INSTITUTE FOR FUSION STUDIES

DOE/ET-53088-516 IFSR #516

Stabilization of the $m = 1$ Tearing Mode by Resonance Detuning

A. THYAGARAJA
Theory Division
Culham Lab.
Abingdon, Oxon, UK
and
R.D. HAZELTINE, and A.Y. AYDEMIR
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

August 1991

THE UNIVERSITY OF TEXAS

AUSTIN

Stabilization of the $m = 1$ Tearing Mode by Resonance Detuning

A. Thyagaraja,[a] R. D. Hazeltine, and A. Y. Aydemir,
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

Abstract

The possibility of stabilizing the $m = 1$ tearing mode by rapidly oscillating the resonance point $r_s(t)$ about its mean position is examined. The calculations are carried out for both externally controlled, coherent oscillations of $r_s(t)$, as well as those resulting from turbulent plasma motions. Complete stabilization is possible in the coherent case, while turbulent fluctuations may yield substantial reduction in the growth rate. The technique seems to apply to any linear mode that depends upon local properties around a resonant surface.

[a] Theory Division, Culham Lab., Abingdon, Oxon, United Kingdom

I. Introduction

Sawtooth oscillations are important in the study of reactor-relevant tokamak physics. Various experimental methods of profile control using a range of heating and current drive techniques have been used to achieve sawtooth stabilization.[1,2] The purpose of the present, theoretical study is to point out a possible new method for stabilizing the $m = 1$ tearing modes.

The $m = 1$ mode, as is well known, is different from higher m tearing modes in several important respects. Its linear stability in a cylinder depends upon the shear parameter $\widehat{s} = r_s q'(r_s)$ (where $q(r_s) = 1$) rather than the global Δ' characteristic of other tearing modes. The possibility exists of stabilizing this mode by reducing the shear[3] either by external means or by the plasma itself via some transport mechanism. It has recently been shown[4] that when the shear parameter $\widehat{s}$ is sufficiently low, and when certain other conditions can be met, the weakly unstable linear resistive internal kink may saturate to form finite amplitude, helical island structures reminiscent of 'snakes.'[5]

In the present work, we investigate a second possibility. Instead of trying to lower $\widehat{s}$, we examine if the resonance at $r = r_s$, where $q = 1$, can be detuned by rapidly *oscillating* the resonant point in time about its mean position. This is analogous physically to frustrating the system from achieving its resonant linear growth rate by rapidly moving the resonant point out of resonance. This process is also analogous to finite Larmor radius stabilization due to the averaging of the potentials seen by gyrating charged particles.

The calculation is carried out for both externally controlled, regular oscillations of $r_s(t)$, as well as those resulting from plasma motions themselves. We discuss two possible experimental scenarios and some typical numbers to illustrate the results. We also present numerical calculations, using the reduced magnetohydrodynamic equations, which confirm and extend approximations used in the analytic theory.

The approach appears to apply in any linear mode for which the dispersion relation and the mode structure are strongly dependent upon local properties around a resonant surface. Extensions to modes other than $m = 1$ resistive tearing will be left to a later work.

II. Formulation of the Problem

Our starting point is the low-beta reduced MHD (RMHD) equations[6] of motion. For the narrow boundary layer associated with localized tearing, the equations are effectively one-dimensional. The linearized form is given by[7]

$$\frac{\partial \psi}{\partial t} + ik_{\|}\phi = \eta\psi \tag{1}$$

$$\frac{\partial \phi''}{\partial t} + ik_{\|}\psi'' = 0 \tag{2}$$

where t is measured in units of poloidal Alfvén time $\tau_A \equiv R/v_A$; $v_A = B_t/\sqrt{4\pi\rho}$, B_t is the toroidal field, ρ is the plasma density, and R is the the major radius. The coördinate $x = r - r_s$ measures the (normalized) distance from the rational surface at r_s, such that $q(r_s) = 1$. Primes denote differentiation with respect to x. The normalized resistivity η is the ratio τ_A/τ_R, where τ_R is the resistive diffusion time-scale defined by $\tau_R^{-1} = c^2\eta^*/4\pi r_s^2$, where η^* is the dimensional (in cgs units) resistivity. In the above equations, ψ and ϕ denote the usual $m = n = 1$ perturbed poloidal flux and velocity stream functions, and $k_{\|} \equiv \frac{1}{q} - 1$ is a function of x, q being the equilibrium safety factor.

