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A hybrid field Ψ, defined as the linear combination of the vorticity and the logarithm of electron density, is a constant of motion along the perturbed orbit in two-dimensional electrostatic turbulences. As a result, the spatial correlation of the fluctuating hybrid field is logarithmically divergent at small distance, which suggests a large wave number asymptote of the correlation proportional to k^{-2}. For the Hasegawa-Mima turbulence it follows that the asymptotic energy spectrum $E_k \sim k^{-4}$.

It is generally interesting to acquire a knowledge of the asymptotic spectrum for a stationary turbulence, such as the well-known Kolmogorov's power law[1] for the Navier-Stokes fluids. For two-dimensional drift waves use has been made of the Gibbs ensemble to obtain the energy spectra, for example, of the Hasegawa-Mima[2] and the Hasegawa-Wakatani[3] turbulence.[4,5] However, the resultant power laws appear to be far away from those obtained by numerical simulations.[6,7,3] In this brief communication a new field Ψ is introduced, which is found to be helpful for studying the spectrum problem.

For a two-dimensional turbulence the electron continuity equation and the vorticity equation are combined to be one equation for $\Psi \equiv \ln\ N - \nabla^2\varphi$, that is a constant of motion along the perturbed orbit due to the $\mathbf{E} \times \mathbf{B}$ nonlinearity, where N is the electron density, $e\varphi/T_e$ is the electrostatic potential, T_e is the electron temperature, e is the electron charge, the radial scale is normalized to ρ_s with $\rho_s = c_s/\omega_{ci}$, c_s is the ion sound speed, and ω_{ci} is the ion cyclotron frequency. Explicitly, the equation is

$$\frac{d\Psi}{dt} = 0 \ , \tag{1}$$

where $d/dt \equiv \partial/\partial t + \mathbf{b} \times \nabla\varphi \cdot \nabla$, $\mathbf{b}$ is the unit vector of the magnetic field, and t is the time normalized to ω_{ci}^{-1}.

For a Gaussian stochastic φ, it follows from Eq. (1) that the time evolution of the correlation function is[8,9]

$$\left(\frac{\partial}{\partial t} - \sum_{i,j=1,2} \nabla_i \cdot \mathbf{D}_{ij} \cdot \nabla_j\right) \langle\psi(\mathbf{r}_1,t)\psi(\mathbf{r}_2,t)\rangle = (\mathbf{D}_{12} + \mathbf{D}_{21}) : \nabla_1\nabla_2\Psi_0(\mathbf{r}_1)\Psi_0(\mathbf{r}_2) \ , \tag{2}$$

where $\langle\ldots\rangle$ is the ensemble average, $\mathbf{D}_{ij} \equiv \int_0^\infty d\tau \ \langle\mathbf{b} \times \nabla_i\varphi(\mathbf{r}_i(t),t)\mathbf{b} \times \nabla_j\varphi(\mathbf{r}_j(t-\tau) \ , t-\tau)\rangle$, with $\mathbf{r}_i(t) = \mathbf{r}_i(-\infty) + \int_{-\infty}^{t} ds \ \mathbf{b} \times \nabla_i\varphi(\mathbf{r}_i(s),s) = \mathbf{r}_i$ (for $i = 1,2$), $\Psi_0 \equiv \ln \ N_0$ for zero equilibrium electrostatic potential, N_0 is the equilibrium density, $\psi \equiv \Psi - \Psi_0 = n - \nabla^2\varphi + \mathcal{O}(n^2)$, and $n = N/N_0 - 1$, the normalized fluctuating electron density. When the homogeneity in space and time is assumed, the *l.h.s.* of Eq. (2) reduces to $-\boldsymbol{\nabla} \cdot \mathbf{D}_-(\mathbf{r}) \cdot \boldsymbol{\nabla} C(\mathbf{r})$, where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $C(\mathbf{r}) = \langle\psi(\mathbf{r}_1)\psi(\mathbf{r}_2)\rangle$, $\nabla \equiv \partial/\partial\mathbf{r}$, and $\mathbf{D}_-(\mathbf{r}) \equiv \mathbf{D}_{11} + \mathbf{D}_{22} - \mathbf{D}_{12} - \mathbf{D}_{21}$. In the small $r = |\mathbf{r}|$ limit $\mathbf{D}_-(\mathbf{r})$ approaches to zero as r^2, whereas the *r.h.s.* of Eq. (2) goes to a constant $2D/L_n^2$, where L_n is the density gradient length, and D is the diffusion coefficient. The solution at small distance for the isotropic turbulence ($D_-^{\theta\theta} = 3D_-^{rr} \sim r^2$, $D_-^{r\theta} = D_-^{\theta r} = 0$, where r, θ are polar coordinates), is found to be $C(\mathbf{r}) \sim \ln \ r$ to the leading order. This is reminiscent of the granulation solutions for similar equations.[8–11] The logarithmic behavior of the correlation function $C(\mathbf{r})$ suggests that the large wave number asymptote of $\overline{C}_\mathbf{k} \equiv \left\langle\psi_\mathbf{k}\psi_\mathbf{k}^*\right\rangle$ be k^{-2}, where $\psi_\mathbf{k}$ is the Fourier transform of $\psi(\mathbf{r})$. To illustrate this assertion, we assume the following isotropic spectrum $\overline{C}_\mathbf{k} \sim 1/(\kappa^2 + \mathbf{k}^2)$. It yields $C(\mathbf{r}) \sim K_0(\kappa r)$, where K_0 is the zeroth order modified Bessel function of the second kind, which is logarithmically divergent at $r = 0$.

With the adiabatic density response $n = \varphi$ for the Hasegawa - Mima equation,[2] the large k asymptote of $|\varphi|^2$ is thus obtained to be k^{-6}, leading to $E_k \sim k^2|\varphi|^2 \sim k^{-4}$, that is same as the results from numerical simulations.[6,7]

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References

[1] A.N. Kolmogorov, DAN SSSR **30**, 299 (1941).

[2] A. Hasegawa and K. Mima, Phys. Fluids **21**, 87 (1978).

[3] M. Wakatani and A. Hasegawa, Phys. Fluids **27**, 611 (1984).

[4] A. Hasegawa, T. Imamuru, K. Mima, and T. Taniuti, J. Phys. Soc. Jpn. **45**, 1005 (1978).

[5] F.Y. Gang, B.D. Scott, and P.H. Diamond, Phys. Fluids **1**, 1331 (1989).

[6] D. Fyfe and D. Montgomery, Phys. Fluids **22**, 246 (1979).

[7] A. Hasegawa, C. Maclennan, and Y. Kodama, Phys. Fluids **22**, 2122 (1979).

[8] T.H. Dupree, Phys. Fluids **15**, 334 (1972).

[9] Y.Z. Zhang, Acta Physica Sinica **30**, 584 (1981).

[10] T.H. Dupree, Phys. Fluids **21**, 783 (1978).

[11] P.H. Diamond, T.H. Dupree, and D.J. Tetreault, Phys. Rev. Lett. **45**, 562 (1982).