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NEOCLASSICAL ENERGY TRANSFER BETWEEN ELECTRONS AND IONS IN A TOKAMAK

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ABSTRACT

Expressions are derived for the rates of energy transfer between electrons and ions associated with the neoclassical, electrostatic and Pfirsch-Schluter contributions to the diffusion and also the anomalous contribution caused by electrostatic drift-wave turbulence. The rates vary in sign as well as magnitude. As a result, even when the resultant diffusion is small, there can be a substantial energy transfer between electrons and ions comparable with the collisional transfer rate.

I. Introduction

Even in an Ohmically-heated Tokamak discharge, in addition to the collisional transfer of energy from electrons to ions, there are several other mechanisms whereby the ions will be heated. In the preceding paper [1] the direct Ohmic heating of the ions due to the ion bootstrap current relative to the impurities was considered. In this paper it is shown that processes associated with neoclassical type diffusion lead to the transfer of energy from electrons to ions. These ion heating mechanisms are of particular interest not only because of the anomalous electron energy loss problem but because of the recently reported experimental evidence for anomalous ion heating as discussed in the preceding paper [1].

In the classic paper on neoclassical transport by Rosenbluth, Hazeltine and Hinton [2] which treats a pure hydrogen plasma in which all forms of diffusion other than the neoclassical component (Γ_{NC}) are taken to be negligibly small, the electron energy balance equation was found to be

$$\frac{\partial}{\partial t}\left(\frac{3}{2}nT_e\right) + \frac{1}{r}\frac{\partial}{\partial r}\, r\left(q_e + \frac{5}{2}\,\Gamma_{NC}T_e\right) = -Q_{ie} + JE_{\varphi} + Q_{NC} \qquad (1)$$

where Q_{ie} is the collisional energy transfer to the ions caused by the temperature difference $(T_e - T_i)$ and where Q_{NC} is given by

$$Q_{NC} = -T_i\Gamma_{NC}\left(\frac{n'}{n} - 0.17\,\frac{T_i'}{T_i}\right) \qquad (2)$$

Here the prime denotes the radial derivative. The ion energy balance equation contains the same term Q_{NC} with opposite sign. This was the first discovery of neoclassical transfer of energy from electrons to ions. Using the poloidal momentum balance condition which, from reference [2], for the banana regime is

$$\overline{V}_{\parallel} = -\frac{T_i}{eB_\theta}\left[\frac{e\Phi'}{T_i} + \frac{n'}{n} - 0.17\,\frac{T_i'}{T_i}\right] , \qquad (3)$$

where $\underset{\sim}{V}$ is the mass velocity of the plasma and the overbar denotes the θ-average, Q_{NC} can be written in the alternative forms

$$\begin{aligned} Q_{NC} &= \Gamma_{NC}\, e\, B_\theta \left(\overline{V}_{\parallel} - \frac{\overline{E}_r}{B_\theta}\right) \\ &= -\frac{\Gamma_{NC}}{n}\, p_i' + \Gamma_{NC}\, e\, \overline{V}_\theta B \\ &= -\frac{\Gamma_{NC}}{n}\, p_i' - \frac{\overline{V}_\theta\, \overline{\left(\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}\right)\sin\theta}}{R} , \end{aligned} \qquad (4)$$

since

$$\overline{V}_\theta = \frac{B_\theta}{B}\,\overline{V}_{\parallel} - \frac{\overline{E}_r}{B} + \frac{p_i'}{neB}$$

and where use has been made of the relationship [3]

$$\Gamma_{NC} = \frac{-\overline{\left(\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}\right)\sin\theta}}{ReB} \qquad (5)$$

(A tilde denotes the part of a quantity which varies with θ.) From Eq. (4) it is seen that if Γ_{NC} is negative - in most experiments the neoclassical pinch effect is dominant [4] - and if $\bar{V}_\theta$ is positive, the Ohmic heating of the electrons is reduced firstly by the term $(\Gamma_{NC}/n)\ p_i'$, which is the energy going to compress the ions, and secondly by the term $-\bar{V}_\theta \overline{(P_{i\parallel}-P_{i\perp})\sin\theta/R}$ which is a measure of the work done by $\nabla\cdot\underset{\approx}{P}_e$ in the presence of the poloidal rotation $\bar{V}_\theta$. The corresponding work done by $\nabla\cdot\underset{\approx}{P}_i$ is $-\bar{V}_\theta \overline{(P_{i\parallel}-P_{i\perp})\sin\theta}$, which from the ambipolarity condition [3], is equal and of opposite sign to the electron term. Thus the pressure anisotropy for each species which must be present for non-zero Γ_{NC}, as seen from Eq. (5), introduces this extra energy transfer term if $\bar{V}_\theta$ is non-zero. The poloidal equilibrium condition, which for the banana regime in pure hydrogen is given by Eq. (3), almost invariably requires non-zero $\bar{V}_\theta$ and with the exception of this one special case $\bar{V}_\theta$ is usually positive.

A good name for the term Q_{NC} is magnetic pumping heating if it is positive, and magnetic pumping cooling if it is negative. The fact that the magnetic pumping effect can cause cooling is because the pressure anisotropy term $\overline{(\tilde{P}_{j\parallel}-\tilde{P}_{j\perp})\sin\theta}$ is driven by other effects as well as $\bar{V}_\theta$ and can have either sign. The term magnetic pumping heating was first used in this context by Bickerton [5], who showed that if a Tokamak plasma undergoes expansion due to neoclassical diffusion, the energy released by the expansion is substantially larger than the energy needed to drive the bootstrap current. (This is in contrast to Pfirsch-Schluter type diffusion where the expansion energy reappears as extra Ohmic heating for the electrons.) Bickerton showed from qualitative estimates that the expansion energy reappeared as magnetic pumping heating of the electrons. In these estimates $\bar{V}_\theta$ was assumed implicitly to be zero and it is seen from Eq. (4) that in this case Q_{NC} reduces to $\Gamma_{NC}\, p_i'/n$ which is precisely the expansion energy for the ions. (The expansion energy of the electrons is cancelled by the corresponding loss term and does not appear in Eq. (1).)

In studies of the energy balance for Tokamak plasmas the term Q_{NC} has largely been ignored. This was understandable in early treatments where a purely neoclassical model was taken for the plasma since if a steady state is assumed with Γ_{NC} zero,then Q_{NC} is also zero. However, when allowance is made for other contributions to the diffusion, the assumption that the total diffusion rate Γ is zero or small will not in general require Γ_{NC} and Q_{NC} to be zero. This can be seen, for example, in the energy balance equations obtained by Hazeltine and Ware [6] for an impure Tokamak plasma, where in addition to Γ_{NC} the electrostatic contribution to the electron diffusion (Γ_{eE}) and the Pfirsch - Schluter contribution (Γ_{ePS}) are retained. The expression obtained for Q_{NC} was

$$Q_{NC} = \left(\Gamma_{eNC} + \Gamma_{eE} - \frac{R\tilde{n}_{zc}}{2r\bar{n}_z}\,\Gamma_{ePS}\right)\left(\bar{V}_{i\theta}\, eB - \frac{p_i'}{n_i}\right) \qquad (6)$$

where $\tilde{n}_{zc}$ is the amplitude of the cosθ Fourier component of $\tilde{n}_z$. Neglecting inertia terms, the definitions of Γ_{jE} and Γ_{jPS} are [3]

$$\Gamma_{jE} = \overline{\tilde{n}_j\tilde{E}_\theta}/B, \quad \Gamma_{jPS} = \overline{\frac{h^2}{Z_j e\bar{B}}\left(\bar{n}_j Z_j e\tilde{E}_\theta - \frac{\partial\tilde{P}_{j\parallel}}{r\partial\theta}\right)} \tag{7}$$

where $h \equiv 1 + (r/R)\cos\theta$ and the approximation $B_\varphi/B \simeq 1$ has been made. (Equation (6) was obtained for the case where the impurity concentration satisfies $n_z Z^2 \gg n_i$, so that the dominant friction force experienced by the electrons is with the impurities.)

