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Electron Diffusion in Tokamaks Due to Electromagnetic Fluctuations

W. Horton and *D. I. Choi*

Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712-1060

P. N. Yushmanov and *V. V. Parail*

Kurchatov Institute of Atomic Energy
Moscow 123-182 USSR

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Abstract

Calculations for the stochastic diffusion of electrons in tokamaks due to a spectrum of electromagnetic drift fluctuations are presented. The parametric dependence of the diffusion coefficient on the amplitude and phase velocity of the spectrum, and the bounce frequency for the electrons is studied. The wavenumber spectrum is taken to be a low order (5×5) randomly-phased, isotropic, monotonic spectrum extending from $k_{\perp \min} \simeq \omega_{ci}/c_s$ to $k_{\perp \max} \simeq 3\omega_{pe}/c$ with different power laws of decrease $\varphi_k \simeq \varphi_1/k^m$, $1 \leq m \leq 3$. A nonlinear Ohm's law is derived for the self-consistent relation between the electrostatic and parallel vector potentials. The parallel structure of the fluctuations is taken to be such that $k_{\|}^{n\ell} v_e < \omega_k$ due to the nonlinear perpendicular motion of the electrons described in the nonlinear Ohm's law. The diffusion coefficient scales approximately as the neo-Alcator and Merezhkin-Mukhovatov empirical formulas for plasma densities above a critical density.

I. Introduction

The drift motions of particles in the low-frequency fluctuations may be the reason for the enhanced particle and energy transport observed in toroidal magnetic traps. For typical experimental conditions, the level of the fluctuations is sufficiently high that the usual quasilinear theory may not be applicable.[1] The nature of the diffusion for high amplitude fluctuations is the stochastic drift motion of the particles. Recently, Horton[2] analyzed the stochastic diffusion of particles in two electrostatic drift waves. Parail and Yushmanov[3] analyze the stochastic diffusion, taking into account both the electrostatic $\mathbf{E} \times \mathbf{B}$ drift and the magnetic drift motion $v_{\|} \delta \mathbf{B}_{\perp}$.

Study of the model of diffusion in one nonlinear mode in Ref. 3 lead to results which were important from the point of view of interpreting the experimental results. One of the principal points of Ref. 3 was that in some range of parameters, the diffusion coefficient does not depend sensitively on the spectrum and amplitude of the fluctuations. The results suggest that it may be possible to obtain theoretically a kind of universal scaling law, within the limits of a broad drift mode type spectrum, which would be applicable to most "quiet" discharges in tokamaks. This theoretical scaling is close to the experimentally observed neo-Alcator or Merezhkin-Mukhovatov scaling laws.

The low-frequency fluctuations in tokamak discharges appear to be in a regime of strong nonlinear perpendicular motion of the particles[4] with $k_{\perp} \tilde{v}_{\perp} > \omega$, where $\tilde{v}_{\perp}$ is the perpendicular drift velocity of the particles in the fluctuations, and $k_{\perp}$, ω are typical perpendicular wavenumbers and frequencies in the fluctuation spectrum. In this regime, particles for a time of one wave period $2\pi/\omega$ move across the ambient magnetic field a distance greater than the perpendicular wavelength. Therefore, one may expect that the drift mode fluctuations are in a regime of strong nonlinearity, describable in terms of a chaotic distribution of vortical flows. Vortical flows centered on the mode-resonant surfaces produce the strongest transport.

The goal of this work is to describe the electron transport in electromagnetic drift modes under conditions of strong nonlinearity $k_{\perp} \tilde{v}_{\perp} > \omega$ using a model of randomly-distributed nonlinear modes located near the resonant surfaces. We present extensive numerical experiments for the particle motion, described in terms of the diffusion coefficient in the stochastic regime. The dependence of the diffusion coefficient on magnetic shear, the mean phase velocity of the fluctuations, and the amplitude and power law index of the wavenumber spectrum are presented.

In Sec. II we discuss the equations of drift motion due to fluctuations in tokamaks. In Sec. III we describe the nonlinear Ohm's law which determines the relation between the magnetic potential $A_{||}$ and the electrostatic potential φ. In Sec. IV the model fluctuation spectrum is given, and the dimensionless parameters in the equations of motion and the diffusion coefficients are defined. The relevant range of the dimensionless parameters are chosen, the results of the numerical experiments presented and the physical processes underlying the results are discussed in Sec. IV. In Sec. V we give the conclusions of our studies and summarize the physical behavior of the stochastic diffusion coefficient.

