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The Shear-Alfvén Harmonic Chain Vortex of Current-Carrying Low-β Plasma Cylinder

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Abstract

Nonlinear shear-Alfvén mode in low-β ($\beta \ll m_e/m_i$) current-carrying plasma cylinder is studied by using two-fluid model. A new type of solitary vortex solution is given. This type of solution consists of monopole and multipole parts, along the azimuthal direction it forms a chain of interwoven cyclones and anticyclones, while the structure globally propagates in the azimuthal direction of plasma with constant angular velocity. Comparing with related structures previously obtained for other nonlinear modes, the equilibrium current of plasma affects the vortex structure. The fact that the radial size of the vortex is comparable to the radius of plasma cylinder may provide some convenience for experimental observation of this coherent structure.

I. Introduction

Starting from the beginning of the last decade, the two-dimensional coherent vortex structure in a magnetized plasma attracted considerable attention and has been widely studied theoretically. The vortex solutions have been constructed for different low frequency nonlinear electrostatic and electromagnetic modes in magnetic confined plasma such as the electrostatic convection cell,[1] the drift vortex,[2,3] exchange flute vortex,[4] the Alfvén vortex,[5,6] the ballooning vortex[7] and the electrostatic and electromagnetic vortex in rotating plasma,[8] only to mention a few. However, besides indirect observation of the drift vortex in rotating neutral fluids[9] there are very few experimental observations on these structures in plasma.[10] Among other reasons which hinder experimentally observation of these vortex structures, two facts may should be mentioned. Firstly, the previous vortex solutions were obtained from the nonlinear equations in which some important physical effects are more or less overlooked, for instance, the combined effects of density gradient and temperature gradient, the kinetic effect, and the magnetic shear effect were not taken into account. Secondly, for all above mentioned vortex solutions, their location in plasma and the size of their core parts were not well defined.

More recently, the theoretical study of vortex in plasma shows noticeable progress in efforts to attack the two problems just mentioned. In the first direction, for example, several analytical and numerical works have discussed drift vortex with the effects of temperature gradient,[11] drift vortex in the plasma with sheared magnetic fields[12] and shear flow,[13] which elucidated the conditions of the existence of the dipole vortex and the relation between the dipole and the monopole vortex. Also the effect of resonant particles on the plasma vortices was discussed.[14] In the second direction, to consider the fact that the laboratory plasmas are of finite size and approximately with cylindrical configuration, the study of global vortex structure has been carried out. Among many works in this direction we may mention the early work on the nonlinear electrostatic drift mode in finite plasma cylinder[15] and recent works of the global vortex structures of nonlinear electrostatic drift mode,[16] exchange mode,[17] and drift-Alfvén mode[18] in rotating plasma cylinder, multipole vortices in electron beams[19] and others.

In this work, following the second direction, we study the possibility of global coherent structure of nonlinear shear-Alfvén mode in low β plasma cylinder. By assuming there is an equilibrium axial velocity for the electron component we consider that the plasma is current-carrying. Using the two-fluid model a set of nonlinear equations which describes shear-Alfvén mode is derived and the global vortex solution is obtained. In contrast to the structure of Larichev-Reznik type vortex,[20] this type of solution consists of monopole and multipole parts, along the azimuthal direction it forms a chain of interwoven cyclones and anticyclones, while the structure globally propagates in the azimuthal direction of plasma with constant angular velocity. Comparing with other related structures previously obtained,[15,16,17,18] the equilibrium current of plasma affects the vortex structure. Specifically, it is the monopole part of the vortex structure depends on the equilibrium current while the multipole part of the solution remains unchanged. The fact that the radial size of the vortex is comparable to the radius of plasma cylinder may provide some convenience for experimental observation.

The article is organized as follows. In Sec. 2 the set of nonlinear equations describing the shear-Alfvén is derived. In Sec. 3 we construct the coherent localized solution of the equation and in the last section we give a brief summary.

