INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-525 IFSR #525

Self-Consistent Radial Sheath in Ignited Plasmas

H. XIAO, R. CARRERA, and R.D. HAZELTINE
Department of Physics and
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

November 1991

THE UNIVERSITY OF TEXAS

AUSTIN

Self-Consistent Radial Sheath in Ignited Plasmas

H. Xiao, R. Carrera,[a] and R.D. Hazeltine
Department of Physics and Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

The radial sheath structure in ignited plasmas is studied based on the self-consistent effects of alpha particle's guiding-center orbits and $\mathbf{E} \times \mathbf{B}$-induced toroidal rotation. The slowing-down alpha particle distribution determines the structure of the radial sheath through finite Larmor radius effects.

[a]Valley Research Corporation, 8868 Research Blvd., Austin, Texas 78758

I. Introduction

In the tokamak plasma edge there is a thin layer in which the plasma density drops dramatically. A strong radial potential sheath has been observed in this layer in several devices.[1] In the H-mode of tokamak operation, a strong radial electric field has been predicted[2] and observed.[3,4] The reason for the formation of an axial sheath in the shadow of a tokamak limiter (scrape-off layer) is that the electrons are lost very quickly by directly following the field line and colliding with the limiter. But the ions are quite different; their gyroradii are much larger than the electron gyroradius, so that they are not lost by hitting the limiter. Instead they form an 'ion halo' which determines the structure of the radial sheath.[5]

In burning plasma the alpha particles produced in D-T fusion reactions with initial energy $E_0 = 3.5\,\mathrm{MeV}$ are the important constituent. After a slowing-down time ($\tau_{s\alpha} \simeq 0.4\,\mathrm{sec}$ for typical ignition conditions $T_i = T_e = 10\,\mathrm{keV}$ and $n_i = n_e = 10^{20}\mathrm{m}^{-3}$), they transfer their energies to the electrons through collisions, yielding the so called slowing-down distribution of alpha particles. After a thermalization time, $\tau_M \simeq 100\tau_{s\alpha}$, the slowing-down alpha particles become fully thermalized. The energy of the slowing-down alphas ranges roughly from 3MeV to 0.5 MeV, and their density can reach 1% of the total plasma density. Since the slowing-down alpha particles have much larger gyroradius (by more than one order of magnitude) than that of the back-ground ion, their effect on the radial sheath should be significant. After the alpha particle thermalization ($T_{\alpha_M} \simeq T_i$) and accumulation, the density of thermal alpha particles can reach 10% of the total plasma density. Thus the Helium ashes also have been considered have some effect on the radial sheath formation.

We consider here both thermal and slowing-down alpha particles in the plasma and use both Maxwellian and slowing-down distributions in the alpha particles in our analysis.[6]

The method used in this letter can also be used in analysis of edge electric field for neutral beam heated tokamak plasma. We don't discuss it here because of the length limitation.

In Sec. II we discuss the guiding-center orbits of the ions and alpha particles in the plasma. The collisional effects of the slowing-down alpha particles are studied in Sec. III. The radial sheath equation is derived from the Poisson equation and orbit squeeze condition in Sec. IV. The solution of the sheath equation is discussed in Sec. V.

II. Guiding-Center Orbits

Because the ion Larmor radius is much larger than the electron Larmor radius ($\rho_i \gg \rho_e$), the ion and the slowing-down alpha particle densities in a magnetized plasma are relatively fuzzy. The ion density cannot follow sharp changes of the electron density, especially in the edge layer. If the plasma induces a large d^2n_e/d^2r at its border, a charge halo will be produced.

For a toroidal magnetically confined plasma, an intensive halo can be induced by banana motion of the ion and alpha particles. This effect should be stronger than the conventional gyro-motion because the banana width is much larger than the gyroradius. The banana radial width can be expressed as

$$\Delta r_b \simeq \left(\frac{r}{R}\right)^{1/2} \rho_p \simeq \rho_p \gg \rho$$

where ρ_p is the gyroradius in the poloidal field, and ρ is the gyroradius in the toroidal field. $\rho_p/\rho = B/B_p = Rq/r \sim 10$ in typical tokamaks. Here R is the major radius, r is the minor radius and q is the safety factor.

