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Kinetic Theory of RF Current Drive and Helicity Injection

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Current drive and helicity injection by plasma waves are examined with the use of kinetic theory. The Vlasov equation yields a general current drive formula which contains resonant and nonresonant (ponderomotive-like) contributions. Standard quasilinear current drive is described by the former, while helicity current drive may be contained in the latter. Since direct analytical comparison of the sizes of the two terms is in general difficult, a new approach is taken. Solution of the drift-kinetic equation shows that the standard Landau damping/transit time magnetic pumping quasilinear diffusion coefficient is the only contribution to steady-state current drive to leading order in $\varepsilon = \rho_L/\ell$, where ρ_L is the Larmor radius and ℓ is the inhomogeneity scale length. All nonresonant contributions, including the helicity, appear at higher order, after averages are taken over a flux surface, over azimuth, and over time. Consequently, at wave frequencies well below the electron cyclotron frequency, a wave helicity flux perpendicular to the magnetic field does *not* influence the parallel motion of electrons to leading order and therefore will not drive a significant current. Any current associated with a wave helicity flux is then either ion current (and thus inefficient) or electron current stemming from effects not included in the drift-kinetic treatment, such as cyclotron, collisional, or nonlinear (i.e. not quasilinear).

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I. INTRODUCTION

Several studies have investigated helicity injection and current drive by plasma waves in bounded and unbounded plasmas.[1-11] Many of these used the magnetohydrodynamic (MHD) model.[2-7] Although initial results,[1-5] which apply to an unbounded plasma, were optimistic, more recent studies show that the current drive is severely limited in a bounded, steady-state plasma with fluctuations driven from outside.[6,7] This is due to two factors.[7] One is that the component of the wave helicity flux perpendicular to the equilibrium magnetic field is proportional to the parallel component of the wave electric field, which is proportional to the resistivity in the MHD model. The other is that the total helicity "dissipation," which causes the bulk of the current in a bounded plasma, is proportional to the resistivity to the two-thirds power in the presence of the Alfvén resonance.

Other studies have considered the two-fluid model.[1,8-10] Like the MHD model, the results of these studies show that the net current averaged over a flux surface (corresponding to a real parallel wave vector) is proportional to a collision frequency, whether it be ion-electron, electron-ion,[8,9] or an effective "rf" collision frequency.[10]

Since these fluid models do not contain the true (collisionless) absorption mechanisms in a hot plasma, a kinetic description of helicity injection and current drive is desirable. Two studies have recently considered the kinetic model.[10,11] One,[11] using the eikonal approximation, showed that the average rf force on a plasma species may be written as the sum of a single resonant and three nonresonant terms. The resonant contribution is equivalent to that considered previously by, for example, Fisch.[12] The nonresonant terms stem from wave dispersion, ponderomotive force, and an internal polarizing force. Based on numerical computations for a specific application, the authors of Ref. 11 speculate that the latter term is related to helicity injection. However, the connection is not clear. Furthermore, applications of this theory are restricted by the eikonal approximation. The other study[10] derived from the Vlasov equation a general and useful form for the net force on a particle species. However, in this form, the resonant and nonresonant contributions are not separate and are therefore not readily compared. In both studies numerical computations indicate that the resonant interaction is dominant in the particular cases considered.

The present study is divided into five parts. Section II re-examines the general form for the force on a plasma species reported in Ref. 10. This result is further generalized to an arbitrary equilibrium magnetic field and written as the sum of resonant-like and nonresonant (ponderomotive-like) contributions. The force is seen to depend on details of the plasma response and gradients of the wave fields. Although amenable to numerical computation, analytical comparison of the sizes of the two terms is in general not easy. Therefore, we take a different approach to the current drive problem. In Secs. III-VI we consider the drift-kinetic equation. In Sec. III a form of the drift-kinetic equation suitable for the study of the current drive problem is developed. In Sec. IV slab geometry and the quasilinear approximation are introduced. Section V contains the solution of the drift-kinetic equation for small Fourier wave electromagnetic fields. Our approach is unique in that it does not make use of Ampere's law, and therefore the susceptibility tensor does not appear in the final result. Also, the ordering of the resonant and nonresonant forces comes directly out of the analysis. It is found, perhaps surprisingly, that *no* nonresonant contribution to the current drive remains to leading order in the expansion parameter $\varepsilon = \rho_L/\ell$, where ρ_L is the Larmor radius and ℓ is the inhomogeneity scale length. A more general derivation of the same result in real space and time is presented in Sec. VI. Here, one can see how the nonresonant contributions vanish upon averaging the fluctuations over a flux surface, over azimuth, and over time. These nonresonant terms are reminiscent of the ponderomotive force. Concluding remarks appear in Sec. VII, and a discussion of the net driven current is presented in the Appendix.

II. VLASOV EQUATION

Collisionless current drive is described by the Vlasov equation,

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\cdot\nabla f_s + \frac{q_s}{m_s}(\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B})\cdot\nabla_v f_s = 0, \tag{1}$$

where the subscript s refers to the particle species. Following the standard quasilinear treatment, we separate the distribution and the fields into a slowly varying part (subscript zero) and a first order rapidly varying part (tilde). Then the slowly varying part is given by

$$\frac{\partial f_{s0}}{\partial t} + \mathbf{v}\cdot\nabla f_{s0} + \frac{q_s}{m_s}(\mathbf{E}_0 + \frac{\mathbf{v}}{c}\times\mathbf{B}_0)\cdot\nabla_v f_{s0} = -\frac{q_s}{m_s}<(\tilde{\mathbf{e}} + \frac{\mathbf{v}}{c}\times\tilde{\mathbf{b}})\cdot\nabla_v \tilde{f}_s>, \tag{2}$$

where <> designates an average. The driven current is related to the $v_{\parallel}$ velocity moment of Eq. (2), where $v_{\parallel} = \mathbf{v}\cdot\hat{\mathbf{n}}_0$ and $\hat{\mathbf{n}}_0 = \mathbf{B}_0/B_0$. This moment may be written, with no further approximation, as

$$\hat{\mathbf{n}}_0\cdot\{m_s n_{s0}[\frac{\partial \mathbf{u}_{s0}}{\partial t} + (\mathbf{u}_{s0}\cdot\nabla)\mathbf{u}_{s0}] + \nabla\cdot\mathbf{P}_{s0}\} = \hat{\mathbf{n}}_0\cdot\{\rho_{s0}\mathbf{E}_0 + <\tilde{\rho}_s\tilde{\mathbf{e}} + \tilde{\mathbf{j}}_s\times\tilde{\mathbf{b}}>\}, \tag{3}$$

where n is the plasma density, $\mathbf{u}$ the fluid velocity, $\mathbf{P}$ the pressure tensor, ρ the charge density, and $\mathbf{j}$ the current density. The right-hand side represents the parallel force on a fluid element due to a parallel steady electric field and fluctuating electromagnetic fields, while the left-hand side represents its change in momentum.

