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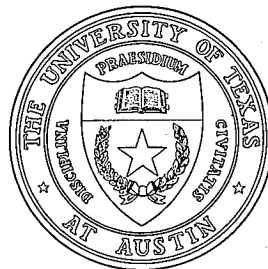
Asymptotic Spectra in Two-Dimensional
Drift Wave Turbulence

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A hybrid field Ψ , defined as the linear combination of the vorticity and the logarithm of electron density, is a constant of motion along the perturbed orbit in two-dimensional electrostatic turbulences. As a result, the spatial correlation of the fluctuating hybrid field is logarithmically divergent at small distance, which suggests a large wave number asymptote of the correlation proportional to k^{-2} . For the Hasegawa-Mima turbulence it follows that the asymptotic energy spectrum $E_k \sim k^{-4}$.

It is generally interesting to acquire a knowledge of the asymptotic spectrum for a stationary turbulence, such as the well-known Kolmogorov's power law¹ for the Navier-Stokes fluids. For two-dimensional drift waves use has been made of the Gibbs ensemble to obtain the energy spectra, for example, of the Hasegawa-Mima² and the Hasegawa-Wakatani³ turbulence.^{4,5} However, the resultant power laws appear to be far away from those obtained by numerical simulations.^{6,7,3} In this brief communication a new field Ψ is introduced, which is found to be helpful for studying the spectrum problem.

For a two-dimensional turbulence the electron continuity equation and the vorticity equation are combined to be one equation for $\Psi \equiv \ln N - \nabla^2 \phi$, that is a constant of motion along the perturbed orbit due to the $\mathbf{E} \times \mathbf{B}$ nonlinearity, where N is the electron density, $e\phi/T_e$ is the electrostatic potential, T_e is the electron temperature, e is the electron charge,

the radial scale is normalized to ρ_s , with $\rho_s = c_s/\omega_{ci}$, c_s is the ion sound speed, and ω_{ci} is the ion cyclotron frequency. Explicitly, the equation is

$$\frac{d\Psi}{dt} = 0, \quad (1)$$

where $d/dt \equiv \partial/\partial t + \mathbf{b} \times \nabla\varphi \cdot \nabla$, \mathbf{b} is the unit vector of the magnetic field, and t is the time normalized to ω_{ci}^{-1} .

For a Gaussian stochastic φ , it follows from Eq. (1) that the time evolution of the correlation function is^{8,9}

$$\left(\frac{\partial}{\partial t} - \sum_{i,j=1,2} \nabla_i \cdot \mathbf{D}_{ij} \cdot \nabla_j \right) \langle \psi(\mathbf{r}_1, t) \psi(\mathbf{r}_2, t) \rangle = (\mathbf{D}_{12} + \mathbf{D}_{21}) : \nabla_1 \nabla_2 \Psi_0(\mathbf{r}_1) \Psi_0(\mathbf{r}_2), \quad (2)$$

where $\langle \dots \rangle$ is the ensemble average, $\mathbf{D}_{ij} \equiv \int_0^\infty d\tau \langle \mathbf{b} \times \nabla_i \varphi(\mathbf{r}_i(t), t) \mathbf{b} \times \nabla_j \varphi(\mathbf{r}_j(t-\tau), t-\tau) \rangle$, with $\mathbf{r}_i(t) = \mathbf{r}_i(-\infty) + \int_{-\infty}^t ds \mathbf{b} \times \nabla_i \varphi(\mathbf{r}_i(s), s) = \mathbf{r}_i$ (for $i = 1, 2$), $\Psi_0 \equiv \ln N_0$ for zero equilibrium electrostatic potential, N_0 is the equilibrium density, $\psi \equiv \Psi - \Psi_0 = n - \nabla^2 \varphi + \mathcal{O}(n^2)$, and $n = N/N_0 - 1$, the normalized fluctuating electron density. When the homogeneity in space and time is assumed, the *l.h.s.* of Eq. (2) reduces to $-\nabla \cdot \mathbf{D}_-(\mathbf{r}) \cdot \nabla C(\mathbf{r})$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $C(\mathbf{r}) = \langle \psi(\mathbf{r}_1) \psi(\mathbf{r}_2) \rangle$, $\nabla \equiv \partial/\partial \mathbf{r}$, and $\mathbf{D}_-(\mathbf{r}) \equiv \mathbf{D}_{11} + \mathbf{D}_{22} - \mathbf{D}_{12} - \mathbf{D}_{21}$. In the small $r = |\mathbf{r}|$ limit $\mathbf{D}_-(\mathbf{r})$ approaches to zero as r^2 , whereas the *r.h.s.* of Eq. (2) goes to a constant $2D/L_n^2$, where L_n is the density gradient length, and D is the diffusion coefficient. The solution at small distance for the isotropic turbulence ($D_{-}^{\theta\theta} = 3D_{-}^{rr} \sim r^2$, $D_{-}^{r\theta} = D_{-}^{\theta r} = 0$, where r, θ are polar coordinates), is found to be $C(\mathbf{r}) \sim \ln r$ to the leading order. This is reminiscent of the granulation solutions for similar equations.⁸⁻¹¹ The logarithmic behavior of the correlation function $C(\mathbf{r})$ suggests that the large wave number asymptote of $\overline{C}_{\mathbf{k}} \equiv \langle \psi_{\mathbf{k}} \psi_{\mathbf{k}}^* \rangle$ be k^{-2} , where $\psi_{\mathbf{k}}$ is the Fourier transform of $\psi(\mathbf{r})$. To illustrate this assertion, we assume the following isotropic spectrum $\overline{C}_{\mathbf{k}} \sim 1/(\kappa^2 + k^2)$. It yields $C(\mathbf{r}) \sim K_0(\kappa r)$, where K_0 is the zeroth order modified Bessel function of the second kind, which is logarithmically divergent at $r = 0$.

With the adiabatic density response $n = \varphi$ for the Hasegawa - Mima equation,² the large k asymptote of $|\varphi|^2$ is thus obtained to be k^{-6} , leading to $E_k \sim k^2|\varphi|^2 \sim k^{-4}$, that is same as the results from numerical simulations.^{6,7}

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