{12}|^2 E_{\rm ww}^{m/n_1}E_{\rm ww}^{m/n_2}(1+E_{\rm ww}^{m/n_1}/2m)(1+E_{\rm ww}^{m/n_2}/2m)}\ ,$$

with

$$a = [1 - f(1 + E_{\rm ww}^{m/n_1}/2m)][1 - f(1 + E_{\rm ww}^{m/n_2}/2m)]$$

$$-|c_{12}|^2(1 + E_{\rm ww}^{m/n_1}/2m)(1 + E_{\rm ww}^{m/n_2}/2m)\ . \tag{42}$$

In most respects, the behavior of the coupled modes is analogous to that of the corresponding uncoupled modes (see Sec. IV.A.1). For instance, as $E_{\rm ww}^{m/n_j}$ (where j is 1 or 2) approaches the vacuum value $-2m$, the associated root of Eq. (41) approaches $\gamma\tau_{\rm w} = -2m$, and the magnetic flux becomes entirely concentrated in the metal sections of the wall. Furthermore, as $E_{\rm ww}^{m/n_j}$ approaches zero, the corresponding root of (41) approaches the marginal value $\gamma\tau_{\rm w} = 0$, and the magnetic flux becomes evenly distributed in the wall. However, mode coupling does affect the onset of ideal growth through the gaps, which now occurs when $a = 0$.

Mode coupling is most effective when the two stability indices are equal; i.e. when $E_{\rm ww}^{m/n_1} = E_{\rm ww}^{m/n_2} = E_{\rm ww}^{m/n}$. For this special case, Eq. (41) reduces to

$$\gamma\tau_{\rm w} = \frac{E_{\rm ww}^{m/n}}{1 - (f \pm |c_{12}|)(1 + E_{\rm ww}^{m/n}/2m)}\ , \tag{43}$$

so the critical stability index for the 'explosion' of the mode through the gaps is

$$(E_{\rm ww}^{m/n})_{\rm crit} = 2m\left(\frac{1}{f + |c_{12}|} - 1\right)\ . \tag{44}$$

It is clear, by comparison with Eq. (38), that mode coupling tends to *reduce* the critical stability index needed for ideal growth. Note that $f + |c_{12}| \leq 1$, so $(E_{\rm ww}^{m/n})_{\rm crit}$ is never negative.

B The interaction with rotating tearing modes

Suppose that a single toroidal harmonic (mode number n, say) has a rational surface lying inside the plasma at minor radius $r_{\rm s}$. In this situation, the stability of the m/n mode is governed by the following coupled equations:[22,29,30]

$$\Delta\Psi_{\rm s}^{m/n} - E_{\rm ss}^{m/n}\Psi_{\rm s}^{m/n} - E_{\rm sw}^{m/n}\Psi_{\rm w}^{m/n} = 0\ , \tag{45a}$$

$$\Delta\Psi_{\rm w}^{m/n} - E_{\rm ww}^{m/n}\Psi_{\rm w}^{m/n} - E_{\rm sw}^{m/n}\Psi_{\rm s}^{m/n} = 0\ , \tag{45b}$$

where $\Psi_{\rm s}^{m/n}$ is the m/n reconnected flux at the rational surface, $\Delta\Psi_{\rm s}^{m/n}$ is the 'jump' in the radial derivative of the m/n flux across the rational surface, $E_{\rm ss}^{m/n}$ is the fixed boundary m/n tearing stability index (calculated assuming zero flux in the wall), and $E_{\rm ss}^{m/n} + (E_{\rm sw}^{m/n})^2/(-E_{\rm ww}^{m/n})$ is the corresponding free boundary stability index (calculated assuming zero eddy currents in the wall). As before, $\Psi_{\rm w}^{m/n}$ is the m/n flux in the wall, $\Delta\Psi_{\rm w}^{m/n}$ is the 'jump' in the radial derivative of the m/n flux across the wall, and $E_{\rm ww}^{m/n}$ is the m/n wall stability index (calculated assuming zero reconnection at the rational surface). The stability of nonresonant modes (i.e. $n' \neq n$) is again governed by

$$\Delta\Psi_{\rm w}^{m/n'} - E_{\rm ww}^{m/n'}\Psi_{\rm w}^{m/n'} = 0\ . \tag{46}$$