The left-hand side can be approximately related to the driven current in steady-state as

$$m_s n_{s0} \hat{\mathbf{n}}_0 \cdot [\frac{\partial \mathbf{u}_{s0}}{\partial t} + (\mathbf{u}_{s0} \cdot \nabla)\mathbf{u}_{s0}] \approx m_s n_{s0} u_{s0//} / \tau_s = m_s J_{s0//} / q_s \tau_s, \tag{4}$$

where τ_s is the slowing-down time of species s on the other species. Thus, neglecting the pressure tensor and omitting the steady electric field, the current driven by the fluctuations is formally given by

$$J_{s0//} = -\frac{q_s \tau_s}{m_s} < \tilde{e}_{//} \nabla \cdot \int dt \, \tilde{\mathbf{j}}_s + [\tilde{\mathbf{j}}_s \times (\nabla \times \int dt \, \tilde{\mathbf{e}})]_{//} >, \tag{5}$$

where Faraday's law and continuity were used. For monochromatic fluctuations ($\sim e^{-i\omega t}$), Eq. (5) may be expressed as the sum of resonant-like and nonresonant (ponderomotive-like) contributions:

$$J_{s0//} = \frac{q_s \tau_s}{i \omega m_s} \hat{\mathbf{n}}_0 \cdot [<(\nabla \tilde{\mathbf{e}}) \cdot \tilde{\mathbf{j}}_s^*> - \nabla \cdot < \tilde{\mathbf{j}}_s^* \tilde{\mathbf{e}} >] + \text{c.c.} \,. \tag{6}$$

For plane-wave fluctuations ($\sim e^{i\mathbf{k}\cdot\mathbf{x}}$), the last term vanishes and Eq. (6) becomes

$$J_{s0//} = \frac{q_s \tau_s k_{//} P_s}{\omega m_s} + \text{c.c.}\,, \tag{7}$$

where P_s is the power absorbed by species s, $\tilde{\mathbf{e}} \cdot \tilde{\mathbf{j}}_s^* + \text{c.c.}$ This form is the same as Eq. (1.3) of Fisch.[12] In general geometry, however, there is nothing intrinsic about the last term in Eq. (6) that should make it vanish. This term could be connected with a helicity flux. In general, the driven current depends on details of the spatial gradients of the fluctuating electric field and current density. Since the driven current ~ $1/m_s$, it is clear the electron contribution will dominate, except possibly under special circumstances. Equation (6) is amenable to numerical calculations since $\tilde{\mathbf{j}}_s$ may be expressed in terms of the fluctuating electric field through the susceptibility tensor:

$$\tilde{\mathbf{j}}_s = \frac{\omega}{4\pi i}\bar{\bar{\chi}}_s \cdot \tilde{\mathbf{e}}. \tag{8}$$

Although further investigation of Eqs. (6) and (8) may be fruitful, the presence of the susceptibility tensor makes direct analytical comparison of the sizes of the two terms difficult. Also, any contribution of the helicity flux to the current drive is not readily apparent. Therefore, we now take a different approach to the current drive problem.

III. DRIFT-KINETIC EQUATION

At frequencies well below the electron cyclotron frequency, electron motion is accurately described by the drift-kinetic equation. In this section, we derive a form of the drift-kinetic equation useful for investigating helicity injection and current drive. We start from a form of the drift-kinetic equation valid in the so-called MHD ordering, where the electric drift v_E is the same order as the thermal speed v_T. In this ordering $E_{\|}/E_{\perp} = O(\varepsilon)$, where $\varepsilon = \rho_L/\ell$, ρ_L is the Larmor radius, and ℓ the inhomogeneity scale length. The proper equation (also called the guiding-center equation) valid to zero order in ε, is[13,14]

$$\frac{\partial f}{\partial t} + (u\hat{\mathbf{n}} + \mathbf{v}_E)\cdot\nabla f + \left[\frac{q}{m}\mathbf{E}\cdot(u\hat{\mathbf{n}} + \mathbf{v}_d) - u\hat{\mathbf{n}}\cdot(\mathbf{v}_E\cdot\nabla)\mathbf{v}_E + \mu\frac{\partial B}{\partial t}\right]\frac{\partial f}{\partial w} = C(f,f), \tag{9}$$

where $f = f(w, \mu, \mathbf{x}, t)$, $w = (\mathbf{v} - \mathbf{v}_E)^2/2$, $\mu = (\mathbf{v}_\perp - \mathbf{v}_E)^2/(2B)$, $\mathbf{v}_E = c\,\mathbf{E} \times \hat{\mathbf{n}}/B$, $\mathbf{v}_d = \hat{\mathbf{n}} \times [\mu\nabla B + u^2(\hat{\mathbf{n}}\cdot\nabla)\hat{\mathbf{n}} + u\partial\hat{\mathbf{n}}/\partial t]/\omega_c$, $\hat{\mathbf{n}} = \mathbf{B}/B$, $u = \mathbf{v}\cdot\hat{\mathbf{n}}$, $\mathbf{v}_\perp = \mathbf{v} - u\hat{\mathbf{n}}$, $\omega_c = qB/(mc)$, and C is a collision operator. Since the driven current is related to the u velocity moment of Eq. (9), it is more convenient and physically relevant to work with a distribution function $g(u, \mu, \mathbf{x}, t)$ than $f(w, \mu, \mathbf{x}, t)$. Because $u = \pm[2(w - \mu B)]^{1/2}$, it is a simple matter to write $f(w, \mu, \mathbf{x}, t) = g(u(w, \mu, \mathbf{x}, t), \mu, \mathbf{x}, t)$ and use the chain rule to convert Eq. (9) into an equation for $g(u, \mu, \mathbf{x}, t)$. The result is

$$\frac{\partial g}{\partial t} + (u\hat{\mathbf{n}} + \mathbf{v}_E)\cdot\nabla g + [\hat{\mathbf{n}}\cdot(\frac{q}{m}\mathbf{E} - \mu\nabla B) + (\frac{q}{m}\mathbf{E}\cdot\mathbf{v}_d - \mu\mathbf{v}_E\cdot\nabla B)/u - \hat{\mathbf{n}}\cdot(\mathbf{v}_E\cdot\nabla)\mathbf{v}_E]\frac{\partial g}{\partial u} = C(g, g). \tag{10}$$