Eq. (6) shows that Q_{NC} does not vanish when $\Gamma_e = \Gamma_{eNC} + \Gamma_{eE} + \Gamma_{ePS}$ is zero. Also, Γ_{eE} is seen to make a similar contribution to that of Γ_{eNC} whereas the Γ_{ePS} term is substantially different. If Γ_{eE} is negative and $\bar{V}_\theta$ is positive the electrostatic term, which is equal to $\bar{V}_\theta\overline{\tilde{n}_e e\tilde{E}_\theta}$ also cools the electrons and the process is essentially inverse Landau damping. (The appropriate neoclassical equations are those for a zero wave-velocity electrostatic wave.) The corresponding heating of the ions is ion Landau damping. Since the wave is not growing the two processes, electron inverse Landau damping and ion Landau damping, combine to give this component of energy transfer from electrons to ions.

The Γ_{ePS} term in Eq. (6) will be found to be related to that part of the Pfirsh-Schluter ion flow $\tilde{V}_\parallel$ which is related to $\bar{V}_\theta$. For positive Γ_{ePS} there is a net expansion cooling of the electrons via the term $\overline{\tilde{P}_{e\parallel} \; \partial\tilde{V}_{\parallel}/\partial x_{\parallel}}$. The electrons assist in the ion accelerations associated with $\tilde{V}_{\parallel}$ via the friction force $\tilde{F}_{e\parallel}$. In the steady state with $\partial\tilde{V}_{\parallel}/\partial t = 0$ the process is an energy transfer with the ions being heated via the term $\overline{\tilde{P}_{i\parallel} \; \partial\tilde{V}_{\parallel}/\partial x_{\parallel}}$.

Taking $\Gamma_e \triangleq 0$, Eq. (6) reduces to

$$Q_{NC} = -\Gamma_{ePS} \left(1 + \frac{R\tilde{n}_{zc}}{r\bar{n}_z}\right) \left(\overline{V}_{i\theta} \; e\overline{B} - \frac{p_i{}'}{n_i}\right) \tag{8}$$

This is typically small in hot Tokamak plasmas because of the smallness of Γ_{ePS}. In Section V below, its magnitude is estimated to be only a few percent of the Ohmic heating.

In addition to the above components of diffusion which occur in a stable plasma, experimental results indicate that there is an anomalous component of outward diffusion Γ_A driven by instabilities. Stambaugh and Rawls [4] have used the predictions of neoclassical theory to deduce the magnitude of Γ_A for a number of experiments. In nearly all the cases studied Γ_A has a substantial positive value corresponding to outward diffusion. The presence of such an extra component of diffusion leads to an enhancement in the magnitude predicted for Q_{NC} since it permits a larger negative value of $(\Gamma_{NC}+\Gamma_E)$ under steady state conditions with $\Gamma \triangleq 0$.

In section II a more general expression is derived for the electron energy balance equation and in particular for Q_{NC} which has the same accuracy as Eq. (6) but is valid for arbitrary impurity concentration. Section III gives energy balance equations for the hydrogen ion thermal energy and for the total positive ion energy $\sum_{i,z} (\frac{1}{2}n_j m_j V_j^2 + \frac{3}{2}p_j)$ where $\underset{\sim}{V}_j$ is the mean velocity of species j. As an example of anomalous diffusion the transport due to drift waves is considered in Section IV. It is shown that such diffusion does not behave like the electrostatic component Γ_E considered above where contributions to Q_{NC} had the same sign as Γ_E. The diffusion due to drift waves (Γ_A) will typically be positive and for the usual cases where the modes are driven by electron terms and suffer ion damping, the turbulence produces its own positive contribution to the energy transfer from electrons to ions further enhancing the total energy transfer. In this case Q_{NC} is estimated to be in the range 10 to 20% of the Ohmic heating which is comparable with the collisional transfer rate.

II. The Electron Thermal Energy Balance

The same notation will be used as in the preceding paper and in particular the electron and hydrogen ion currents will be defined relative to the local mean velocity of the impurity ions; $J_{ez} \equiv -n_e e(\underset{\sim}{V}_e - \underset{\sim}{V}_z)$, etc. The starting point is the heat balance equation, namely Eq. (21) of the preceding paper, which for the electrons after taking the volume average between adjacent magnetic surfaces is

$$\frac{3}{2}\frac{\partial \bar{p}_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\left[q_e + \frac{3}{2}T_e\Gamma_e + (\overline{\tilde{E}_\theta \tilde{P}_{e\perp}}/B)\right] = -\left\langle \underset{\approx}{P}_e : \nabla \underset{\sim\sim}{V}_e \right\rangle_v + \left\langle Q_e \right\rangle_v \tag{9a}$$

Here and throughout the following sections the overbar will be omitted from the quantities R, B, B_θ, E_r and T_j for convenience; the θ-average is intended in each case. As noted in reference (1) the $\overline{\tilde{E}_\theta \tilde{P}_{e\perp}}/B$ term appears explicitly because it is omitted from the defintion of q_{eE}, the electrostatic part of q_e; this term will cancel with part of $\underset{\approx}{P}_e : \nabla \underset{\sim\sim}{V}_e$. For preciseness the electrons and hydrogen ions will be assumed to be in the plateau regime and the energy balance equation will be determined to second order in each of the small parameters $(\rho_{e\theta}/L)$ and (r/R) but the equation will be applicable to the banana or Pfirsch-Schluter regimes as well. This is because terms small in (ν_e/ω_{be}) are not neglected. However, classical transport contributions which are smaller by the extra factor $1/q^2$, where q is the safety factor, will be omitted.

Time derivatives for n_e and T_e are assumed to be of order $(\rho_{e\theta}/L)^2(r/R)^2$, corresponding to the slow diffusion time scale, and hence to order $(\rho_{e\theta}/L)^2(r/R)$

$$\underset{\sim}{\nabla}\cdot n_e \underset{\sim}{V}_e = 0 \qquad (10)$$

An alternative form for Eq. (9a) is

$$\frac{3}{2}\frac{\partial \bar{p}_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\left[q_e + \frac{5}{2} T_e\Gamma_e + (\overline{\tilde{E}_\theta \tilde{P}_{e\perp}}/B)\right] = \langle Q_e\rangle_v + \langle \underset{\sim}{V}_e\cdot(\underset{\sim}{F}_e - ne\underset{\sim}{E})\rangle_v \qquad (9b)$$

This form can be obtained by adding to Eq. (9a), before volume averaging, the scalar product of $\underset{\sim}{V}_e$ with the electron momentum balance equation, namely,

$$-n_e e\ [\underset{\sim}{E}+\underset{\sim}{V}_e\times\underset{\sim}{B}] - \underset{\sim}{\nabla}\cdot\underset{\approx}{P}_e+\underset{\sim}{F}_e=0\ .$$

(It should be noted, as discussed below, that V_{er} is second order in $\rho_{e\theta}/L$ and that $\underset{\approx}{P}_e$ is isotropic in zeroth order; as a result the extra term which would appear in the square bracket on the left hand side of Eq. (9b) namely $\overline{h\sum_x V_{er}\ P_{erx}}$ reduces to $\overline{h\ V_{er}\ P_{eo}} \cong \overline{h\ n_{eo}\ V_{er}}\ T_e \equiv \Gamma_e T_e$, since $\tilde{p}_{eo} = \bar{n}_e e\tilde{\Phi} = \tilde{n}_{eo}T_e$).
In the form Eq. (9b) the $\overline{\tilde{E}_\theta\tilde{P}_{e\perp}}/B$ term will cancel with the term $\langle n_e e\ \underset{\sim}{V}_e\cdot\underset{\sim}{\nabla}\tilde{\Phi}\rangle_v$ which is included on the right hand side[6].