II. Electron Equation of Motion

The motion of an electron in fluctuations with frequency components $\omega \ll \omega_{ce}$ ($\omega_{ce} = eB/m_e c$ is the electron cyclotron frequency) and perpendicular wavelengths $\lambda_\perp \gg \rho_e$ ($\rho_e = v_\perp/\omega_{ce}$ is the electron gyroradius) is described by the drift motion of the guiding center. In the magnetic field of a tokamak with concentric circular magnetic surfaces

$$\mathbf{B} = \left(B_0\hat{\phi} + B_\theta(r)\hat{\theta}\right)(1 - \varepsilon\cos\vartheta), \tag{1}$$

where $\varepsilon = r/R$ and $B_\theta = \varepsilon B_0/q, q$ is safety factor, the equations of motion are

$$\frac{dv_{||}}{dt} = -\frac{e}{m}E_{||} - \frac{\mu B_0 \varepsilon}{qR}\sin\vartheta, \tag{2}$$

$$\frac{d\vec{r}}{dt} = \frac{c\vec{E}\times\hat{b}}{B_0} + v_{||}\hat{b} + v_d\left(\hat{\theta}\cos\vartheta + \hat{r}\sin\vartheta\right), \tag{3}$$

where $\mu = v_\perp^2/2B$ is the magnetic moment, $v_d = \left(v_\perp^2 + 2v_{||}^2\right)\Big/2\omega_{ce}R$ is the toroidal drift velocity and

$$\hat{b} = \hat{b}_0 + \frac{\vec{B}_\perp}{B_0} = \hat{\phi} + \frac{\varepsilon}{q}\hat{\theta} + \frac{\delta\vec{B}_\perp}{B_0} \tag{4}$$

the unit vector along magnetic field line.

The low frequency, strongly elongated ($k_{\|} \ll k_{\perp}$) electromagnetic fluctuations can be described by the potentials[1] φ and A such that

$$\vec{E} = -\nabla\varphi - \frac{\hat{b}}{c}\frac{\partial A}{\partial t}, \tag{5}$$

$$\delta\vec{B}_{\perp} = \nabla A \times \hat{b}. \tag{6}$$

Let us compare terms in the right-hand side of Eq. (2). For typical conditions of tokamak plasma fluctuations $\omega \sim k_{\perp}v_{de}$, $k_{\|} \sim 1/qR$ ($v_{de} = Tc/eBr_n$ —the electron diamagnetic velocity) we have

$$\frac{e\nabla_{\|}\varphi}{m} \Big/ \frac{\mu B_0 \varepsilon}{qR} \sim \frac{e\tilde{\varphi}}{T\varepsilon}$$

$$\frac{e}{mc}\frac{\partial A}{\partial t} \Big/ \frac{\mu B_0 \varepsilon}{qR} \sim \frac{k_{\perp}TA}{mBr_n} \Big/ \frac{T\varepsilon}{mqR} \sim \frac{\tilde{B}}{B_0}\frac{qR}{\varepsilon r_n}.$$

It is easy to see that for not extremely high fluctuations amplitudes $e\tilde{\varphi}/T \lesssim 10^{-2}$ and $\tilde{B}/B_0 \lesssim 10^{-3}$, which are satisfied in tokamak devices,[4] the parallel acceleration from the oscillations $eE_{\|}/m$ are negligible, except for the small region of marginally-trapped particles at $v_{\|} = \sqrt{\varepsilon}v_{\perp}$. In this case, the longitudinal motion (Eq. (2) and the parallel component of Eq. (3)) decouples from the perpendicular motion. The equilibrium toroidal mirror force separates the electron population into passing $v_{\|} > \varepsilon^{1/2}v_{\perp}$ and trapped $v_{\|} < \varepsilon^{1/2}v_{\perp}$ particles with

$$v_{\|}(t) = \begin{cases} v_m\, dn(u,\kappa) \approx v_{\|}(1 + \delta\cos\omega_t t) & \text{for} \quad v_{\|} > \varepsilon^{1/2}v_{\perp} \\ v_m cn(u,\kappa) \approx v_{\|}\cos\omega_b t & \text{for} \quad v_{\|} < \varepsilon^{1/2}v_{\perp} \end{cases} \tag{7}$$