Using the definitions of $\mathbf{v}_d$ and $\mathbf{v}_E$, the last three terms in the square brackets of Eq. (10) may be converted into a more recognizable form,

$$\frac{\partial g}{\partial t} + (u\hat{\mathbf{n}} + \mathbf{v}_E)\cdot\nabla g + \{\hat{\mathbf{n}}\cdot(\frac{q}{m}\mathbf{E} - \mu\nabla B) + \mathbf{v}_E\cdot[\frac{\partial\hat{\mathbf{n}}}{\partial t} + u(\hat{\mathbf{n}}\cdot\nabla)\hat{\mathbf{n}} + (\mathbf{v}_E\cdot\nabla)\hat{\mathbf{n}}]\}\frac{\partial g}{\partial u} = C(g, g). \tag{11}$$

It is interesting to note that apart from the v_E drift appearing in the definition of μ, du/dt (the term in curly braces) in Eq. (11) agrees with that given by Eq. (1.20) of Northrop.[15]

As discussed in the Appendix, the driven current is proportional to $\int du d\mu u \mathcal{J} g$, where $\mathcal{J}$ is the Jacobian relating d^3v to $du d\mu d\zeta$, and ζ is the gyrophase. (Here $\mathcal{J} = B + O(\varepsilon)$). Since subsequently we expand the distribution function and fields in powers of the fluctuating field amplitudes (in which case $\mathcal{J}$ must also be expanded), it is more convenient to work with an equation for the variable $G = \mathcal{J}g$ rather than Eq. (11). Due to the properties of $\mathcal{J}$, this equation is the conservative analog of Eq. (11) (see Littlejohn,[16] for example):

$$\frac{\partial G}{\partial t} + \nabla\cdot[(u\hat{\mathbf{n}} + \mathbf{v}_E)G]$$
$$+ \frac{\partial}{\partial u}\left\{\{\hat{\mathbf{n}}\cdot(\frac{q}{m}\mathbf{E} - \mu\nabla B) + \mathbf{v}_E\cdot[\frac{\partial\hat{\mathbf{n}}}{\partial t} + u(\hat{\mathbf{n}}\cdot\nabla)\hat{\mathbf{n}} + (\mathbf{v}_E\cdot\nabla)\hat{\mathbf{n}}]\}G\right\} = C(G, G). \qquad (12)$$

Our goal is now to solve Eq. (12) under conditions approximating steady fluctuations driven from a plasma boundary.

IV. PLANE SLAB MODEL

Following the standard quasilinear treatment, we separate the distribution and the fields into a slowly varying equilibrium part (subscript zero) and a first order rapidly varying part (tilde). As a simple geometrical model, we consider the sheet pinch in which equilibrium quantities vary only in the x-direction and

$$\mathbf{B}_0 = (0, B_{0y}(x), B_{0z}(x)), \qquad (13)$$

$$G_0 = G_0(u, \mu, x). \qquad (14)$$

[Although our choice of velocity-space coordinates may be unconventional, the proper equilibrium moments result from Eq. (14), as shown in the Appendix.] Expanding the total fields in powers of the fluctuating field amplitudes (designated by λ), we find

$$B^{-1} = (\mathbf{B}\cdot\mathbf{B})^{-1/2} = B_0^{-1} - \lambda\, \tilde{b}_{\parallel}B_0^{-2} - \lambda^2(\tfrac{1}{2}\tilde{b}_{\perp}^{\,2} - \tilde{b}_{\parallel}^{\,2})B_0^{-3} + \ldots \qquad (15)$$

$$\hat{\mathbf{n}} = \mathbf{B}B^{-1} = \hat{\mathbf{n}}_0 + \lambda\,\tilde{\mathbf{b}}_\perp B_0^{-1} - \lambda^2[\tilde{\mathbf{b}}\tilde{b}_\parallel + \hat{\mathbf{n}}_0(\tfrac{1}{2}\tilde{b}_\perp{}^2 - \tilde{b}_\parallel{}^2)]B_0^{-2} + \ldots \tag{16}$$

$$\mathbf{v}_\mathrm{E} = \lambda c\,\tilde{\mathbf{e}}\times\hat{\mathbf{n}}_0 B_0^{-1} + \lambda^2 c\,(\tilde{\mathbf{e}}\times\tilde{\mathbf{b}}_\perp - \tilde{b}_\parallel\tilde{\mathbf{e}}\times\hat{\mathbf{n}}_0)B_0^{-2} + \ldots \tag{17}$$

$$\hat{\mathbf{n}}\cdot\mathbf{E} = \lambda\,\tilde{e}_\parallel + \lambda^2\tilde{\mathbf{e}}\cdot\tilde{\mathbf{b}}_\perp B_0^{-1} + \ldots \tag{18}$$

$$\hat{\mathbf{n}}\cdot\nabla B = \lambda\,(\tilde{b}_x B_0^{-1}\frac{\partial B_0}{\partial x} + \hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_\parallel)$$
$$+ \lambda^2(\tfrac{1}{2}\hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_\perp{}^2 + \tilde{\mathbf{b}}_\perp\cdot\nabla\,\tilde{b}_\parallel - \tilde{b}_\parallel\tilde{b}_x B_0^{-1}\frac{\partial B_0}{\partial x})B_0^{-1} + \ldots \tag{19}$$

