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Excitation of Toroidicity-Induced Shear Alfvén Eigenmode by Fusion Alpha Particles in an Ignited Tokamak

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Abstract

The toroidicity-induced shear Alfvén eigenmode is found to be destabilized by fusion alpha particles in an ignited tokamak plasma.

In a reacting deuterium-tritium tokamak plasma, fusion-product alpha particles are born with an energy of 3.52 MeV and a density that is sharply peaked at the center of the plasma due to the sensitive dependence of the fusion reaction rate on the plasma temperature. Thus, drift-type instabilities can occur, with the alpha particle density inhomogeneity as the free energy source. In particular, shear Alfvén waves can resonantly interact with the transiting alpha particles via inverse Landau damping to tap this free energy source and become unstable.

The destabilization of shear Alfvén waves was first studied qualitatively by Mikhailovskii,[1] using a local dispersion relation. By including finite Larmor radius effects to discretize the continuum, Rosenbluth and Rutherford[2] found that the Kinetic Alfvén Wave can be destabilized by alpha particles. Tsang *et al.*[3] studied the details of the same problem numerically and reached a similar conclusion. Recently, Li *et al.*[4] found that the Global Alfvén Eigenmode (GAE) in cylindrical geometry can also be destabilized in a similar manner. More recently, however, it has been found that GAE modes can be readily stabilized when toroidicity effects are taken into account.[5–7]

In this letter, we study the possibility of alpha particle destabilization of another type of low-mode-number global Alfvén wave called the Toroidicity-Induced Shear Alfvén Eigenmode (TAE),[8] whose frequency lies within gaps in the Alfvén frequency continuum. Cheng *et al.*[8,9] were the first to show the existence of both low-mode-number and high-mode-number TAE modes, in the ideal MHD limit, without alpha particles. Here we show that low-mode-number TAE modes can be strongly excited by fusion alpha particles in a burning tokamak plasma. This result is relevant to plasma confinement in proposed ignition experiments.

Simply described, the TAE mode is a type of shear Alfvén wave that can exist only in toroidal geometry. For example, if the cylindrical geometry Alfvén continua corresponding to modes with toroidal and poloidal mode numbers (n, m) and $(n, m+1)$ intersect at $r =$

r_0, where $q(r_0) = (m + 1/2)/n$, then toroidicity resolves this degeneracy with a "gap" in the coupled toroidal continuum. Frequencies within this gap are forbidden (analogous to Brillouin zones for the motion of electrons in a periodic lattice potential), except for a certain discrete frequency, which constitutes the TAE mode. In as much as this is a global mode (i.e., low mode number and radially non-localized), its stability can be a significant issue. Here, we show that the presence of highly energetic alpha particles, as in an ignited tokamak plasma, can strongly destabilize the TAE mode.

The theory to describe this physics becomes analytically tractable by adopting a low beta, large aspect ratio, circular flux surface tokamak equilibrium. Following Rosenbluth and Rutherford,[2] we describe the dynamics of the shear Alfvén wave with alpha particles by the linearized drift kinetic equation, using ϕ_1 the electrostatic potential and $A_{\|1}$ the parallel vector potential to represent the perturbed electromagnetic fields (this implies that $B_{\|1} = 0$ where $B_\|$ is the parallel magnetic field). We integrate the linearized drift kinetic equation over all velocities, multiply it by the charge e_s, and sum over all species (indexed by "s"), thus obtaining a moment equation for the perturbed current density:

$$\mathbf{b} \cdot \nabla j_{\|1} + \mathbf{b}_1 \cdot \nabla j_\| + \sum_s e_s \int d^3v \mathbf{v}_{ds} \cdot \nabla f_{1s} = -\nabla \cdot \left[\frac{i\omega c^2}{4\pi V_A^2} \nabla_\perp \phi_1 \right] \tag{1}$$