We use the large aspect ratio approximation, $r/R \ll 1$, and suppose that the poloidal sheath potential variation is relatively small. In this case we can following the previous analysis of the tokamak orbits.[7,8]

The angular momentum of the guiding-center of the alpha particles is

$$p_\zeta = mRv_{||} - \frac{ZeX}{c} \tag{1}$$

where ζ denotes the toroidal angle, m is the particle mass, $v_\parallel$ is the parallel velocity, Z is the ion charge number, χ is the poloidal flux and $B_p = \chi'/R$. In an axisymmetric system p_ζ is invariant. In the large aspect ratio approximation we have:

$$v_{\parallel\zeta} \simeq v_\parallel \ .$$

Poloidal guiding-center motion results from streaming as well as poloidal $\mathbf{E} \times \mathbf{B}$ drifts,

$$v_p = \left(\frac{B_p}{B}\right) v_\parallel + V_E = \left(\frac{B_p}{B}\right) v_\parallel - \frac{cE_r}{B} \ .$$

The $\mathbf{V}_E \equiv c\mathbf{E} \times \mathbf{B}/B$ term is retained here because the $v_\parallel$ is reduced by an inverse aspect ratio.

At the bounce points the trapped particles satisfy $v_p(r_b, \theta_b) = 0$, where r_b and θ_b are the radius and the poloidal angle at the bounce point respectively. We define

$$U(r) = \frac{cE_r}{B_p}.$$

then

$$v_\parallel(r_b, \theta_b) = U(r_b) \ .$$

Using the condition:

$$\Delta p_\zeta = 0 \tag{2}$$

and the local approximation

$$-\frac{Ze\Delta\phi}{m} = -\frac{Ze}{m}\left(\phi_b + \phi_b'\Delta r + \frac{\phi_b''\Delta r^2}{2} + \cdots\right) = \Omega_p \left(U_b \Delta r + \frac{U_b'\Delta r^2}{2} + \cdots\right) \tag{3}$$

we obtain the equation:

$$\left(\Delta r - \frac{v_\parallel \Delta r}{\Omega_p R}\right)^2 - \frac{U_b'}{\Omega_p}\Delta r^2 = \frac{2\Delta r}{m\Omega_p^2 R}\left(\mu B + \frac{1}{2} m v_\parallel^2\right) \ . \tag{4}$$

Let Δr_0 denote the conventional banana width:

$$\Delta r_0 = \Delta r(U_b' = 0) \ .$$

The squeezed orbit is described by

$$\Delta r^2 = \frac{\Delta r_0^2}{S}$$

with

$$S = \left|1 - \frac{U_b'}{\Omega_p}\right| . \tag{5}$$

For deuterium ions $\Omega_{pi} = \Omega_{p\alpha}$, so that $S_i = S_\alpha = -1 - \frac{\rho_{pi}^2 Z_i e\phi''}{2T_i}$.

III. Collisional Effect

Coulomb collisions have important effects on the formation of the radial sheath. There are two types of effects: orbit randomization (when effective collision frequency ν_{eff} exceeds the banana bounce frequency ω_b) and rotational relaxation (which more importantly determines the space charge even when $\nu_{\text{eff}} \ll \omega_b$).

To investigate the rotation effect on the slowing-down alpha particle we follow the argument of Hinton and Wong.[9] First consider the zeroth order Fokker-Planck equation in a reference frame with velocity $\mathbf{u}_0$:

$$\frac{\partial f_0}{\partial t_0} + \left(v_\parallel \hat{b} + \mathbf{u}_0\right) \cdot \nabla f_0 - \frac{e}{m} \hat{b} \cdot \nabla \Phi_0 + \left[\frac{\partial \mathbf{u}_0}{\partial t_0} + \left(v_\parallel \hat{b} + \mathbf{u}_0\right) \cdot \nabla \mathbf{u}_0\right] \cdot \hat{b} \frac{\partial f_0}{\partial v_\parallel}$$

$$+ \frac{v_\perp^2}{2} \left(\nabla \cdot \hat{b}\right) \left[\frac{\partial f_0}{\partial v_\parallel} - \frac{v_\perp}{v_\parallel} \frac{\partial f_0}{\partial v_\perp}\right] - \frac{v_\perp}{2} \left[\nabla \cdot \mathbf{u}_0 - \hat{b} \cdot \nabla \mathbf{u}_0 \cdot \hat{b}\right] \frac{\partial f_0}{\partial v_\perp} = \mathrm{C}(f_0) \tag{6}$$

where $\hat{b} = \mathbf{B}/B$ and $\mathrm{C}(f_0)$ is the lowest order collision operator for alpha particle with distribution f_0. For the slowing-down distribution, f_0 can be written as:[10]