II. Derivation of the Nonlinear Equations

Suppose a homogeneous plasma cylinder with radius R is immersed into a strong constant axial magnetic field $\mathbf{B} = B_0\hat{\mathbf{e}}_z$, where $\hat{\mathbf{e}}_z$ is the unit vector in axial direction. Also we suppose there is an equilibrium motion of the electron component of the plasma relative to the ion component, therefore the plasma is current carrying. To avoid the complexity of the equilibrium magnetic field we further require the equilibrium velocity $\mathbf{V}_0 = V_0\hat{\mathbf{e}}_z$ is slow enough that the magnitude of the azimuthal magnetic field B_θ generated by it is much smaller than B_0. We also assume that β of the plasma is in the regime $\beta \ll m_e/m_i$, where m_e, m_i are the mass of electron and ion, respectively.

For the plasma in above assumed β regime, it is well known that the inertia term in the electron momentum equation is much important than the thermal pressure term. Therefore, in two-fluid model the equations describing shear Alfvén wave are the quasineutrality condition and the longitudinal electron momentum equation[6,21]:

$$\nabla_\perp \cdot \mathbf{j}_\perp + \partial_z j_{\|} = 0 \; , \tag{1}$$

$$\partial V_{ez} + (V_0 + V_{ez})\partial_z V_{e\perp} + \mathbf{V}_{e\perp} \cdot \nabla V_{ez} = -\frac{e}{m_e} E_z - \frac{e}{m_e c} (\mathbf{V}_{e\perp} \times \mathbf{B}) \cdot \hat{\mathbf{e}}_z \; , \tag{2}$$

where $\mathbf{j}_\perp$, $j_{\|}$ are perpendicular and parallel components of the current, e is electron charge, c is the speed of light.

To describe the shear-Alfvén wave in low-β plasma one can neglect the compressible component of the perturbed magnetic field, using the perturbed electromagnetic scalar potential $\phi(\mathbf{r}, t)$ and parallel component of vector potential $A_z(\mathbf{r}, t)$ the perturbed electromagnetic fields are expressed as

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\,\partial_t A_z \hat{\mathbf{e}}_z \; , \tag{3}$$

$$\mathbf{B}_\perp = \nabla A_z \times \hat{\mathbf{e}}_z \; . \tag{4}$$

Since the equilibrium density of plasma is homogeneous, the density perturbation is not involved. In drift approximation, i.e., $\partial_t \ll \omega_{ci} = eB_0/m_i c$, the perpendicular components of perturbed velocity for electron and ion are

$$\mathbf{V}_{e\perp} = \mathbf{V}_E + V_0 \frac{\mathbf{B}_\perp}{B_0} + V_{ez} \frac{\mathbf{B}_\perp}{B_0} \; , \tag{5}$$

$$\mathbf{V}_{i\perp} = \mathbf{V}_E + \mathbf{V}_p \ , \tag{6}$$

where

$$\mathbf{V}_E = \frac{c}{B_0} \hat{\mathbf{e}}_z \times \nabla\phi \ , \tag{7}$$

$$\mathbf{V}_p = -\frac{c}{B_o \omega_{ci}} (\partial_t + \mathbf{V}_E \cdot \nabla)\nabla\phi \ , \tag{8}$$

are $\mathbf{E} \times \mathbf{B}$ drift velocity and polarization drift velocity, respectively. In the right-hand side of Eq. (5) the second term represents the contribution to perturbed electron velocity from coupling of equilibrium velocity with perturbed magnetic field while the third term expresses the nonlinear coupling between longitudinal perturbed electron motion and perturbed magnetic field. Neglecting parallel motion of ion and displacement current, the longitudinal component of Ampere's law gives the relation between A_z and parallel component of perturbed velocity of electron as

$$V_{ez} = \frac{c}{4\pi e n_0} \nabla_\perp^2 A_z \ . \tag{9}$$

Substituting Eqs. (3)–(9) into Eqs. (1)–(2), and assuming that $\mathbf{V}_{e\perp} \cdot \nabla \gg V_{ez}\partial_z$, a set of coupled nonlinear equations describing the shear-Alfvén wave is obtained in the dimensionless form as