$$\mathbf{v}_\mathrm{E}\cdot[\frac{\partial\hat{\mathbf{n}}}{\partial t} + u(\hat{\mathbf{n}}\cdot\nabla)\hat{\mathbf{n}} + (\mathbf{v}_\mathrm{E}\cdot\nabla)\hat{\mathbf{n}}]$$
$$= \lambda^2\{\frac{\partial}{\partial t}(-\tfrac{1}{2}u\tilde{b}_\perp{}^2) + c\hat{\mathbf{n}}_0\cdot\nabla[\tfrac{1}{2}c\tilde{e}_\perp{}^2 - u\,\hat{\mathbf{n}}_0\cdot(\tilde{\mathbf{e}}\times\tilde{\mathbf{b}})]\}B_0^{-2} + \ldots \tag{20}$$

where the subscripts $\parallel$ and $\perp$ of the field quantities are defined relative to $\hat{\mathbf{n}}_0$ (unlike the velocity-space coordinates, which are defined relative to $\hat{\mathbf{n}}$). To obtain Eq. (20), we have used $\nabla\cdot\mathbf{B} = 0$, Faraday's law, and have consistently discarded higher order terms, which stem from the ratio $\tilde{e}_\parallel/\tilde{e}_\perp$. [Note $\tilde{e}_\parallel = \tilde{\mathbf{e}}\cdot\hat{\mathbf{n}}_0 = \tilde{\mathbf{e}}\cdot(\hat{\mathbf{n}} - \lambda\,\tilde{\mathbf{b}}_\perp B_0^{-1} + \ldots)$ so $\tilde{e}_\parallel/\tilde{e}_\perp = O(\varepsilon) + O(\lambda)$.] Then the fluctuating distribution is given by

$$\frac{\partial\tilde{G}}{\partial t} + u\hat{\mathbf{n}}_0\cdot\nabla\tilde{G} - C(\tilde{G}, G_0) - C(G_0, \tilde{G})$$
$$= -\nabla\cdot(\tilde{\mathbf{v}}_\mathrm{D}G_0) - \frac{\partial}{\partial u}(\tilde{a}_\parallel G_0), \tag{21}$$

where

$$\tilde{\mathbf{v}}_\mathrm{D} = u\tilde{\mathbf{b}}_\perp B_0^{-1} + c\,\tilde{\mathbf{e}}\times\hat{\mathbf{n}}_0 B_0^{-1}, \tag{22}$$

$$\tilde{a}_\parallel = \frac{q}{m}\tilde{e}_\parallel - \mu(\hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_\parallel + \tilde{b}_x B_0^{-1}\frac{\partial B_0}{\partial x}), \tag{23}$$

and the slowly varying distribution satisfies

$$
\begin{aligned}
C(G_0,G_0) &= \nabla\cdot\{<\tilde{\mathbf{v}}_D\tilde{G}> + <-\tilde{\mathbf{v}}_D\tilde{b}_{/\!/} + (c\tilde{\mathbf{e}}\times\tilde{\mathbf{b}}_\perp - \tfrac{1}{2}u\hat{\mathbf{n}}_0\tilde{b}_\perp{}^2)B_0{}^{-1}>B_0{}^{-1}G_0\} \\
&+ \frac{\partial}{\partial u}\{<\tilde{a}_{/\!/}\tilde{G}> + <\frac{q}{m}\tilde{\mathbf{e}}\cdot\tilde{\mathbf{b}}_\perp - \mu(\tfrac{1}{2}\hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_\perp{}^2 + \tilde{\mathbf{b}}_\perp\cdot\nabla\,\tilde{b}_{/\!/} - \tilde{b}_{/\!/}\tilde{b}_xB_0{}^{-1}\frac{\partial B_0}{\partial x}) \\
&\quad + \frac{\partial}{\partial t}(-\tfrac{1}{2}u\tilde{b}_\perp{}^2B_0{}^{-1}) + cB_0{}^{-1}\hat{\mathbf{n}}_0\cdot\nabla[\tfrac{1}{2}c\tilde{e}_\perp{}^2 - u\,\hat{\mathbf{n}}_0\cdot(\tilde{\mathbf{e}}\times\tilde{\mathbf{b}})]>B_0{}^{-1}G_0\}. \qquad (24)
\end{aligned}
$$

V. FOURIER MODES

We now consider the fluctuations to be produced by an external antenna, represented by boundary conditions at $x = \pm a$, and we Fourier analyze the fluctuations into components,

$$\tilde{\mathbf{b}}(\mathbf{x},t) = \mathbf{b}(x)e^{i\psi} + \text{c.c.} \qquad (25)$$

$$\tilde{\mathbf{e}}(\mathbf{x},t) = \mathbf{e}(x)e^{i\psi} + \text{c.c.} \qquad (26)$$

$$\tilde{G} = h(x)e^{i\psi} + \text{c.c.} \qquad (27)$$

where $\psi = k_y y + k_z z - \omega t$. Landau damping and transit time magnetic pumping (TTMP) are correctly treated by letting $\mathbf{k}$ be real and $\omega = \omega_0 + i\gamma$ with $\gamma \to 0^+$. For the collision frequency much smaller than ω, the fluctuating distribution, from Eq. (21), is given by

$$h = \frac{1}{i(\omega - uk_{/\!/})}[\tilde{\nabla}\cdot(\mathbf{v}_D G_0) + \frac{\partial}{\partial u}(a_{/\!/}G_0)], \qquad (28)$$

where

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + i\mathbf{k} \tag{29}$$

$$\mathbf{v}_\mathrm{D} = u\,\mathbf{b}_\perp B_0^{-1} + c\,\mathbf{e}\times\hat{\mathbf{n}}_0 B_0^{-1} \tag{30}$$

$$a_{/\!/} = \frac{q}{m}e_{/\!/} - \mu(ik_{/\!/}b_{/\!/} + b_x B_0^{-1}\frac{\partial B_0}{\partial x}), \tag{31}$$

and the slowly varying distribution, from Eq. (24), is given by

$$C(G_0,G_0) = \frac{\partial}{\partial x}\left\{v_{Dx}h^* - G_0 B_0^{-1} v_{Dx} b_{/\!/}^*\right\} + \text{c.c.}$$
$$+ \frac{\partial}{\partial u}\left\{a_{/\!/}h^* + G_0 B_0^{-1}[\frac{q}{m}\,\mathbf{e}\cdot\mathbf{b}_\perp^* - \mu(\mathbf{b}_\perp\cdot\nabla^* b_{/\!/}^* - b_x b_{/\!/}^* B_0^{-1}\frac{\partial B_0}{\partial x})]\right\} + \text{c.c.} \tag{32}$$