$$z(t) = \int^t v_{\|}\, dt' \approx \begin{cases} v_{\|}\left(t + \delta\omega_t^{-1}\sin\omega_t t\right) & \text{for} \quad v_{\|} > \varepsilon^{1/2}v_{\perp} \\ v_{\|}\omega_b^{-1}\sin\omega_b t & \text{for} \quad v_{\|} < \varepsilon^{1/2}v_{\perp} \end{cases} \tag{8}$$

$$\theta(t) = z(t)/qR, \tag{9}$$

where $v_m = \left(v_{\|}^2 + \varepsilon v_{\perp}^2\right)^{1/2}$, $\kappa^2 = 2\varepsilon v_{\perp}^2/v_m^2$, $\omega_t \approx v_{\|}/qR$ and $\omega_b \approx v_{\perp}\sqrt{\varepsilon}/qR$.

In the presence of collisions, Eq. (2) becomes a Langevin equation with the orbits (7) and (8) modified by $v_{\|}^2(t) \to v_{\|}^2(t) + v_e^2\nu_{\text{eff}}t$ from pitch angle scattering. Here we restrict consideration to the banana regime where $\nu_{\text{eff}}\tau_c \ll \varepsilon$, reserving for future studies the finite $\nu_{\text{eff}}\tau_c \sim \nu_{\text{eff}}/\omega_b$ regime.

We now proceed to the perpendicular drift motion of electrons. We take $x = r - r_0$ and $y = r_0\theta$ to be local coordinates perpendicular to the equilibrium magnetic field at $r = r_0$. The cross-field motion from Eq. (3) becomes

$$\frac{dx}{dt} = -\frac{c}{B}\frac{\partial \varphi}{\partial y} + \frac{v_{\parallel}}{B}\frac{\partial A}{\partial y} + v_d \sin\theta \tag{10}$$

$$\frac{dy}{dt} = \frac{c}{B}\frac{\partial \varphi}{\partial x} - \frac{v_{\parallel}}{B}\frac{\partial}{\partial x}(A + A_s) + v_d \cos\theta, \tag{11}$$

where

$$A_s = \frac{1}{2}\frac{B_0}{R}\frac{r_0 d}{dr}\left(\frac{1}{q}\right)x^2,$$

and all dependences on z and θ are substituted as functions of t from Eqs. (8) and (9). The use of local coordinates x, y, z rather than global toroidal coordinates is justified by the small space scale c/ω_{pe} of the diffusion over a correlation time.

Equations (10) and (11) may be represented in Hamiltonian form. The effective $1\frac{1}{2}$-D Hamiltonian for the cross-field motion is

$$\begin{aligned} H(x,y,t) &= H_{\text{eff}}(x,y,t) + H_d(x,y,t) \\ &= \frac{c}{B}\left[\varphi(x,y,t) - \frac{v_{\parallel}(t)}{c}A(x,y,t)\right] - \frac{v_{\parallel}(t)}{B}A_s(x) + v_d r \cos\left(\theta(t) + \frac{y}{r_0}\right), \end{aligned} \tag{12}$$

and the equations of motion are

$$\frac{dx}{dt} = -\frac{\partial H}{\partial y} \quad \text{and} \quad \frac{dy}{dt} = \frac{\partial H}{\partial x}.$$

To understand the physical picture of the stochastic diffusion, let us consider the simplest case $A_s = 0$, $A = 0$ and $v_d = 0$. Then for sinusoidal oscillations

$$\varphi(x,y,t) = \sum_{\mathbf{k}} \varphi_{\mathbf{k}} \sin(k_x x + \alpha_{\mathbf{k}}) \cos(k_y y - \omega_{\mathbf{k}} t + \beta_{\mathbf{k}}), \tag{13}$$

the problem is mathematically the same as that analyzed by Horton,[2] showing that the motion becomes stochastic in the presence of two drift waves when the circulation (or $\mathbf{E} \times \mathbf{B}$ trapping) frequency $\Omega_k = ck_x k_y \varphi_{\mathbf{k}}/B$ becomes comparable to the dispersion in the wave frequencies $\Delta\omega$. In the present problem with finite $A_{\parallel}$ ($v_{\parallel}A_{\parallel} \lesssim c\varphi$), the dominant time dependence of H_{eff} arises from $v_{\parallel}(t)$ given in Eq. (7). Therefore, for $A_{\parallel}(x,y,t)$