2.1 Compression and Viscous Heating

Our prime concern is with the term $\langle \underset{\approx}{P}_e:\underset{\sim}{\nabla}V_e\rangle_v$ in Eq. (9a) which generates the neoclassical heat transfer but parts of of this term cancel with components of the Q_e term and so the Q_e term will be determined also in the next sub-section.

If $\underset{\approx}{P}_e$ and $\underset{\sim}{V}_e$ are expanded in the small parameter $\rho_{e\theta}/L$, the order of a component being denoted by a subscript, the lowest non-zero order in $\underset{\sim}{V}_e$ is $\underset{\sim}{V}_{e1}$. Hence the highest order term required in $\underset{\sim}{P}_e$ is $\underset{\sim}{P}_{e1}$. In zeroth order, $\underset{\approx}{P}_e$ is isotropic and given by

$$p_{eo} = \bar{p}_{eo} + \tilde{p}_{eo} = \bar{p}_{eo} + \bar{n}_e e\tilde{\Phi} \quad .$$

To first order $\underset{\approx}{P}_e$ is diagonal [6] and is also first order in (r/R);

$$\underset{\approx}{P}_{e1} = P_{e\|}\underset{\sim}{i}_{\|}\ \underset{\sim}{i}_{\|} + \left(\underset{\approx}{I} - \underset{\sim}{i}_{\|}\ \underset{\sim}{i}_{\|}\right) P_{e\perp}$$

where $i_{\|} \equiv \underset{\sim}{B}/B$.

The p_{eo} contributions to the pressure tensor term are

$$-\left\langle \underset{\approx}{P}_{eo}:\nabla\underset{\sim}{V}_e \right\rangle_v = -\bar{p}_{eo}\left\langle \underset{\sim}{\nabla}\cdot\underset{\sim}{V}_e\right\rangle_v - \left\langle \tilde{p}_{eo}\underset{\sim}{\nabla}\cdot\underset{\sim}{V}_e\right\rangle_v$$

$$= -\frac{\bar{p}_{eo}}{r}\frac{\partial}{\partial r} r\overline{hV}_{er} + \overline{\frac{\tilde{p}_{eo}}{\bar{n}_e}\left(\bar{V}_{eo}\frac{\partial\tilde{n}_e}{r\partial\theta} + \tilde{V}_{er}\,\bar{n}_e'\right)}$$

$$= -\frac{\bar{p}_{eo}}{r}\frac{\partial}{\partial r}r\left(\frac{\bar{n}_e\overline{hV_r}+\overline{\tilde{n}_e\tilde{V}_{er}}}{\bar{n}_e}\right) + \frac{\bar{p}_{eo}}{\bar{n}_e r}\frac{\partial}{\partial r}r\overline{\tilde{n}_e\tilde{V}_{er}} + \overline{e\tilde{\Phi}\tilde{V}_{e\theta}\frac{\partial\tilde{n}_e}{r\partial\theta}}$$

$$= -\frac{\bar{p}_{eo}}{r}\frac{\partial(r\Gamma_e/\bar{n}_e)}{\partial r} + \frac{T_e}{r}\frac{\partial r\overline{\tilde{n}_e\tilde{V}_{er}}}{\partial r} + \Gamma_{eE}\ e\ B\ \bar{V}_{e\theta} \qquad (11)$$

,

using Eqs. (10) and (7), the relationship $\Gamma_e = \overline{hn_e V_{er}}$ and the fact that V_{er} is second order in $(\rho_{e\theta}/L)$ since

$$V_{er} - \frac{1}{n_e eB}\left[\frac{\partial \tilde{P}_{e\perp}}{r\partial\theta} + \frac{(P_{e\parallel} - P_{e\perp})\sin\theta}{R}\right] \tag{12}$$

The contribution from $\underset{\approx}{P}_{e1}$ is given by

$$- \left\langle \underset{\approx}{P}_{e1} : \nabla \underset{\sim}{V}_e \right\rangle_v = - \left\langle (\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}) \underset{\sim}{i}_\parallel \cdot \underset{\sim}{i}_\parallel \cdot \nabla \underset{\sim}{V}_e \right\rangle_v - \left\langle P_{e\perp} \nabla \cdot \underset{\sim}{V}_e \right\rangle_v$$

$$= - \overline{(\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}) \left(\Theta \frac{\partial \tilde{V}_{e\parallel}}{r\partial\theta} - \frac{\overline{V}_{e\perp}\sin\theta}{R}\right)} + \overline{\tilde{P}_{e\perp} \frac{\overline{V}_{e\theta}}{\overline{n}_e} \frac{\partial \tilde{n}_{eo}}{r\partial\theta}} \tag{13}$$

since to order $(\rho_{e\theta}/L)$ (r/R) Eq. (10) gives

$$\nabla \cdot n_e \underset{\sim}{V}_e = \overline{n}_e \nabla \cdot \underset{\sim}{\tilde{V}}_e + \overline{V}_{e\theta} \frac{\partial n_{eo}}{r\partial\theta} = 0 \tag{14}$$

and where $\overline{V}_{e\perp} = - (\overline{p}_e' + \overline{n}_e eE_r)/\, \overline{n}_e eB$.

Eq. (14) can be written in the alternative form

$$\Theta_o \frac{\partial \tilde{V}_{e\parallel}}{r\partial\theta} = \frac{\left(\Theta_o \overline{V}_{e\parallel} + 2\overline{V}_{e\perp}\right)\sin\theta}{R} - \left(\overline{V}_{e\parallel} - \frac{E_r}{B_\theta}\right) \frac{1}{\overline{n}_e} \frac{\partial \tilde{n}_{eo}}{r\partial\theta} + \frac{\partial \tilde{\Phi}}{r\partial\theta} \frac{\overline{n}_e'}{B\overline{n}_e}$$

$$= \overline{V}_{e\theta} \left(\frac{\sin\theta}{R} - \frac{1}{\overline{n}_e r} \frac{\partial \tilde{n}_{eo}}{\partial\theta}\right) + \frac{V_{e\perp}\sin\theta}{R} - \frac{T_e'}{T_e B} \frac{\partial \tilde{\Phi}}{r\partial\theta} \tag{15}$$

where $\overline{V}_{e\theta} = \Theta_o \overline{V}_{e\parallel} + \overline{V}_{e\perp}$. From Eqs. (13) and (15)

$$- \left\langle \underset{\approx}{P}_{e1} : \nabla \underset{\sim}{V}_e \right\rangle_V = \overline{\frac{\left(\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}\right) \overline{V}_{e\theta} \sin\theta}{R}} + \overline{\tilde{P}_{e\parallel} \frac{\overline{V}_{e\theta}}{\overline{n}_e r} \frac{\partial \tilde{n}_{eo}}{\partial \theta}}$$

$$- \overline{\frac{\tilde{\Phi}\, T_e'}{T_e B} \frac{\partial}{r \partial \theta} \left(\tilde{P}_{e\parallel} - \tilde{P}_{e\perp}\right)} \qquad (16)$$