To obtain this result, the term containing $(\mathbf{e}\times\mathbf{b}_\perp^*)_x$ was dropped since it is proportional to $e_{/\!/}$ and therefore higher order. With $\nabla\cdot\mathbf{B} = 0$ and Faraday's law, we find

$$\nabla\cdot\mathbf{v}_\mathrm{D} = [i(\omega - uk_{/\!/})b_{/\!/} - v_{Dx}\frac{\partial B_0}{\partial x} + c\mathbf{e}\cdot(\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{x}})]B_0^{-1}. \tag{33}$$

But since $\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{x}} = \hat{\mathbf{n}}_0(\hat{\mathbf{n}}_0\times\frac{\partial\hat{\mathbf{n}}_0}{\partial x})_x$ (true for arbitrary $\hat{\mathbf{n}}_0$ if $\hat{\mathbf{n}}_0\cdot\hat{\mathbf{x}} = 0$), the last term in Eq. (33) is higher order and should be neglected. Then

$$h = b_{/\!/}B_0^{-1}G_0 + \frac{1}{i(\omega - uk_{/\!/})}[v_{Dx}B_0\frac{\partial}{\partial x}(G_0 B_0^{-1}) + a_{/\!/}\frac{\partial G_0}{\partial u}], \tag{34}$$

and Eq. (32) becomes

$$C(G_0,G_0) = \frac{\partial}{\partial x}\{\frac{iv_{Dx}v_{Dx}^*}{\omega^* - uk_{\|}}B_0\frac{\partial}{\partial x}(G_0B_0^{-1}) + \frac{iv_{Dx}a_{\|}^*}{\omega^* - uk_{\|}}\frac{\partial G_0}{\partial u}\} + \text{c.c.}$$
$$+ \frac{\partial}{\partial u}\{\frac{ia_{\|}a_{\|}^*}{\omega^* - uk_{\|}}\frac{\partial G_0}{\partial u} + \frac{ia_{\|}v_{Dx}^*}{\omega^* - uk_{\|}}B_0\frac{\partial}{\partial x}(G_0B_0^{-1})$$
$$+ G_0B_0^{-1}(\frac{q}{m}\mathbf{e} - \mu\nabla b_{\|})\cdot\mathbf{b}^*\} + \text{c.c.} \tag{35}$$

This equation may be further reduced by writing v_{Dx}, with the help of Faraday's law, as

$$v_{Dx} = \frac{c}{\omega B_0}[(\omega - uk_{\|})e_p + uk_{\perp}e_{\|}], \tag{36}$$

where $e_p = \mathbf{e}\cdot\hat{\mathbf{p}}$ and $\hat{\mathbf{p}} = \hat{\mathbf{n}}_0 \times \hat{\mathbf{x}}$. Again the last term is higher order and may be neglected relative to the first. Then Eq. (35) becomes, after some rearrangement,

$$C(G_0,G_0) = \frac{\partial}{\partial u}\{\frac{ia_{\|}a_{\|}^*}{\omega^* - uk_{\|}}\frac{\partial G_0}{\partial u}$$
$$+ G_0B_0^{-1}[(\frac{q}{m}\mathbf{e} - \mu\nabla b_{\|})\cdot\mathbf{b}^* - \frac{ic}{\omega_0}\frac{\partial}{\partial x}(a_{\|}e_p^*)]\} + \text{c.c.} \tag{37}$$

The first term on the right-hand side is purely resonant, since

$$\frac{ia_{\|}a_{\|}^*}{\omega^* - uk_{\|}} + \text{c.c.} = \lim_{\gamma\to 0^+}\frac{-2\gamma a_{\|}a_{\|}^*}{(\omega_0 - uk_{\|})^2 + \gamma^2} = -2\pi a_{\|}a_{\|}^*\delta(\omega_0 - uk_{\|}). \tag{38}$$

This term describes the effects of Landau damping and TTMP and has been thoroughly examined in the literature.[12,17] The right-hand side of Eq. (38) is simply the quasilinear diffusion coefficient, when the cyclotron resonance is unimportant. The other terms in Eq. (37) are purely nonresonant and are of our main concern regarding helicity injection and current drive. [The $\mathbf{e}\cdot\mathbf{b}^*$ term is related to the wave helicity flux,[7] whereas the nonresonant terms containing the magnetic moment have not been considered in previous studies.] However, quite surprisingly, it may be shown, with the use of $\nabla\cdot\mathbf{B} = 0$ and Faraday's law, that

$$(\frac{q}{m}\mathbf{e} - \mu\nabla b_{/\!/})\cdot\mathbf{b}^* \; + \; \text{c.c.} = \frac{ic}{\omega_0}\frac{\partial}{\partial x}(a_{/\!/}e_p^{\;*}) \; + \; \text{c.c.} \; + \; O(\varepsilon) \; + \; O(\lambda). \tag{39}$$

Consequently the nonresonant terms cancel and Eq. (37) becomes

$$C(G_0,G_0) = \frac{\partial}{\partial u}\left\{-2\pi a_{/\!/}a_{/\!/}^{\;*}\delta(\omega_0 - uk_{/\!/})\frac{\partial G_0}{\partial u}\right\}. \tag{40}$$

This is quite significant and is the main result of the present work. It shows that a helicity flux perpendicular to the magnetic field, represented by $\frac{ic}{\omega_0}\frac{\partial}{\partial x}(\frac{q}{m}e_{/\!/}e_p^{\;*}) + \text{c.c.}$, does *not* influence the parallel motion of an electron (or ion) to leading order in ε. It says that a perpendicular helicity flux may influence the parallel electron (or ion) motion only when cyclotron ($O(\varepsilon)$) effects are important. Therefore, near the ion cyclotron resonance, when a perpendicular helicity flux is efficiently absorbed by the plasma,[8,9] only the ion motion is affected. This leads one to believe that current drive by helicity injection may be inefficient. This conjecture is generally supported by the numerical calculations of Ref. 10. The result also agrees with that of Ref. 11, where it may be verified that the electron contribution to the internal (polarizing) force in the eikonal approximation is $O(\omega/\omega_{ce})$, when the wave amplitudes are averaged over a flux surface (using the mobility tensor of Ref. 18).