According to standard Rutherford island theory,[31] the nonlinear evolution of the m/n tearing mode satisfies

$$\tau_R\,\frac{d}{dt}\left(\frac{W}{r_{\rm s}}\right) = r_{\rm s}\Delta_{\rm s}' \equiv E_{\rm ss}^{m/n} + E_{\rm sw}^{m/n}\,{\rm Re}\left(\frac{\Psi_{\rm w}^{m/n}}{\Psi_{\rm s}^{m/n}}\right)\ , \tag{47}$$

where W is the island width, $\tau_R = 0.8227\,\mu_0 r_{\rm s}^2/\eta_\parallel(r_{\rm s})$ is the resistive diffusion time scale, and $\eta_\parallel$ is the parallel plasma resistivity. The nonlinear toroidal electromagnetic torque acting at the rational surface due to eddy currents flowing in the wall is given by Refs. 22, 29, and 30

$$\delta T_\phi(r_{\rm s}) = \frac{2n\pi^2 R_0}{\mu_0} \times E_{\rm sw}^{m/n}\,{\rm Im}(\Psi_{\rm w}^{m/n}\Psi_{\rm s}^{m/n\,*})\ . \tag{48}$$

Consider the simplest possible case, where the wall stability indices of the nonresonant modes do not differ appreciably from the vacuum value $-2m$. This situation is analogous to that studied in Sec. IV.A.1 provided $E_{\rm ww}^{m/n} \rightarrow E_{\rm ww}^{m/n} + E_{\rm sw}^{m/n}\Psi_{\rm s}^{m/n}/\Psi_{\rm w}^{m/n}$ and $\gamma \rightarrow -{\rm i}\,\omega$. Here,

ω is the angular rotation frequency of the m/n magnetic island. It is assumed that the m/n wall mode is intrinsically stable, so that $E_{\rm ww}^{m/n} < 0$. If follows from Eq. (36) that

$$\frac{\Psi_{\rm w}^{m/n}}{\Psi_{\rm s}^{m/n}} = \frac{(1 - {\rm i}\,\omega\tau_{\rm w} f/2m)\, E_{\rm sw}^{m/n}}{-{\rm i}\,\omega\tau_{\rm w}[1 - f(1 + E_{\rm ww}^{m/n}/2m)] - E_{\rm ww}^{m/n}}\,. \tag{49}$$

Equations (31) and (32) yield

$$|\Psi_{\rm w\,gap}| = \frac{(1 - {\rm i}\,\omega\tau_{\rm w}/2m)\, \Psi_{\rm w}^{m/n}}{(1 - {\rm i}\,\omega\tau_{\rm w}/2m) f + (1 - f)}\,, \tag{50a}$$

$$|\Psi_{\rm w\,mtl}| = \frac{\Psi_{\rm w}^{m/n}}{(1 - {\rm i}\,\omega\tau_{\rm w}/2m) f + (1 - f)}\,, \tag{50b}$$

while Eqs. (47) and (48) imply that

$$r_{\rm s}\Delta_{\rm s}' = E_{\rm ss}^{m/n} + (E_{\rm sw}^{m/n})^2 \times \frac{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^{m/n}/2m] f/2m - E_{\rm ww}^{m/n}}{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^{m/n}/2m)]^2 + (E_{\rm ww}^{m/n})^2}\,, \tag{51a}$$

$$\delta T_\phi(r_{\rm s}) = \frac{2n\pi^2 R_0}{\mu_0} \times |\Psi_{\rm s}^{m/n}|^2 (E_{\rm sw}^{m/n})^2 \times$$

$$\frac{\omega\tau_{\rm w}(1 - f)}{(\omega\tau_{\rm w})^2[1 - f(1 + E_{\rm ww}^{m/n}/2m)]^2 + (E_{\rm ww}^{m/n})^2}\,. \tag{51b}$$