Combining Eqs. (11) and (16), noting from Eq. (12) that to first order in (r/R)

$$\tilde{V}_{er2} = \frac{1}{\overline{n}_e e B} \frac{\partial \tilde{P}_{e\perp 1}}{r \partial \theta} \quad , \qquad (17)$$

$$- \left\langle P_e : \nabla \underset{\sim}{V}_e \right\rangle_V = - \frac{\overline{P}_{eo}}{r} \frac{\partial}{\partial r} \left(\frac{r \Gamma_e}{\overline{n}_e} \right) + \frac{1}{r} \overline{\frac{\partial r \tilde{p}_{eo}}{\partial r} \tilde{V}_{er}}$$

$$+ \left(\Gamma_{eNC} + \Gamma_{eE}\right) e\, \overline{V}_{e\theta}\, B + \tilde{P}_{e\parallel 1} \frac{\partial \tilde{\Phi}}{r \partial \theta} \left(\frac{T_e'}{T_e B} + \frac{e \overline{V}_{e\theta}}{T_e} \right) \qquad (18)$$

with Γ_{eNC} defined by Eq. (5).

2.2 Frictional Heating

From Eq. (25) of the preceding paper

$$\langle Q_e \rangle_v = - \langle Q_{ie} + Q_{ze} \rangle_v + \langle \underset{\sim}{F}_{ez} \cdot (\underset{\sim}{V}_z - \underset{\sim}{V}_e) \rangle_v + \langle \underset{\sim}{F}_{ei} \cdot (\underset{\sim}{V}_i - \underset{\sim}{V}_e) \rangle_v$$

$$\hat{=} \bar{Q}_{ie} + \bar{Q}_{ze} + \langle F_{e\parallel} (V_{z\parallel} - V_{e\parallel}) \rangle_v + \langle F_{ei\parallel} (V_{i\parallel} - V_{z\parallel}) \rangle_v \tag{19}$$

where $\bar{Q}_{ie}$ and $\bar{Q}_{ze}$ have the form $Q_{je} = 3(m_e/m_j)\bar{n}_e \bar{\nu}_{ej}(T_e - T_j)$ and $\underset{\sim}{F}_e = \underset{\sim}{F}_{ei} + \underset{\sim}{F}_{ez}$. Considering the third term on the right of Eq. (19) and substituting from the electron momentum balance parallel to $\underset{\sim}{B}$

$$\langle F_{e\parallel} \left(V_{z\parallel} - V_{e\parallel}\right) \rangle_v = \langle F_{e\parallel} \rangle_a \left(\overline{hV}_{z\parallel} - \overline{hV}_{e\parallel}\right) + \overline{\left(\frac{\Theta_o \widetilde{F}_{e\parallel}}{\Theta}\right) \left(\widetilde{hV}_{z\parallel} - \widetilde{hV}_{\theta\parallel}\right)}$$

$$\hat{=} \frac{J_{ez}}{n_e e} \left[\bar{n}_e e E_\varphi - \frac{\Theta_o \overline{(\widetilde{P}_{e\parallel} - \widetilde{P}_{e\perp}) \sin\theta}}{R} + \Theta_o \overline{\widetilde{n}_e e \widetilde{E}_\theta}\right] + \overline{\widetilde{F}_{e\parallel} \left(\widetilde{hV}_{z\parallel} - \widetilde{hV}_{e\parallel}\right)} \tag{20}$$

The fourth term in Eq. (19) yields

$$\langle F_{ei\parallel} \left(V_{i\parallel} - V_{z\parallel}\right) \rangle_v \hat{=} \left(\frac{\bar{n}_i}{\bar{n}_i + \bar{n}_z Z^2}\right) \langle F_{e\parallel} \rangle_a \left(\overline{hV}_{i\parallel} - \overline{hV}_{z\parallel}\right) + \overline{\widetilde{F}_{ei\parallel} \left(\widetilde{hV}_{i\parallel} - \widetilde{hV}_{z\parallel}\right)}$$

$$= \frac{J_{iz}}{\bar{n}_e Z_{eff}} \left[\bar{n} e E_\varphi - \frac{\Theta_o \overline{(\widetilde{P}_{e\parallel} - \widetilde{P}_{e\parallel}) \sin\theta}}{R} + \Theta_o \overline{\widetilde{n}_e e \widetilde{E}_\theta}\right] + \overline{\widetilde{F}_{ei\parallel} \left(\widetilde{hV}_{i\parallel} - \widetilde{hV}_{z\parallel}\right)} \tag{21}$$

Expressions for $\widetilde{hV}_{j\parallel}$ can be obtained from the species continuity equation to order $(\rho_{j\theta}/L)(r/R)$ which gives

$$h\tilde{V}_j = \frac{2r}{R}\cos\theta\left(\frac{E_r}{B_\theta} - \frac{\bar{p}_j{}'}{\bar{n}_j Z_j e B_\theta}\right) - \left(\bar{V}_{j\|} - \frac{E_r}{B_\theta}\right)\frac{\tilde{n}_{jo}}{\bar{n}_j} + \frac{\tilde{\Phi}}{B_\theta}\frac{\bar{n}_j{}'}{\bar{n}_j} \qquad (22)$$

From Eq. (19) - (22)

$$\left\langle Q_e \right\rangle_v = -\bar{Q}_{ie} - \bar{Q}_{ze} + \left(J_{ez} + \frac{J_{iz}}{Z_{eff}}\right)\left[E_\varphi + \frac{\left(\Gamma_{eNC}+\Gamma_{eE}\right)B_\theta}{\bar{n}_e}\right]$$

$$+\sum_{i,z}\tilde{F}_{ej\|}\left[-\left(\bar{V}_{j\|} - \frac{E_r}{B_\theta}\right)\frac{\tilde{n}_{jo}}{\bar{n}_j} + \left(\bar{V}_{e\|} - \frac{E_r}{B_\theta}\right)\frac{\tilde{n}_{eo}}{\bar{n}_e} - \frac{2r\cos\theta}{eRB_\theta}\left(\frac{\bar{p}_j{}'}{Z_j\bar{n}_j} + \frac{\bar{p}'_e}{\bar{n}_e}\right) + \frac{\tilde{\Phi}}{B_\theta}\left(\frac{\bar{n}_j{}'}{\bar{n}_j} - \frac{\bar{n}_e{}'}{\bar{n}_e}\right)\right] \qquad (23)$$

2.3 Resultant Electron Heating

The expressions obtained in Eqs. (18) and (23) are now combined to give the resultant electron heating term, noting the relationships $\Theta_o \partial P_{e\parallel 1}/r\partial\theta = \tilde{F}_{e\parallel}$, $\Gamma_{eps} = r\tilde{F}_{e\parallel} c/ReB_\theta$, $\overline{V}_{e\theta} = \Theta_o \overline{V}_{e\parallel} - (E_r/B_\theta) - \partial\overline{p}_e/\overline{n}_e eB\partial r$, $\overline{V}_{e\parallel} = \overline{V}_{z\parallel} - (J_{ez}/\overline{n}_e e)$, giving

$$- \left\langle \underset{\approx}{P}_e : \nabla \underset{\sim\sim}{V}_e \right\rangle_v + \left\langle Q_e \right\rangle_v = -\overline{Q}_{ie} - \overline{Q}_{ze} - \frac{1}{r}\frac{\partial r\Gamma_e T_e}{\partial r} + \overline{\frac{1}{r}\frac{\partial r\tilde{p}_{eo}\tilde{V}_{er}}{\partial r}}$$

$$+ \left(J_{ez} + \frac{J_{iz}}{Z_{eff}}\right) E_\varphi + \left(\Gamma_{eNC} + \Gamma_{eE}\right)\left(V_{z\parallel} - \frac{E_r}{B_\theta} + \frac{J_{iz}}{\overline{n}_e e Z_{eff}}\right) eB_\theta$$

$$+ \sum_{i,z} \overline{\tilde{F}_{ej} \left[-\left(\overline{V}_{j\parallel} - \frac{E_r}{B_\theta}\right) \frac{\tilde{n}_j}{\overline{n}_j} + \frac{\tilde{\Phi}}{B_\theta}\frac{\overline{n}_j'}{\overline{n}_j} - \frac{2r\cos\theta\, p_j'}{\overline{n}_j Z_j eRB_\theta} \right]} \tag{24}$$

Finally, Eq. (24) can be used to replace the right hand side of the energy balance equation, Eq. (9a), yielding

$$\frac{3}{2}\frac{\partial\overline{p}_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r \left(q_e + \frac{5}{2} T_e \Gamma_e\right) = \left[J - J_{iz}\left(1 - \frac{1}{Z_{eff}}\right) \right] E_\varphi$$

$$- \overline{Q}_{ie} - \overline{Q}_{ze} + \left(\Gamma_{eNC} + \Gamma_{eE}\right)\left(\hat{V}_\parallel - \frac{E_r}{B_\theta}\right) eB_\theta$$

$$+ \sum_{i,z} \tilde{F}_{ej} \left[-\left(\overline{V}_{j\parallel} - \frac{E_r}{B_\theta}\right) \frac{\tilde{n}_j}{\overline{n}_j} + \frac{\tilde{\Phi}\overline{n}_j'}{B_\theta \overline{n}_j} - \frac{2r\cos\theta}{\overline{n}_j Z_j eRB_\theta}\frac{\partial\overline{p}_i}{\partial r} \right] \tag{25}$$

where the total toroidal current $J \equiv J_{ez} + J_{iz}$ has been introduced and where $\hat{V}_{\|}$ is the friction weighted mean ion velocity parallel to $\underset{\sim}{B}$ given by

$$\hat{V}_{\|} \equiv \bar{V}_{z\|} + \frac{J_{iz}}{\bar{n}_e e Z_{eff}} \tag{26}$$

Note that the third term on the right of Eq. (24) has been transferred to the left hand side in Eq. (25) making the appropriate coefficient $\frac{5}{2}$. Also the fourth term on the right of Eq. (24) has cancelled with the $\overline{\tilde{E}_\theta \tilde{P}_{e\perp}/B}$ term in Eq. (9a) since, using Eq. (17),

$$\overline{\tilde{p}_{eo} \tilde{V}_{er}} = \overline{\left(\bar{n}_e e\tilde{\Phi}\right) \left(\frac{1}{\bar{n}_e eB} \frac{\partial \tilde{P}_{e\perp}}{r\partial\theta}\right)}$$

$$= \overline{\tilde{E}_\theta \tilde{P}_{e\perp}}/B .$$

III. Ion Energy Balance

A similar analysis to that given in section II leads to the corresponding hydrogen ion energy balance equation which is

$$\frac{3}{2}\frac{\partial \bar{p}_i}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r \left(q_i + \frac{5}{2} T_i \Gamma_i\right)$$

$$= J_{iz}\left[E_\varphi\left(1 - \frac{1}{Z_{eff}}\right) - \frac{\left(\Gamma_{eNC}+\Gamma_{eE}\right)B_\theta}{Z_{eff}\,\bar{n}_e}\right] + \bar{Q}_{ie} - \bar{Q}_{zi}$$

$$- \left(\Gamma_{iNC}+\Gamma_{iE}\right)\left(\bar{V}_{z\parallel} - \frac{E_r}{B_\theta}\right) eB_\theta$$

$$+ \overline{\tilde{F}_{iz}\left[-\left(\bar{V}_{z\parallel} - \frac{E_r}{B_\theta}\right)\frac{\tilde{n}_z}{\bar{n}_z} + \frac{\tilde{\Phi}}{B_\theta}\frac{\bar{n}_z{}'}{\bar{n}_z} - \frac{2r\cos\theta p_z{}'}{\bar{n}_z ZeB_\theta}\right]} \qquad (27)$$

The treatment of the frictional heating term has been somewhat more accurate here than in reference [1], and has yielded the extra term $-J_{iz}\left(\Gamma_{eNC}+\Gamma_{eE}\right)B_\theta/Z_{eff}\bar{n}_e$.

Of greater interest is the energy balance equation for the total positive ion population, hydrogen plus impurities, and including the energy of the mass motion which can be appreciable for the impurity ions. For the hydrogen or impurity ions separately the form of the total energy equation is [8]

$$\frac{\partial}{\partial t}\left\langle \tfrac{1}{2}n_j m_j V_j^{\,2}+\tfrac{3}{2}n_j T_j \right\rangle_V + \frac{1}{r}\frac{\partial}{\partial r}\, r\left[\overline{hV_{jr}\left(\tfrac{1}{2}n_j m_j V_j^{\,2}+\tfrac{5}{2}n_j T_j\right)+q_j+A}\right]$$

$$= \left\langle \underset{\sim}{V}_j \cdot (n_j Z_j e\underset{\sim}{E}+\underset{\sim}{F}_j) + Q_j \right\rangle_V \qquad (28)$$

where the usual volume average has been taken and where the term

$$A \equiv \frac{\overline{\tilde{E}_\theta \tilde{P}_{j\perp}}}{B} - \frac{\overline{h\tilde{E}_\theta\left(\mathbb{P}_{j\parallel}-\mathbb{P}_{j\perp}\right)}}{B} \qquad (29)$$

has been omitted from the definition of q_j. This more general form of the omitted term [c.f.Eq. (9)] allows for the fact that for the Z-ions $(\overline{\mathbb{P}}_{z\parallel}-\overline{\mathbb{P}}_{z\perp})\simeq \bar{n}_z m_z \overline{V}_{z\parallel}^{\,2}$ can be significant. As a result the form of V_{zr} is

$$\tilde{V}_{zr} = \frac{1}{\bar{n}_z ZeB}\left(-\frac{\partial \bar{p}_z}{r\partial\theta} + \bar{n}_z Ze\tilde{E}_\theta - \frac{n_z m_z \overline{V}_{\parallel}^{\,2}\sin\theta}{R}\right)$$

The term involving A will cancel with the term $-\left\langle n_z \underset{\sim}{V}_z \cdot Ze\underset{\sim}{\nabla}\tilde{\Phi}\right\rangle$ which is contained on the right of Eq. (28).