Finally, we note that, had the term containing the helicity flux not cancelled out, it would have been of the same order as the Landau damping term. Using $\int_{-\infty}^{\infty} du\, u \frac{\partial F}{\partial u} = - \int_{-\infty}^{\infty} du F$ we have

$$2\pi \frac{q^2}{k_{\parallel} m^2} e_{\parallel} e_{\parallel}^* \frac{\partial G_0}{\partial u}\Big|_{u = \omega_0/k_{\parallel}} \sim \frac{q}{m} \frac{ic}{\omega_0 B_0} \left(\int du\, G_0\right) \frac{\partial}{\partial x} (e_{\parallel} e_p^* - \text{c.c.}) , \tag{41}$$

or, assuming $u \sim v_T \sim \omega_0/k_{\parallel}$,

$$e_{\parallel} \sim \frac{\omega_0}{2\pi k_{\parallel} L \omega_{ce}} e_p , \tag{42}$$

which is consistent with the ordering. Since $e_{\parallel}$ appears in the term containing the helicity flux, the perpendicular helicity flux is small, within the present ordering. But, had it not cancelled out, the helicity term might have generated current comparable to the Landau damping term. Thus, the cancellation of the helicity term is not simply due to $e_{\parallel}/e_{\perp} = O(\varepsilon)$. At $O(\varepsilon)$, we find terms proportional to the helicity flux multiplied by gradients of the equilibrium, as well as a host of others. However, these are small compared to the resonant term in Eq. (40).

VI. REAL SPACE AND TIME

It is possible to see the cancellation of the nonresonant terms without the Fourier decomposition of the fluctuating fields. The derivation may be carried out formally by defining the operator

$$\mathcal{L} = \left(\frac{\partial}{\partial t} + u \hat{\mathbf{n}}_0 \cdot \nabla\right). \tag{43}$$

Then Eq. (21) becomes, with the neglect of the collision terms,

$$\mathcal{L}\tilde{G} = -\nabla\cdot(\tilde{\mathbf{v}}_{\mathrm{D}}G_0) - \frac{\partial}{\partial u}(\tilde{a}_{/\!/}G_0). \tag{44}$$

From $\nabla\cdot\mathbf{B} = 0$ and Faraday's law, we find, neglecting higher order terms,

$$\nabla\cdot\tilde{\mathbf{v}}_{\mathrm{D}} = -\mathcal{L}(\tilde{b}_{/\!/}B_0^{-1}) - \tilde{v}_{\mathrm{D}x}B_0^{-1}\frac{\partial B_0}{\partial x} \tag{45}$$

and

$$\mathcal{L}(\tilde{\mathbf{e}}\times\hat{\mathbf{n}}_0) = \frac{B_0}{c}\frac{\partial\tilde{\mathbf{v}}_{\mathrm{D}}}{\partial t} + u\tilde{e}_p\hat{\mathbf{p}}\Big(\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{n}}_0\Big)_x. \tag{46}$$

Therefore, defining $\dfrac{\partial\tilde{\boldsymbol{\xi}}}{\partial t} = \tilde{\mathbf{e}}$, we have

$$\tilde{\mathbf{v}}_{\mathrm{D}} = \frac{c}{B_0}\Big[\mathcal{L}(\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0) - u\tilde{\xi}_p\hat{\mathbf{p}}\Big(\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{n}}_0\Big)_x\Big], \tag{47}$$

and so Eq. (44) may be formally solved for $\tilde{G}$, yielding

$$\tilde{G} = \tilde{b}_{/\!/}B_0^{-1}G_0 - c\tilde{\xi}_p\frac{\partial}{\partial x}(B_0^{-1}G_0) - \mathcal{L}^{-1}(\tilde{a}_{/\!/}\frac{\partial G_0}{\partial u}). \tag{48}$$

With this form for the perturbed distribution, Eq. (24) becomes

$$C(G_0,G_0) = <-\nabla\cdot\Big\{\tilde{\mathbf{v}}_{\mathrm{D}}\Big[c\tilde{\xi}_p\frac{\partial}{\partial x}(B_0^{-1}G_0) + \mathcal{L}^{-1}(\tilde{a}_{/\!/})\frac{\partial G_0}{\partial u}\Big] \\ + \big(\tfrac{1}{2}u\hat{\mathbf{n}}_0\tilde{b}_{\perp}^{\,2} - c\,\tilde{\mathbf{e}}\times\tilde{\mathbf{b}}_{\perp}\big)G_0B_0^{-2}\Big\}$$

$$+ \frac{\partial}{\partial u}\{-\tilde{a}_{\parallel}\mathcal{L}^{-1}(\tilde{a}_{\parallel})\frac{\partial G_0}{\partial u} - \tilde{a}_{\parallel}c\tilde{\xi}_p\frac{\partial}{\partial x}(B_0^{-1}G_0)$$

$$+ [\frac{q}{m}\tilde{\mathbf{e}}\cdot\tilde{\mathbf{b}} - \mu(\tfrac{1}{2}\hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_{\perp}^{\,2} + \tilde{\mathbf{b}}\cdot\nabla\,\tilde{b}_{\parallel})$$

$$+ \frac{\partial}{\partial t}(-\tfrac{1}{2}u\tilde{b}_{\perp}^{\,2}B_0^{-1}) + cB_0^{-1}\hat{\mathbf{n}}_0\cdot\nabla(\tfrac{1}{2}c\tilde{e}_{\perp}^{\,2} - u\,\hat{\mathbf{n}}_0\cdot(\tilde{\mathbf{e}}\times\tilde{\mathbf{b}}))]B_0^{-1}G_0\}>. \tag{49}$$

The term containing $<-\tilde{a}_{\parallel}\mathcal{L}^{-1}(\tilde{a}_{\parallel})>$ is the resonant term describing Landau damping and TTMP. Recall that this was the only term which survived in the analysis of Sec. V. Our goal here is to see what form the other terms take. The terms on the last line of Eq. (49) vanish for steady fluctuations and when the fluctuations are averaged over a flux surface. Consequently they do not contribute to steady-state current drive. For the other terms not to contribute to current drive, they should be expressible in similar forms. In this spirit, for example, we note from Faraday's law that

$$\tilde{\mathbf{e}}\cdot\tilde{\mathbf{b}} = -\frac{c}{2}[\frac{\partial}{\partial t}(\tilde{\boldsymbol{\xi}}\cdot\nabla\times\tilde{\boldsymbol{\xi}}) + \nabla\cdot(\tilde{\boldsymbol{\xi}}\times\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t})]. \tag{50}$$