$$\partial_t \nabla_\perp^2 \Phi + \partial_z \nabla_\perp^2 \Psi = [\Psi,\ \nabla_\perp^2 \Psi] - [\Phi,\ \nabla_\perp^2 \Phi] \ , \tag{10}$$

$$(1 - \lambda^2 \nabla_\perp^2)\partial_t \Psi + \partial_z \Phi - \lambda^2 v_0 \partial_z \nabla_\perp^2 \Psi = [\Psi,\ \Phi] + \lambda^2 [\Phi - v_0 \Psi\ ,\ \nabla_\perp^2 \Psi] \ , \tag{11}$$

where

$$[f,\ g] = \hat{\mathbf{e}}_z \cdot (\nabla f \times \nabla g)$$

and

$$\Phi \equiv \frac{e\phi}{T_e}\ ,\ \Psi \equiv \frac{v_A}{c}\frac{eA_z}{T_e}\ ,\ v_0 = \frac{V_0}{v_A}\ ,\ v_A = \sqrt{\frac{B_0^2}{4\pi m_i n_0}}\ (\text{Alfvén speed})\ ,$$

$$\lambda = \frac{\lambda_s}{\rho_s} \ , \ \rho_s = \frac{c_s}{\omega_{ci}} \ , \ \lambda_s = \frac{c}{\omega_{pe}} \ \text{(electron collisionless skin depth)} \ ,$$

$$c_s = \sqrt{\frac{T_e}{m_i}} \ \text{(ion acoustic speed)} \ , \ \omega_{pe} = \sqrt{\frac{4\pi e^2 n}{m_e}} \ \text{(electron plasma frequency)} \ .$$

Also variable r, z, t are normalized as $r \to r/\rho_s$, $z \to z/(c/\omega_{pi})$, $t \to t/(1/\omega_{ci})$, where $\omega_{pi} = \sqrt{\frac{4\pi e^2 n_0}{m_i}}$. Equations (10) and (11) compose the set of nonlinear equations describing the nonlinear shear-Alfvén waves in current-carrying plasma. When $v_0 = 0$ the equations reduced to the same equations for plasma without equilibrium current.[5,21]

III. Stationary Coherent Structures

In this section we seek the two-dimensional localized solution of Eqs. (10)–(11) which propagates with angular speed ω in the azimuthal direction. Suppose the perturbed potential functions Φ, Ψ are in the form of traveling wave, i.e.,

$$\Phi(r,\theta,t) = \Phi(r,\eta) = \Phi(r,\theta - \Omega t + \alpha z) \ , \quad \Psi(r,\theta,t) = \Psi(r,\eta) = \Psi(r,\theta - \Omega t + \alpha z) \ ,$$

where α is the slanted angle between the wave front and the $r-\theta$ plane. Then with the transformation that $\partial_\theta = \partial_\eta$, $\partial_t = -\Omega \partial_\eta$, $\partial_z = \alpha \partial_\eta$ in the $r-\eta$ plane Eqs. (10)–(11) can be rewritten as

$$[\tilde{\Phi}, \quad \nabla^2\Phi] - [\tilde{\Psi}, \quad \nabla^2\Psi] = 0 \ , [\tilde{\Phi}, \quad \tilde{\Psi} - \lambda^2\nabla^2\Psi] + v_0\lambda^2[\tilde{\Psi}, \quad \nabla^2\Psi] = 0 \ ,$$

where

$$\tilde{\Phi} = \Phi - \frac{1}{2}\Omega r^2, \quad \tilde{\Psi} = \Psi - \frac{1}{2}\alpha r^2 \ .$$