non-parallel to $\varphi(x, y, t)$, i.e., $\hat{b} \cdot \nabla\varphi \times \nabla A \neq 0$, the presence of $v_{\|}(t)A$ makes the single-wave Hamiltonian have stochastic motion when the circulation frequency Ω_k is comparable to the electron bounce frequency $\omega_b = \varepsilon^{1/2} v_{\perp}/qR$. For the mixing length level of saturation $e\varphi_k/T \sim 1/k_{\perp} r_n$, the single-wave stochasticity condition is satisfied for $k_{\perp}\rho_s > \varepsilon^{3/2} q^{-1} \left(m_i/m_e\right)^{1/2} \sim 1$. This condition is typically satisfied for the tokamak fluctuation spectrum. In the presence of a spectrum of waves, global stochasticity appears when this threshold condition is satisfied due to the overlapping of the nonlinear drift resonances in the manner calculated in Refs. 2 and 3.

III. Nonlinear Ohm's Law

The principal part of the analysis, before proceeding to the numerical solution of Eqs. (10) and (11), is to establish the relation between φ and A. These relations may be obtained from the two equations describing longitudinal oscillations in strongly-magnetized plasma $\partial n/\partial t + e^{-1}\nabla_{\|} j_{\|} = 0$ and $j_{\|} = \sigma E_{\|}$. As suggested by Parail and Yushmanov,[3] we use the longitudinal Ohm's law to describe the connection between A and φ. Let us first show that the nonlinear drift motion does not change the hydrodynamic Ohm's law. Under the conditions of small density fluctuations ($\tilde{n} \ll n_0$) and constant temperature $\left(\nabla_{\|} T_e = 0\right)$, one can obtain nonlinear longitudinal Ohm's law from the equation of motion of electron fluid parallel, $j_{\|} = -en_e u_{\|}$, to the magnetic field

$$m_e n_e \frac{du_{\|}}{dt} = -en_e E_{\|} - T_e \nabla_{\|} n_e \tag{14}$$

and continuity equation

$$\frac{\partial n_e}{\partial t} + \frac{c}{B}[\varphi, n_e] - \frac{1}{e}\nabla_{\|} j_{\|} = 0, \tag{15}$$

where $[A, B]$ is the Poisson bracket, $E_{\|} = -\nabla_{\|}\varphi - \frac{1}{c}\frac{\partial A}{\partial t}$ is the parallel component of the electric field, and

$$\nabla_{\|} = \frac{\partial}{\partial z} - \frac{1}{B}[A, \ ]. \tag{16}$$

In the case of quiet modes with prescribed helicity α and phase velocity u, the dependence of all variables is of the form $A \equiv A(x, \eta)$, $\eta = y + \alpha z - ut$, and after dividing $n(x, \eta)$ in two parts

$$n(x, \eta) = n_0(x) + \tilde{n}(x, \eta)$$

with $\tilde{n} \ll n_0$, one can rewrite Eqs. (14) and (15) in the form

$$-u\frac{\partial \tilde{n}}{\partial \eta} - \frac{c}{B}\frac{dn_0}{dx}\frac{\partial \varphi}{\partial \eta} + \frac{c}{B}[\varphi, \tilde{n}] - \frac{1}{e}\alpha\frac{\partial j_{\|}}{\partial \eta} + \frac{1}{eB}[A, j_{\|}] = 0 \tag{17}$$

$$\begin{aligned} m_e n_0 \left(-u\frac{\partial u_{\|}}{\partial \eta} + \frac{c}{B}\left[\varphi, u_{\|}\right]\right) =& e n_0 \left(\alpha\frac{\partial \varphi}{\partial \eta} - \frac{1}{B}[A, \varphi] - \frac{u}{c}\frac{\partial A}{\partial \eta}\right) \\ & - T_e\left(\alpha\frac{\partial \tilde{n}}{\partial \eta} - \frac{1}{B}[A, n]\right). \end{aligned} \tag{18}$$

The linear solution of these equations, when one neglects nonlinear terms, is

$$j_{\|} = \frac{n_0 e^2}{m}\frac{(u - v_{de})(\alpha\varphi - uA/c)}{u^2 - \alpha^2 v_T^2} \tag{19}$$

$$\tilde{n} = -\frac{c}{uB_0}\frac{dn_0}{dx}\varphi - \frac{\alpha}{eu}j_{\|} \tag{20}$$

where

$$v_{de} = -\frac{cT_e}{eBn_0}\frac{dn_0}{dx} \quad \text{and} \quad v_T^2 = T_e/m_e.$$