Oscillations in the resonant point are introduced by letting

$$k_{\|}(x,t) = k_{\|}' \{x - \xi(t)\} \ , \tag{3}$$

where $\xi(t)$ is the motion of the resonance, and $k_{\|}' \equiv dx/k_{\|}$ at $r = r_s$. Below we let $s \equiv -k_{\|}'$.

A basic assumption of our analysis is that motion of the rational surface is rapid compared to the classical growth rate:

$$\omega \gg \gamma_c \ , \tag{4}$$

where ω is a characteristic "jitter" frequency, $\omega \sim d\ln\xi/dt$, and $\gamma_c^3 = s^2\eta$ is the standard $m = 1$ tearing mode growth rate in the absence of $\xi(t)$. The jitter-modified growth rate will turn out to be complex, $\gamma = \gamma_r + i\gamma_i$. While $\gamma_r \le \gamma_c$ also satisfies (4), the oscillation frequency γ_i need not: $\gamma_i \sim \omega$.

We assume that the fields ψ and ϕ have the form

$$f(t) = e^{\gamma t}(\overline{f} + \widetilde{f}) \ , \tag{5}$$

where $\overline{f}$ is independent of time. We also assume the existence of an 'intermediate' time-scale T such that

$$\omega \gg \frac{1}{T} \gg \gamma_c \ , \tag{6}$$

and define the time-average of the function $f(t)$ by

$$\langle f(t) \rangle \equiv \frac{1}{T}\int_{-T/2}^{T/2} f(t)dt \ . \tag{7}$$

In the coherent case we can choose $T = 1/\omega$. Note that $\langle f(t) \rangle \neq \overline{f}$.

III. Analysis of the Coherent Case

With the notations introduced, the equations of motion take the following form:

$$\gamma(\overline{\psi} + \widetilde{\psi}) + \frac{\partial\widetilde{\psi}}{\partial t} - isx(\overline{\phi} + \widetilde{\phi}) - \eta(\overline{\psi}'' + \widetilde{\psi}'') = -is\xi(\overline{\phi} + \widetilde{\phi}) \tag{8}$$

$$\gamma(\overline{\phi}'' + \widetilde{\phi}'') + \frac{\partial\widetilde{\phi}''}{\partial t} - isx(\overline{\psi}'' + \widetilde{\psi}'') = -is\xi(\overline{\psi}'' + \widetilde{\psi}'') \ . \tag{9}$$

After time averaging we get the 'slow time' equations,

$$\gamma\overline{\psi} - isx\overline{\phi} - \eta\overline{\psi}'' = -is\,\langle\xi\phi\rangle \tag{10}$$

$$\gamma\overline{\phi}'' - isx\overline{\psi}'' = -is\,\langle\xi\psi''\rangle \ . \tag{11}$$

The problem consists in evaluating the averages $\langle\xi\phi\rangle$ and $\langle\xi\psi''\rangle$ in terms of $\overline{\phi}, \overline{\psi}'', \omega$ and s, thus closing the system (10) and (11). By subtracting (10) and (11) from (8) and (9), respectively, we obtain the exact equations:

$$\frac{\partial\widetilde{\psi}}{\partial t} + \gamma\widetilde{\psi} - isx\widetilde{\phi} - \eta\widetilde{\psi}'' = -is(\xi\phi - \langle\xi\phi\rangle) \tag{12}$$

$$\frac{\partial\widetilde{\phi}''}{\partial t} + \gamma\widetilde{\phi}'' - isx\widetilde{\psi}'' = -is(\xi\psi'' - \langle\xi\psi''\rangle)\ . \tag{13}$$