By using the property of Poisson bracket that if $[f, \ g] = 0$, then $f = f(g)$, where $f(g)$ is the arbitrary function of g, the nonlinear equations (12)–(13) can be integrated to give

$$\nabla^2\Phi = f_1(\tilde{\Phi}) - b_1\tilde{\Psi} + c_1 \ , \tag{12}$$

$$\nabla^2\Psi = b_1\tilde{\Phi} - \frac{1 - b_1 v_0 \lambda^2}{\lambda^2}\tilde{\Psi} + c_2 \ , \tag{13}$$

where $f_1(\widetilde{\Phi})$ is arbitrary continuous function of $\widetilde{\Phi}$. For simplicity we choose a linear function which leads to

$$\nabla^2\Phi = a_1\Phi - b_1\Psi + \frac{d_2}{2}r^2 + c_1' \ , \tag{14}$$

$$\nabla^2\Psi = b_1\Phi + d_1\Psi + \frac{d_3}{2}r^2 + c_2' \ , \tag{15}$$

where

$$d_1 = \frac{1 - b_1 v_0 \lambda^2}{\lambda^2} \ ,$$

$$d_2 = \alpha b_1 - \Omega a_1 \ ,$$

$$d_3 = -\left[b_1\Omega + \frac{(1 - b_1 v_0 \lambda^2)}{\lambda^2}\right] \ ,$$

and a_1, b_1, c_1', c_2' are constants to be determined.

The general solutions of the linear partial differential equations (15)–(16) are

$$\Phi(r,\eta) = \Phi_m^{(0)}[J_m(p_1 r) + J_m(p_2 r)]\cos(m\eta) + \frac{\alpha_3}{2\alpha_2}r^2 + \frac{2\alpha_1\alpha_3 - \alpha_2\alpha_4}{\alpha_2^2} \ , \tag{16}$$

$$\Psi(r,\eta) = \frac{1}{b_1}\left\{\Psi_m^{(0)}[(a_1 + p_1^2)J_m(p_1 r) + (a_1 + p_2^2)J_m(p_2 r)]\cos(m\eta) + \frac{1}{2}\left(d_2 + \frac{a_1 d_3}{\alpha_2}\right)r^2 \right.$$

$$\left. + \frac{1}{\alpha_2^2}[(2\alpha_1\alpha_3 + \alpha_2\alpha_4)a_1 - 2\alpha_2\alpha_3] + c_1'\right\} \ , \tag{17}$$

where

$$p_1 = \frac{1}{2}\left(-\alpha_1 + \sqrt{\alpha_1^2 - 4\alpha_2}\right) \ ,$$

$$p_2 = \frac{1}{2}\left(-\alpha_1 - \sqrt{\alpha_1^2 - 4\alpha_2}\right) \ ,$$

$$\alpha_1 = a_1 + d_1 \ ,$$

$$\alpha_2 = b_1^2 + a_1 d_1 \ ,$$

$$\alpha_3 = d_1 d_2 + b_1 d_3 \ ,$$

$$\alpha_4 = c_1' d_1 + c_2' b_1 - 2d_2 \ ,$$

$J_m(x)$ is the m-th order Bessel function ($m = 1, 2, \cdots$, positive integers). In solutions (18)–(19) we already assumed that

$$\alpha_1 < 0 \ , \qquad \alpha_1^2 \geq 4\alpha_2 \ . \tag{18}$$

From the requirement that at the boundary of plasma cylinder there should no plasma radial motion, the boundary conditions for Φ and Ψ at $r = R$ can be set as[15]

$$\partial_\eta \Phi(r, \eta)\Big|_{r=R} = 0 \ , \tag{19}$$

$$\Phi(r, \eta)\Big|_{r=R} = \Psi(r, \eta)\Big|_{r=R} = 0 \ . \tag{20}$$

It is obvious that boundary condition Eq. (20) is satisfied as long as $p_1 R$ and $p_2 R$ are zeros of $J_m(x)$. For simplicity we suppose that

$$p_1 R = p_2 R = \gamma_{m,n} \ , \tag{21}$$

where $\gamma_{m,n}$ is the n-th zero of $J_m(x)$. This leads to

$$p_1 = p_2 = \frac{\gamma_{m,n}}{R} \ . \tag{22}$$

Considering the relation between $p_{1,2}$, $\alpha_{1,2}$ and a_1, b_1 given above, the parameters a_1 and b_1 are determined as

$$a_1 = -\frac{1}{1 \pm v_0}\left(\frac{1}{\lambda^2} + \frac{\gamma_{m,n}^2}{R^2}\right) - \frac{\gamma_{m,n}^2}{R^2} \ , \tag{23}$$