If one now calculates the nonlinear terms of Eqs. (17) and (18)

$$\begin{aligned} \left[\varphi, j_{\|}\right] &= -\frac{u}{c}\cdot\frac{ne^2}{m}\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}[\varphi, A] \\ [\varphi, \tilde{n}] &= -\frac{\alpha}{eu}\left[\varphi, j_{\|}\right] = \frac{\alpha}{ec}\frac{ne^2}{m}\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}[\varphi, A] \\ \left[A, j_{\|}\right] &= -\alpha\frac{ne^2}{m}\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}[\varphi, A] \\ [A, \tilde{n}] &= \left(\frac{c}{uB}\frac{dn_0}{dx} + \frac{\alpha^2}{eu}\frac{ne^2}{m}\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}\right)[\varphi, A], \end{aligned}$$

then it is easy to see all the nonlinear terms in Eqs. (17) and (18) cancel. In Eq. (17):

$$\frac{c}{B}[\varphi, \tilde{n}] + \frac{1}{eB}\left[A, j_{\|}\right] = 0,$$

in Eq. (18):

$$\begin{aligned} -\frac{mc}{eB}\left[\varphi, j_{\|}\right] + \frac{en_0}{B}[A, \varphi] - \frac{T_e}{B}[A, \tilde{n}] =& \frac{n_0 eu}{B}\cdot\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}[\varphi, A] - \frac{en_0}{B}[\varphi, A] \\ & - \frac{T_e c}{B^2 u}\frac{dn_0}{dx}[\varphi, A] - \frac{T_e\alpha^2}{Beu}\frac{n_0 e^2}{m}\frac{u - v_{de}}{u^2 - \alpha^2 v_T^2}[\varphi, A] = 0. \end{aligned}$$

Therefore, when nonlinear motion is studied one may use the linear form of Ohm's law (19), which together with Ampere's law $\Delta_\perp A = -4\pi j_\| / c$, gives

$$A - \frac{c^2}{\omega_{pe}^2} \frac{u^2 - \alpha^2 v_T^2}{u\,(u - v_{de})} \Delta_\perp A = \frac{\alpha c}{u} \varphi \tag{21}$$

for the relationship between the magnetic and scalar potentials.

The limiting case $u \ll \alpha v_T$ of Eq. (21) corresponds to fast thermal motion of electrons with respect to wave phase, which give rise to the Boltzmann equilibrium of electrons $\tilde{n} \sim n_0 e\varphi/T$. The opposite limiting hydrodynamic case ($u \gg \alpha v_T$) corresponds to neglect of thermal motion. For linear waves, the last situation may exist only very close to the resonant surface $k_\| v_{Te} < \omega$. But if one takes into account the nonlinear motion of the electrons in a wave structure of sufficiently high amplitude $k_\perp \tilde{v}_\perp > k_\| v_{Te}, \omega$, where $\tilde{v}_\perp$ is the characteristic perpendicular particle drift velocity in the oscillation, one may use the hydrodynamic limit for a wide parameter range. This is due to the fact that under the condition $k_\perp \tilde{v}_\perp > k_\| v_{Te}, \omega$ most of the electrons become $\mathbf{E} \times \mathbf{B}$ trapped[2] in potential cells. In this regime, the longitudinal motion does not change the phase of the wave, and there is no reason for the Boltzmann averaging.[3]

Using these arguments for neglecting the thermal motion of electrons in a high amplitude regime, one may also obtain Ohm's law in a simpler way while taking into account the ballooning structure $\varphi_0(z)$ of the mode potential

$$\varphi = \varphi_0(z) f(x, y + \alpha z - ut). \tag{22}$$

The longitudinal equation of motion without parallel velocity may be rewritten as

$$\frac{\delta v_\|}{\delta t} = \frac{e}{m} \left[\frac{(\mathbf{B}_0 + \mathbf{B}_s) \cdot \nabla \varphi}{B_0} + \frac{1}{c} \frac{\delta \tilde{A}}{\delta t} \right],$$

where $\frac{\delta}{\delta t} = \frac{\partial}{\partial t} + \frac{c}{B}[\varphi, \ ]$ is the time derivative along hydrodynamic trajectory. Substituting $v_\| = -j_\|/en = \frac{c}{4\pi e n} \Delta_\perp A$, the solution of this equation may be written as follows.

