INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-529 IFSR #529

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December 1991

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Abstract

Models of ion temperature gradient-driven turbulence are reexamined in terms of the structure of the turbulent spectrum for radially localized modes to explain the significant difference between the radial profiles of ion heat conductivity inferred from local turbulence models and those observed in experiments. The strong radial inhomogeneity of the effective density of the turbulence spectrum in **k** is shown to produce a significant increase of the fluctuation level and ion thermal diffusion toward the plasma edge as compared with the local models.

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I. Introduction

Anomalous diffusion in tokamaks is often attributed to turbulent processes caused by unstable small-scale perturbations. The η_i-mode turbulence is widely regarded as a likely model for theoretical description of the anomalous ion thermal diffusion. The main reason for this is a relatively good (order of magnitude) quantitative agreement between theoretical predictions of this model and actually observed levels of anomalous transport in large tokamaks.[1] However, the model always underestimates the transport in colder regions of plasma column (near the edge). The reason for this is that the usual assumption of a locally homogeneous turbulence leads to the so-called gyro-reduced Bohm scaling $\chi_i \propto T^{3/2}/B^2$ of the theoretical ion heat conductivity χ_i with temperature T and magnetic field intensity B. Such scaling makes χ_i drop with decreasing temperature, while the actual experimental levels of turbulence and transport usually increase toward the edge.[2] This discrepancy can be explained in L-mode regimes as being due to another kind of driving instability, namely, the resistive ballooning modes. In the H-mode and hot-ion regimes, however, such treatment cannot be applied because the edge temperature is too high for resistive modes to be effective in providing the large, radially increasing χ_i required by the ion power balance.

In this article we propose an alternative treatment of the η_i-mode turbulence that explains the high effective levels of fluctuations and transport in the outer half of plasma radius without invoking additional instabilities. We do not to revise earlier linear and nonlinear studies of the topic, but rather make a new interpretation of the previous research, so that our result can be expressed as a correction factor $F(r)$, which can be applied to most previous estimates of $\chi_i(r)$ from the η_i-modes.

We now proceed to the description of theoretical foundations of our approach. Most of the relevant general results are already published in our earlier paper,[3] while here we concentrate on its applicability and predictions in the particular case of the η_i-mode turbulence.

II. Approximation of Independent Subsystems in a Turbulent State

Let us assume that the anomalous transport coefficient χ^a can be represented as a sum over the turbulent spectrum, such as

$$\chi^a = \sum_{\mathbf{k}} g(\mathbf{k}) |\phi_{\mathbf{k}}|^2 \ , \tag{1}$$

where $\mathbf{k}$ is the wave-vector, $\phi_{\mathbf{k}}$ is the wave amplitude, and g describes the phase shift between $\phi_{\mathbf{k}}$ and $T_{\mathbf{k}}$. If this is the case then for roughly equivalent waves it can be rewritten as

$$\chi^a = \left\langle g(\mathbf{k}) |\phi_{\mathbf{k}}|^2 \right\rangle \cdot N_\chi \ , \tag{2}$$

where N_χ is the effective number of waves in the spectrum and the angular brackets denote the mean value of quantity over all $\mathbf{k}$'s. Our aim is to concentrate on the role of the factor N_χ, which can be important for radially localized modes.

From expression (2) one can see that if the wave amplitudes $\phi_{\mathbf{k}}$ are given independently, then $\chi^a \propto N_\chi$, and the anomalous transport should grow rapidly with an increase of the density of the spectrum. However, this is not so if we are considering a turbulent system where saturated amplitudes are determined by the nonlinear transfer of energy in the spectrum. In such a case the total energy in fluctuations is usually bounded from above so that

$$E = \sum_{\mathbf{k}} \mathbf{k}^2 |\phi_{\mathbf{k}}|^2 = \left\langle \mathbf{k}^2 |\phi_{\mathbf{k}}|^2 \right\rangle \cdot N_E \le E_t \ . \tag{3}$$

Comparing expressions (2) and (3) we note that the number of modes N will not enter into the final result if we use the limit E_t and the equipartition of energy among modes as an estimate for the level of turbulence ($\tilde{\phi} = \langle |\phi_{\mathbf{k}}|^2 \rangle^{1/2} \le (E_t/N_E)^{1/2}$). This is why the density of the spectrum is usually considered unimportant for mixing-level estimates of χ^a.