$$b_1 = \frac{1}{1 \pm v_0}\left(\frac{1}{\lambda^2} + \frac{\gamma_{m,n}^2}{R^2}\right) \ . \tag{24}$$

c_1', c_2' are determined by Eq. (20), which gives

$$c_1' = 2\frac{d_1 d_2 + b_1 d_3}{b_1^2 + a_1 d_1} - \frac{d_2 R^2}{2} , \tag{25}$$

$$c_2' = \frac{2d_2}{b_1}\left[1 - \frac{(a_1 + 2d_1)(d_1 d_2 + b_1 d_3)}{d_2(b_1^2 + a_1 d_1)}\right] - \frac{d_3 R^2}{2} . \tag{26}$$

Substitute (24)–(27) into Eqs. (18)–(19), finally we find the solution of nonlinear Eqs. (10)–(11) are

$$\Phi(r,\eta) = \Phi_m^{(0)} J_m\left(\frac{\gamma_{m,n} r}{R}\right)\cos(m\eta) + \Phi_m^{(1)}(r^2 - R^2) , \tag{27}$$

$$\Psi(r,\eta) = \Psi_m^{(0)} J_m\left(\frac{\gamma_{m,n} r}{R}\right)\cos(m\eta) + \Psi_m^{(1)}(r^2 - R^2) , \tag{28}$$

where

$$\Psi_m^{(0)} = \frac{a_1 R^2 + \gamma_{m,n}^2}{b_1 R^2}\,\Phi_m(0) , \tag{29}$$

$$\Phi_m^{(1)} = \frac{1}{2}\,\frac{d_1 d_2 + b_1 d_3}{b_1^2 + a_1 d_1} , \tag{30}$$

$$\Psi_m^{(1)} = \frac{b_1(a_1 d_3 + b_1 d_2) + 2a_1 d_1 d_2}{b_1(b_1^2 + a_1 d_1)} , \tag{31}$$

From the expression of the solutions one can see that this kind of structure consists of two parts, the azimuthal angle dependent $2m$-pole vortex part and monopole part which is only r dependent. Since parameters d_1 and d_3 depend on the equilibrium electron velocity v_0, the expressions (28)–(32) indicate that this velocity only affect the structure of the monopole, while the global structure of the solution depends on azimuthal mode number m and the choice of zeros of $J_m(x)$. Fig. 1 gives the illustration of the solution (28).

IV. Summary

In this work we studied the nonlinear shear Alfvén wave in low β, current carrying plasma cylinder and the global harmonic multipole vortex solution is obtained. This type of solution consists of monopole and multipole parts, along the azimuthal direction it forms a chain interwoven with cyclone and anticyclone, while the structure globally propagates in the azimuthal direction of plasma with constant angular velocity. Compared with similar structures previously obtained for other nonlinear modes, the equilibrium current of plasma affects both the vortex structure. The fact that the radial size of the vortex is comparable to the radius of plasma cylinder may provide some convenience for experimental observation of this coherent structure.