$$A - \frac{c^2}{\omega_{pe}^2} \Delta_\perp A = \frac{c}{B_0} \int\limits_{-\infty}^{t} (\mathbf{B}_0 + \mathbf{B}_s) \cdot \nabla \varphi \delta t', \tag{23}$$

where the integration is taken along hydrodynamic trajectory (inverse to $\delta/\delta t$). Taking into account the ballooning representation (22), one has

$$\begin{aligned}(\mathbf{B}_0 + \mathbf{B}_s) \cdot \nabla\varphi &= B_0 \frac{\partial \varphi_0}{\partial z} f + B_s \varphi_0 \frac{\partial f}{\partial y} \\ &= B_0 \frac{\partial \varphi_0}{\partial z} f - \frac{B_s}{u} \frac{\partial f}{\partial t} \varphi_0 \\ &= B_0 \frac{\partial \varphi_0}{\partial z} f - \frac{B_s \varphi_0}{u} \frac{\delta f}{\delta t}.\end{aligned} \tag{24}$$

Substituting now this expression into Eq. (23), we find

$$A - \frac{c^2}{\omega_{pe}^2} \Delta_\perp A = -\frac{\bar{B}_s c}{B_0 u} \varphi + c \int_{-\infty}^{t} \frac{\partial \varphi_0}{\partial z} f \delta t', \tag{25}$$

where $\bar{B}_s$ means averaging over the hydrodynamic trajectory.

Let us analyze the last term of Eq. (25) from the point of view of neglecting of thermal motion. We may not take into account longitudinal motion if it does not change the phase of the wave. But the "ballooning-phase" (which means the z position of particle) does not become fixed in nonlinear case. Therefore, longitudinal thermal motion of electrons average the "ballooning phase" and makes the last term of Eq. (25) negligible. Therefore in what follows, we shall use the simplest form of Ohm's law

$$A - \frac{c^2}{\omega_{pe}^2} \Delta_\perp A = \frac{\bar{B}_s c}{B_0 u} \varphi \tag{26}$$

with φ of the form given in Eq. (22).

IV. Numerical Experiments

The stochastic drift orbits of thermal electrons are evaluated using the reduced equations of motion (10), (11), the electrostatic field in Eq. (13), and the associated A from Eq. (26). Using Eqs. (13) and (22) in Eq. (26), the effective Hamiltonian becomes

$$H_{\text{eff}} = \frac{cT_e}{eB}\sum_{\mathbf{k}_\perp}\frac{e\varphi_{\mathbf{k}}\left(z\left(\omega_b t\right)\right)}{T_e}\left[1+\frac{\gamma_0+\gamma_1\sin\omega_b t}{1+k_\perp^2c^2/\omega_{pe}^2}\right]\sin\left(k_x x+\alpha_k\right)\cos\left[k_y(y-ut)+\beta_k\right] \tag{27}$$

where we neglect any dispersion in the frequency spectrum and use $u \gg \alpha v_T$. In Eq. (27), $\bar{k}_\parallel v_\parallel(t)/\omega = \bar{B}_s v_\parallel(t)/B_0 u = \gamma_0 + \gamma_1 \sin(\omega_b t)$ where for passing electrons, $\gamma_0 \gg \gamma_1$ and the motion is integrable for $\tilde{v}_\perp \gg u$ and for trapped electrons $\gamma_0 = 0$ and $\gamma_1 = \bar{k}_\parallel v_\parallel/\omega \lesssim \omega_b/\omega \lesssim 10$. The dependence on $\varphi_{\mathbf{k}}\left(z\left(\omega_b t\right)\right)$ describes ballooning effects.

We measure x and y in units of c/ω_{pe} and t in units of $\omega_b^{-1} = qR/v_\perp\varepsilon^{1/2} \sim qR/v_e\varepsilon^{1/2}$. We express the spectrum $\varphi_{\mathbf{k}}$ in terms of the strength parameter ϕ_1 and power law index p defined through

$$\frac{1}{\omega_b}\frac{\omega_{pe}^2}{c^2}\frac{cT_e}{eB}\frac{e\varphi_{\mathbf{k}}}{T_e} = \frac{\phi_1}{k^{2+p}}, \tag{28}$$

where $k = ck_\perp/\omega_{pe}$. For $p=0$, the circulation frequency $\Omega_k = ck_\perp^2\varphi_{\mathbf{k}}/B$ from Eq. (28) is constant over the spectrum, and equal to $\phi_1\omega_b$. We have varied p over $p=-1,0,+1$. We characterize the diffusion coefficient by its dependence on the root-mean-square fluctuation level $\tilde{\varphi} = \left\langle\varphi^2\right\rangle^{1/2}$ and the correlation length ℓ_c, which are functions of ϕ_1, p, $k_{\min}$ and $k_{\max}$. To take $\tilde{\varphi}$ as the independent variable we choose the amplitude ϕ_1 of the spectrum by

$$\phi_1 = 2\tilde{\varphi}\bigg/\left[\sum_{\mathbf{k}} k^{-4-2p}\right]^{1/2}.$$