where the subscript "1" denotes perturbed quantities, $\mathbf{b}$ is the unit vector along the magnetic field, $j_\|$ is the plasma parallel current, e_s is the particle charge, f_{1s} is the perturbed distribution function, B is the magnetic field strength, $V_A = B/(4\pi n_i m_i)^{1/2}$ is the Alfvén speed, and $\mathbf{v}_{ds} = m_s c(\mu B + v_\|^2)/(e_s B^2)\mathbf{b} \times \nabla B$ is the magnetic curvature drift velocity (in the low beta limit) with $\mu = v_\perp^2/2B$ the magnetic moment. The alpha particle contribution to the polarization current has been neglected, owing to the orderings $n_\alpha \ll n_i$ and $\beta_\alpha \ll \beta_i$. The contribution of the alpha particles to the quasi-neutrality condition can also be neglected due to the same ordering. However, the perturbed alpha current caused by the drift velocity $\mathbf{v}_{d\alpha}$ due to the gradient and curvature of the equilibrium magnetic field is retained, because of the very high energy of the alpha particles. The perturbed electron drift current will also be retained, whereas that of the plasma ions may be neglected because $v_i \lll V_A$ where $v_i = (2T_i/m_i)^{1/2}$ is the ion thermal speed. Finally, we have also neglected the finite ion gyro-radius term contribution in the polarization current since the TAE mode width is much larger than the ion gyro-radius. With the help of Ampére's law and using the quasi-neutrality condition to eliminate $A_{\|1}$ in term of ϕ_1, one can rewrite Eq. (1) as follows:

$$\mathbf{b}\cdot\nabla\nabla_\perp^2\mathbf{b}\cdot\nabla\phi_1 - \frac{\mathbf{b}\times\nabla(\mathbf{b}\cdot\nabla\phi_1)}{B}\cdot\nabla\left(\frac{4\pi}{c}j_\|\right) + \nabla\cdot\frac{\omega^2}{v_A^2}\nabla_\perp\phi_1$$
$$= \sum_s \frac{i4\pi\omega}{c^2}e_s\int d^3v\mathbf{v}_{ds}\cdot\nabla f_{1s} \tag{2}$$

For simplicity, we assume concentric magnetic flux surfaces. Expanding the toroidicity effect to first order in the inverse aspect ratio $a/R < 1$ and retaining only the two dominant poloidal modes for the TAE mode, we then arrive at the following two coupled second-order eigenmode equations for the poloidal electrical field $E \propto \phi_1/r$:

$$\left[\frac{d}{dr}r^3\left(\frac{\omega^2}{v_A^2} - k_{\|m}^2 + \sum_s A_{s,m}\right)\frac{d}{dr} - (m^2-1)r\left(\frac{\omega^2}{v_A^2} - k_{\|m}^2 + \sum_s A_{s,m}\right)\right.$$
$$\left. + \left(\frac{\omega^2}{v_A^2}\right)' r^2 + \sum_s mB'_{s,m}r^2\right]E_m + \left[\varepsilon\frac{d}{dr}\frac{\omega^2}{v_A^2}\frac{r^4}{a}\frac{d}{dr}\right]E_{m+1} = 0 \tag{3}$$

$$\left[\frac{d}{dr}r^3\left(\frac{\omega^2}{v_A^2} - k_{\|m+1}^2 + \sum_s A_{s,m+1}\right)\frac{d}{dr} - \left((m+1)^2-1\right)r\left(\frac{\omega^2}{v_A^2} - k_{\|m+1}^2 + \sum_s A_{s,m+1}\right)\right.$$
$$\left. + \left(\frac{\omega^2}{v_A^2}\right)' r^2 + \sum_s (m+1)B'_{s,m+1}r^2\right]E_{m+1} + \left[\varepsilon\frac{d}{dr}\frac{\omega^2}{v_A^2}\frac{r^4}{a}\frac{d}{dr}\right]E_m = 0 \tag{4}$$

where $\varepsilon = 3a/2R$, the prime denotes radial differentiation, and the subscripts m and $m+1$ are the two dominant poloidal mode number. Also, $k_{\|m} = (n - m/q)/R$ is the parallel wavenumber, with R the major radius and q the safety factor. The quantities $A_{s,m}$ and $B_{s,m}$

represent the kinetic response arising from the perturbed curvature drift of species s with poloidal mode number m and are given by

$$A_{s,m} = Q_{s,m-1} + Q_{s,m+1} \tag{5}$$

$$B_{s,m} = Q_{s,m-1} - Q_{s,m+1} \tag{6}$$

$$Q_{s,m\pm1} = -i\frac{\beta_s}{2R^2}\left(P_{s,m\pm1} - \frac{\omega_{*s,m}}{\omega} R_{s,m\pm1}\right) \tag{7}$$