Although linear, these equations are difficult to solve exactly due to the shear (isx) and resistive (η) terms. However, we anticipate here that $\widetilde{\phi}$ and $\widetilde{\psi}$ will derive their spatial structure from that of the averaged fields, $\overline{\phi}$ and $\overline{\psi}$. Thus, for example, in (12),

$$\eta\widetilde{\psi}'' \sim \left(\frac{\eta}{w^2}\right)\widetilde{\psi}\ ,$$

where w is the layer width associated with the linear tearing mode. Recalling that $\gamma_c \sim \eta/w^2$, we see that (4) allows us to ignore resistivity in (12). A similar argument shows the shear terms to be of order γ_c/ω also. Hence we consider the lowest order system:

$$\frac{\partial\widetilde{\psi}}{\partial t} \simeq -\gamma\widetilde{\psi} - is(\xi\phi - \langle\xi\phi\rangle) \tag{14}$$

$$\frac{\partial\widetilde{\phi}}{\partial t} \simeq -\gamma\widetilde{\phi} - is(\xi\psi - \langle\xi\psi\rangle)\ . \tag{15}$$

Note that (15) is derived from (13) by a double spatial integration.

To uncouple these equations, we introduce the fields,

$$u = \psi + \phi \tag{16}$$

and

$$v = \psi - \phi\ . \tag{17}$$

Then we find that $\widetilde{u}, \widetilde{v}$ satisfy,

$$\frac{\partial\widetilde{u}}{\partial t} = -(\gamma + is\xi)\widetilde{u} - is(\xi\overline{u} - \langle\xi u\rangle), \tag{18}$$

$$\frac{\partial \tilde{v}}{\partial t} = -(\gamma - is\xi)\tilde{v} + is(\xi\overline{v} - \langle \xi v \rangle) \ . \tag{19}$$

It is sufficient to solve for u since v can be obtained from u using symmetry: $v = u(s \to -s, \overline{u} \to \overline{v})$. We introduce the function $X(t)$ such that

$$\frac{dX}{dt} = \xi(t) \ . \tag{20}$$

Then Eq. (18) is straightforwardly integrated and we find

$$\tilde{u}(t) + \overline{u} = A(t)(is \langle \xi u \rangle + \gamma \overline{u}) \tag{21}$$

where

$$A(t) \equiv \int_{-\infty}^{t} dt' \exp\left[is(X(t') - X(t)) + \gamma(t' - t)\right] \ . \tag{22}$$

Now we average Eq. (21) to get

$$\overline{u} = is \langle \xi u \rangle \langle A(t) \rangle + \gamma \langle A(t) \rangle \overline{u} \tag{23}$$

or

$$\langle \xi u \rangle = -\frac{i\overline{u}}{s} \frac{(1 - \gamma \langle A \rangle)}{\langle A \rangle} \ . \tag{24}$$

From symmetry, we have

$$\langle \xi v \rangle = \frac{i\overline{v}}{s} \frac{(1 - \gamma \langle A \rangle)}{\langle A \rangle} \ . \tag{25}$$

Using Eqs. (16) and (17) in (24) and (25) gives

$$\left\langle \xi\tilde{\phi} \right\rangle = -\frac{i}{s} \left[\frac{(1 - \gamma \langle A \rangle)}{\langle A \rangle} \right] \overline{\psi} \tag{26}$$

$$\left\langle \xi\tilde{\psi} \right\rangle = -\frac{i}{s} \left[\frac{(1 - \gamma \langle A \rangle)}{\langle A \rangle} \right] \overline{\phi} \ . \tag{27}$$