The average of the first term vanishes for steady fluctuations, but the second term is finite if $(\tilde{\boldsymbol{\xi}}\times\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t})_x$ is nonzero. This represents a transverse wave helicity flux. Cancellation of this term with part of $\nabla\cdot\{\tilde{\mathbf{v}}_{\mathrm{D}}\mathcal{L}^{-1}(\tilde{a}_{\parallel})\frac{\partial G_0}{\partial u}\}$ occurs when we write

$$\tilde{\mathbf{v}}_{\mathrm{D}}\mathcal{L}^{-1}(\tilde{a}_{\parallel}) = \mathcal{L}^{-1}(\tilde{\mathbf{v}}_{\mathrm{D}}\tilde{a}_{\parallel}) - \mathcal{L}^{-1}(\tilde{\mathbf{v}}_{\mathrm{D}})\tilde{a}_{\parallel}, \tag{51}$$

and use Eq. (47). We find

$$\tilde{\mathbf{v}}_D \mathcal{L}^{-1}(\tilde{a}_{/\!/}) = \mathcal{L}^{-1}(\tilde{\mathbf{v}}_D\tilde{a}_{/\!/}) - \frac{c}{B_0}\frac{q}{m}\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t}\cdot\hat{\mathbf{n}}_0\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0$$
$$+ \frac{c}{B_0}\mu(\hat{\mathbf{n}}_0\cdot\nabla\tilde{b}_{/\!/} + \tilde{b}_x B_0^{-1}\frac{\partial B_0}{\partial x})\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0$$
$$+ \frac{c}{B_0}\mathcal{L}^{-1}(u\tilde{\xi}_p)\tilde{a}_{/\!/}\hat{\mathbf{p}}(\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{n}}_0)_x. \tag{52}$$

If we further write

$$\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t}\cdot\hat{\mathbf{n}}_0\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0 = \tfrac{1}{2}\{\tilde{\boldsymbol{\xi}}\times\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t} - \hat{\mathbf{n}}_0\hat{\mathbf{n}}_0\cdot(\tilde{\boldsymbol{\xi}}\times\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t}) + \frac{\partial}{\partial t}(\tilde{\boldsymbol{\xi}}\cdot\hat{\mathbf{n}}_0\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0)\}, \tag{53}$$

we see that the first term of Eq. (53) partially cancels with the last term of Eq. (50), generating another term, $\frac{q}{m}\frac{c}{2}(\tilde{\boldsymbol{\xi}}\times\frac{\partial\tilde{\boldsymbol{\xi}}}{\partial t})_x\frac{\partial}{\partial x}(B_0^{-1}\frac{\partial G_0}{\partial u})$. We find that this new term cancels with yet another term. In this way, Eq. (49) may be eventually reduced to

$$C(G_0,G_0) = <-\frac{\partial}{\partial u}\{\tilde{a}_{/\!/}\mathcal{L}^{-1}(\tilde{a}_{/\!/})\frac{\partial G_0}{\partial u}\}$$
$$-\nabla\cdot\{\mathcal{L}^{-1}(\tilde{a}_{/\!/}\tilde{\mathbf{v}}_D)\frac{\partial G_0}{\partial u} + \tfrac{1}{2}c^2 B_0^{-1}\frac{\partial}{\partial x}(B_0^{-1}G_0)\,\mathcal{L}(\tilde{\xi}_p\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0)\}$$
$$-\tfrac{1}{2}G_0 B_0^{-2}\mathcal{L}(\tilde{b}_\perp{}^2)$$
$$-\tfrac{1}{2}B_0^{-1}\frac{\partial G_0}{\partial u}\frac{\partial}{\partial t}\{\frac{q}{m}c\,[\tilde{\boldsymbol{\xi}}\cdot(\nabla\times\tilde{\boldsymbol{\xi}}) - \nabla\cdot(\tilde{\xi}_{/\!/}\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0)] + uB_0^{-1}\tilde{b}_\perp{}^2\}$$
$$-B_0^{-1}\frac{\partial G_0}{\partial u}\hat{\mathbf{n}}_0\cdot\nabla\{cB_0^{-1}[u\hat{\mathbf{n}}_0\cdot(\tilde{\mathbf{e}}\times\tilde{\mathbf{b}}) - \tfrac{1}{2}c\tilde{e}_\perp{}^2] + \tfrac{1}{2}\frac{q}{m}c\,\hat{\mathbf{n}}_0\cdot(\tilde{\boldsymbol{\xi}}\times\tilde{\mathbf{e}})$$
$$-\tfrac{1}{2}\mu c^2\frac{\partial}{\partial x}(\tilde{\xi}_p{}^2 B_0^{-1}\frac{\partial B_0}{\partial x}) - \tfrac{1}{2}\mu\tilde{b}_\perp{}^2 - \mu c(\tilde{\boldsymbol{\xi}}\times\hat{\mathbf{n}}_0)\cdot\nabla\tilde{b}_{/\!/}\}$$
$$-cB_0^{-1}\hat{\mathbf{p}}\cdot\nabla\{\tfrac{1}{2}c\hat{\mathbf{n}}_0\cdot(\tilde{\boldsymbol{\xi}}\times\mathcal{L}\tilde{\boldsymbol{\xi}})\frac{\partial}{\partial x}(B_0^{-1}G_0)$$
$$+ [\tilde{a}_{/\!/}\mathcal{L}^{-1}(u\tilde{\xi}_p)(\frac{\partial\hat{\mathbf{n}}_0}{\partial x}\times\hat{\mathbf{n}}_0)_x + \mu\tilde{\xi}_x\tilde{b}_x B_0^{-1}\frac{\partial B_0}{\partial x}]\frac{\partial G_0}{\partial u}\}>. \tag{54}$$

Except for the first term on the right-hand side, which is the resonant term discussed earlier, and the second term, containing $\mathcal{L}^{-1}(\tilde{a}_{\parallel}\tilde{\mathbf{v}}_D)$, all terms vanish for steady fluctuations ($\partial/\partial t = 0$), when averaged over a flux surface ($\hat{\mathbf{n}}_0 \cdot \nabla = 0$) and when averaged over "azimuth" ($\hat{\mathbf{p}} \cdot \nabla = 0$). These nonresonant terms are reminiscent of the conservative ponderomotive force.[19,20] Although the second term rigorously vanishes in the analysis of Sec. V, it remains to be shown whether this is true in general.