$$P_{s,m\pm1} = \frac{\pi\omega}{v_s^4}\int d^3v \left(v_\perp^2/2 + v_\parallel^2\right)^2 \left(-T_s\frac{\partial f_s}{\partial\epsilon}\right)\delta\left(\omega - k_{\parallel_{m\pm1}}v_\parallel\right) \tag{8}$$

$$R_{s,m\pm1} = \frac{\pi\omega}{v_s^4}\int d^3v \left(v_\perp^2/2 + v_\parallel^2\right)^2 f_s\delta(\omega - k_{\parallel_{m\pm1}}v_\parallel) \tag{9}$$

where β_s is the beta value of species s, $\omega_{*s,m}$ is the diamagnetic drift frequency, and $v_s^2 = 2T_s/m_s$ is the thermal speed. The forms for $A_{s,m}$ and $B_{s,m}$ can be derived by solving the linearized drift kinetic equation.[4] These two quantities originate from the drift current due to the perturbed electrical field with toroidal and poloidal mode number (n, m), when only the resonant part of the response is retained. The two contributions to $A_{s,m}$ and $B_{s,m}$, from the sideband resonances $\omega = k_{\parallel m\pm1}v_\parallel$, result from the poloidal variation of the particle drift velocity. Because of this, the resonant part of the perturbed particle distribution has poloidal variation $\exp[i(m \pm 1)\theta]$ in response to the perturbed field $E_m \exp(im\theta)$, although the perturbed drift current has the poloidal variation $\exp(im\theta)$, at least to lowest order in the inverse aspect ratio. Finally we remark that the mode coupling due to this drift current is small by the ordering $\beta_s \ll 1$.

First, let us consider the MHD limit of Eqs. (3) and (4), dropping the kinetic terms A_s and B_s temporarily. In cylindrical geometry ($\varepsilon = 0$), the two poloidal modes E_m and E_{m+1} are decoupled. Then Eqs. (3) and (4) are singular at $\omega_1^2 = k_{\parallel m}^2 v_A^2$ and $\omega_2^2 = k_{\parallel m+1}^2 v_A^2$, respectively, which give the two cylindrical shear Alfvén continua. In tokamak geometry, Eqs. (3) and (4) are coupled due to the finite toroidicity, and the poloidal mode numbers are no longer good "quantum" numbers. The toroidal shear Alfvén continuum can be obtained by setting the determinant of the coefficients of the second-order derivative terms equal to zero. This yields the following two branches of the toroidal continuum:

$$\omega_\pm^2 = \frac{k_{\|m}^2 v_A^2 + k_{\|m+1}^2 v_A^2 \pm \sqrt{\left(k_{\|m}^2 v_A^2 - k_{\|m+1}^2 v_A^2\right)^2 + 4\varepsilon^2 x^2 k_{\|m}^2 v_A^2 k_{\|m+1}^2 v_A^2}}{2(1-\varepsilon^2 x^2)} \tag{10}$$

where $x = r/a$ is the normalized radius. In particular, at the crossing point of the two cylindrical continua where $k_{\|m} = -k_{\|m+1}$, or $q = (2m+1)/2n$, a gap appears whose width is given by

$$\Delta\omega = \omega_+ - \omega_- \cong 2qx \left(|k_{\|m} v_A|\right)_{q=(2m+1)/2n} . \tag{11}$$

Figure 1 shows the toroidal continuum (solid curves) given by Eq. (10) for $n = -1, m = -2$, and $\varepsilon = 0.375$ with a constant density profile and $q(r) = 1 + (r/a)^2$. The cylindrical continua (dashed curves) are also shown in Fig. 1. The corresponding TAE mode structure is shown in Fig. 2; its eigenfrequency $\omega = 0.93 \left(|k_{\|m} v_A|\right)_{q=1.5}$ lies inside the continuum gap. The eigenmode was obtained by numerically solving Eqs. (3) and (4) with the shooting method. We observe that the mode is peaked at the location of the gap, i.e., near the crossover point of the cylindrical continua.