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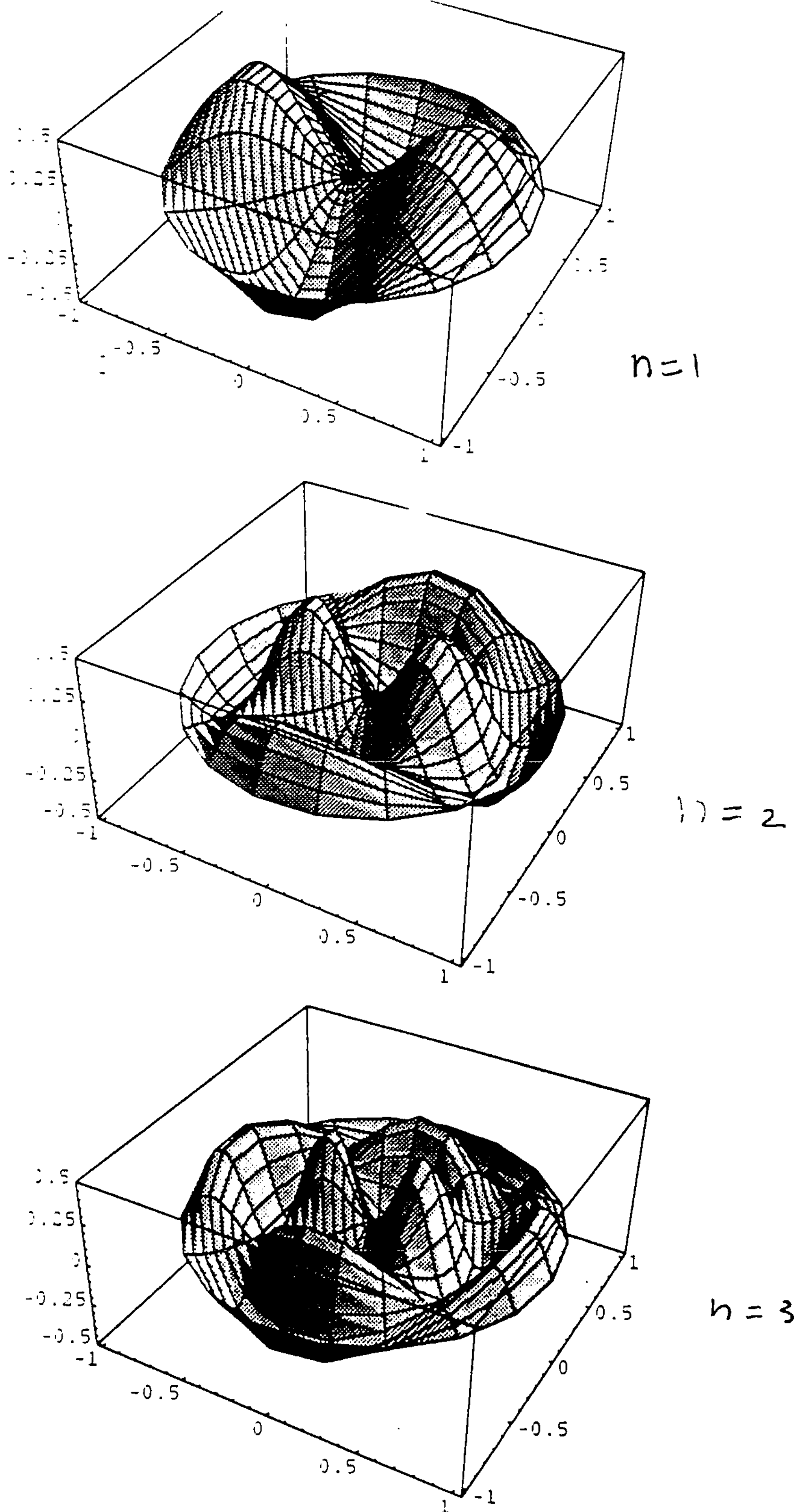
Figure Captions

1. a) Scheme of $\Phi_m/\Phi_m^{(0)}$ where $m = 1$, $n = 1, 2, 3, 4$, $\Phi_m^{(1)}\, R^2/\Phi_m^{(0)} = 0.1$.

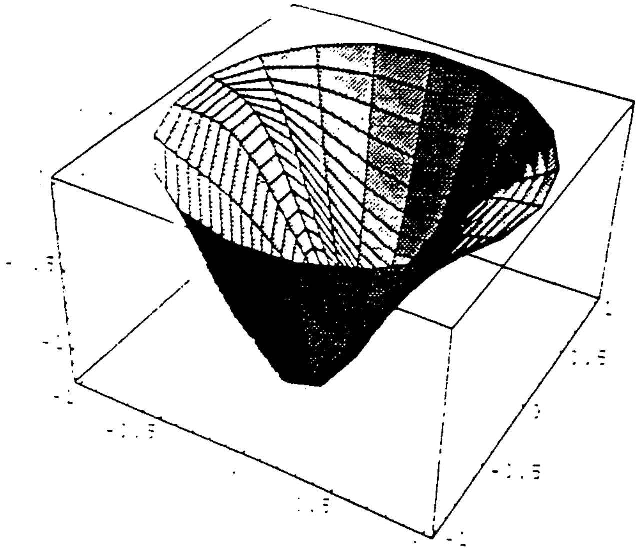
   b) Scheme of $\Phi_m/\Phi_m^{(0)}$ where $m = 2$, $n = 1, 2, 3$, $\Phi_m^{(1)}\, R^2/\Phi_m^{(0)} = 0.1$.