VII. CONCLUSIONS

In the first part of this study we presented a general form for collisionless current drive which contains resonant-like and nonresonant (ponderomotive-like) contributions. Standard quasilinear current drive is given by the former, while helicity current drive may be contained in the latter. Although amenable to numerical calculation, direct analytical comparison of the sizes of the two terms is in general difficult. Therefore, a new approach was taken. We showed by solution of the drift-kinetic equation that the standard resonant Landau damping/TTMP quasilinear diffusion coefficient is the only contribution to the current drive to leading order in $\varepsilon = \rho_L/\ell$. All nonresonant contributions, including the helicity flux, appear at higher order. Consequently, at wave frequencies $\omega << \omega_{ce}$, a wave helicity flux perpendicular to the magnetic field does *not* influence the parallel motion of electrons to leading order and therefore will not drive a significant steady current. Any current associated with a wave helicity flux is thus either ion current (and therefore inefficient) or electron current stemming from effects not included in the drift-kinetic treatment, such as cyclotron, collisional ($\omega \sim \nu_{coll}$), or nonlinear (i.e., not quasilinear).

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APPENDIX: MOMENTS OF $G(u, \mu, \mathbf{x}, t)$

The use of the velocity-space coordinate u (instead of w) and the fact that the velocity space coordinates (u, μ) are defined relative to $\hat{\mathbf{n}}$ (rather than $\hat{\mathbf{n}}_0$) may raise questions concerning the validity of the analysis. In this Appendix, we show that, because of the definition of $G(u, \mu, \mathbf{x}, t)$, there are no inconsistencies.

First we note that the density

$$\begin{aligned} n(\mathbf{x}, t) &= \int d^3v\, f(\mathbf{v}, \mathbf{x}, t) \\ &= \int du d\mu d\zeta\, \mathcal{J}(u, \mu, \zeta, \mathbf{x}, t) g(u, \mu, \zeta, \mathbf{x}, t) \\ &= \int du d\mu\, \mathcal{J}(u, \mu, \mathbf{x}, t) g(u, \mu, \mathbf{x}, t) + O(\varepsilon^2) \\ &= \int du d\mu G(u, \mu, \mathbf{x}, t) + O(\varepsilon^2), \end{aligned} \tag{A1}$$

where $-\infty < u < \infty$ and $0 < \mu < \infty$. Then the equilibrium density $n_0(x)$ can only be given by

$$n_0(x) = \int du d\mu G_0(u, \mu, x), \tag{A2}$$

which is consistent with Eq. (14). [It is interesting to note that because of the choice of velocity-space coordinates, the Jacobian is always a function of $(u, \mu, \mathbf{x}, t)$ and so $G_0(u, \mu, x) = \mathcal{J}(u, \mu, \mathbf{x}, t)g_0(u, \mu, \mathbf{x}, t)$.]

Furthermore, the parallel current is given by

$$\begin{aligned}
J_{/\!/}(\mathbf{x}, t) &= qn(\mathbf{x}, t)\hat{\mathbf{n}}(\mathbf{x}, t)\cdot\mathbf{u}(\mathbf{x}, t) \\
&= q\hat{\mathbf{n}}(\mathbf{x}, t)\cdot\int d^3v\, \mathbf{v}f(\mathbf{v}, \mathbf{x}, t) \\
&= q\int d^3v\, v_{/\!/}f(\mathbf{v}, \mathbf{x}, t) \\
&= q\int dud\mu d\zeta u\mathcal{J}(u, \mu, \zeta, \mathbf{x}, t)g(u, \mu, \zeta, \mathbf{x}, t) \\
&= q\int dud\mu u\mathcal{J}(u, \mu, \mathbf{x}, t)g(u, \mu, \mathbf{x}, t) + O(\varepsilon^2) \\
&= q\int dud\mu uG(u, \mu, \mathbf{x}, t) + O(\varepsilon^2).
\end{aligned} \tag{A3}$$

Then the "equilibrium" current $J_{/\!/0}(x)$ is given by

$$J_{/\!/0}(x) = q\langle n\hat{\mathbf{n}}\cdot\mathbf{u}\rangle_0(x) = q\int dud\mu uG_0(u, \mu, x), \tag{A4}$$

which is again consistent with Eq. (14). This current is related to the net current driven along the equilibrium magnetic field by

$$\begin{aligned}
J_{/\!/}^{(0)} &= q\langle n\hat{\mathbf{n}}_0\cdot\mathbf{u}\rangle \\
&= q\langle n(\hat{\mathbf{n}} - \tilde{\mathbf{n}})\cdot\mathbf{u}\rangle \\
&= J_{/\!/0}(x) - q\langle\tilde{\mathbf{n}}\cdot\int dud\mu\mathbf{v}G(u, \mu, \mathbf{x}, t)\rangle,
\end{aligned} \tag{A5}$$

where $\tilde{\mathbf{n}} = \hat{\mathbf{n}} - \hat{\mathbf{n}}_0$ and $\mathbf{v} = u\hat{\mathbf{n}} + \mathbf{v}_E + O(\varepsilon)$. With the help of Eqs. (16) and (17), Eq. (A5) may be further reduced to

$$J_{\parallel}^{(0)} = J_{\parallel 0}(x)\left(1 - \tfrac{1}{2}\lambda^2 <\tilde{b}_{\perp}^{\,2}> B_0^{-2}\right) + \lambda^2 q c n_0\, \hat{\mathbf{n}}_0 \cdot <\tilde{\mathbf{e}} \times \tilde{\mathbf{b}}> B_0^{-2} . \tag{A6}$$

Then, to order λ, we find $J_{\parallel}^{(0)} = J_{\parallel 0}(x)$. It may also be noted that the last term in Eq. (A6) contributes no current when summed over species; it comes from the second-order $\mathbf{v}_E$ drift. Furthermore, it may be shown that the $O(\lambda^2)$ terms in Eq. (A6) contribute a current which is smaller than the Landau damping/TTMP quasilinear current by a factor $(\omega\tau_s)^{-1}$, where τ_s is the slowing-down time of electrons on ions.

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