Next we consider the kinetic effects of alpha particles and electrons on the TAE modes. The resonant contributions of these fast particles can be included perturbatively by assuming that the imaginary part of the frequency is small compared to the real part. To begin with, we expand the solution of Eqs. (3) and (4) as $E_m = E_{0,m} + \delta E_m$ and $\omega = \omega_0 + \delta\omega$, where $E_{0,m}$ and ω_0 are the MHD eigenfunction and eigenfrequency, respectively. We expand the coupled equations to first order in β_s (where it is assumed that $\beta_s < \varepsilon$). Exploiting the self-adjointness of the coupled equations, we obtain the change in frequency due to the kinetic terms as follows:

$$\frac{\delta\omega}{\omega_0} = -\frac{v_{A_0}^2}{2\omega_0^2}$$

$$\cdot \frac{\sum_{m,s}\left\langle r^3 E_{0,m}'^2 A_{s,m} + \left[(m^2-1) r A_{s,m} - r^2 A_{s,m}' - m r^2 B_{s,m}'\right] E_{0,m}^2 \right\rangle}{\left\langle \sum_m \left[(r^3/\tilde{v}_A^2)\, E_{0,m}'^2 + (m^2-1)\,(r/\tilde{v}_A^2)\, E_{0,m}'^2 - (1/\tilde{v}_A^2)' r^2 E_{0,m}^2\right] + 2q\varepsilon\,(r^4/a\tilde{v}_A^2)\, E_{0,m}' E_{0,m+1}' \right\rangle} \quad (12)$$

where $\tilde{v}_A$ is the Alfvén velocity normalized to its value at the center of the plasma. Using Eq. (12), we calculate the growth rate for the TAE mode of Fig. 2. Figure 3 shows the growth rate (normalized to the real frequency) as a function of the alpha particle density scale length L_α, for the typical ignition tokamak parameters of $a/R = 1/4$, $\rho_{\alpha 0}/a = 0.05$, $v_{\alpha_0} = 2v_A$, $\beta_e(0) = 6\%$, $\beta_\alpha(0) = 3\%$, and $T_\alpha = 1\,\mathrm{MeV}$ and for the profiles $\beta_\alpha = \beta_\alpha(0)\exp(-r^2/L_\alpha^2)$ and $\beta_e = \beta_e(0)(1 - r^2/a^2)^3$. A maximum growth rate of $(\gamma/\omega_0)_{\max} = 2.5 \times 10^{-2}$ is obtained at $L_\alpha = 0.5a$. Over the range of $0.20a < L_\alpha < 0.87a$, the TAE mode is unstable.

In the limit of large aspect ratio, the TAE mode is highly peaked around the $q = (2m + 1)/2n$ surface. Then a simple analytical form for the growth rate of the TAE mode can be obtained by evaluating A_s at this surface:

$$\gamma/\omega_0 \cong \frac{9}{4}\left[\beta_\alpha\left(\frac{\omega_{*,\alpha}}{\omega_0} - \frac{1}{2}\right) F\left(\frac{v_A}{v_\alpha}\right) - \beta_e \frac{v_A}{v_e}\right] \quad (13)$$

where the function F is defined as $F(x) = x(1 + 2x^2 + 2x^4)e^{-x^2}$. All the radially dependent quantities are evaluated at the $q = (2m+1)/2n$ surface, where the TAE mode is peaked. The first term in the square bracket on the right-hand side of Eq. (13) comes from the alpha particles and is destabilizing if $\omega_{*,\alpha}/\omega_0 > 1$, whereas the second term is due to electron Landau damping and is always stabilizing since $|\omega_{*,e}/\omega_0| \ll 1$. Thus we have two conditions for the TAE modes to be unstable. The first condition requires $\omega_{*,\alpha}/\omega_0 > 1$, i.e., that the alpha density scale length is small enough. The second condition requires that the alpha particle destabilization overcomes the electron damping effect. Balancing these two terms gives a critical alpha-particle beta value for the TAE mode to be unstable:

$$\beta_{\alpha,\mathrm{crit}} = \frac{v_\alpha}{v_A}\sqrt{\frac{m_e}{m_i}}\,\beta_e\,\frac{1}{\left(\frac{\omega_{*,\alpha}}{\omega_0} - 1\right)} . \tag{14}$$

Using the parameters of Fig. 3, we obtain $\beta_{\alpha,\mathrm{crit}}(0) = 0.3\%$ from Eq. (14). We also calculate the growth rate, using Eq. (13), and obtain $(\gamma/\omega_0)_{\max} = 2.5 \times 10^{-2}$ at $L_\alpha = 0.59a$, which agrees with the numerical results in Fig. 3 very well.