Adding together the appropriate versions of Eq. (28) for the two species of ions, the right hand side gives

$$\sum_{i,z}\left\langle Q_j+\underset{\sim}{V}_j\cdot(n_j Z_j e\underset{\sim}{E}+F_j)\right\rangle_V = \overline{Q}_{ie} + \overline{Q}_{ze} + \left\langle F_{iz\parallel}\left(V_{z\parallel}-V_{i\parallel}\right)\right\rangle_V$$

$$+\sum_{i,z}\left(\overline{hn_j V_{jr} ZeE_r} - \left\langle n_j \underset{\sim}{V}_j Z_j e \underset{\sim}{\nabla\tilde{\Phi}}\right\rangle_v\right) + \left\langle V_{i\parallel}\left(n_i eE_\varphi + F_{ie\parallel} + F_{iz\parallel}\right)\right.$$

$$\left. + V_{z\parallel}\left(n_z ZeE_\varphi + F_{ze\parallel} + F_{zi\parallel}\right)\right\rangle_v$$

$$= \bar{Q}_{ie} + \bar{Q}_{ze} + \left(\Gamma_i + Z\Gamma_z\right)eE_r - \sum_{i,z}\frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{n}_j\overline{\tilde{V}_{jr}Z_j e\tilde{\Phi}}\right)$$

$$+ \overline{hV}_{z\parallel}\left\langle n_e eE_\varphi - F_{e\parallel}\right\rangle_a + \left(\overline{hV}_{i\parallel} - \overline{hV}_{z\parallel}\right)\left\langle n_i eE_\varphi - F_{ie}\right\rangle_a$$

$$+\sum_{i,z}\overline{\left(\widetilde{hV}_{j\parallel}\right)\tilde{F}_{je\parallel}} \qquad (30)$$

Hence, writing $\mathcal{E}$ for $\sum\left(\frac{1}{2}n_j m_j V_j{}^2 + \frac{3}{2}n_j T_j\right)$, the resultant combined total ion energy equation is

$$\frac{\partial}{\partial t}\langle\mathcal{E}\rangle + \frac{1}{r}\frac{\partial}{\partial r}\, r\sum_{i,z}\left[\overline{hV_j r\left(\frac{1}{2}n_j m_j V_j{}^2 + \frac{5}{2}n_j T_j\right) + q_j}\right]$$

$$= \bar{Q}_{ie} + \bar{Q}_{ze} + \Gamma_e eE_r - \left(\Gamma_{eNC} + \Gamma_{eE}\right)e\,\hat{V}_\parallel\, B_\theta$$

$$+ J_{iz}\, E_\varphi\left(1 - \frac{1}{Z_{eff}}\right) + \sum_{i,z}\overline{\left(\widetilde{hV}_{j\parallel}\right)\tilde{F}_{je\parallel}}\,, \qquad (31)$$

using the equalities $\Gamma_i + Z\Gamma_z = \Gamma_e$, $\left\langle n_e eE_\varphi - F_e\right\rangle_a = -\left(\Gamma_{eNC} + \Gamma_{eE}\right)e\, B_\theta$ and

Eq. (26). Finally if the expressions for $\left(h\tilde{V}_{j\parallel}\right)$ are substituted from Eq. (22), parts of the $\tilde{F}_{je\parallel}$ term cancel with the Pfirsch-Schluter part of $\Gamma_e eE_r$, since $\Gamma_{eps} = \overline{h^2\tilde{F}_{e\parallel}}/e\,B_\theta$. The right hand side of Eq. (31) is then the negative of the right hand side of the electron energy equation, Eq. (25), with the single term JE_φ omitted.

IV. Drift Wave Diffusion

A single electrostatic wave with wave vector $\underset{\sim}{k}$ is considered which is taken as being representative of many modes making up drift wave turbulence. Since such modes typically satisfy $v_{Te} \gg |\omega/k_{\parallel}| \gg v_{Ti}$ with $\omega \sim \omega^* \sim \Theta(\rho_{i\theta}/L) k\, v_{Ti}$ it follows that

$$\frac{|k_{\parallel}|}{k} \ll \frac{\omega}{kv_{Ti}} \sim \frac{\rho_{i\theta}}{L}\,\Theta \ll \Theta \qquad (32)$$

Taking for simplicity $\overline{V}_{i\parallel} \doteq \overline{V}_{z\parallel} = \overline{V}_{\parallel}$, if in the absence of $\overline{V}_{\parallel}$ and E_r the solution for the k-mode has the form $\exp(-i\omega t + \underset{\sim}{k}\cdot\underset{\sim}{r})$, then in the presence of non-zero but small E_r and $\overline{V}_{\parallel}$ with the usual ordering $E_r/B \sim \Theta(\rho_{i\theta}/L)v_{Ti}$ and $\overline{V}_{\parallel} \sim (\rho_{i\theta}/L)v_{Ti}$, the only effect on the mode which is first order in $\rho_{i\theta}/L$ is a Doppler shift in the frequency so that ω is changed to $\hat{\omega}$ where

$$\hat{\omega} = \omega + k_{\parallel}\overline{V}_{\parallel} - k_{\perp}\frac{E_r}{B} \doteq \omega - k_{\perp}\frac{E_r}{B} \qquad (33)$$

This result follows from the fact that in the drift-kinetic equation for f_k, the perturbation in the distribution function f (for ions or electrons) due to the wave k, the only extra terms first order in $(\rho_{i\theta}/L)$ which appear are

$$-\frac{Er}{B}(ik_{\perp}f_k) \quad \text{and} \quad \overline{V}_{\parallel}(ik_{\parallel}f_k)$$

which can be combined with the $\partial f_k/\partial t$ term to give

$i\left(\hat{\omega}-k_\parallel \bar{V}_\parallel + k_\perp \frac{E_r}{B}\right) f_k$ and the solution for $\left(\hat{\hat{\omega}}-k_\parallel \bar{V}_\parallel + k_\perp \frac{E_r}{B}\right)$ is the same as the solution for ω in the absence of $\bar{V}_\parallel, E_r$.

Turning to the electron energy balance equation in the form Eq. (9b), the radial component of the total energy flux due to the mode k has the usual electrostatic form considered in Section II and is given by

$$\overline{\left(\frac{3}{2}\,\tilde{p}_{ek} + \tilde{P}_{e\perp k}\right)\left(-\frac{ik_\perp \tilde{\Phi}_k}{B}\right)}$$

where the overbar here denotes an average over both θ and φ and where

$$\frac{3}{2}\,\tilde{p}_{ek} = \int \frac{mv^2}{2}\, f_{ek}\, d^3v\ ,\ \tilde{P}_{e\perp k} = \int \mu B f_{ek} d^3v$$

Once again the divergence of the $\tilde{P}_{e\perp k}$ term will cancel with an energy source term on the right hand side of Eq. (9b). Hence, q_{ek} is defined such that

$$q_{ek} + \frac{5}{2}\,\Gamma_{ek}\,T_e \equiv \frac{3}{2}\,\overline{\tilde{p}_{ek}(-ik_\perp \tilde{\Phi}_k)}/B \qquad (34a)$$

where

$$\Gamma_{ek} \equiv \overline{\tilde{n}_{ek}\,(-ik_\perp \tilde{\Phi}_k)}/B \qquad (34b)$$

(Eq. (34a) is, in fact, the usual form taken for the total energy flux due to electrostatic drift wave turbulence[9].)