   c) Scheme of $\Phi_m/\Phi_m^{(0)}$ where $m =$, $n = 1, 2, 3$, $\Phi_m^{(1)}\, R^2/\Phi_m^{(0)} = 0.1$.



• **Figure 1. 3D plot of $\Phi_m/\Phi_m^{(0)}$ for different value of $\Phi_m^{(1)}/\Phi_m^{(0)}$.**

[Fig.1a]: $\Phi_m^{(1)}/\Phi_m^{(0)} = 0.01, m = 2,\ n = 1, 2, 3.$



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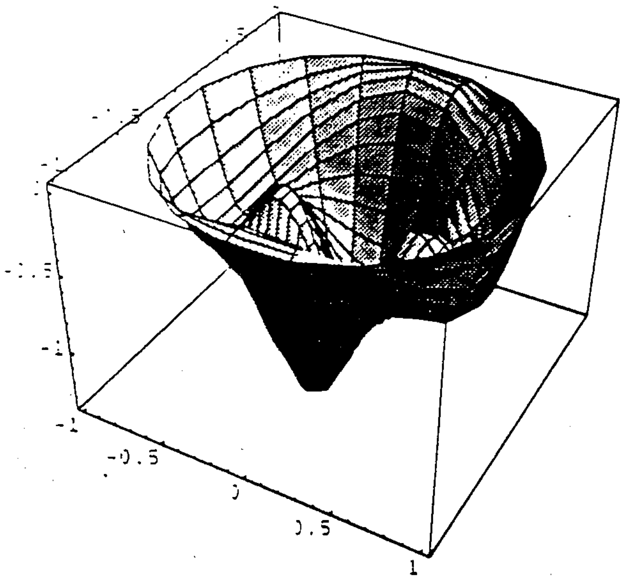
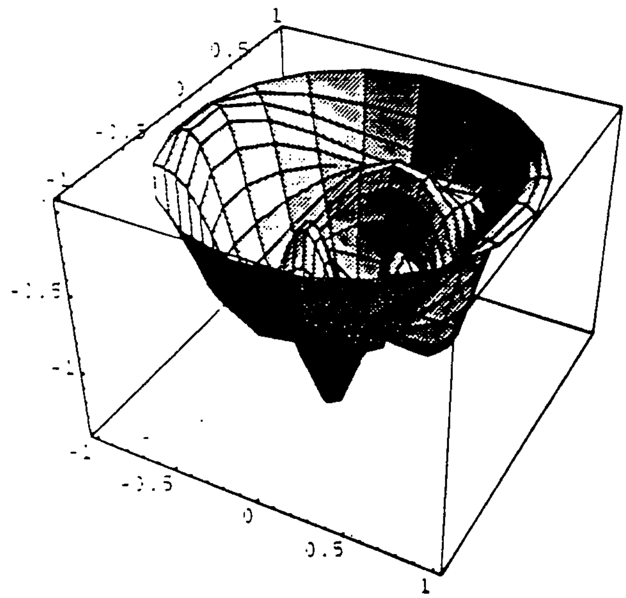