We conclude that the low-n Toroidicity-Induced Shear Alfvén Eigenmode (TAE) can be destabilized by fusion alpha particles in an ignited tokamak plasma. The same result, obtained here with a model equilibria, has been confirmed in a two-dimensional, finite aspect ratio, finite beta, toroidal equilibrium.[7] Regarding their possible detrimental effects on the plasma confinement, nonlinear investigation of TAE modes is required. A preliminary analysis of TAE nonlinear saturation has been recently carried out by Breizman and Berk.[10] Experimental studies are also needed .

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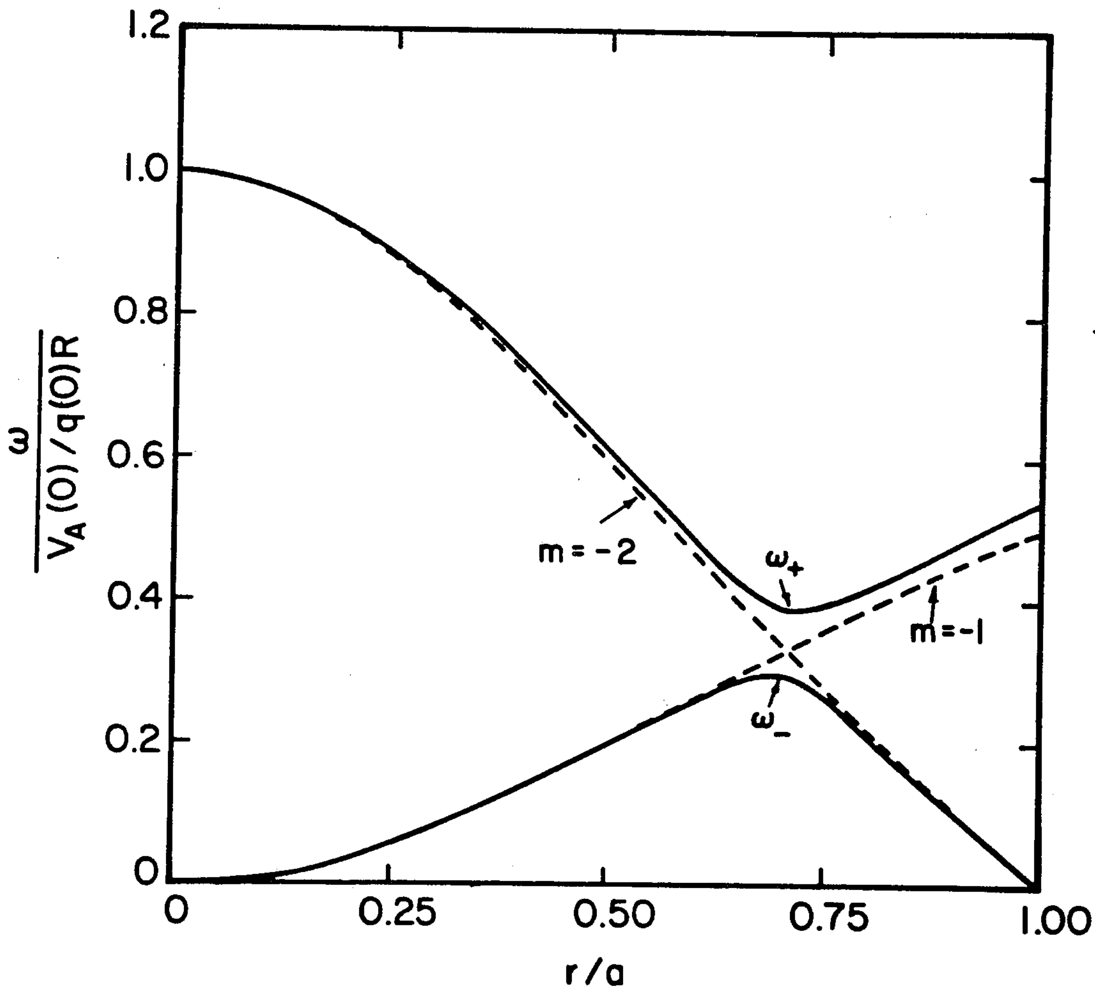
Figure Captions

1. Toroidal shear Alfvén continuous spectrum with gap, for safety factor profile $q = 1 + (r/a)^2$ and a constant density profile; the cylindrical spectra (dashed curves) for $m = -1$ and $m = -2$, with $n = -1$, cross at the flux surface where $q = 1.5$.