Turning to the heating term on the right of Eq. (9b), namely $\langle \underset{\sim}{V}_e \cdot (\underset{\sim}{F}_e - n_e e \underset{\sim}{E}) \rangle_v$, we are concerned with contributions to this term from the mode k which, when summed over k are at least comparable with $Q_{NC} \sim \Gamma_{eNC}\, e\, V_\theta B \sim \Gamma_{NC}\, T/L$, it being assumed that $\Gamma_A \equiv \sum_k \Gamma_{ek} \sim \Gamma_{eNC}$. Firstly, the contribution to $\overline{F}_{e\|}$ due to the mode k is

$$(\overline{F}_{e\|})_k = \overline{-\tilde{n}_{ek} e\, ik_\| \tilde{\Phi}_k}$$

and hence both of the terms $\overline{V}_\| \sum_k (\overline{F}_{e\|})_k$ and $(J/n_e e) \sum_k (\overline{F}_{e\|})_k$ are small compared with the corresponding neoclassical terms since for example

$$\left| \overline{V}_\| \sum_k (F_{e\|})_k \right| = \left| \Theta\, \overline{V}_\| \sum_k \frac{-k_\|}{\Theta k_\perp} \Gamma_{ek}\, e\, B \right|$$

$$\ll \left| V_\theta\, \Gamma_A\, eB \right| \quad .$$

The contributions from $\tilde{F}_{e\|k}$ is also small, since noting that the magnitude of $\tilde{V}_{e\|k}$ is

$$(\hat{\omega}/k_\|)(\tilde{n}_{ek}/\overline{n}_e) \sim (\hat{\omega}/k_\|)\ (e\tilde{\Phi}_k/T_e),$$

the term

$$\overline{\tilde{V}_{e\|k}\tilde{F}_{e\|k}} \sim \bar{n}_e m_e \nu_e \left(\frac{\hat{\omega}}{k_\|}\right)^2 \left(\frac{e\Phi_k}{T_e}\right)^2 .$$

Comparing this term with $\Gamma_{ek} e V_\theta B$, since $V_\theta \sim \omega^*/k_\perp \sim \hat{\omega}/k_\perp$,

$$\frac{\overline{\tilde{V}_{e\|k}\tilde{F}_{e\|k}}}{\Gamma_{ek} e V_\theta B} \sim \frac{n_e m_e \nu_e (\hat{\omega}/k_\|)^2 (e\Phi_k/T_e)^2}{(\hat{\omega}/k_\perp) e \, \overline{\tilde{n}_{ek} \; ik_\perp \Phi_k}}$$

$$\sim \left(\frac{\hat{\omega}}{k_\| v_{Te}}\right) \frac{\nu_e}{\hat{\omega}} \ll 1,$$

provided $\nu_e \gtrless \hat{\omega}$, which is assumed to be the case. [The part of $\tilde{n}_{ek}$ in phase with $i\Phi_k$ has been taken to be of order $\bar{n}_e(\hat{\omega}/k_\| v_{Te})(e\Phi_k/T_e)$.] The components of $\underset{\sim}{F}_e$ not parallel to $\underset{\sim}{B}$ produce even smaller contributions.

Hence the heating term due to the turbulence is

$$\sum_k \left\langle \underset{\sim}{V}_e \cdot (\underset{\sim}{F}_e - n_e e \underset{\sim}{E}) \right\rangle_v \Big|_k$$

$$= \sum_k \left[-\overline{\tilde{n}_{ek}\tilde{V}_{erk}} e E_r + \left\langle (n_e \underset{\sim}{V}_e)_k \cdot e \underset{\sim}{\nabla} \tilde{\Phi}_k \right\rangle_v \right]$$

$$= -\Gamma_A e E_r + \sum_k \left(\frac{1}{r}\frac{\partial}{\partial r} r\bar{n}_e e \overline{\tilde{V}_{erk}\tilde{\Phi}_k} + \overline{e\tilde{\Phi}_k \frac{\partial \tilde{n}_{ek}}{\partial t}} \right) \qquad (35)$$

to second order accuracy in the k-terms. The third term in Eq. (35) can be rewritten

$$\sum_k e\tilde{\Phi}_k \frac{\partial \tilde{n}_{ek}}{\partial t} = -\sum_k \overline{e\tilde{\Phi}_k\, i\left(\omega_k - k_\perp \frac{E_r}{B}\right) \tilde{n}_{ek}}$$

$$= \Gamma_A\, eE_r - \sum_k \overline{e\tilde{\Phi}_k (i\omega_k \tilde{n}_{ek})} \qquad (36)$$

so that Eq. (35) becomes

$$\sum_k \left\langle \underset{\sim}{V}_e \cdot (\underset{\sim}{F}_e - n_e e\underset{\sim}{E}) \right\rangle_v \Big|_k = \sum_k \left[\frac{1}{r}\frac{\partial}{\partial r}\, r\bar{n}_e e\overline{\tilde{V}_{erk}\tilde{\Phi}_k} - \overline{e\tilde{\Phi}_k (i\omega_k \tilde{n}_{ek})}\right] \qquad (37)$$

Since $\tilde{V}_{erk} = ik_\perp \left(\tilde{P}_{e\perp k} - \bar{n}_e e\tilde{\Phi}_k\right)/\bar{n}_e eB$, the first term on the right of Eq. (37) cancels with divergence of the $\tilde{P}_{e\perp k}$ part of the total electron energy flux referred to above.

Thus in the electron energy balance equation the extra terms which appear due to the drift wave turbulence are (i) the addition contributions to q_e and Γ_e given by $q_A = \sum_k q_{ek}$ and $\Gamma_A = \sum_k \Gamma_{ek}$ and (ii) the extra heating term on the right hand side given by

$$P_A = -\sum_k \overline{e\tilde{\Phi}_k \left(i\omega_k \tilde{n}_{ek}\right)} \qquad (38)$$

This term is a measure of the resultant heating of the electrons due to the drift waves. For drift waves driven by the electrons it will be negative, the energy going to ion mass motion in the waves and ion heating via ion-Landau damping or other ion viscosity. The term P_A will be positive only for waves driven by the ions and which are experiencing electron damping. Thus the presence of a substantial anomalous component of diffusion due to drift waves Γ_A, which cancels the negative contributions due to Γ_{eNC} and Γ_{eE}, does not involve a cancellation of Q_{NC}. In fact, for electron-driven waves the energy transfer to the ions is enhanced by the term P_A. (This energy transfer caused by drift waves has been considered by Gladd and Horton [10]).

V. Summary and Conclusion.

The results of the preceding three sections can be summarized in the electron energy balance equation which in the absence of radiation losses is

$$\frac{3}{2}\frac{\partial \overline{p}_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r\left(q_e + \frac{5}{2}\Gamma_e T_e\right) - JE_\varphi = - J_{iz}\left(1 - \frac{1}{Z_{eff}}\right)E_\varphi$$

$$-(Q_{ie}+Q_{ze}) + Q_{NC} + P_A \qquad (39)$$

where

$$q_e = q_{eNC}+q_{eE}+q_{ePS}+q_A \ ,$$

$$\Gamma_e = \Gamma_{eNC}+\Gamma_{eE}+\Gamma_{ePS}+\Gamma_A \ . \qquad (40)$$

Here q_A, Γ_A are the anomalous contributions to the average radial electron heat conduction and diffusion due to instabilities or turbulence. Definitions of the other contributions to q_e and Γ_e are given in the preceding paper [1].

On the right of Eq. (39), the J_{iz} term was the subject of the preceding paper [1], and Q_{ie}, Q_{ze} are the well known collisional energy transfer terms also defined in reference [1]. P_A is the rate energy is received per unit volume by the electrons due to the instabilities and Q_{NC}, the neoclassical energy transfer, is given by

$$Q_{NC} = \left(\Gamma_{eNC} + \Gamma_{eE}\right)\left(\hat{V}_{\parallel} - \frac{E_r}{B_\theta}\right) eB_\theta + \sum_{i,z} \tilde{F}_{ej\parallel}\left[-\left(\overline{V}_{j\parallel} - \frac{E_r}{B_\theta}\right)\frac{\tilde{n}_j}{\overline{n}_j} + \frac{\tilde{\Phi}\overline{n}_j{}'}{B_\theta \overline{n}_j} - \frac{2r\cos\theta\,\overline{p}_j{}'}{\overline{n}_j Z_j eRB_\theta}\right] \qquad (41)$$

This expression is valid for arbitrary impurity concentration and will apply in the limiting cases of either a pure hydrogen plasma or for $\overline{n}_z Z^2 \gg \overline{n}_i$. In these limits the remaining component of friction $\tilde{F}_{ej}$ can be written in terms of Γ_{ePS}, since

$$\Gamma_{ePS} = r\,\tilde{F}_{e\parallel c}/RB_\theta \ ,$$

provided the $\sin\theta$ parts of $\tilde{F}_{e\parallel}$, $\tilde{n}_j$ and $\tilde{\Phi}$ are higher order than the $\cos\theta$ parts, which is often the case [6].