Substituting in Eqs. (12) and (13), we finally get

$$\frac{1}{\langle A \rangle} \overline{\psi} - isx\overline{\phi} - \eta\overline{\psi}'' = 0 \tag{28}$$

$$\frac{1}{\langle A \rangle} \overline{\phi} - isx\overline{\psi}'' = 0 \ . \tag{29}$$

Thus, the new dispersion relation is extremely simple: It is obtained by replacing γ by $1/\langle A\rangle$ in the old one. Thus, the modified dispersion relation (for arbitrary $\xi(t)$) is simply

$$\frac{1}{\langle A\rangle} = \gamma_c \, . \tag{30}$$

We now evaluate $\langle A\rangle$ in the simple case where $\xi(t) = \xi_0 \cos\omega t$:

$$A(t) = \int_{-\infty}^{t} dt' \exp\left[\frac{is\xi_0}{\omega}(\sin\omega t' - \sin\omega t) + \gamma(t'-t)\right] . \tag{31}$$

Letting $\lambda \equiv s\xi_0/\omega$, and using the standard Bessel function relation,

$$e^{i\lambda \sin\omega u} = \sum_{n=-\infty}^{\infty} J_n(\lambda) e^{inu}, \tag{32}$$

we readily obtain,

$$\langle A(t)\rangle = \sum_{n=-\infty}^{\infty} J_n^2(\lambda)\left(\frac{i}{\omega}\right)\left(n + \frac{i\gamma}{\omega}\right)^{-1} \tag{33}$$

or

$$\langle A(t)\rangle \simeq \frac{J_0^2(\lambda)}{\gamma}\left[1 + \mathcal{O}\left(\frac{\gamma}{\omega}\right)\right] . \tag{33'}$$

It finally follows that

$$\gamma \simeq \gamma_c J_0^2(\lambda) \, , \quad \lambda \equiv \frac{s\xi_0}{\omega} \, . \tag{34}$$

Because $\gamma_i \simeq \omega$ in some cases, the exact form of (33) is generally needed.

Complete stabilization occurs for $\lambda = 2.4$. For typical large-tokamak parameters ($B_t \simeq 30\,\text{kG}$, $n \simeq 5\times10^{13}/\text{cm}^3$, $R = 300\,\text{cm}$, $r_s q' \simeq 0.1$), the jitter frequency ω needed to completely stabilize the $m = 1$ mode is on the order of a few kHz, for a jitter amplitude of $\xi_0 \lesssim 5\%$ of the minor radius.

IV. The 'Random Function' Approach

Now we turn to a more general case where $\xi(t)$ consists of an ensemble of functions $\xi(t) = \xi_0 \cos(\omega t + \theta)$ where the phases θ are uniformly distributed in $(0, 2\pi)$. We still assume that $\omega \gg \gamma_c$. Interpreting the time averages now to be ensemble averages, we find that the preceding analysis applies provided we set $\langle \quad \rangle \equiv 1/(2\pi) \int_0^{2\pi} d\theta$. For the present ξ_0 will be fixed. The equation for $A(t, \theta)$ thus becomes,

$$A(t,\theta) = \int_{-\infty}^{t} dt' \exp\left[i\lambda\left(\sin(\omega t' + \theta) - \sin(\omega t + \theta)\right) + \gamma(t' - t)\right] , \tag{35}$$

and we have

$$\langle A(t,\theta)\rangle_\theta = \frac{J_0^2}{\gamma} \left(\frac{s\xi_0}{\omega}\right) , \tag{36}$$

with no approximation.