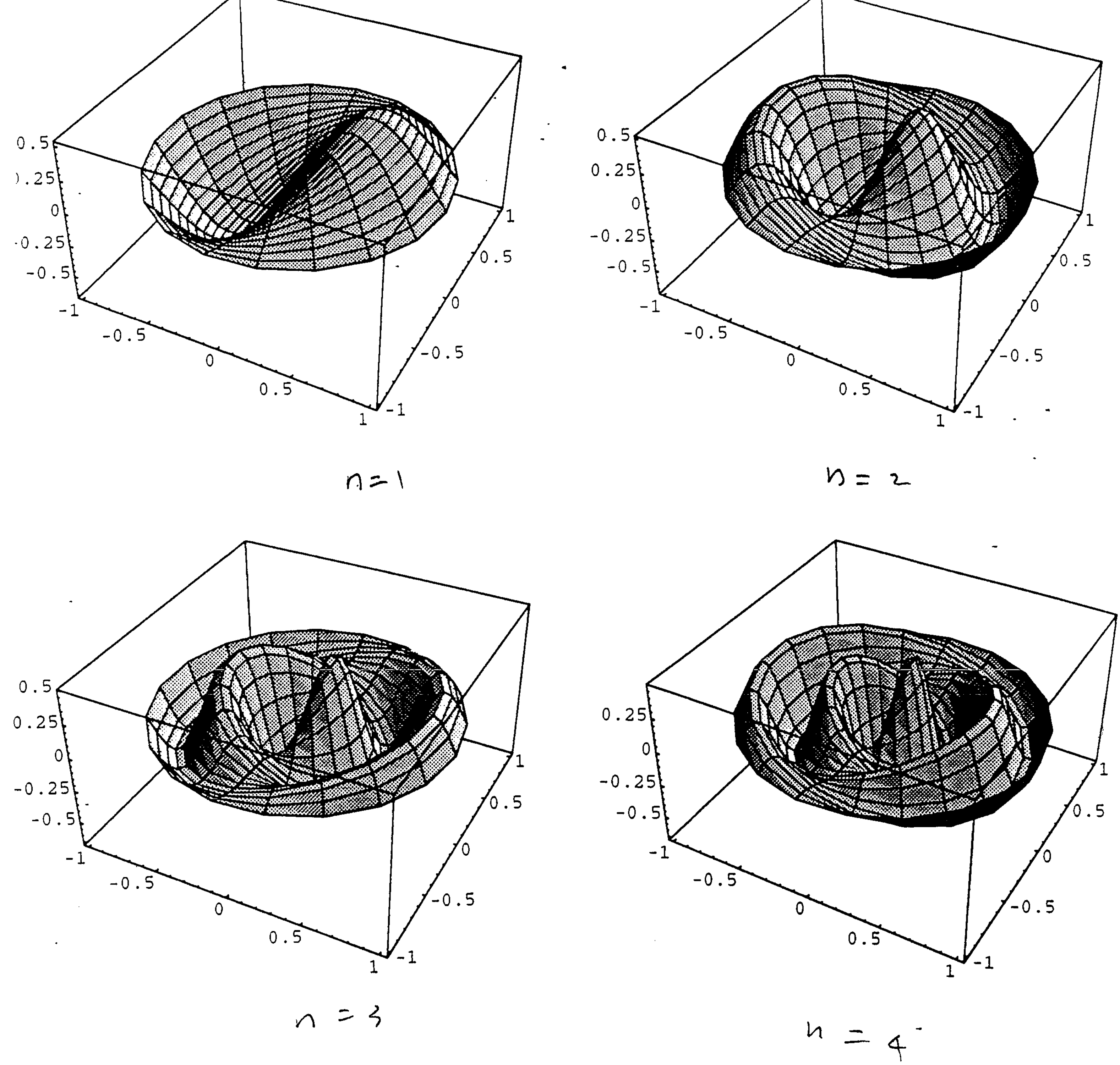
n=2



n=3

• **Figure 1. 3D plot of $\Phi_m/\Phi_m^{(0)}$ for different value of $\Phi_m^{(1)}/\Phi_m^{(0)}$.**

[Fig.1b]: $\Phi_m^{(1)}/\Phi_m^{(0)} = 1$, m and n are same as in Fig.1a



• Figure 2. 3D plot of $\Phi_m/\Phi_m^{(0)}$ for different $\gamma_{m,n}$.

$$\Phi_m^{(1)}/\Phi_m^{(0)} = 0.1,\ m = 1,\ n = 1, 2, 3, 4$$