$$f_0(v) = \frac{n_\alpha(r)}{\ln\left(1 + \frac{v_\alpha^3}{v_c^3}\right)} \frac{H(v_\alpha - v)}{v^3 + v_c^3} \tag{7}$$

where v is the magnitude of the particle velocity, $v_\alpha = (2E_\alpha/m_\alpha)^{1/2}$, $v_e = (2T_e/m_e)^{1/2}$, $v_c = (3\pi^{1/2} m_e Z_1/4m_\alpha)^{1/3} v_e$ is the so-called critical velocity, where $Z_j \equiv \Sigma_{j\neq e}(n_j m_\alpha Z_j^2/n_e m_j)$, Z_j is the charge number, and $H(x)$ is the usual Heaviside step function.

Since $\partial f_0/\partial t_0$, $\mathbf{C}(f_0) = 0$, and

$$\frac{\partial f_0}{\partial v_\parallel} - \frac{v_\perp}{v_\parallel}\frac{\partial f_0}{\partial v_\perp} = 0$$

we can use the relation

$$\frac{1}{f_0}\frac{\partial f_0}{\partial v} = \frac{3v^2}{v^3 + v_c^3}$$

to rewrite the Fokker-Planck equation as

$$\frac{1}{n_\alpha}\left(v_\parallel \widehat{b}\cdot\nabla n_\alpha + \mathbf{u}_0\cdot\nabla n_\alpha\right) - \left\{\left[\frac{e}{m}\widehat{b}\cdot\nabla\Phi_0 + \left(v_\parallel\widehat{b} + \mathbf{u}_0\right)\cdot\nabla\mathbf{u}_0\cdot\widehat{b}\right]v_\parallel\right.$$

$$\left. + \frac{v_\perp^2}{2}\left[\nabla\cdot\mathbf{u}_0 - \widehat{b}\cdot\nabla\mathbf{u}_0\cdot\widehat{b}\right]\right\}\frac{3v}{v^3 + v_c^3} = 0\ . \tag{8}$$

By comparing the coefficients of the v^0, $v_\parallel$, $v_\parallel v$, $v_\parallel^2 v$, v^3 and $v_\parallel^3 v$ terms, we obtain:

$$\mathbf{u}_0\cdot\nabla n_\alpha = 0 \tag{9}$$

$$\widehat{b}\cdot\nabla n_\alpha = 0 \tag{10}$$

$$\frac{e}{m}\widehat{b}\cdot\nabla\Phi_0 + \mathbf{u}_0\cdot\nabla\mathbf{u}_0\cdot\widehat{b} = 0 \tag{11}$$

$$\widehat{b}\cdot\nabla\mathbf{u}_0\cdot\widehat{b} + \frac{1}{3}\nabla\cdot\mathbf{u}_0 = 0 \tag{12}$$

$$\nabla\cdot(n_\alpha\mathbf{u}_0) = 0\ . \tag{13}$$

Assuming the form $\mathbf{u}_0 = \omega R\widehat{e}_\phi + F\mathbf{B}$, it is easy to prove that $F = 0$. Thus we have:

$$\mathbf{u}_0 = \omega(\Psi)R\widehat{e}_\phi\ . \tag{14}$$

Therefore, in the slowing-down distribution the zeroth-order flow velocity is only allowed in the toroidal direction, $\mathbf{u}_0 = U(r)$.

Equation (6) is zeroth-order with respect to the ratio of the radial orbit width to the gradient scale length. Thus the function $\omega(\Psi)$ in Eq. (14) must be nearly constant over the width of a banana orbit. It will appear that this circumstance is consistent with orbital squeezing.

IV. Radial Sheath Equation in an Ignited Plasma

The density of the ions and the alpha particles near the plasma edge can be obtained following Ref. 5. From the orbit equation and the local approximation we have

$$r - r_0 = \pm \frac{v'_{\parallel}}{\Omega_p S} \tag{15}$$

where $v'_{\parallel} = v_{\parallel} - U(r)$. Then the normalized velocity limit can be found as

$$\begin{aligned} x_m &= |a-r|S/\rho_{pi} && \text{for ions} \\ y_M &= |a-r|S/\rho_{ps} && \text{for slowing-down } \alpha\text{-particles} \\ x_M &= |a-r|S/\rho_{pM} && \text{for thermal } \alpha\text{-particles} \end{aligned}$$

where $\rho_{\alpha s}$ and ρ_{α_M} are the poloidal Larmor radii of the slowing-down and thermal alpha particles respectively.

The ion and alpha particle densities can be written as:

$$n = \int g(\mathbf{v}, r) d\mathbf{v} \ . \tag{16}$$

For simplicity, we consider:

$$g = \begin{cases} 0 & \text{for } r > a \\ \dfrac{a-r}{r_n} f_0 & \text{for } r < a \end{cases}$$

with

$$f_0 = \frac{n_0}{\pi^{\frac{3}{2}} v_{thi}^3} \exp\left(-\frac{v^2}{v_{thi}^2}\right) + \frac{n_{M0}}{\pi^{\frac{3}{2}} v_{th\alpha}^3} \exp\left(-\frac{v^2}{v_{th\alpha}^2}\right) + \frac{3}{4\pi} \frac{n_{s0}}{\ln(1+v_\alpha^3)} \frac{H(v_\alpha - v)}{v^3 + v_c^3} \tag{17}$$

where $v = \sqrt{v_\perp^2 + v_\parallel'^2}$, n_0, n_{s0} and n_{M0} are the total densities of the ions, the slowing-down alpha particles and the thermal alpha particles, respectively. Here we assumed that ions and thermal alpha particles satisfy the Maxwellian distribution with toridal rotation[5,9] and the slowing-down alpha particles satisfy the slowing-down distribution with toridal rotation obtained in Eq. (14). The total density can be written as

$$n = n_i + 2n_{\alpha s} + 2n_{\alpha_M}$$

where the ion density is[5]

$$n_i = \frac{n_0}{2r_n}\left[a - r + |a-r|\,\mathrm{erf}\,(Sx) + \frac{\rho_{pi}}{S\sqrt{\pi}}\exp(-S^2x^2)\right] \tag{18}$$

the slowing-down alpha particle density is

$$n_{\alpha s} = \frac{3n_{s0}(a-r)}{4\pi r_n \ln(1+y_\alpha^3)}\int_{y_m}^{y_\alpha} dy\frac{2\pi y^2}{1+y^3}\int_{\frac{y_m}{y}}^{1} d\zeta\left(1-\frac{y}{y_m}\zeta\right) \tag{19}$$

and the thermal alpha particle density is

$$n_{\alpha_M} = \frac{n_{M0}}{2r_n}\left[a - r + |a-r|\,\mathrm{erf}\,(x_M) + \frac{\rho_{pM}}{S\sqrt{\pi}}\exp(-{x_M}^2)\right] \tag{20}$$

with $y = v/v_c$, $\zeta = v_{||}/v$, $y_\alpha = v_\alpha/v_c$, $x = |a-r|/\rho_{pi}$, $x_M = (\rho_{pi}/\rho_{\alpha_M})Sx$ and $y_m = y_\alpha y_M = (\rho_{pi}/\rho_{p\alpha})y_\alpha Sx$. Where

$$\mathrm{erf}\,(x) = \left(\frac{2}{\sqrt{\pi}}\right)\int_0^x \exp(-u^2)du$$

is the error function.

We use the following model for the electron density distribution profile[5]:

$$n_e = (n_0 + 2n_{M0})\frac{a-r}{r_n}\Theta(a-r)\ . \tag{21}$$

This density distribution is chosen to ensure the plasma quasi-neutrality inside of the plasma far from the plasma edge ($|a-r| \gg r_n$). Using Poisson's equation

$$\phi'' = -4\pi\, e(n_i + 2n_{\alpha s} + 2n_{\alpha_M} - n_e) \tag{22}$$

and the expression:

$$S = -1 - \frac{\rho_{pi}^2\, Z_i e\phi''}{2T_i}$$

we obtain the sheath equation

$$\delta S(S+1) = \frac{1}{\sqrt{\pi}}\exp(-S^2x^2) - Sx\,\mathrm{erf}\,(Sx)$$

$$+\frac{3n_{s0}}{n_0}\frac{\rho_{\alpha s}}{\rho_{pi}}\frac{1}{\ln(1+y_\alpha^3)}\left[\frac{2y_m}{3}\ln\frac{1+y_\alpha^3}{1+y_c^3} + \frac{1}{6}(1+y_m^2)\ln\frac{(1+y_\alpha)^2(y_m^2-y_m+1)}{(1+y_m)^2(y_\alpha^2-y_\alpha+1)}\right.$$

$$-y_\alpha + y_m + \frac{1}{\sqrt{3}}\left(\arctan\frac{2y_\alpha - 1}{\sqrt{3}} - \arctan\frac{2y_m - 1}{\sqrt{3}}\right)\Bigg]$$

$$+\frac{2n_{M0}}{n_0}\frac{\rho_{pM}}{\rho_{pi}}\left[\frac{1}{\sqrt{\pi}}\exp(-x_M^2) - x_M\,\mathrm{erf}\,(x_M)\right] \qquad (23)$$

where δ is the sheath parameter:

$$\delta = \left(\frac{4r_n}{\rho_{pi}}\right)\left(\frac{\lambda_D}{\rho_{pi}}\right)^2\left(\frac{T_i}{T_e}\right)$$

and λ_D is the Debye length with density n_0

$$\lambda_D^2 = \frac{T_e}{4\pi n_0 e^2}\ .$$

We can obtain $S = S(x)$ from the solution of Eq. (23).

V. Discussion

Solving the Eq. (23) numerically and using $S = -1-\rho_{pi}^2\, Z_i e\phi''/2T_i$, we obtain the electric field and the sheath potential near the plasma border. Calculation with $\delta = 10^{-2}$, $n_{\alpha s}/n_0 = 10^{-2}$, $n_{\alpha M}/n_0 = 10^{-1}$, $\rho_{\alpha s}/\rho_{pi} = 10^2$ and $\rho_{\alpha M}/\rho_{pi} = 1/\sqrt{2}$. Since no experimental results are available on the ignited tokamak plasmas, the boundary conditions have been chosen as $\phi(0) = 11$ and $E(0) = 0$ in arbitrary units. Some results are shown in Fig. 1 and 2, which represent the profiles of the potential ϕ, electric field E and squeezing factor S when slowing-down and thermal alpha particles (Fig. 1) and only slowing-down alpha particles (Fig. 2) are taken into account in the calcultion. We find that the alpha particles, especially the slowing-down alpha particles, have a substantial effect on the radial sheath of an ignited tokamak plasma. The appearance of alpha particles in the ignited plasma increases the squeezing effect of the guiding-center orbits of both ions and alpha particles, changes the edge radial sheath potential profile and increases the edge radial electric field (in absolute value). The major reason for the increase in the squeezing factor is that the slowing-down alpha particles have much larger gyroradius than the ions thus they interact with the radial electric field more effectively. This result is important because the squeezing of the alpha particle orbits can directly affect the wall bombardment and the increase of the radial electric field near the plasma border can weaken the alpha particle radial loss.

From the calculation we also find that the thermal alpha particles have very little effect on the radial sheath, because their gyroradius is almost the same as the ion gyroradius, while their density is much lower. We find that when the density slowing-down alpha particles is a few percent of total density, its effect is on the radial sheath is substantial. If the density is very low, for instance, less than 0.1% of the total density, the effect is weak.

K. Itoh *et al.* recently have studied the effect of a radial electric field near the plasma edge on the loss of the ripple trapped alpha particles in a burning tokamak.[11] They have found that when the potential difference is of the order of the plasma temperature, the peaking of the localized heat deposition on the first wall becomes weaker, owing to the energy distribution of the ripple-trapped loss particles. Since the loss of alpha particles will increase the radial electric field, the self-consistent treatment of the radial electric field and alpha particle transport is needed for the better understanding of alpha particle transport in the ignited tokamak plasma.

References

1. Ritz, Ch P., Brower, D.L., Rhodes, T.L., Bengtson, R.D., Levinson, S.J., Luhmann, N.C., Jr., Peebles, W.A., Powers, E.J., Nucl. Fusion **27**, 1125 (1987).

2. Itoh, S.-I., Itoh, K., Phys. Rev. Lett. **60**, 2276 (1988).

3. Groebner, R.J., Burrel, K.H., Seraydarian, R.P., Phys. Rev. Lett. **64**, 3015 (1990).

4. Ida, K., Hidekuma, S., Miura, Y., Fijita, T., Mori, M., Hoshino, K., Suzuki, N., Yamaguchi, T., JFT-2M Group, Phys. Rev. Lett. **65**, 1364 (1990).

5. Hazeltine, R.D., Phys. Fluids B **1**, 2031 (1989).

6. Rewoldt, G., Phys. Fluids **31**, 3727 (1988).

7. Berk, H.L., Galeev, A.A., Phys. Fluids **10**, 441 (1967).

8. Furth, H.P., and Rosenbluth, M.N., in *Plasma Physics and Controlled Nuclear Fusion Research* (IAEA, Vienna, 1969) Vol. 1, p. 821.

9. Hinton, F.L., Wong, S.K., Phys. Fluids **28**, 3062 (1985).

10. Graffey, J.D. Jr., J. Plasma Phys. **16**, 149 (1976).

11. Itoh, K., Sanuki, H., Itoh, S.-I., Tani, K., Effect of Radial Electric Field on α-Particle Loss in Tokamaks, NIFS-66, National Institute For Fusion Science, Nagoya, Japan (1990).

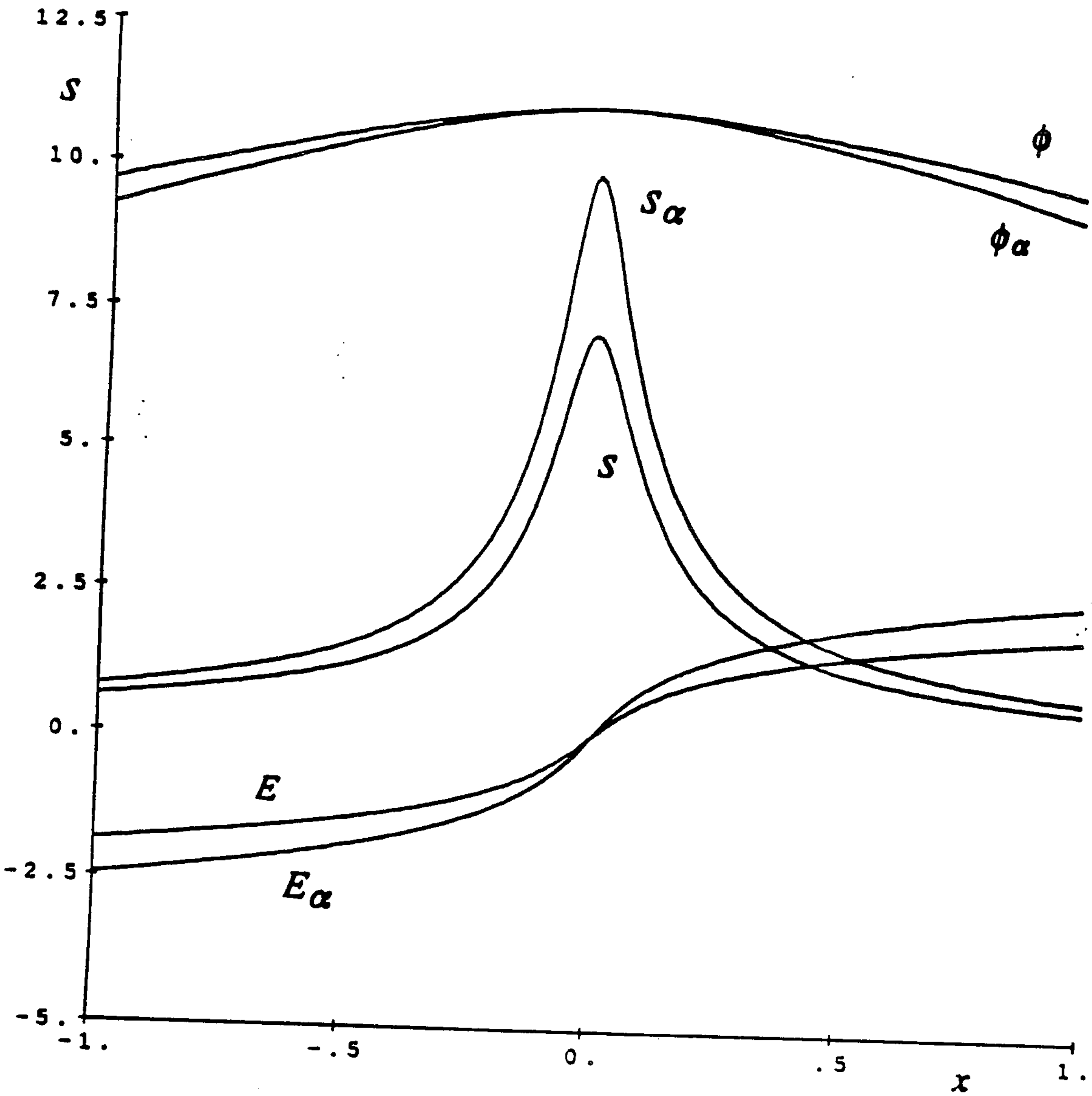

Fig. 1

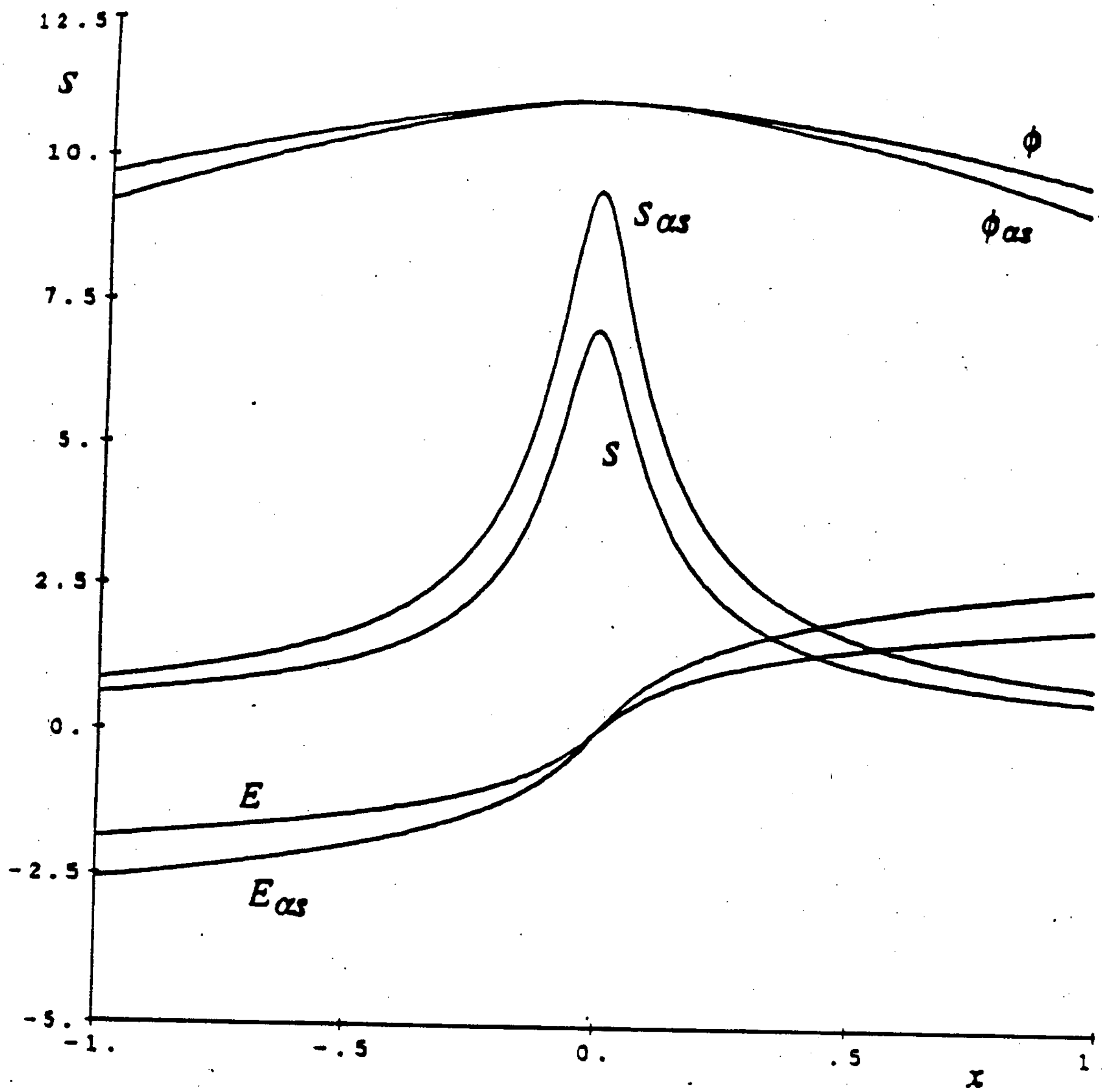

Fig. 2

INSTITUTE FOR FUSION STUDIES
THE UNIVERSITY OF TEXAS AT AUSTIN
RLM 11.218
AUSTIN, TEXAS 78712-1060
U.S.A.