Nonuniform systems with complex turbulent behavior may retain traces of the N-dependence in the following manner. Suppose that in the same medium there are F independent turbulent processes, each of which is saturated by a nonlinear interaction. Then the turbulent heat conductivity is a sum of partial heat conductivities induced by each process separately. An obvious example is a coexistence of drift, plasma and Alfvén waves in the same volume, each saturating by its own physical process while contributing to the overall transport. A less obvious, but also possible case, is when the independent turbulent processes are caused by different subsets of modes of the same physical origin that for some reason are weakly interacting between each other. Now the nonlinear saturation is produced by the energy transfer within the given subsystem of modes, while such transfer between different subsystems is weak. We argue that due to the radial localization of different helical components around corresponding rational surfaces this is the case for drift-wave turbulence.

Separating in Eq. (1) groups of terms corresponding to different subsystems and performing partial summation within each group we get

$$\chi^a = \sum_\alpha \chi_\alpha \sim \langle \chi_\alpha \rangle \cdot F \ , \tag{4}$$

where χ_α are partial transport coefficients caused by independent turbulent processes, labeled by α. Since saturation occurs independently within each group, the saturation level (3) is relevant only for determination of each $\chi_\alpha = \sum_{\mathbf{k}_\alpha} g_\alpha(\mathbf{k})|\phi_\mathbf{k}|^2$, while the overall transport χ^a may be F times higher.

At this point we need to explicitly formulate our assumptions about the η_i-mode turbulence, which can be summarized as follows.

1. The unstable excited η_i-modes are radially localized. It means that all perturbations can be decomposed into a series of helical modes (with given poloidal and toroidal wavenumbers m and n), each of which is non-zero only in vicinity of the corresponding rational magnetic surface, where the safety factor $q(r) = m/n$.

2. Radial localization of modes means that for any given magnetic surface with radius radius r there is only a finite number of modes with non-zero amplitudes. The spectrum of turbulence thus contains only a few excited modes at any given radius and is far from

being continuous. (Of course, at other radii the sets of non-zero modes are different and the total number of excited modes is huge.)

3. We assume that possible poloidal correlations due to ballooning effects between different helical waves is broken.[4]

4. Finally, we postulate that the nonlinear interactions between different helical modes are much weaker than those between waves of the same helical symmetry ($m' = km$, $n' = kn$, which are sitting on the same resonant surface, $q(r) = m/n = km/kn$, $k = 0, \pm 1, \pm 2, \pm 3, \ldots$). At this time we do not have direct numerical evidence for validity of this last assumption, but there are some qualitative arguments in its favor, which are discussed below.

When these assumptions about the η_i-mode turbulence are satisfied then we can use Eq. (4) for the anomalous transport coefficient, where α will now label nonlinear perturbations of different helical symmetry, or $\alpha = m/n$. One implication of this formula is that the effective number of helical waves F is proportional to the radial density of resonant magnetic surfaces and is, typically, a sharply rising function of minor radius (in a tokamak). If $\langle \chi_\alpha \rangle$ is given by the usual mixing-level estimate, then F is the new correction factor, which should be introduced for description of three-dimensional turbulence.

