INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-526 IFSR #526

Edge Turbulence Scaling with Shear Flow

Y.Z. ZHANG and S.M. MAHAJAN
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

November 1991

THE UNIVERSITY OF TEXAS

AUSTIN

Edge Turbulence Scaling with Shear Flow

Y.Z. Zhang and S.M. Mahajan
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

A formula relating turbulence levels with arbitrary shear flow is derived. When the diffusion coefficient is made a functional of the corresponding turbulence level, it is found that the scaling laws governing turbulence suppression are considerably modified. The results are compared with known formulas in various limiting cases, indicating that turbulence suppression mainly pertains in the moderate shear flow regime. The results also show that a flattened (steep) radial equilibrium gradient tends to enhance (eliminate) turbulence suppression due to the shear flow.

One of the most important problems in current fusion research is to understand the physics of the H-mode, a state of improved confinement in tokamaks.[1–3] It is generally believed that conditions favorable for the existence of H-mode are created by the suppression of turbulence levels at the plasma edge through the agency of the radial shear in v_E, the $\mathbf{E} \times \mathbf{B}$ poloidal fluids velocity.[4–6] In the literature there exist formulas relating the suppression of turbulence levels to the strength of shear flow. By studying the "averaged" orbit equations of relative motion of two fluid elements, two distinct groups[7–9] have come to different conclusions on the nature of the suppression. Biglari *et al.*,[8] who primarily deal with the "large" shear regime, obtain the suppression of turbulence that scales as $|dv_E/dr|^{-2/3}$, while Shaing *et al.* find that the shear reduces the fluctuation level by a term proportional to $|dv_E/dr|^2$. This latter result is applicable only in the small shear regime.[9]

In order to find a scaling for arbitrary shear flow, we reinvestigate the orbit equations by introducing the self-consistent constraint that diffusion coefficient D with (without) shear flow must be a functional of the corresponding turbulence levels with (without) shear flow. We also take into account the effects of a possible shear induced change in spectral shape. It will be seen that the theory of turbulence suppression can be cast in terms of only two independent parameters: (1) γ, defined by $D = D^* \left\langle |\delta\hat{\xi}|^2 \right\rangle^\gamma$, where $\left\langle |\delta\hat{\xi}|^2 \right\rangle$ is the ensemble average of fluid fluctuations normalized to its equilibrium value, and D^* is the part independent of turbulence, which relates D to the strength of the fluctuations; and (2) $W \equiv |dv_E/dr| t_{c0}/\alpha \left\langle \hat{k}_\perp^2 \right\rangle^{1-\gamma}$, where t_{c0} is the decorrelation time without shear flow (henceforth, we shall use the subscript 0 to denote physical quantities in absence of shear flow), α measures the anisotropy of the k-spectrum, and $\left\langle \hat{k}_\perp^2 \right\rangle$ is the ratio of the averaged square of the perpendicular wave number with shear flow to that without shear flow. The analysis presented in this letter follows the methodology adopted by the previous authors.[7–9]

The set of differential equations governing the evolution of the two-point correlations is

$$\frac{\partial}{\partial t}\left\langle X_-^2(t)\right\rangle = 2\left\langle D_-^{11}\right\rangle \ ,$$
$$\frac{\partial}{\partial t}\left\langle Y_-^2(t)\right\rangle = 2\frac{dv_E}{dr}\left\langle X_-(t)Y_-(t)\right\rangle + 2\left\langle D_-^{22}\right\rangle \ , \qquad (1)$$
$$\frac{\partial}{\partial t}\left\langle X_-(t)Y_-(t)\right\rangle = 2\frac{dv_E}{dr}\left\langle X_-^2(t)\right\rangle + \left\langle D_-^{21}\right\rangle + \left\langle D_-^{12}\right\rangle \ ,$$

where

$$\langle D_-^{\mu\nu}\rangle \equiv \int d\mathbf{k}d\omega(\mathbf{b}\times\mathbf{k})^{\mu}(\mathbf{b}\times\mathbf{k})^{\nu}\left\langle|\delta\varphi|^2_{\mathbf{k},\omega}\right\rangle\left[k_x^2\left\langle X_-^2(t)\right\rangle + k_y^2\left\langle Y_-^2(t)\right\rangle + 2k_xk_y\left\langle X_-(t)Y_-(t)\right\rangle\right]iG_{\mathbf{k},\omega} \ ,$$

describes the relative diffusion of the two fluid elements, $\left\langle X_-^2(t)\right\rangle$, $\left\langle Y_-^2(t)\right\rangle$, and $\langle X_-(t)Y_-(t)\rangle$ are the orbit correlations for the relative distance of the two fluid elements in the radial and poloidal direction respectively,[10] $\left\langle|\delta\varphi|^2_{\mathbf{k},\omega}\right\rangle$ is the spectrum of the fluctuating field, and $G_{\mathbf{k},\omega}$ is the one-point Green's function in the Fourier representation. Defining $\Pi(\mathbf{k}) \equiv \int d\omega\left\langle|\delta\varphi|^2_{\mathbf{k},\omega}\right\rangle iG_{\mathbf{k},\omega}$, introducing an elliptical k-spectrum for Π, i.e., $\Pi(\mathbf{k}) = \Pi(k_x^2 + \alpha^2 k_y^2)$, and carrying out the k integrals, we can write Eq. (1) in the dimensionless form:

$$\left(\frac{\partial}{\partial \tau} - 1\right)\left\langle X_-^2(\tau)\right\rangle = 3\left\langle \overline{Y}_-^2(\tau)\right\rangle \ ,$$
$$\left(\frac{\partial}{\partial \tau} - 1\right)\left\langle \overline{Y}_-^2(\tau)\right\rangle = 3\left\langle X_-^2(\tau)\right\rangle + 8\sigma\left\langle X_-(\tau)\right\rangle\left\langle \overline{Y}_-(\tau)\right\rangle \ , \qquad (2)$$
$$\left(\frac{\partial}{\partial \tau} + 2\right)\left\langle X_-(\tau)\overline{Y}_-(\tau)\right\rangle = 8\sigma\left\langle X_-^2(\tau)\right\rangle \ ,$$

where $\tau \equiv tD\langle k_\perp^2\rangle$, $\sigma \equiv (dv_E/dr)/4\alpha D\langle k_\perp^2\rangle$, $\overline{Y}_- \equiv Y_-/\alpha$, D is the radial diffusion coefficient, and $\langle k_\perp^2\rangle \equiv \int_0^\infty dkk^5\Pi(k)/\int_0^\infty dkk^3\Pi(k)$ is a measure of the decorrelation length for a given turbulence spectrum. Equation (2) is a well-defined initial value problem with the given initial conditions: $\left\langle X_-^2(\tau)\right\rangle|_{\tau=0} = X_-^2$, $\left\langle \overline{Y}_-^2(\tau)\right\rangle|_{\tau=0} = \overline{Y}_-^2$, and $\langle X_-(\tau)\overline{Y}_-(\tau)\rangle|_{\tau=0} = X_-\overline{Y}_-$. All the three orbit correlations obey the same differential equation

$$\left[(\partial/\partial\tau + 2)^2(\partial/\partial\tau - 4) - 3(8\sigma)^2\right]\Psi(\tau) = 0 \ .$$

The corresponding characteristic equation is

$$\left(\Lambda+\frac{1}{2}\right)^2(\Lambda-1)=3\sigma^2\ , \tag{3}$$

yielding three solutions for $\Lambda \equiv 1/4\tau_c$, (where τ_c is the e-folding inverse dimensionless decorrelation time) of which only the positive real solution is physically interesting. In order to determine the influence of the shear flow on turbulence levels, Eq. (3) should be coupled to the equation for the correlation function, which can be written schematically as $t_c^{-1}\left\langle|\delta\widehat{\xi}|^2\right\rangle = 2D/L_\xi^2$, where L_ξ is the radial equilibrium gradient length. This equation relates Λ to the turbulence level $[\Lambda = [2\left\langle|\delta\widehat{\xi}|^2\right\rangle L_\xi^2\,\langle k_\perp^2\rangle]^{-1}]$, implying that Eq. (3) could be solved directly for $\left\langle|\delta\widehat{\xi}|^2\right\rangle$ after assuming a model for $D[\left\langle|\delta\widehat{\xi}|^2\right\rangle]$.

Let us first discuss the relative change of turbulence levels just before and after suppression [L to H mode transition, for example] assuming an invariant radial gradient length L_ξ. This is plausible for a sudden change of shear flow in a time scale much faster than the transport time scale. Defining $\widehat{\Theta} \equiv \left\langle|\delta\widehat{\xi}|^2\right\rangle / \left\langle|\delta\widehat{\xi}|^2\right\rangle_0$ to be the ratio of turbulence levels with and without shear flow, we obtain an equation for $P \equiv [\widehat{\Theta}\left\langle\widehat{k}_\perp^2\right\rangle]^{-1}$ with $\left\langle\widehat{k}_\perp^2\right\rangle \equiv \langle k_\perp^2\rangle / \langle k_\perp^2\rangle_0$ describing the change of the decorrelation length due to shear flow (indeed, there is experimental evidence for the change of decorrelation length due to shear flow [6]),

$$\left(P+\frac{1}{2}\right)^2(P-1)=3W^2P^{2\gamma}\ . \tag{4}$$

The solution of Eq. (4) is shown in Fig. 1 where we plot $\left\langle\widehat{k}_\perp^2\right\rangle\left\langle|\delta\widehat{\xi}|^2\right\rangle / \left\langle|\delta\widehat{\xi}|^2\right\rangle_0$ versus $W \equiv |dv_E/dr|t_{c0}\,/\alpha\left\langle\widehat{k}_\perp^2\right\rangle$ for various γ. Since $\gamma = 1$ is favored by the weak turbulence theory, and $\gamma = 0.5$ is favored by the strong turbulence theory, the most probable range for γ is 0.5 to 1.

In the small shear limit the simple analytical solution of Eq. (4),

$$\left\langle\widehat{k}_\perp^2\right\rangle\widehat{\Theta} = 1-\frac{4}{3}\left(\frac{|dv_E/dr|t_{c0}}{\alpha\left\langle\widehat{k}_\perp^2\right\rangle^{1-\gamma}}\right)^2\ , \tag{5}$$

is insensitive to γ, and reproduces the scaling given by Shaing *et al.* in the corresponding limit. In the large shear limit the solution is

$$\left\langle \hat{k}_\perp^2 \right\rangle \hat{\Theta} = \left(\frac{\sqrt{3}|dv_E/dr|t_{c0}}{\alpha \left\langle \hat{k}_\perp^2 \right\rangle^{1-\gamma}} \right)^{-\frac{2}{3-2\gamma}} , \tag{6}$$

which is consistent with the results of Biglari *et al.* only for $\gamma = 0$. In particular, for $\gamma = 1$ the scaling behaves like $|dv_E/dr|^{-2}$, implying a much faster suppression (in the large shear limit) than predicted by Biglari *et al.* If γ becomes greater than 1.5 there are generally two solutions for turbulence levels at a given small shear flow, and no solution for W greater than 0.57. For the interesting case of $\gamma = 1$, we can find a simple interpolated formula (valid for arbitrary shear),

$$[\left\langle \hat{k}_\perp^2 \right\rangle \hat{\Theta}]^{-1} = 1 + 2 \left(\frac{|dv_E/dr|t_{c0}}{\alpha} \right)^2 , \tag{7}$$

which is also shown in Fig. 1 by the dashed line. This interpolated formula is a good approximation to the numerical solution, and happens to be similar to the formula given by Shaing[9] (even in the large shear flow regime), and also to that used in Hinton's bifurcation model.[11]

It can be readily seen from Fig. 1 that for γ greater than 0.5, the turbulence suppression takes place mainly in the moderate shear flow regime, e.g. $|dv_E/dr|t_{c0} \sim O(1)$. This trend could be even strengthened by considering that γ could also change with turbulence levels, i.e., γ may evolve to a greater value, when the turbulence drops into the weak turbulence regime ($\gamma = 1.0$) from the strong turbulence regime ($\gamma = 0.5$), the regime more likely to pertain before the suppression. This analysis suggests that the regime of comparatively large shear flow may be uninteresting for practical purposes, both for its small contribution to the improvement of confinement as well as for its possible role in triggering the Kelvin-Helmholtz instabilities.

In the remaining part of this letter, we consider the solution for $\left\langle |\delta\hat{\xi}|^2 \right\rangle$ with the shear flow in a steady state, which will be obtained by solving Eq. (3) with the given power laws for the diffusion coefficient. Without shear flow the solution is straightforward, $\left\langle |\delta\hat{\xi}|^2 \right\rangle_0 =$

$1/2 \langle k_\perp^2 \rangle_0 L_\xi^2$. This result is independent of the functional form of the diffusion coefficient. It is not surprising, therefore, that the formula is consistent with the so-called mixing length result.[8] In a similar manner, we can set up an, implicit, interpolated formula for $\left\langle |\delta\hat{\xi}|^2 \right\rangle$ with shear flow,

$$\left\langle |\delta\hat{\xi}|^2 \right\rangle^{-1} = \left\langle k_\perp^2 \right\rangle L_\xi^2 \left(2 + \left[\left| \frac{dv_E}{dr} \right| L_\xi^2 \frac{\left\langle |\delta\hat{\xi}|^2 \right\rangle}{\alpha D} \right]^2 \right) , \tag{8}$$

where D is still a function of $\left\langle |\delta\hat{\xi}|^2 \right\rangle$. Equation (8) has rather simple forms for $\gamma = 1$, and 0.5. Solving these two particular cases, we then construct an explicit interpolated formula that matches the solution at $\gamma = 1$ and at $\gamma = 0.5$,

$$\left\langle |\delta\hat{\xi}|^2 \right\rangle^{-1} = 2 \left\langle k_\perp^2 \right\rangle L_\xi^2 + \left(\frac{|dv_E/dr| L_\xi^2}{\alpha D^*} \right)^{2\gamma} \left(\left\langle k_\perp^2 \right\rangle L_\xi^2 \right)^\gamma . \tag{9}$$

which is consistent with Eq. (7) in the corresponding scenario. This formula approximately describes the situation as the equilibrium gradient length L_ξ evolves due to improved confinement; a smaller (greater) L_ξ makes the shear suppression less (more) effective, because the suppression is controlled by the parameter $|dv_E/dr|^\gamma L_\xi^{3\gamma-1}$, and not by $|dv_E/dr|$ alone. This L_ξ-enhanced shear suppression may be related to the second stage of turbulence suppression, recently discovered on DIII-D,[12] which is found to occur at the flattened density region of the H-mode phase on a transport time scale. The second stage of suppression is important, as pointed out by Ref. 12, because the turbulence in the L-mode phase is located in this region, which was affected little by the first stage suppression due to L-H transition occurring at the steep gradient region of the H-mode phase. At this stage, we would like to draw the reader's attention that Eq. (9) does not necessarily mean that a steep equilibrium gradient would bring the turbulence levels back to its original one. It may be possible that at the steep gradient region the spectrum is simultaneously shifted towards shorter wave lengths[6] so that the low turbulence levels may survive even for small L_ξ. This H-mode phase could be destroyed only if the long wave length modes are triggered and developed. If this were to happen, the weakened effective shear strength (for small L_ξ) will be unable to stop their growth to moderate values. The growth of fluctuation would lead to the reemergence of the L-phase with a large characteristic L_ξ switching on once again the effectiveness of shear stabilization. Detailed comparison of the speculation with experiments requires more information on the $k_\perp$-spectrum localized at the steep gradient region of a quiescent H-phase, which seems to be crucial for acceptance of the present methodology (the local two-point correlation theory)[7–9] as the theoretical basis on which the mechanism of shear suppression on turbulences is engineered.

In summary, using the same methodology as the previous authors,[7–9] but introducing the self-consistent constraint on the diffusion coefficient to be a functional of appropriate turbulence levels, we have reinvestigated the turbulence suppression due to shear flow. It is found that: (1) Shaing *et al.*'s results follow in the small shear limit, (2) the $(-2/3)$ scaling found by Biglari *et al.*, is correct only if the diffusion coefficient is independent of turbulence levels; (3) the turbulence suppression mainly pertains in the moderate shear flow regime; and (4) the shear flow suppression becomes much more (less) effective for flattened (steep) equilibrium gradient.

Acknowledgment

The work was supported by the U.S. Department of Energy contract no. DE-FG05-80ET-53088 and no. DE-FG05-80ER-53266.

References

1. ASDEX TEAM, Nucl. Fusion **29**, 1959 (1989).

2. R.D. Stambaugh, S.M. Wolfe, R.D. Hawryluk, J.H. Harris, H. Biglari, S.C. Prager, R.J. Goldston, R.J. Fonck, T. Ohkawa, B.G. Logan, and E. Oktay, Phys. Fluids B **2**, 2941 (1990).

3. K.H. Burrel, T.N. Carlstron, E.J. Doyle, P. Gohil, R.J. Groebner, T. Lehecka, N.C. Luhmann, H. Matsumoto, T.H. Osborne, W.A. Peebles, and R. Philipona, Phys. Fluids B **2**, 1405 (1990).

4. R.J. Taylor, M.L. Brown, B.D. Fried, H. Grote, J.R. Liberati, G.J. Morales, and P. Pribyl, Phys. Rev. Lett. **63**, 2365 (1990).

5. R.J. Groebner, K.H. Burrel, and R.P. Seraydarian, Phys. Rev. Lett. **64**, 3015 (1990).

6. Ch.P. Ritz, H. Lin, T.L. Rhodes, and A.J. Wootten, Phys. Rev. Lett. **65**, 2543 (1990).

7. K.C. Shaing, G.S. Lee, B.A. Carreras, W.A. Houlberg, E.C. Crume, Jr., in *Proceedings of the Twelfth International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Nice, 1988* (International Atomic Energy Agency, Vienna, 1989), Vol. **II**, p. 13.

8. H. Biglari, P.H. Diamond, and P.W. Terry, Phys. Fluids B **2**, 1 (1990).

9. K.C. Shaing, E.C. Crume, Jr., and W.A. Houlberg, Phys. Fluids B **2**, 1492 (1990).

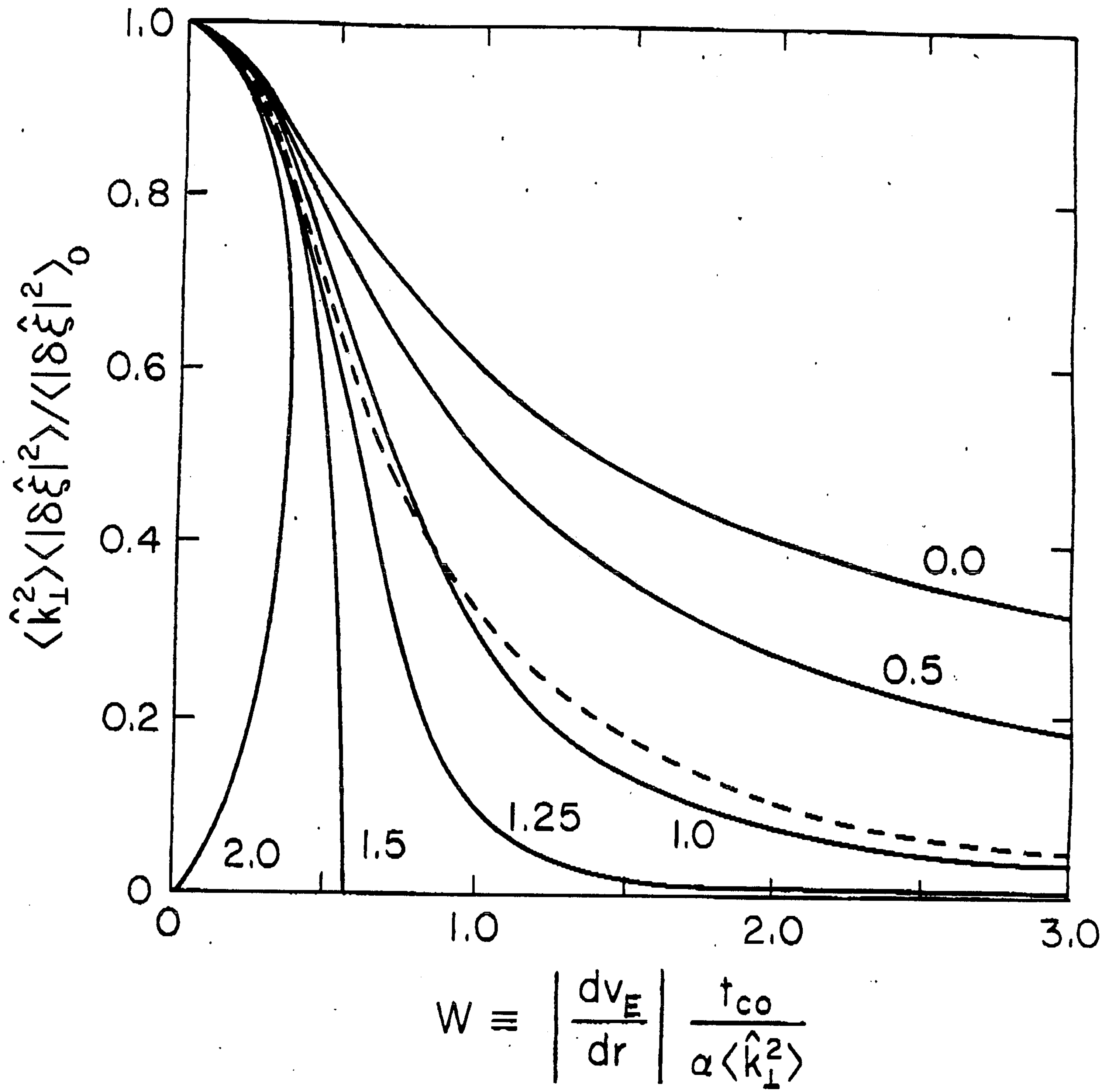
10. T.H. Dupree, Phys. Fluids **15** , 334 (1972).

11. F.L. Hinton, Phys. Fluids B **3**, 696 (1991).

12. R. Philipona and C.L. Rettig, private communication.

Figure Caption

1. The numerical solution of Eq. (4) for $P^{-1} \equiv \left\langle \hat{k}_\perp^2 \right\rangle \left\langle |\delta\hat{\xi}|^2 \right\rangle / \left\langle |\delta\hat{\xi}|^2 \right\rangle_0$ versus $W \equiv |dv_E/dr|t_{c0}/\alpha \left\langle \hat{k}_\perp^2 \right\rangle$ for various γ's (shown by the corresponding solid lines). The dashed line is the interpolated solution, given by Eq. (7) for $\gamma = 1$ only.



1.0
0.8
0.6
0.4
0.2
0
0
1.0
2.0
3.0
0.0
0.5
1.0
1.25
1.5
2.0
$\langle \hat{k}_\perp^2 \rangle \langle |\delta\hat{\xi}|^2 \rangle / \langle |\delta\hat{\xi}|^2 \rangle_0$
$W \equiv \left| \frac{dv_E}{dr} \right| \frac{t_{co}}{\alpha \langle \hat{k}_\perp^2 \rangle}$

INSTITUTE FOR FUSION STUDIES
THE UNIVERSITY OF TEXAS AT AUSTIN
RLM 11.218
AUSTIN, TEXAS 78712-1060
U.S.A.