Now we allow ξ_0^2 to be a random variable, distributed exponentially with mean σ^2. Thus, $P(\xi_0^2) \equiv e^{-\xi_0^2/\sigma^2}/\sigma^2$. The resulting integral can be evaluated to give

$$\langle A(t,\theta,\xi_0)\rangle_{\theta,\xi_0} = \frac{1}{\gamma}\, e^{-\frac{s^2\sigma^2}{2\omega^2}}\, I_0\left(\frac{s^2\sigma^2}{2\omega^2}\right). \tag{37}$$

Thus,

$$\gamma = \gamma_c \exp\left\{-\frac{s^2\sigma^2}{2\omega^2}\right\} I_0\left(\frac{s^2\sigma^2}{2\omega^2}\right) . \tag{38}$$

Finally, asymptotically expanding $I_0(z)$ for large $\frac{s^2\sigma^2}{2\omega^2}$, we obtain

$$\gamma = \gamma_c \left(\frac{\omega}{s\sigma}\right) \frac{1}{\sqrt{\pi}} . \tag{39}$$

We observe that to get total stabilization, a coherent amplitude (except for possibly random phasing) is necessary.

We could alternatively consider $\xi(t)$ to be of the 'random Fourier series' form,

$$\xi(t) = \sum_{n=1}^{\infty} \xi_n \cos(\omega_n t + \theta_n) \tag{40}$$

where $\{\omega_n\}$ is an increasing sequence of frequencies. Provided ξ_n, ω_n are not random variables, Eq. (36) may be generalized to

$$\langle A(t;\theta_1,\ldots,\theta_n)\rangle_{\theta_1,\ldots} = \prod_{n=1}^{\infty} \frac{J_0^2\left(\frac{s\xi_n}{\omega_n}\right)}{\gamma} . \tag{41}$$

Finally and briefly, we consider the general case where $\xi(t)$ is a Gaussian random function of time, with a correlation time τ, and a correlation function $C(u-u')$ given by

$$C(u-u') = \langle \xi(u)\xi(u') \rangle \equiv \left\langle \xi_0^2 \right\rangle \exp\left(-\frac{|\Delta u|}{\tau}\right) \cos\omega\Delta u \ , \tag{42}$$

$$\Delta u = u - u' \ .$$

It follows from conventional cumulant expansion that

$$\langle \exp\left[is\left(X(t') - X(t)\right)\right]\rangle = \exp\left[-\frac{s^2}{2}\left\langle (X'-X)^2 \right\rangle\right] \tag{43}$$

$$\langle (X'-X) \rangle^2 = \int du \int du' \, C(u-u') = \alpha \, g\left(\frac{\Delta t}{\tau}\right) \ , \tag{44}$$

where

$$\alpha = 2s^2 \left\langle \xi^2 \right\rangle \tau^2 \left(1+\omega^2\tau^2\right)^{-1} \ , \quad \Delta t \equiv t - t' \ , \tag{45}$$

and

$$g(w) \equiv w + e^{-w}\left(1+\omega^2\tau^2\right)^{-1}\left[\left(1-\omega^2\tau^2\right)\cos\omega\tau w - 2\omega\tau\sin\omega\tau w\right] - \frac{(1-\omega^2\tau^2)}{(1+\omega^2\tau^2)} \ . \tag{46}$$

We observe that $g(w) > 0$, and $g(w) \sim w^2$ for $w \to 0$. It follows that

$$\langle A \rangle = \tau \int_0^\infty dw \exp\left[-\gamma\tau w\right] \exp\left[-\alpha g(w)\right] \ . \tag{47}$$

Note that

$$\frac{\langle A \rangle}{\tau} = F\left(\gamma\tau \ , \ s^2\left\langle \xi^2 \right\rangle \tau^2 \ , \ \omega\tau\right) \ . \tag{48}$$

This form shows that τ can be scaled out of the dispersion relation. The 'reduced' dispersion relation takes the form,

$$G(\widehat{\gamma}, \widehat{k}, \widehat{\omega}) = \widehat{\gamma}_c \ , \tag{49}$$

where $\widehat{\omega} = \omega\tau$, $\widehat{\gamma} = \gamma\tau$, $\widehat{\gamma}_c = \gamma_c\tau$, and $\widehat{k} = \left[\alpha(1+\omega^2\tau^2)\right]^{1/2}$. If time is measured in units of τ, the dispersion properties are independent of τ, which merely amounts to recalibration of the clock.