Assuming the energy of the fluctuating fields $\tilde{E}$ and $\tilde{B}$ in the instabilities is small, all the terms on the right hand side of Eq. (39) will appear with the opposite sign in the total ion energy balance equation,[see Eq. (31) for the case of a stable plasma]. The J_{iz} term is direct Ohmic heating of the hydrogen ions; the electrons never receive this part of JE_φ. The other three terms involve energy transfer between electrons and ions.

In the case of electrostatic drift wave turbulence

$$\Gamma_A = \sum_k \overline{\tilde{n}_{ek} \; (-ik_\perp \tilde{\Phi}_k)}/B$$

$$q_A + \frac{5}{2}\Gamma_A T_e = \sum_k \frac{3}{2}\overline{\tilde{p}_{ek}(ik_\perp \tilde{\Phi}_k)}/B \qquad (42)$$

and

$$P_A = -\sum_k \overline{e\tilde{\Phi}_k(i\omega_k \tilde{n}_{ek})} \qquad (43)$$

where ω_k is the complex frequency of the wave measured in the moving frame with velocity $V_\perp = -E_r/B$ as shown in section IV. (The frequency in the rest frame is $\left[\omega_k - k_\perp (E_r/B)\right]$.)

The important feature of the above results is that the various contributions to Γ_e make substantially different contributions to the energy transfer. Just as was found in reference [6], the steady state condition $\Gamma_e \hat{=} 0$ does not reduce Q_{NC} to zero. In particular, although the energy transfer due to $(\Gamma_{eNC} + \Gamma_{eE})$ is proportional to $[\hat{V}_\parallel - (E_r/B_\theta)]$, the energy transfer due to electrostatic drift wave turbulence is independent of $\hat{V}_\parallel$ and E_r for normal neoclassical magnitudes for these quantities. Hence, if $(\Gamma_{eNC} + \Gamma_{eE})$ is balanced by Γ_A with Γ_{ePS} assumed small, Q_{NC} will not be cancelled by P_A. In general Q_{NC} will be negative $(\Gamma_{eNC} + \Gamma_{eE} \lessdot 0)$ and for waves driven by the electrons and damped by the ions P_A will also be negative, so that both Q_{NC} and P_A involve energy transfer from electrons to ions.

Two example cases are considered to illustrate the magnitude of Q_{NC}. Firstly, a completely stable plasma is considered with $\Gamma_A = q_A = P_A = 0$. If such a plasma is in an approximate steady state, which incidentally would imply a higher β than observed in most present experiments [4], then

$$\Gamma_{eNC} + \Gamma_{eE} \doteq -\Gamma_{ePS} = -r\tilde{F}_{e\|c}/ReB_\theta \ .$$

Taking for simplicity $\bar{n}_z Z^2 \gg \bar{n}_i$, and noting that $\tilde{n}_z$ and $\tilde{\Phi}$ are then proportional to $\cos\theta$ [6], Q_{NC} reduces to

$$Q_{NC} \doteq -\tilde{F}_{ez_\| c} \left[\left(\frac{r}{R} + \frac{\tilde{n}_{zc}}{2\bar{n}_z} \right) \frac{\bar{V}_{z\theta}}{\Theta} - \frac{\tilde{\Phi}_c}{2B_\theta} \frac{\bar{n}_z{}'}{\bar{n}_z} \right]$$

where the $p_z{}'$ terms have been neglected because of the small factor $1/Z$. If $\tilde{n}_z$ is large, the dominant term in $\tilde{F}_{ez\|}$ is $(\tilde{n}_z/\bar{n}_z)\ \bar{n}_e e\eta J \doteq (\tilde{n}_z/\bar{n}_z)\ \bar{n}_e eE_\varphi$, giving

$$Q_{NC} \doteq -n_e eE_\varphi \left(\frac{\tilde{n}_z}{\bar{n}_z} \right) \left[\left(\frac{r}{R} + \frac{\tilde{n}_z}{2\bar{n}_z} \right) \frac{\bar{V}_{z\theta}}{\Theta} - \frac{\tilde{\Phi}_c \bar{n}_z{}'}{2B_\theta \bar{n}_z} \right]$$

Taking $V_{z\theta} \sim \Theta v_{Tz}$ (the value $1{\cdot}5\ \Theta v_{Tz}$ was measured on LT-3[11]) and since $J \sim n_e e v_{Ti}$

$$Q_{NC} \sim \frac{1}{8} \left(\frac{\tilde{n}_z}{\bar{n}_z} \right)^2 JE_\varphi$$

for oxygen impurity. Hence, even for $\tilde{n}_z/\bar{n}_z \sim (r/R)^{\frac{1}{2}}$ which can occur when $\bar{V}_{z\theta}$ is close to the wave velocity for the slow compressional wave [12], Q_{NC} is only a few percent of JE_φ.

A larger value is obtained for Q_{NC} if $(\Gamma_{eNC}+\Gamma_{eE})$ is assumed to be balanced by Γ_A, as suggested by the experimental results [4]. Assuming the plasma to be in the banana regime with the neoclassical pinch effect dominating the neoclassical diffusion terms

$$(\Gamma_{eNC}+\Gamma_{eE}) \simeq -n_{eT}E_\varphi/B_\theta$$

where n_{eT} is the density of trapped electrons which, for $\tilde{\Phi}$ proportional to $+\cos\theta$, is given approximately by

$$n_{eT} \doteq \bar{n}_e \left[\frac{2\left(\frac{r}{R} + \frac{e\tilde{\Phi}_c}{T_e}\right)}{1 + \left(\frac{r}{R} + \frac{e\tilde{\Phi}_c}{T_e}\right)} \right]^{\frac{1}{2}} \tag{44}$$

Neglecting the $\tilde{F}_{e\parallel}$ terms, Q_{NC} is given by

$$Q_{NC} = n_{eT}eE_\varphi \left(\hat{V}_\parallel - \frac{E_r}{B_\theta}\right) .$$

If as before $\left(\hat{V}_\parallel - \frac{E_r}{B_\theta}\right) \sim v_{Tz} \sim v_{Ti}/4 \sim J/4\bar{n}_e e$

$$Q_{NC} \sim \frac{1}{4}\left(\frac{n_{eT}}{\bar{n}_e}\right) JE_\varphi \tag{45}$$

Using Eqs. (44) and (45) and taking $e\tilde{\Phi}_c/T_e \sim r/2R$, Q_{NC} varies from 10% of JE_φ at $r/R = 1/20$ to 20% at $r/R = 1/3$. Hence, in this case Q_{NC} has similar magnitude to the collisional transfer $(Q_{ie} + Q_{ze})$.

Finally it can be noted that during a poloidal rotation instability - a saw-tooth disruption - as reported in reference [12], both $[\hat{V}_{\|} - (E_r/B_\theta)]$ and E_φ will increase substantially, making Q_{NC} much larger during the disruption.

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