In the high frequency limit, $\omega\tau_{\rm w} \gg 1$, Eqs. (49)–(51) reduce to:

$$\frac{\Psi_{\rm w}^{m/n}}{\Psi_{\rm s}^{m/n}} \simeq \frac{E_{\rm sw}^{m/n} f/2m}{[1 - f(1 + E_{\rm ww}^{m/n}/2m)]}\,, \tag{52a}$$

$$\frac{|\Psi_{\rm w\,gap}|}{\Psi_{\rm s}^{m/n}} \simeq \frac{E_{\rm sw}^{m/n}/2m}{[1 - f(1 + E_{\rm ww}^{m/n}/2m)]}\,, \tag{52b}$$

$$\frac{|\Psi_{\rm w\,mtl}|}{|\Psi_{\rm w\,gap}|} \simeq {\rm i}\,\frac{2m}{\omega\tau_{\rm w}}\,, \tag{52c}$$

$$r_{\rm s}\Delta_{\rm s}' \simeq E_{\rm ss}^{m/n} + \frac{(E_{\rm sw}^{m/n})^2 f/2m}{[1 - f(1 + E_{\rm ww}^{m/n}/2m)]}\,, \tag{52d}$$

$$\delta T_\phi(r_s) \simeq \frac{2n\pi^2 R_0}{\mu_0} \frac{|\Psi_s^{m/n}|^2 (E_{sw}^{m/n})^2 (1-f)}{(\omega\tau_w)[1-f(1+E_{ww}^{m/n}/2m)]^2} . \tag{52e}$$

Equation (52a) indicates that a wall with gaps is unable to completely shield the perturbed flux due to a rapidly rotating tearing mode island from the region outside the wall. According to Eqs. (52b) and (52c) the rotating flux is able to penetrate through the wall by 'squeezing' through the gaps. Of course, in the limit where the gaps become very narrow (i.e. $f \to 0$) the amount of flux which gets through the wall becomes negligible. Equation (52d) shows that the tearing mode stability index asymptotes to the fixed boundary value as the gaps become very narrow (i.e. $f \to 0$), and asymptotes to the free boundary value as the gaps become very wide (i.e. $f \to 1$). Finally, Eq. (52e) shows that the torque exerted on the rotating tearing mode island by eddy currents induced in the wall asymptotes to zero as the gaps become very wide (i.e. $f \to 1$).

The above results suggest that the interaction of a rapidly rotating tearing mode island with a wall possessing thin toroidal gaps is very similar to the corresponding interaction with a uniform wall, except that in the former case a small amount of rotating magnetic flux gets through the wall, and the slowing down torque exerted on the island is slightly reduced. A thin toroidal limiter (radius r_L, say) can be modelled as a wall with a very large gap (i.e. f just less than unity). According to the above analysis, such a limiter is ineffective at shielding magnetic flux from the region $r > r_L$, and only exerts a comparatively weak slowing down torque on any rotating islands inside the plasma. It should be noted, however, that the constraint (12) limits the applicability of the above statements to gaps and limiters which are significantly wider than $2\pi\, r_w/m$ in the toroidal direction.

V The Effect of a Modular Wall

A The stability of wall modes

Consider a wall made up of alternate thick and thin sections of time constants τ_{w_1} and τ_{w_2}, respectively ($\tau_{w_1} > \tau_{w_2}$). For this simple case, the single mode dispersion relation (35) reduces to

$$\gamma^2 \tau_{w_1}\tau_{w_2} + \gamma\left[2m\left\{f_1\tau_{w_1} + f_2\tau_{w_2}\right\} - E_{ww}^{m/n}\left\{f_2\tau_{w_1} + f_1\tau_{w_2}\right\}\right] - 2m\, E_{ww}^{m/n} = 0\ , \tag{53}$$

where f_1 is the total angular extent of the thick sections, and $f_2 \equiv 1 - f_1$ is the total extent of the thin sections. According to Eq. (32), the ratio of the flux in the thin sections of the wall to that in the thick sections is

$$\left|\frac{\Psi_{w2}}{\Psi_{w1}}\right| = \frac{2m + \gamma\tau_{w_1}}{2m + \gamma\tau_{w_2}}\ . \tag{54}$$