The general integral in (46) is not elementary. We consider two illustrative special cases:

(a) $\widehat{\omega} \to 0$, $\widehat{k} \to 0$ ($\widehat{\gamma}$-finite):

$$\langle A \rangle \simeq \tau e^{\alpha} \int_0^1 dw\, w^{\alpha+\gamma\tau-1} \exp(-\alpha w) ,$$

which leads to $\langle A \rangle \simeq \tau(1+\alpha)/(\alpha+\gamma\tau)$, and

$$\gamma = \gamma_c - 2s^2 \left\langle \xi^2 \right\rangle \tau . \tag{50}$$

The growth rate is moderately reduced.

(b) $\tau \to \infty$, keeping ω fixed:

$$\widehat{\omega} \to \infty\,,\ \widehat{k} \to \infty \quad \gamma/\omega \ll 1\,,\ \alpha/\omega\tau \ll 1\,.$$

After some manipulations, we find

$$\langle A \rangle = \left(\gamma + \frac{\alpha}{\tau}\right)^{-1} e^{-\alpha}\, I_0(\alpha) \tag{51}$$

and

$$\gamma = \gamma_c\, I_0(\alpha)\, e^{-\alpha-\frac{\alpha}{\tau}} . \tag{52}$$

This is a mild generalization of (38).

V. Comparison with Numerical Simulations

Now we turn to a comparison of our analytic results with numerical solutions of the reduced MHD equations as an initial value problem. For brevity, only the 2-D coherent jitter case is considered. We let the equilibrium poloidal flux be given by

$$\psi_0(r,t) \equiv \psi_0^s(r) + \frac{\epsilon}{2}(1-r^2)\cos\omega t , \tag{53}$$

which leads to

$$k_{\|}(x,t) \equiv k_{\|}^{s}(x) + \epsilon \cos \omega t \ . \tag{54}$$

The static part of the equilibrium, determined by ψ_0^s above, is chosen such that $q_0 = 0.9, \quad q_l = 3$, and $k_{\|}'(r_s) = -0.97$. Note that the quantity $\epsilon = k_{\|}'\xi_0$ of the previous sections. The calculated growth rate of the $m = 1$ mode as a function of the jitter amplitude is shown in Fig. 1. A combination of two different jitter frequencies, and two values of γ_c, the classical growth rate, are considered: a) $\eta = 10^{-7}, \ \ \omega = 4\times 10^{-2}$, b) $\eta = 10^{-7}, \ \ \omega = 2\times 10^{-2}$, and c) $\eta = 10^{-6}, \ \ \omega = 4 \times \ 10^{-2}$. Since the dispersion relation of (30) and (33) has only a weak dependence on ω for $\omega \gg \gamma_c$, the results from these three different scans are expected to lie quite close to the same curve, as seen in Fig. 1.

Also shown in Fig. 1 is part of the numerical solution of the analytic dispersion relation in Eqs. (30) and (33). Evidently, for a given λ and ω, the dispersion relation has an infinite number of solutions of the form $\gamma = \gamma_n + i\ n\omega, \quad n = 0, 1, 2, \ldots \infty$. Initial value solutions of the RMHD equations, which converge on the fastest growing mode, find only the two lowest order branches ($n = 0$, and $n = 1$), as seen in the figure. For $\lambda \lesssim 0.7$, the $n = 0$ branch dominates, whereas for $0.7 \lesssim \lambda \lesssim 1.2$, the $n = 1$ branch has the higher growth rate. No unstable modes are found for $\lambda \gtrsim 1.2$ with the initial value code. There seems to be an approximate factor of two difference between the stability boundary calculated numerically and the one predicted by theory. We do not have an explanation for this anomaly. Note that this factor has been incorporated into the numerical solution of the dispersion relation in Fig. 1.