2. Radial profiles of the dominant poloidal harmonics for the $n = -1$ TAE mode, for the same equilibrium as in Fig. 1.

3. Growth rate (normalized to the real frequency ω_r) for the $n = -1$ TAE mode as a function of the alpha-particle density gradient scale length.



1.2
1.0
0.8
0.6
0.4
0.2
0
$\frac{\omega}{V_A(0)/q(0)R}$
m = -2
ω_+
ω_-
m=-1
0
0.25
0.50
0.75
1.00
r/a

Fig. 1

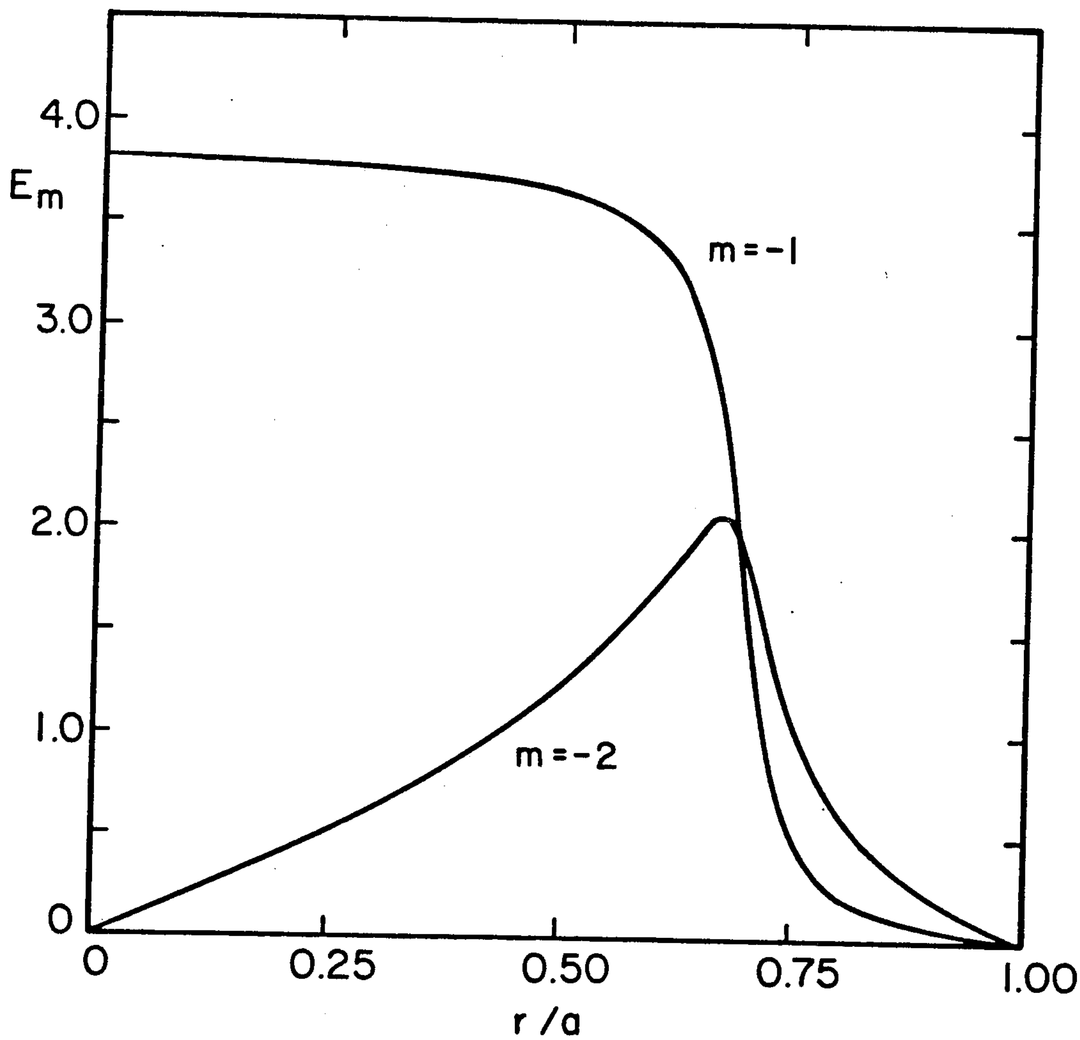


Fig. 2

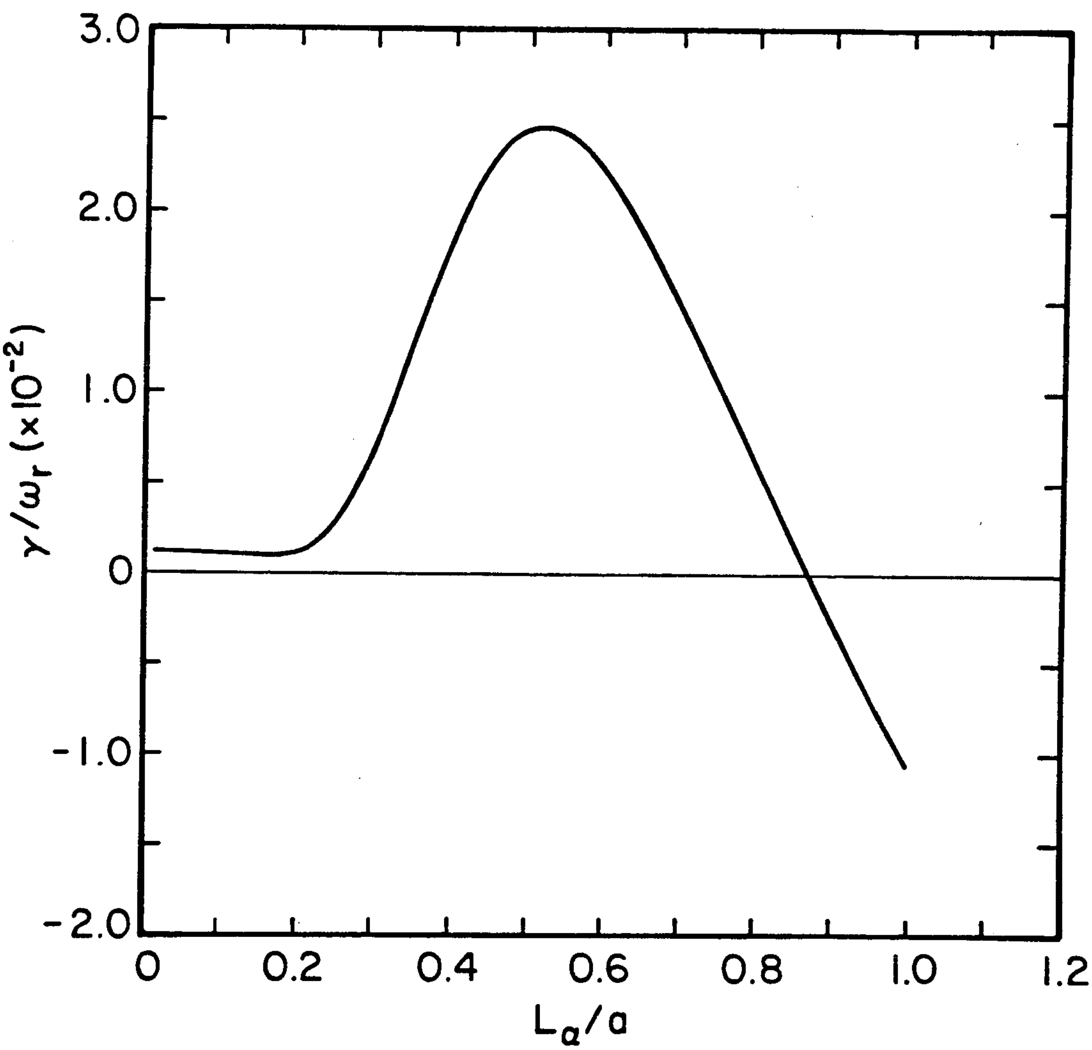

3.0
2.0
1.0
0
-1.0
-2.0
0
0.2
0.4
0.6
0.8
1.0
1.2
$\gamma/\omega_r\ (\times 10^{-2})$
L_α/a

Fig. 3