In the limit $\tau_{w_2} \ll \tau_{w_1}$, Eq. (53) possesses the following asymptotic solutions:

$$\gamma\tau_{w_1} \simeq \frac{E_{ww}^{m/n}}{1 - f_2(1 + E_{ww}^{m/n}/2m)}\ , \tag{55a}$$

$$\left|\frac{\Psi_{w2}}{\Psi_{w1}}\right| \simeq 1 + \frac{\gamma\tau_{w_1}}{2m}\ , \tag{55b}$$

for $E_{ww}^{m/n} \ll 2m\,(1/f_2 - 1)$, and

$$\gamma\tau_{w_2} \simeq f_2 E_{ww}^{m/n}\ , \tag{56a}$$

$$\left|\frac{\Psi_{w2}}{\Psi_{w1}}\right| \simeq \frac{\tau_{w_1}}{\tau_{w_2}}\ , \tag{56b}$$

for $E_{ww}^{m/n} \gg 2m\,(1/f_2 - 1)$. It can be seen, by comparison with the results of Sec. IV.A.1, that for a stable or moderately unstable mode the thin sections of the wall act rather like gaps. However, for a very unstable mode the finite conductivity of the thin sections limits the accumulation of magnetic flux there, which has the effect of limiting the mode growth rate. In fact, the mode is unable to evolve on a faster time scale than the time constant of the thin sections of the wall.

B The interaction with rotating tearing modes

Suppose that a single toroidal harmonic (mode number n, say) has a rational surface lying inside the plasma, and that the stability indices of the nonresonant harmonics do not differ appreciably from the vacuum value $-2m$. In the high frequency limit, $\omega\tau_{\rm w_1} \gg 1$ and $\omega\tau_{\rm w_2} \gg 1$, a similar analysis to that of Sec. IV.B yields:

$$\frac{\Psi_{\rm w}^{m/n}}{\Psi_{\rm s}^{m/n}} \simeq \frac{\rm i}{\omega}\left(\frac{f_2}{\tau_{\rm w_2}} + \frac{f_1}{\tau_{\rm w_1}}\right) E_{\rm sw}^{m/n}\,, \tag{57a}$$

$$\frac{\Psi_{\rm w_1}}{\Psi_{\rm s}^{m/n}} \simeq {\rm i}\,\frac{E_{\rm sw}^{m/n}}{\omega\tau_{\rm w_1}}\,, \tag{57b}$$

$$\frac{\Psi_{\rm w_2}}{\Psi_{\rm s}^{m/n}} \simeq {\rm i}\,\frac{E_{\rm sw}^{m/n}}{\omega\tau_{\rm w_2}}\,, \tag{57c}$$

$$r_{\rm s}\Delta_{\rm s}' \simeq E_{\rm ss}^{m/n}\,, \tag{57d}$$

$$\delta T_\phi(r_{\rm s}) \simeq \frac{2n\pi^2 R_0}{\mu_0}\,\frac{|\Psi_{\rm s}^{m/n}|^2 (E_{\rm sw}^{m/n})^2}{\omega}\left(\frac{f_2}{\tau_{\rm w_2}} + \frac{f_1}{\tau_{\rm w_1}}\right). \tag{57e}$$

The above results indicate that a modular wall is able to shield the flux due to a rapidly rotating tearing island from the region beyond the wall. Equations (57b) and (57c) show that the residual flux tends to concentrate in the thin sections of the wall. According to Eq. (57d), the tearing mode stability index asymptotes to the fixed boundary value. Finally, Eq. (57c) shows that the slowing torque exerted on a rotating island is the same as that exerted by a uniform wall with the same average resistance as the modular wall.

VI Summary and Discussion

In Sec. II a general method is developed for investigating the influence of a nonuniform resistive wall on the stability of MHD modes. The analysis is performed in the cylindrical tokamak limit, assuming that the wall lies in the 'thin-shell' regime. The dispersion relation for wall modes (i.e. modes which do not reconnect magnetic flux inside the plasma) reduces to a matrix eigenvalue problem [see Eqs. (6) and (10)]. The eigenvalue determines the mode growth rate, and the eigenvectors determine the Fourier harmonics of either the magnetic flux in the wall or the stream function of the wall eddy current. In general, the dispersion relation requires numerical solution. However, an analytic solution is obtainable for the special case of a toroidally nonuniform wall where the variation scale length of the resistance is much larger than the poloidal spacing of the eddy current vortices (see Sec. III).

Consider the stability of a set of coupled wall modes with common poloidal mode number m. The MHD free energy associated with each mode is parameterized by a wall stability index $E_{\rm ww}^{m/n}$ (where n is the toroidal mode number — see Sec. II.D). In a general tokamak plasma, the stability indices of most modes lie close to the vacuum value, $-2m$. Suppose that only one mode (toroidal mode number n, say) has a stability index which differs appreciably from the vacuum value. In this situation, the wall mode dispersion relation for a toroidally nonuniform wall satisfying the constraint (12) reduces to the particularly simple form

$$\oint \frac{2m + E_{\rm ww}^{m/n}}{2m + \gamma\tau_{\rm w}(\phi)}\,\frac{d\phi}{2\pi} = 1\;, \tag{58}$$

and the perturbed poloidal flux at the wall radius is given by

$$\Psi_{\rm w}(\theta,\phi) \propto \frac{\exp[{\rm i}\,(m\theta - n\phi)]}{2m + \gamma\tau_{\rm w}(\phi)} \tag{59}$$

[see Eqs. (29)–(32)]. Here, γ is the growth rate, $\tau_{\rm w}(\phi)$ is the toroidally varying wall time constant, and $E_{\rm ww}^{m/n}$ is the 'special' stability index. If there are N modes with 'special' stability indices, then the wall mode dispersion relation takes the form of an $N \times N$ matrix equation [see Eq. (29)].

Toroidal gaps are incorporated into tokamak vacuum vessels in order to suppress eddy currents and thereby reduce the penetration time scale for the vertical magnetic field. However, *helical* eddy currents are not suppressed because, unlike the nonhelical (i.e. $m/n = 0/0$) current, they are able to turn around before reaching the gaps (see Fig. 2). Of course, large gaps attenuate helical eddy currents to some extent because they reduce the effective area of the wall.

The influence of toroidal gaps on wall mode stability is investigated in Sec. IV.A. For the case where only one mode has a stability index which differs appreciably from the vacuum value, it is found that if the mode is stable the perturbed poloidal flux tends to concentrate in the metal sections of the wall, if the mode is marginally stable the flux becomes evenly distributed between the metal and gap sections, and if the mode is unstable the flux tends to concentrate in the toroidal gaps. In fact, if the stability index exceeds a critical value [see Eq. (38)] the flux becomes entirely concentrated in the gap regions, and the mode can then 'explode' through the gaps with an ideal growth rate. If more than one mode possesses a stability index which differs from the vacuum value, then the critical stability index needed for the ideal growth of a given mode is reduced (see Sec. IV.A.2). Finally, a wall possessing toroidal gaps is unable to completely shield the flux of a rapidly rotating tearing island from the region beyond the wall because the flux is able to 'squeeze' through the gaps (see Sec. IV.B).

The analytical results obtained in this paper are only valid under an extremely restricted set of circumstances (i.e. for toroidally nonuniform walls where the width of any gap or metal sections is much greater than $2\pi\, r_{\rm w}/m$; $r_{\rm w}$ is the wall minor radius). Nevertheless, an important insight has been gained into the physics of the interaction of MHD modes with a nonuniform wall. It is found that nonuniform wall eddy currents enable unstable modes to transfer some of their free energy to stable modes of different helicities, so as to build up a mode structure which minimizes the induced eddy currents. For a wall possessing vacuum gaps, a sufficiently unstable mode can modify its structure so as to induce no eddy currents at all in the wall. At this point the wall clearly loses any moderating influence on the mode growth rate. It follows that the structure of an unstable mode interacting with a nonuniform wall cannot be assumed *a priori*, but must instead be solved for in a self consistent manner along with the growth rate. This conclusion has obvious implications for the design of tokamaks with incomplete stabilizing shells. There are also ramifications for the design of MHD feedback systems, since an incomplete resistive wall is equivalent to a low gain feedback system with finite coils. It seems clear that a sufficiently unstable mode can completely defeat a feedback system by 'squeezing' between the coils. This is not a 'phase instability', but rather a 'spectrum instability' in which the mode changes its structure under the influence of the applied feedback signals. This effect cannot easily be counteracted, since it is a function of the number and extent of the feedback coils rather than the strength or phasing of the feedback signals.

In conclusion, the investigation of the influence of a wall with toroidal structure on tokamak stability has yielded a number of useful and interesting results. The techniques developed in this paper can be extended to deal with the intrinsically more difficult problems of the influence of a wall with poloidal structure on tokamak stability and MHD feedback.

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Figure Captions

1. Wall eddy current vortices with 'square' ends.

2. Realistic wall eddy current vortices.

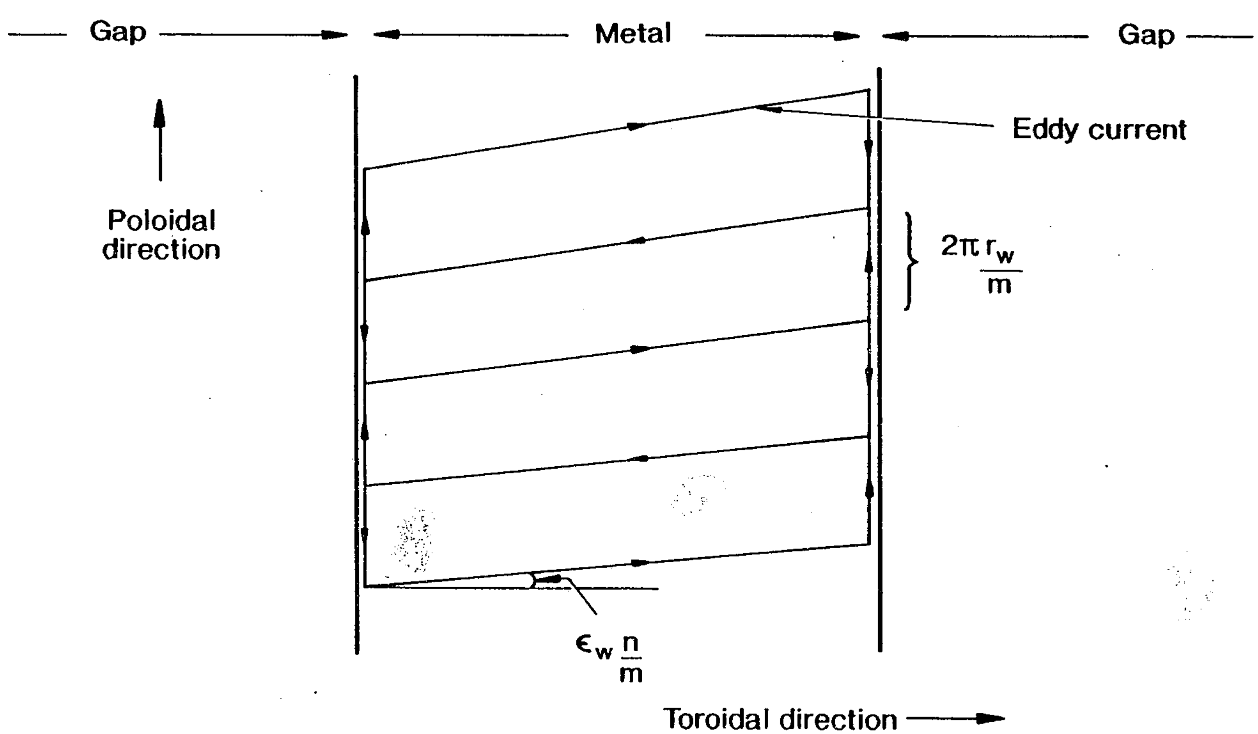
Gap
Metal
Gap
Eddy current
Poloidal direction
$2\pi r_w / m$
$\epsilon_w n / m$
Toroidal direction

Fig. 1.

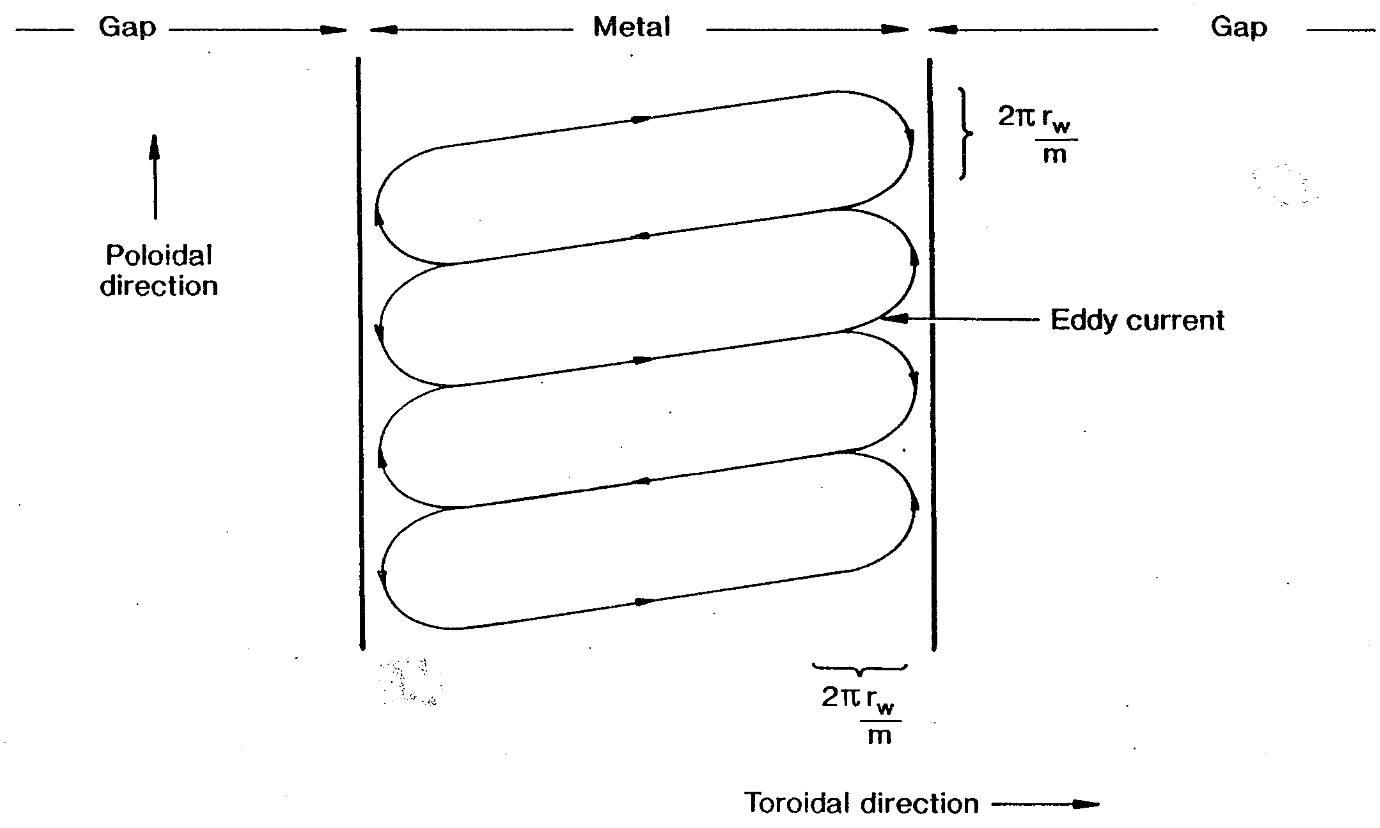
Gap
Metal
Gap
Poloidal direction
$2\pi \frac{r_w}{m}$
Eddy current
$2\pi \frac{r_w}{m}$
Toroidal direction

Fig. 2.