VI. Conclusions

We have used linear reduced MHD to study the effects of motion of the rational surface on $m = 1$ tearing stability. It is found that the growth rate of the tearing mode can be reduced, even in principle to zero, by coherent surface jitter at the appropriate frequency. Random motion of the rational surface is similarly, if less dramatically, stabilizing. The mechanism in both cases is detuning of the $k_{\|} = 0$ resonance, closely analogous to the finite Larmor radius (FLR) detuning of electrostatic modes, due to particle gyromotion. Our methods combine analytical (quasilinear) and numerical treatments, which are found to be in good agreement.

It is clear that the present investigation is neither exact nor exhaustive. In particular, while treating with case the $m = 0$, $n = 0$ component of an externally applied field perturbation, we have neglected other components, some of which might have serious nonlinear consequences. Perhaps more importantly the (low-beta) reduced MHD model neglects several effects, including plasma pressure corrections, FLR terms and kinetic processes. Nonetheless we find the results sufficiently striking, and the analysis sufficiently straightforward, to suggest further theoretical and experimental study.

In one respect, the conclusions presented here seem remarkably general. Notice that the quasilinear treatment given by Eqs. (14)–(22) is nearly model-independent. The particular resistive and shear terms that affect linear stability could be modified freely, with the addition perhaps of kinetic and FLR corrections, without changing the main conclusion: that γ, in the averaged equations of motion, is to be replaced by $1/\langle A \rangle$. What is essential is that the stability of the linear mode of interest be determined *locally*, by conditions near the resonant rational surface. This circumstance suggests that stabilization by resonance detuning might pertain to a number of instabilities, including for example, drift-tearing modes and even drift waves.

The most important problem not addressed in this paper is the practical issue of varying the safety factor so as to move the $q = 1$ surface in the manner required. We first note that there are always natural perturbations in the plasma (effects of global modes and local turbulence) which lead to some jitter. The results of the "stochastic/incoherent" case of our model suggest that such naturally occurring motions of the $q = 1$ resonance, although of possible experimental importance, cannot be expected to provide complete stabilization of the linear $m = 1$ tearing mode. Therefore we consider whether suitable particle/energy/current sources can be applied to the plasma by external means: rf heating/current drive suitably amplitude modulated and localized near $q = 1$.

An approach to this question uses conventional fluid equations, *given* the appropriate (possibly anomalous) transport coefficients and suitable fluctuating sources. It is sufficient to consider the linearized response to an oscillatory source. Since this system is nonresonant $(m = 0, n = 0)$ it is in principle possible to calculate the function $q(\gamma, t) \equiv q_0(r) + \delta q(\gamma, t) \equiv q_0(\gamma) + \delta\widehat{q}(\gamma)\cos\omega t$. The perturbation $\delta\widehat{q}(\gamma)$ will depend upon the transport properties and equilibrium profiles and of course will be limited by plasma dissipation; on the other hand, we emphasize that $\delta\widehat{q} \sim 10^{-2}$ seems sufficient for dramatic growth rate reduction. This problem can be investigated numerically.

It might be desirable to test the basic results derived here on resonance detuning directly by using, for example, an ecrh current source $\delta j(r, t)$ suitably localized near $q = 1$ and amplitude-modulated to give a harmonic perturbation to the $q = 1$ radius of a suitable amplitude. If our model is correct, this should detune the $m = 1$ resonance and thereby lower the growth rate of the resistive internal kink. We emphasize that there is no need to *eliminate* the mode. If the growth rate of the linear mode is sufficiently reduced, it is known[4] that nonlinear effects can lead to saturated island states.

References

1. D. J. Campbell, and JET Group, Phys. Rev. Lett. **60**, 2148 (1988).

2. F. X. Söldner, and ASDEX Group, Phys. Rev. Lett. **57**, 1137 (1986).

3. K. Hanada, H. Tanaka, M. Iida, S. Ide, T. Minami, M. Nakamura, T. Maekawa, Y. Terumichi, S. Tanaka, M. Yamada, J. Manickam, and R. B. White, Phys. Rev. Lett. **66**, 1974 (1991).

4. A. Thyagaraja and F. A. Haas, Phys. Fluids B **3**, 580 (1991).

5. A. Weller, A. D. Cheetham, A. W. Edwards, R. D. Gill, A. Gondhalekar, R. S. Granetz, J. Snipes, and J. A. Wesson, Phys. Rev. Lett. **59**, 2303, (1987).

6. H. R. Strauss, Phys. Fluids **19**, 134 (1976).

7. See for example, R. D. Hazeltine and J. D. Meiss, Physics Reports **121**, 90 (1985).

Figure Caption

1. Normalized growth rate of the $m = 1$ mode as a function of the jitter amplitude. The curves labelled $a - c$ are results of the initial value calculations, whereas d, e show the solutions of the analytic dispersion relation, Eqs. (30)–(33). a) $\eta = 10^{-7}$, $\omega = 4\times 10^{-2}$, b) $\eta = 10^{-7}$, $\omega = 2 \times 10^{-2}$, and c) $\eta = 10^{-6}$, $\omega = 4 \times 10^{-2}$, d) $n = 0$ branch of the analytic dispersion dispersion relation, e) $n = 1$ branch.

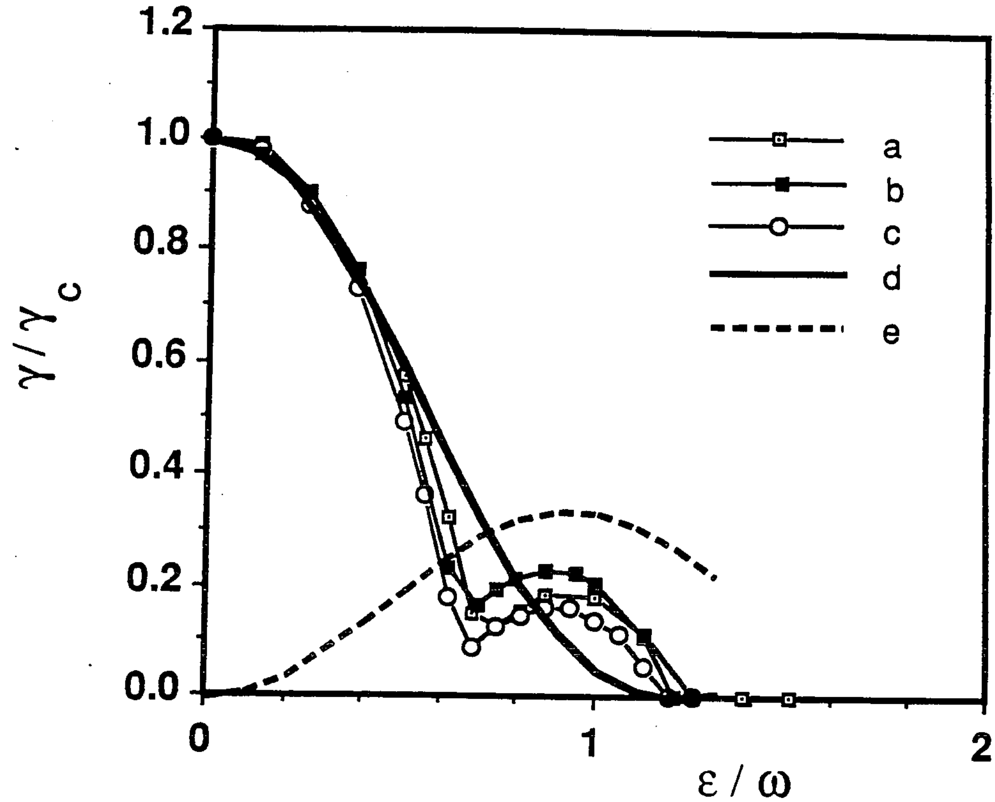

Fig. 1