The correlation length ℓ_c is the scale for the fall-off of $\langle\varphi(\mathrm{x}+\mathrm{r})\varphi(\mathrm{x})\rangle$ which is approximately the size of the potential cells.

In terms of these dimensionless variables, Eqs. (10) and (11), expressed in the frame moving with the fluctuations, are

$$\begin{aligned}\frac{dx}{dt} &= -\frac{\partial h}{\partial y} + v_d\sin\theta\\ \frac{dy}{dt} &= +\frac{\partial h}{\partial x} + v_d\cos\theta + v_s + u\end{aligned} \tag{29}$$

with

$$h(x,y,t) = \phi_1 \sum_{\mathbf{k}} \frac{1}{k^{2+p}} \left[1 + \frac{\gamma_0 + \gamma_1 \sin t}{1+k^2} \right] \sin\left(k_x x + \alpha_{\mathbf{k}}\right) \cos\left(k_y y + \beta_{\mathbf{k}}\right),$$

where the dimensionless u, v_d and v_s are given by

$$u = \frac{u\omega_{pe}}{c\omega_b} \simeq q \left(\frac{\beta}{\varepsilon^3}\right)^{1/2} \leq 4$$

$$v_d = \frac{v_d\omega_{pe}}{c\omega_b} \simeq q \left(\frac{\beta}{\varepsilon}\right)^{1/2} \lesssim 1,$$

and there are two models for v_s given by

$$v_s^{(1)} = \frac{\bar{k}_{\|} v_{\|} \omega_{pe}}{\omega_b c k_y} \lesssim \varepsilon^{-1/2} \frac{\omega_{pe}\rho_s}{c} \sim 10,$$

where $\bar{k}_{\|} = 1/qR = \text{const.}$, or

$$v_s^{(2)} = \frac{1}{\varepsilon^{1/2}} \left(\frac{rq'}{q}\right) x \sim x,$$

where $k_{\|} = k_{\|}' x$.

We use a low-order square $\mathbf{k}$ space with $\mathbf{k} = (nk_1, mk_1)$, taking $n, m = 1,2,3,4,5$ and $k_1 = c\omega_{ci}/c_s\omega_{pe} = \left(2m_e/m_i\beta_e\right)^{1/2} \simeq 0.3$, in the experiments presented here. The contours of constant $\varphi(x,y,t=0)$ are shown in Fig. 1 for the three different spectral laws $p = 0, \pm 1$. For these spectra, the correlation lengths are $\ell_c(p=+1) = 12$, $\ell_c(p=0) = 7$, and $\ell_c(p=-1) = 5$. The relation between the parameter ϕ_1 and the root-mean-square fluctuation level is $\tilde{\varphi}(p=+1) = 14.2\phi_1$, $\tilde{\varphi}(p=0) = 7.1\phi_1$, and $\tilde{\varphi}(p=-1) = 4.76\phi_1$.

We introduce N particles with $x_i(0) = x_0$ and $y_i(0) = (2\pi/k_1)(i/N)$ and calculate the position of the particles at $t_n = 2\pi n$ corresponding to multiples of the bounce period. We define the diffusion coefficient by

$$D(t) = \frac{1}{2tN} \sum_{i=1}^{N} \left(x_i(t) - x_i(0)\right)^2 \tag{30}$$

and look for well-defined values for $N = 10$ to 128 and $t/2\pi \sim 100 - 200$. When $D(t)$ appears convergent we define

$$D(t_0) = \frac{1}{T} \int_{t_0 - T}^{t_0} D(t)\, dt \tag{31}$$

with $T \sim t_0/2$. The error bars in the figures are computed from the standard deviation of $D(t)$ from the mean value $D(t_0)$.

For integrating the equation we use a 5–6 order adaptive Runge-Kutta subroutine (DVERK) on the *MFE CRAY* computer. