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Abstract

A new method of analyzing the toroidicity-induced Alfvén eigenmode (TAE) from kinetic theory is presented. The analysis includes electron parallel dynamics non-perturbatively, an effect which is found to strongly influence the character and damping of the TAE - contrary to previous theoretical predictions. The normal electron Landau damping of the TAE is found to be higher than previously expected, and may explain recent experimental measurements of the TAE damping coefficient.

Toroidicity-induced Alfvén eigenmodes (TAE) are currently of great interest because they may destroy the confinement of fast ions in a burning tokamak plasma.[1-7] Their excitation depends critically on the difference between the growth rate due to the fast ions and the damping rate, mainly due to electrons. Past theories have predicted a very low Landau damping for the TAE,[2,3] and have determined the dominant form of damping of the TAE to be Landau damping due to the magnetic curvature drift of the electrons.[2] Perhaps stimulated by recent Tokamak Fusion Test Reactor (TFTR) results[4] showing a higher excitation threshold than expected, recent theoretical studies have focused on alternate damping mechanisms, such as continuum damping,[5,6] and trapped electron effects.[7] In contrast, the present study attempts to demonstrate that a non-perturbative treatment of electron parallel dynamics (the source of normal Landau damping) yields intrinsic Landau damping of the TAE significantly higher than originally thought. The theory also points out interesting relationships between the TAE and the global Alfvén eigenmode (GAE).

Damping calculations, i.e. analytical estimates and numerical values obtained by direct integration of the basic equations, are in good agreement for a wide range of parameters including those relevant for the TFTR TAE experiment. In the latter case the calculated damping is quite close to the experimental estimate. Numerical results indicate that there is only one mode when no kinetic effects are present. It is found that equilibrium current (essential for the GAE[8-10]) and toroidal coupling are both essential for the formation of this "MHD" (magnetohydrodynamic) TAE. Kinetic effects alter the mode structure of the MHD TAE, shift its frequency, and cause it to damp. Kinetic effects also introduce a countable infinity of new modes (also like the GAE[8]), one of which may lie within the gap. Quite surprisingly, depending on the plasma parameters, we find that this "kinetic TAE" may have a lower damping coefficient than the MHD TAE. This is the case for the TFTR TAE experiment,[4] and for a TFTR deuterium-tritium (D-T) burning experiment (discussed subsequently). The modes further outside the gap generally have high damping coefficients. They correspond to the continuum, which has been discretized by the electron dynamics. Further such details will be given in a future publication.

We consider a TAE formed by the coupling between two poloidal harmonics m_1 and m_2. As our model, we use an equation describing Alfvén waves in an inhomogeneous, current carrying, cylindrical plasma, corrected by toroidal coupling to first order in inverse aspect ratio $\varepsilon(r) = r/R$,

$$\left[rK_1^2 \frac{d}{dr}\frac{\Delta_1}{rK_1^2}\frac{d}{dr}r - rK_1^2(\Delta_1 - G_1) + K_1 k_1 \left(\frac{d}{dr}r\frac{d}{dr} - rK_1^2\right)\frac{\sigma_1}{K_1 k_1}\left(\frac{1}{r^2}\frac{d}{dr}r\frac{d}{dr}r - K_1^2\right)\right]E_1$$
$$= -\left(\frac{d}{dr}\varepsilon\frac{\omega^2}{v_A^2}\frac{d}{dr} - K_2^2\varepsilon\frac{\omega^2}{v_A^2}\right)rE_2 \,. \tag{1}$$

In this equation, the poloidal electric field $E(r) \sim e^{i(m\theta+n\phi-\omega t)}$, the poloidal wavenumber $K = (q^2+\varepsilon^2)^{-1/2}(mq+\varepsilon^2 n)/r$, the parallel wavenumber $k = (q^2+\varepsilon^2)^{-1/2}(m-nq)/R$, while $\Delta = \omega^2/v_A^2 - k^2$, $G = (dA/dr)/K - A^2/K^2$, $\sigma = k^2\rho_s^2/[1+\zeta Z(\zeta)]$. In addition, $A = \varepsilon(2qk - \varepsilon K)/[r(q^2+\varepsilon^2)]$, $\rho_s = c_s/\omega_{ci}$, $\zeta = \omega/(|k|v_e)$, and Z is the plasma dispersion function. The coupled system is completed with the equation formed by $1 \leftrightarrow 2$. The left-hand side of this equation was derived in Ref. 8 and used to examine kinetic Alfvén waves (KAW) and GAE.[8–10] It stems from the well-known system of equations describing Alfvén waves derived in Ref. 11. Electron kinetics are described by the term containing σ, while G embodies the effect of equilibrium current ($G \sim \pm s$ where $s = dlnq/dlnr$). [To leading order in ε, $G = (dk^2/dr)/(rK^2)$.] We note that the toroidal coupling term on the right-hand side of Eq. (1) is the same as Eq. (30) of Ref. 1. Neglecting the kinetic term, Eq. (1), apart from the last term on the right-hand side, is the same as Eq. (35) of Ref. 6. Equation (1) may also be reduced to Eq. (2) of Ref. 5 under appropriate limits.

The essential features of the TAE may be obtained from Eq. (1) and its counterpart by expanding $\Delta(r)$ in powers of r about the position r_0 where $\Delta_1 = \Delta_2$. For simplicity we assume the radial variation in all other quantities (except E) about r_0 is unimportant. We take $\phi(r) = rE(r)$, $r = r_0 + x$, $\Delta_1 = \Delta - \alpha_1 x$, $\Delta_2 = \Delta + \alpha_2 x$, where $\alpha_1 = -d\Delta_1/dr|_{r_0}$, $\alpha_2 = d\Delta_2/dr|_{r_0}$ and obtain

$$\left[\frac{d}{dx}(\Delta - \alpha_2 x)\frac{d}{dx} - K_1^2(\Delta - \alpha_1 x - G_1) + \sigma\left(\frac{d^2}{dx^2} - K_1^2\right)^2\right]\phi_1 = -\varepsilon\frac{\omega^2}{v_A^2}\left(\frac{d^2}{dx^2} - K_2^2\right)\phi_2 \, . \tag{2}$$

The other equation has $1 \leftrightarrow 2$ and the opposite sign of α. Here, the quantities Δ, σ, ε, v_A, K_1, K_2, G_1, G_2 are all evaluated at $r = r_0$ and are therefore constants. (The subscripts were dropped on Δ and σ since $1 = 2$.)

Equation (2) and its counterpart are conveniently analyzed in Fourier space. Parseval's theorem implies that any function localized (square integrable) in x will also be localized in the conjugate Fourier variable. We take

$$\phi(x) = \int_{-\infty}^{\infty} dp\, \tilde{\phi}(p)e^{ipx} \tag{3a}$$

$$\tilde{\phi} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dx\, \phi(x)e^{-ipx} \tag{3b}$$

and Eq. (2) becomes

$$\left[\frac{d}{dp} + \frac{i\Delta}{\alpha_1} + \frac{p}{p^2 + K_1^2} - \frac{iK_1^2 G_1}{\alpha_1(p^2 + K_1^2)} - \frac{i\sigma}{\alpha_1}(p^2 + K_1^2)\right]\tilde{\phi}_1 = -\frac{i\varepsilon}{\alpha_1}\frac{\omega^2}{v_A^2}\frac{p^2 + K_2^2}{p^2 + K_1^2}\tilde{\phi}_2 \, . \tag{4}$$

This equation and its counterpart may be symmetrized by defining the new functions

$$\psi_1 = \tilde{\phi}_1\left[\alpha_1 e^{i\eta}(p^2 + K_1^2)\right]^{1/2} \tag{5a}$$

$$\psi_2 = \tilde{\phi}_2\left[\alpha_2 e^{i\eta}(p^2 + K_2^2)\right]^{1/2} \tag{5b}$$

where $\eta = (\alpha_1^{-1} - \alpha_2^{-1})(\Delta p - \frac{1}{3}\sigma p^3) - \sigma(K_1^2/\alpha_1 - K_2^2/\alpha_2)p - K_1 G_1 \alpha_1^{-1}\mathrm{atan}(p/K_1) + K_2 G_2 \alpha_2^{-1}\mathrm{atan}(p/K_2)$. Then Eq. (4) and its counterpart become

$$\left[\frac{d}{dy} + ih(y)\right]\psi_1 = -\hat{\varepsilon}\psi_2 f(y)\ , \tag{6a}$$

$$\left[\frac{d}{dy} - ih(y)\right]\psi_2 = \frac{i\hat{\varepsilon}\psi_1}{f(y)}\ , \tag{6b}$$

where we have introduced the quantities: $y = p/\kappa$, $\kappa = [\alpha(K_1^2/\alpha_1 + K_2^2/\alpha_2)]^{1/2}$, $\alpha = (\alpha_1^{-1} + \alpha_2^{-1})^{-1}$, $h(y) = \widehat{\Delta} - \widehat{G}(y) - \widehat{\sigma}(y^2+1)$, $\widehat{\Delta} = \frac{1}{2}\kappa\Delta/\alpha$, $\widehat{G}(y) = \frac{1}{2}[\widehat{G}_1/(1+y^2/y_1^2) + \widehat{G}_2/(1+y^2/y_2^2)]$, $\widehat{G}_1 = \kappa G_1/\alpha_1$, $\widehat{G}_2 = \kappa G_2/\alpha_2$, $y_1 = K_1/\kappa$, $y_2 = K_2/\kappa$, $\widehat{\sigma} = \frac{1}{2}\sigma\kappa^3/\alpha$, $\widehat{\varepsilon} = \varepsilon\kappa(\alpha_1\alpha_2)^{-1/2}\omega^2/v_A^2$, $f(y) = [(y^2 + y_2^2)/(y^2 + y_1^2)]^{1/2}$. We have reduced the TAE problem to a coupled pair of linear, first order, ordinary differential equations with the eigenvalue ω entering through $\widehat{\Delta}(\omega)$, $\widehat{\sigma}(\omega)$, and $\widehat{\varepsilon}(\omega)$. For the TAE, we expect $\widehat{\Delta}(\omega)$ to be small ($\widehat{\Delta} \sim \widehat{\varepsilon}$), and so it is a good approximation to put $\widehat{\sigma}(\omega) = \widehat{\sigma}(kv_A)$ and $\widehat{\varepsilon}(\omega) = \widehat{\varepsilon}(kv_A)$ and treat $\widehat{\Delta}$ as the eigenvalue. It is clear from Eqs. (6a) and (6b) that since $\widehat{\sigma}$ is small, it will influence only the high frequency components of the wave function, as expected.

Before solving Eqs. (6a) and (6b), it is illuminating to examine some properties of the system with $\widehat{\sigma} = 0$. Then these equations may be combined into

$$\left\{\frac{d}{dy} f \frac{d}{dy} + f\left[(\widehat{\Delta} - \widehat{G})^2 - \widehat{\varepsilon}^2\right] - i\frac{d}{dy}\left[f(\widehat{\Delta} - \widehat{G})\right]\right\}\psi_2 = 0 . \tag{7}$$

Since $f(y), \widehat{G}(y), \widehat{\varepsilon}$ are all real, we expect $\widehat{\Delta}$ to be real. Since $f(y) \to 1$ and $\widehat{G}(y) \to 0$ as $y \to \infty$, asymptotically Eq. (7) becomes

$$\left[\frac{d^2}{dy^2} + (\widehat{\Delta}^2 - \widehat{\varepsilon}^2)\right]\psi_2 = 0 . \tag{8}$$

Consequently for a bounded solution, $\widehat{\Delta}^2 - \widehat{\varepsilon}^2 < 0$. Taking the inner product $\langle\ \ \rangle = \int_{-\infty}^{\infty} dy$ of Eq. (7) with ψ_2^* and adding the result to its complex conjugate, we find

$$(\widehat{\Delta}^2 - \widehat{\varepsilon}^2)\left\langle f|\psi_2|^2\right\rangle = \left\langle f|d\psi_2/dy|^2\right\rangle - \left\langle f\widehat{G}(\widehat{G} - 2\widehat{\Delta})|\psi_2|^2\right\rangle . \tag{9}$$

Since $f(y)$ is positive for all y, this (virial-type) equation shows that finite $\widehat{G}(y)$ is required to make $\widehat{\Delta}^2 - \widehat{\varepsilon}^2 < 0$ and thus to create a localized mode. The function $f(y)$ plays essentially no role in the formation of the mode. Notice also that since $\widehat{\varepsilon}$ is small, it is quite likely that there is only a single mode — one with no nodes (zero crossings) in ψ_2. This is because as one creates a node, $|d\psi_2/dy|^2$ increases, requiring $|\widehat{\Delta}|$ to increase (on the right-hand side), but thereby preventing $\hat{\Delta}^2 - \hat{\varepsilon}^2 < 0$. It is also seen that a larger $|\hat{G}(y)|$ requires a larger $|d\psi_2/dy|^2$ and a smaller $|\psi_2|^2$, which implies a more localized mode in y-space (and thus a broader mode in x-space). These tendencies are born out in the numerical solutions of Eqs. (6a) and (6b).

If the K_1^2 and K_2^2 terms are dropped in the coupling terms on the right-hand side of Eq. (2) and its counterpart (often used as an approximation), a similar virial-type construction shows that a finite $\hat{G}(y)$ is *not* required to make $\hat{\Delta}^2 - \hat{\varepsilon}^2 < 0$. It indicates, incorrectly, that a TAE may be formed by toroidal coupling alone. Since it is more accurate to keep the K_1^2 and K_2^2 terms, this suggests that many terms are of the same order and so one must be cautious when dropping various terms. Finally, we point out that with $\hat{\sigma} \neq 0$ the condition $\hat{\Delta}^2 - \hat{\varepsilon}^2 < 0$ is no longer necessary to permit a localized solution.

We now analytically derive the dispersion relation and damping coefficient using a variational technique. Two derivations are presented. One includes $\hat{\sigma}$ perturbatively and is valid when $\hat{\sigma}$ is sufficiently small. Here the damping scales as $\mathrm{Im}(\hat{\sigma})$ as expected, but is enhanced significantly by $\hat{\varepsilon}$ and $\hat{G}$. The other includes $\hat{\sigma}$ non-perturbatively. Here the dependence of $\hat{\Delta}$ on $\hat{\sigma}$ is in general quite complicated, but a simple iterative formula holds in the regime of interest. Results are compared with values obtained by direct numerical integration of the basic equations.

Recognizing from the previous arguments that $f(y)$ plays a minimal role in the form of the TAE, for simplicity we set $f(y) = 1$ in Eqs. (6a) and (6b). We further make the simplification $\hat{G}(y) \cong \hat{G}_0/(y^2+1)$, where $\hat{G}_0 = \frac{1}{2}(\hat{G}_1 + \hat{G}_2)$ (generally $\hat{G}_0 > 0$). We write $\psi_1 = \psi_1^{(0)} + \psi_1^{(1)}$, and the same for ψ_2 and $\hat{\Delta}$, where the superscript$^{(1)}$ terms are of $\mathcal{O}(\hat{\sigma})$. Then to leading order

$$\left[\frac{d}{dy} + i\left(\hat{\Delta}^{(0)} - \frac{\hat{G}_0}{y^2+1}\right)\right]\psi_1^{(0)} = -i\hat{\varepsilon}\psi_2^{(0)} , \tag{10a}$$

$$\left[\frac{d}{dy} - i\left(\hat{\Delta}^{(0)} - \frac{\hat{G}_0}{y^2+1}\right)\right]\psi_2^{(0)} = i\hat{\varepsilon}\psi_1^{(0)} , \tag{10b}$$

from which it is clear that $\psi_2^{(0)} = \psi_1^{(0)*}$. With $\psi_2^{(0)} = \psi_R + i\psi_I$,

$$\frac{d\psi_I}{dy} = \left[\hat{\varepsilon} + \hat{\Delta}^{(0)} - \frac{\hat{G}_0}{(y^2+1)}\right]\psi_R \,, \tag{11}$$

and

$$\frac{d}{dy}\frac{y^2+1}{y^2+\mu}\frac{d\psi_R}{dy} + \left[\hat{\Delta}^{(0)^2} - \hat{\varepsilon}^2 + \frac{\hat{G}_0(\hat{\varepsilon} - \hat{\Delta}^{(0)})}{y^2+1}\right]\psi_R = 0 \,, \tag{12}$$

where $\mu = (\hat{\varepsilon} - \hat{\Delta}^{(0)^2} + \hat{G}_0)/(\hat{\varepsilon} - \hat{\Delta}^{(0)})$. Taking the inner product of Eq. (12) with ψ_R gives

$$-\left\langle \frac{y^2+1}{y^2+\mu}\left(\frac{d\psi_R}{dy}\right)^2\right\rangle + \left(\hat{\Delta}^{(0)^2} - \hat{\varepsilon}^2\right)\left\langle\psi_R^2\right\rangle + \hat{G}_0(\hat{\varepsilon} - \hat{\Delta}^{(0)})\left\langle\frac{\psi_R^2}{y^2+1}\right\rangle = 0 \,. \tag{13}$$

As our variational procedure, we take the trial function $\psi_R = e^{-\lambda y^2/2}$ with the parameter λ, substitute into Eq. (13), and carry out the integrals. This gives (approximately)

$$-\lambda/2 + \hat{\Delta}^{(0)^2} - \hat{\varepsilon}^2 + (\pi\lambda)^{1/2}\hat{G}_0(\hat{\varepsilon} - \hat{\Delta}^{(0)}) = 0 \,. \tag{14}$$

The parameter λ is determined by finding the extremum of Eq. (14), which corresponds to

$$\lambda = \pi\left[\hat{G}_0(\hat{\varepsilon} - \hat{\Delta}^{(0)})\right]^2 \,. \tag{15}$$

Substituting this into Eq. (14) gives the leading order dispersion relation

$$\hat{\Delta}^{(0)} = -\hat{\varepsilon}\,\frac{1 - \pi\hat{G}_0^2/2}{1 + \pi\hat{G}_0^2/2} \,. \tag{16}$$

Thus $\hat{\Delta}^{(0)^2} < \hat{\varepsilon}^2$ as expected. At first order we have

$$\left[\frac{d}{dy} + i\left(\hat{\Delta}^{(0)} - \frac{\hat{G}_0}{y^2+1}\right)\right]\psi_1^{(1)} + i\hat{\varepsilon}\psi_2^{(1)} = -i\left[\hat{\Delta}^{(1)} - \hat{\sigma}(y^2+1)\right]\psi_1^{(0)} \,, \tag{17a}$$

$$\left[\frac{d}{dy} - i\left(\hat{\Delta}^{(0)} - \frac{\hat{G}_0}{y^2+1}\right)\right]\psi_2^{(1)} - i\hat{\varepsilon}\psi_1^{(1)} = i\left[\hat{\Delta}^{(1)} - \hat{\sigma}(y^2+1)\right]\psi_2^{(0)} \,, \tag{17b}$$

These equations may be combined into a single equation for $\psi_2^{(1)}$. Taking the inner product of the resulting equation with $\psi_2^{(0)}$, integrating by parts, substituting the leading order equation for $\psi_2^{(0)}$, and using the symmetry properties of ψ_R and ψ_I leads to

$$\widehat{\Delta}^{(1)}\left\langle\psi_I^2+\psi_R^2+i(\psi_I^2-\psi_R^2)\left(\widehat{\Delta}_0-\frac{\widehat{G}_0}{y^2+1}\right)\right\rangle$$
$$=\widehat{\sigma}\left\langle(\psi_I^2+\psi_R^2)(y^2+1)+i(\psi_I^2-\psi_R^2)\left[\widehat{\Delta}^{(0)}(y^2+1)-\widehat{G}_0\right]\right\rangle\ . \tag{18}$$

This may be simplified by noting that $\langle\psi_I^2\rangle \sim \widehat{\varepsilon}\,\langle\psi_R^2\rangle$, $\widehat{\Delta}^{(0)} \sim -\widehat{\varepsilon}$, $\widehat{\varepsilon} \ll 1$, $\widehat{G}_0 \ll 1$. Then $\widehat{\Delta}^{(1)} \cong \widehat{\Delta}^{(1)} \cong \widehat{\sigma}\,\langle\psi_R^2(y^2+1)\rangle\,/\,\langle\psi_R^2\rangle$, which may be evaluated with our trial function and Eq. (15) simply as

$$\widehat{\Delta}^{(1)} \cong \frac{\widehat{\sigma}}{8\pi(\widehat{\varepsilon}\widehat{G}_0)^2}\ . \tag{19}$$

Thus the damping coefficient is proportional to $\mathrm{Im}(\widehat{\sigma})$ and is enhanced by the small parameters $\widehat{\varepsilon}$ and $\widehat{G}_0$.

If $\widehat{\sigma}$ is sufficiently large, the perturbation scheme fails. The scaling of the dispersion relation with $\widehat{\sigma}$ in this case may be found from a non-perturbative variational analysis. We point out, however, that as $\widehat{\sigma}$ increases, the wave function becomes more complicated. This cannot be accounted for in our simple variational procedure. Consequently one should ultimately verify the results by direct numerical integration of Eqs. (6a) and (6b).

Again for simplicity we set $f(y) = 1$ and $\widehat{G}(y) \cong \widehat{G}_0/(y^2+1)$ and then Eqs. (6a) and (6b) may be combined into a single (Schrödinger) equation for ψ_2,

$$\left[\frac{d^2}{dy^2}+h_0^2(y)-\widehat{\varepsilon}^2-i\,\frac{dh_0(y)}{dy}\right]\psi_2=0\ , \tag{20}$$

where $h_0(y) = \widehat{\Delta} - \widehat{G}_0/(y^2+1) - \widehat{\sigma}(y^2+1)$. It is interesting to note the similarity between $h_0(y)$ and the effective potential for the GAE discussed in Ref. 8 [c.f. Eqs. (23) and (24) and Figs. 1 and 2]. However, the correspondence is not complete since in the present case the effective potential is $h_0^2 - \widehat{\varepsilon}^2 - i\,dh_0/dy$, making the problem significantly more complicated. Taking the inner product of Eq. (20) with ψ_2, using the trial function $\psi_2 = e^{-\lambda\psi^2/2}$ with the parameter λ (now complex), and carrying out the integrals leads to

$$-\frac{1}{2}\lambda+\widehat{\Delta}^2-\widehat{\varepsilon}-(\pi\lambda)^{1/2}\widehat{G}_0\left(2\widehat{\Delta}-\frac{1}{2}\widehat{G}_0\right)-\widehat{\sigma}\left[2(\widehat{\Delta}-\widehat{G}_0)+\widehat{\Delta}/\lambda-\frac{3}{4}\frac{\widehat{\sigma}}{\lambda^2}\right]=0\ . \tag{21}$$

The parameter λ corresponds to the extremum of Eq. (21). A perturbative analysis of this equation (neglecting $\widehat{\sigma}$, calculating $\widehat{\Delta}$, then putting $\widehat{\sigma}$ back and finding the correction to $\widehat{\Delta}$) gives results substantively similar to Eqs. (16) and (19). However, when the effect of $\widehat{\sigma}$ is stronger, (enhanced by small λ), the $\widehat{\sigma}\widehat{\Delta}/\lambda$ term dominates and we can write

$$-\frac{1}{2}\lambda+\widehat{\Delta}^2-\widehat{\varepsilon}^2-\frac{\widehat{\sigma}\widehat{\Delta}}{\lambda}=0 \; . \tag{22}$$

This has the extremum $\lambda=(2\widehat{\sigma}\widehat{\Delta})^{1/2}$ and gives the dispersion relation

$$\widehat{\Delta}^2-\widehat{\varepsilon}^2=(2\widehat{\sigma}\widehat{\Delta})^{1/2} \; . \tag{23}$$

This equation is easily solved iteratively. It is a result very different from the perturbative analysis. There are three fundamental differences: 1) $\widehat{\Delta}$ is independent of $\widehat{G}_0$, 2) $\mathrm{Re}(\widehat{\Delta}^2)>\widehat{\varepsilon}^2$, and 3) $\mathrm{Re}(\widehat{\Delta})>0$. The second point does not necessarily imply that the mode lies outside the gap. Because of the normalization, the value of $\widehat{\Delta}$ corresponding to the gap boundary is approximately given by $\widehat{\Delta}_{\mathrm{gap}}=\pm\frac{1}{2}\widehat{\varepsilon}(\alpha_1\alpha_2)^{1/2}(\alpha_1^{-1}+\alpha_2^{-1})\simeq\pm\widehat{\varepsilon}\frac{m_1+m_2}{2\sqrt{m_1m_2}}$. The mode corresponding to Eq. (23) is the "kinetic TAE" described in the introductory remarks. It may, depending on the plasma parameters, have a lower damping coefficient than the MHD TAE.

The dispersion relations given by Eqs. (16) and (19) or Eq. (23) are valid only when the wave function is reasonably close to the assumed form $\psi=e^{-\lambda y^2/2}$. To verify this, $\widehat{\Delta}$, ψ_1, and ψ_2 were found by direct numerical integration of Eqs. (6a) and (6b) with a shooting code (originally developed by J. Sedlak). The equations were solved as the coupled system with the WKB-type boundary conditions

$$\frac{\psi_1}{\psi_2}=\frac{\pm i[\widehat{\varepsilon}^2-h^2(y)]^{1/2}-h(y)}{\widehat{\varepsilon}/f(y)} \tag{24}$$

at $y\gg 1$ and $y\ll -1$ respectively. The code input $\widehat{G}_1$, $\widehat{G}_2$, y_1^2, y_2^2, $\widehat{\sigma}$, $\widehat{\varepsilon}$ (which were calculated from their definitions). The solutions were rapidly convergent and robust.

Numerically computed eigenvalues for the MHD and kinetic TAE modes for TFTR plasma parameters are shown in Table 1. Also shown are the damping coefficients γ/ω corresponding to the lowest-damped mode and, for comparison, values of $(\gamma/\omega)_{\rm mcd}$ due to the magnetic curvature drift of the electrons [calculated from Eq. (10) of Ref. 2]. The plasma density and q profiles were chosen to scale parabolically with r/r_p, while the temperature $T_e \sim [1-(r/r_p)]^2$. Two cases are considered. The first corresponds to the TFTR TAE experiment discussed in Ref. 4. Here, $R = 2.4\,{\rm m}$, $r_p = 0.75\,{\rm m}$, $T_e = 1.7\,{\rm keV}$, $n = 2.7 \times 10^{13}\,{\rm cm}^{-3}$, $B = 1.1\,{\rm T}$, $q_{\rm cntr} = 1.04$, and $q_{\rm edge} = 2.8$. The second case corresponds to a D-T burning experiment with $R = 2.5\,{\rm m}$, $r_p = 0.8\,{\rm m}$, $T_e = 10\,{\rm keV}$, $n = 10^{14}\,{\rm cm}^{-3}$, $B = 5\,{\rm T}$, $q_{\rm cntr} = 1.04$, and $q_{\rm edge} = 3.1$. In both cases an effective mass of 2.5 was used. For each case, we consider a low mode number: $n = (1,1)$, $m = (1,2)$ with $q = 1.5$, and a higher mode number: $n = (2,2)$, $m = (2,3)$ with $q = 1.25$. The latter case fits the experimentally measured $q \sim 1.3$ of Ref. 4 with $n = 2$. As shown, the damping coefficients for the MHD TAE are a bit larger than those for the "kinetic TAE." Note that the damping coefficients are higher for the higher mode numbers. This is due to larger values of $\hat{\sigma}$, which scales as $\kappa^2(\sim m_1 m_2)$ relative to the other normalized parameters. The predicted damping coefficient for the TAE experiment is quite close to the experimentally measured value (of Ref. 4) of $\sim 3\%$. In each case $\gamma/\omega > (\gamma/\omega)_{\rm mcd}$.

Figure 1 shows the wave function $\psi_1(y)$ corresponding to the "kinetic TAE" for the first row in Table 1. For $n = 2, \psi_1(y)$ has a slightly more oscillatory character due to the larger value of $\hat{\sigma}$. The wave function for the MHD TAE is more oscillatory than the "kinetic TAE" for these plasma parameters. The wave functions for the D-T burning case are qualitatively similar. For other plasma parameters, e.g. for DIII-D, we find the situation reversed, with the MHD TAE having a smooth profile and a lower damping coefficient. Generally, $\psi_2(y) \sim -\psi_1(-y)$.

Predictions of the real part of $\hat{\Delta}$ from Eq. (16) (the perturbative derivation) are found to lie within 50% of the values shown in Table 1 for the MHD TAE. In contrast, predictions of the imaginary part of $\widehat{\Delta}$ from Eq. (19) are small by up to an order of magnitude. The largeness of $\widehat{\sigma}$ for these cases causes the perturbation scheme to fail. We find better agreement between the perturbative predictions and the numerical results for other plasma parameters, when $\widehat{\sigma}$ is smaller. Values obtained iteratively from Eq. (23) are generally in close agreement with those for the "kinetic TAE" in Table 1. The real part of $\widehat{\Delta}$ is within 20% for all cases, while the imaginary part is within a factor of two. Better agreement should not be expected because of the difference between the actual shape of the wave function and our trial function.

In conclusion, a non-perturbative treatment of electron parallel dynamics (the source of normal Landau damping) predicts a non-negligible intrinsic Landau damping of the TAE. This, combined with other damping mechanisms, including mode coupling to the kinetic Alfvén wave (continuum damping[5,6]), could render the TAE harmless in a reactor environment.

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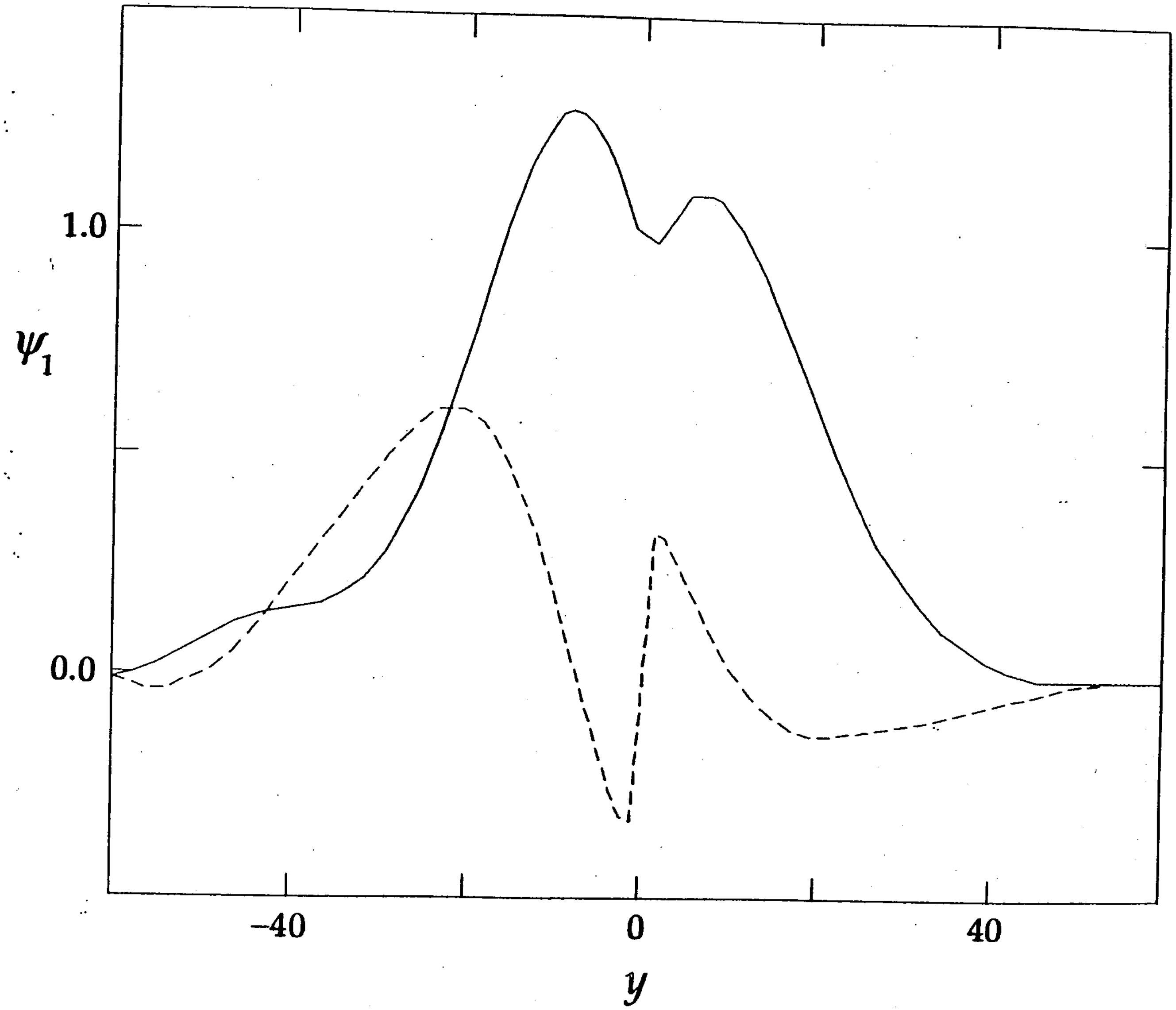
Table I: eigenvalues and damping rates

TFTR case	n	$\widehat{\Delta}_{\mathrm{MHD}}$	$\widehat{\Delta}_{\mathrm{KIN}}$	(γ/ω)	$(\gamma/\omega)_{\mathrm{mcd}}$
TAE	1	$(-4.4 - 1.6i) \times 10^{-2}$	$(12. - 1.2i) \times 10^{-2}$	1.4×10^{-2}	3.9×10^{-3}
	2	$(-7.0 - 9.6i) \times 10^{-2}$	$(21. - 4.3i) \times 10^{-2}$	2.8×10^{-2}	5.1×10^{-3}
D-T	1	$(-4.3 - 0.74i) \times 10^{-2}$	$(9.5 - 0.63i) \times 10^{-2}$	7.3×10^{-3}	4.3×10^{-3}
	2	$(-7.0 - 5.3i) \times 10^{-2}$	$(16. - 2.0i) \times 10^{-2}$	1.3×10^{-2}	5.4×10^{-3}

Figure Caption

1. Numerically computed wave function $\psi_1(y)$ in Fourier space for the $n = 1$, $m = 1, 2$ ($q = 1.5$) "kinetic TAE" and for plasma parameters corresponding to the TFTR TAE experiment (Ref. 4).



1.0
0.0
ψ1
-40
0
40
y