Figure 1 illustrates two possible limits involved in the calculation of the sum $\sum_\alpha \chi_\alpha$ for radially localized modes. In area (A) the helical modes do not overlap, so that $F < 1$, and this regime is characterized by quasilinear saturation. In area (B) the radial density of turbulent states is high, so that the overall level of transport may be above the mixing-level estimates due to the significant overlapping. It is clear that without overlapping different helical perturbations do not interact and assumption (4) is strictly satisfied. In the strongly overlapping regime there is a limit when interaction with the bath of other helical states becomes comparable to nonlinear interactions within the single-helicity subsystems. If the difference of interaction efficiency can be described by a small parameter $\epsilon \ll 1$ then the number of effectively independent turbulent states F cannot exceed $F_{\max} = 1/\epsilon \gg 1$. (Since if the number of turbulent states exceeds $F_{\max}$ then the energy transfer from any given helical state to all others will exceed the energy transfer within the subsystem and thus the levels of nonlinear saturation will not be independent.)

Figure 2 serves as a qualitative argument for the existence of such a small parameter for the η_i-modes. These modes are radially localized ($\delta x \ll r$), which means that any given wave may effectively interact only with those others, which have non-zero amplitudes within the area of support[5] (δx) of the first mode. In the graph each wave is represented by the crossing of lines corresponding to given integer wavenumbers m and n. Overlapping modes should fit into a narrow sector, so that $n \cdot [q(r_o) - q'\delta r] \leq m \leq n \cdot [q(r_o) + q'\delta r]$, where $\delta r = \delta x/2$ is the mode width and q' is the radial derivative of $q(r)$. One can see that with these restrictions it is hard to satisfy the three-wave interaction condition ($m' = m + m''$, $n' = n + n''$) unless the modes have the same helical symmetry, that is unless the $\mathbf{k}$ vectors are collinear and modes are on the same line $m = nq(r)$.

III. Enhanced Diffusion Due to η_i-modes

The formula expressing the effective turbulent transport through the radial density of resonant rational surfaces and the local diffusivity has been derived in Ref. 3:

$$\chi^a(r) = \frac{6|q'|}{\pi^2 q^2} \sum_{m=m_i}^{m_f} m\overline{\chi(r,m)}\delta x(r,m) \ . \tag{5}$$

Here m_i and m_f are the lower and upper boundaries of the excited spectrum of modes producing significant transport; $\overline{\chi(r,m)}$ is the averaged value of transport caused by a single helical subset of modes within its area of support, viz., $\delta x(r,m)$. Both $\overline{\chi}$ and δx retain dependence on the lowest poloidal wavenumber m in the corresponding helical subset. Equation (5) is a good approximation of the actual sum, Eq. (4) in the overlapping limit ($F > 1$). A similar treatment of the sum over $\mathbf{k}$ for radially localized modes has been used by Diamond and Rosenbluth.[6]

For evaluation of expression (5) we need knowledge of explicit dependencies $\overline{\chi(m)}$, $\delta x(m)$, and m_i, m_f. A reasonable approximation for these can be found in the following way. According to Ref. 7 the heat flux from one mode behaves as

$$Q_i(k_y) \;\propto\; Ak_y|\phi_k|^2 \,, \tag{6}$$

while the saturated amplitude is taken as the mixing-level estimate

$$c^2k_y^2|\phi_k|^2/B^2 \;\propto\; C(\eta_i)\cdot v_{*i}^2 \,. \tag{7}$$

Here $k_y = m/r$ is the poloidal component of the wave-vector; v_{*i} is the ion diamagnetic velocity; ϕ is the normalized electrostatic potential; A and $C(\eta_i)$ are some functions of η_i and other plasma parameters but are independent of k_y.

Combining Eqs. (6) and (7) we get $\overline{\chi_i(m)} \propto 1/m$ or

$$\overline{\chi_i(m)} = \overline{\chi_i(m_i)}\frac{m_i}{m} \,. \tag{8}$$

At least for $k_y\rho_i \geq 1$ (where ρ_i is the ion Larmor radius) the typical η_i-mode spectrum decreases with k_y faster than $|\phi_k| \sim k_y^{-1}$, which is assumed above. The mode-coupling model solved in Ref. 7 suggests that within the scale-range $\varepsilon_n/q \leq k_y\rho_i \leq 1$ ($\varepsilon_n = 1/(|\nabla \ln n|R)$, R is the major radius) the form of the spectrum follows $|\phi_k| \sim k_y^{-1}$, while in the region $k_y\rho_i \geq 1$ it drops as $\phi_k| \propto k_y^{-3}$, producing negligible transport. If we take the short-wave spectrum in this form, that will be essentially equivalent to setting the upper limit of the spectrum to $m_f \approx r/\rho_i$, since the contribution from short-wave perturbations in Eq. (5) is much smaller than that from perturbations with $k_y \leq \rho_i^{-1}$.

We consider two different approximations for the $\delta x(m)$-dependence. The first (a) is $\delta x \propto 1/m$, which is based on the assumption $k_x \sim k_y$, and the second (b) is $\delta x \sim$ const.

Since δx and $\overline{X}$ enter in Eq. (5) only as a product, the m-dependence of case (a) may also be interpreted as $\delta x \sim$ const. but $|\phi_k| \sim k_y^{-1.5}$.

The lower boundary of the spectrum, viz., m_i, can be estimated from the condition of zero linear growth rate, when the growth rate[8] of the linear mode γ is balanced by the parallel damping

$$\gamma \sim |k_y|(v_{*i} v_{Di} \eta_i)^{1/2} \;\approx\; |k_{\|}| v_{Ti} \;, \tag{9}$$

where v_{Di} is the toroidal magnetic drift frequency; $\eta_i = \partial \ln T_i / \partial \ln n$ is the ratio of logarithmic derivatives of the ion temperature and density; $k_{\|}$ is the component of the wave-vector along the magnetic field; v_{Ti} is the ion thermal velocity. Since $k_{\|} = k_y \cdot (xs/Rq)$, where x is the distance from the resonant surface, $s = rq'/q$, and R is the major radius, relation (9) gives the radial width of the marginally stable mode with broadest radial extent. Invoking $k_x \sim k_y$ again, we get $m_i \approx r/x$ and

$$m_i \;\approx\; \frac{rs}{Rq} \frac{v_{Ti}}{(v_{*i}\, v_{Di} \eta_i)^{1/2}} \;. \tag{10}$$

Now we can substitute the m-dependence of $\overline{X}$ and δx in expression (5), which yields $\chi^a(r) = F(r) \cdot \chi^0$, where $\chi^0 = \overline{X(m_i)}$, and the "density of states" correction F becomes

$$F_a \;\approx\; 0.6 \frac{|q'|}{q^2}\, \delta x(m_i) m_i^2 \ln\left(\frac{m_f}{m_i}\right) \;, \tag{11}$$

for the case 'a' with $\delta x \propto 1/m$, and

$$F_b \;\approx\; 0.6 \frac{|q'| \delta x}{q^2}\, m_i (m_f - m_i) \;, \tag{12}$$

if we take $\delta x \sim$ const. (case 'b'). Using obtained $m_i \approx \sqrt{\varepsilon_n} sr/q\rho_i$ and $m_f \approx r/\rho_i$ we arrive at the final form of correction factors

$$F_a = 0.6 \sqrt{\varepsilon_n} \frac{rs^2}{q^2 \rho_i} \ln\left(\frac{q}{s\sqrt{\varepsilon_n}}\right) \;, \tag{13}$$

and

$$F_b = 0.6 \frac{sr}{q\rho_i} \;. \tag{14}$$

Here $\varepsilon_n = 1/(|\nabla \ln n|R)$.

Figure 3 shows the effect of obtained correction factors on the agreement between theoretical and experimental profiles of χ_i. The dot-dashed curve represents the "experimental" χ_i, i.e. that derived from the ion power-balance analysis of the experimental data from the Tokamak Fusion Test Reactor (TFTR) supershot 44669 before (A) and after (B) pellet injection. These two discharge states have been used in Ref. 2, where the corresponding heat diffusivity (filled circles) has been found from the local kinetic theory and conventional mixing-length formulas. We use this result in place of χ^0, so that the two upper curves in each graph correspond to this result multiplied by F_a (diamonds) and F_b (open circles). One can see that case 'b', when the transport is dominated by the short-wave end of the spectrum, significantly overestimates the transport, while case 'a', where the dominant contribution comes from long-wavelength modes, actually improves agreement of theory with experiment. Also note that assumptions about the form of turbulent spectrum and the radial extent of excited modes are not refined, and thus the fine-tuning of these parameters may change the correspondence within the factor of 5 difference between corrected theory and experiment, but the tendency of the correction factors F to increase the heat conductivity in the outer part of the plasma column is robust. This is the physical consequence of assumptions (1)-(4) (formulated in Sec. II) concerning the nature of the drift-wave turbulence.

IV. Conclusions

In this work we note that the conventional analysis of the radial dependence of the anomalous ion heat conductivity based on local models of the η_i-mode turbulence has been incomplete. In particular, with the separate treatment of two-dimensional single-helicity self-interactions and essentially three-dimensional nonlinear interactions of different helical waves, the strong radial variation of the density of mode rational surfaces becomes important and leads to a qualitatively new description of the thermal diffusivity.

Results in this article follow from assumptions given in items (1)–(4) in Sec. II, concerning the nature of the η_i-mode fluctuations and the hierarchy of nonlinear interactions, and can be summarized as follows:

1. There is a general effect that causes the weak-turbulence transport coefficients to increase with radial density of excited localized modes;

2. The density of states is an important parameter of turbulence, which should be consistently included in 3-D numerical simulations even if the turbulence regime is not weak;

3. Correction factors, calculated for the η_i-turbulence, change the scaling of transport estimates from the gyro-reduced Bohm $\sim T_i^{3/2}/B^2$ to Bohm-like $\sim T_i/B$, but with a complicated dependence on current, temperature and density profiles;

4. With the additional assumption that the anomalous transport is caused predominantly by the long-wave end of the turbulence spectrum, the found correction to the turbulent transport appears to improve the correlation between theoretical and experimental profiles of $\chi_i(r)$ for TFTR supershots.

Acknowledgments

This work is supported by the U.S. Department of Energy contract no. DE-FGO5-80ET-53088.

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Figure Captions

1. Helical subsystems of the turbulent spectrum are localized in radius to the vicinity of corresponding mode rational surfaces. Relative position of these surfaces may cause non-overlapping (A) and overlapping (B) regimes. δx is the area of support of the mode.

2. The wave-number plane, where the overlapping modes are represented by crossings within the narrow sector (shown by dashed lines), which corresponds to the area of support of one mode. Vectors show possible three-wave interactions within the single-helicity subset of modes.

3. Anomalous ion heat conductivity versus minor radius as found in experiment and estimated from different theoretical models for the TFTR supershot 44669, before (A) and after injection of the pellet, (B). The experimental profiles of χ_i found from the power-balance are shown by dash-dotted lines; predictions of the local kinetic formula (Ref. 2) are represented by filled circles, these results multiplied by the correction factors F_a and F_b are shown by diamonds and open circles respectively.

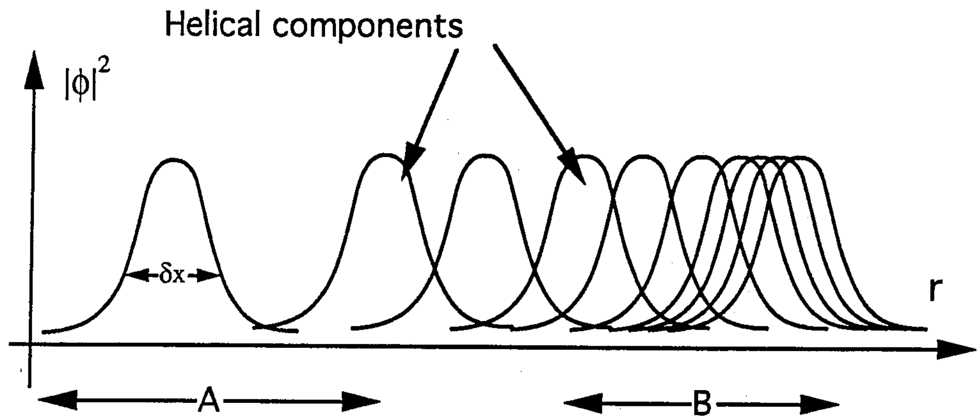

Fig. 1

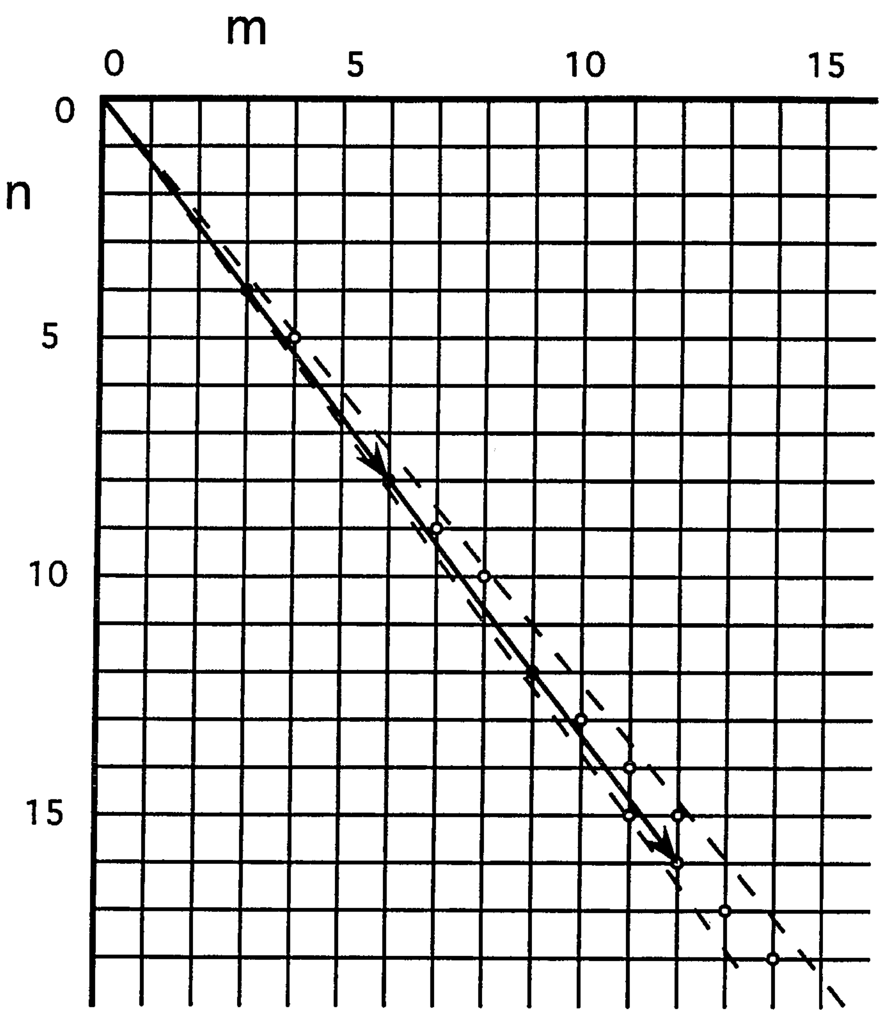

Fig. 2

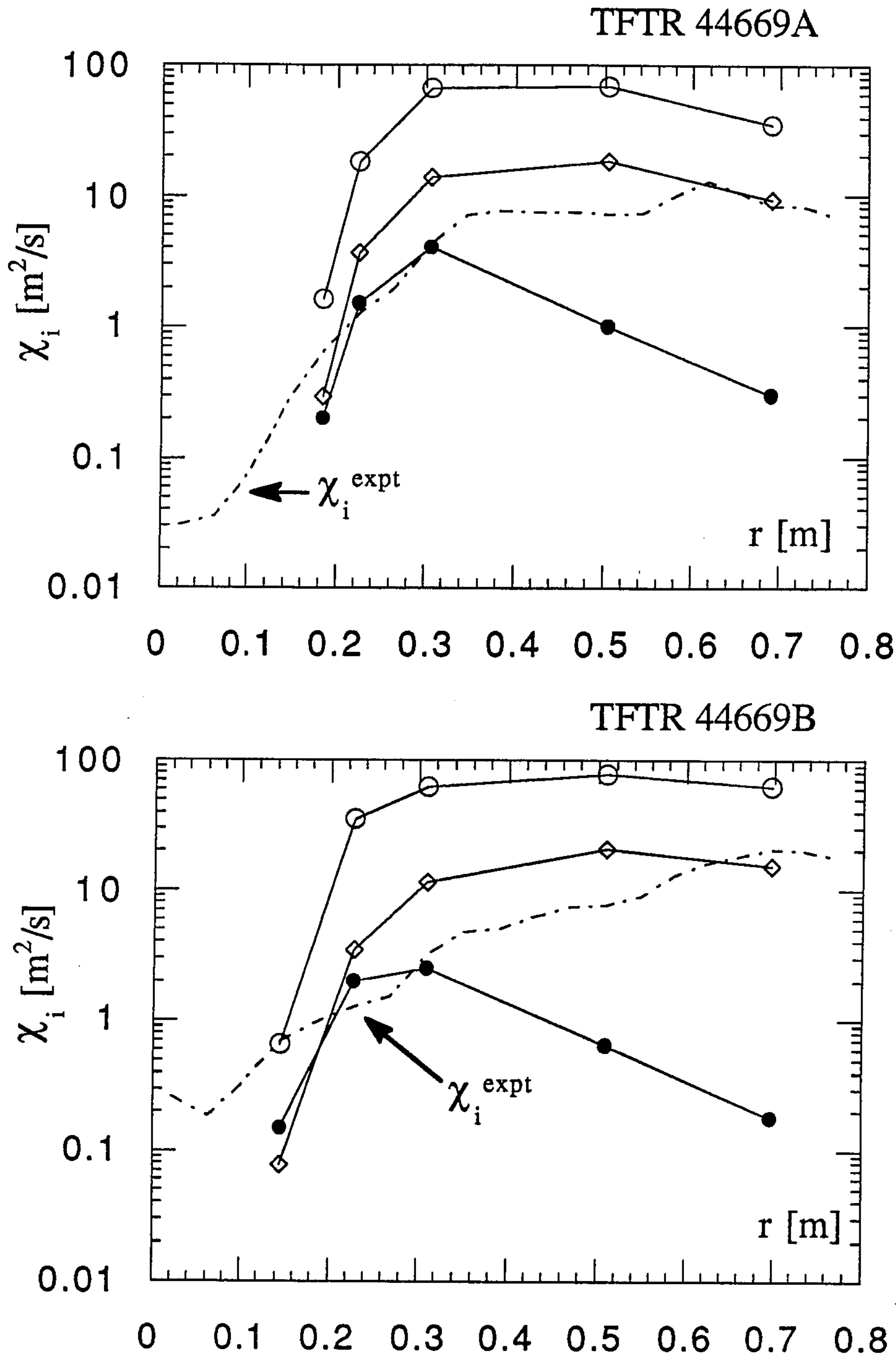
TFTR 44669A
100
10
1
0.1
0.01
χi [m2/s]
χi expt
r [m]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
TFTR 44669B
100
10
1
0.1
0.01
χi [m2/s]
χi expt
r [m]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Fig. 3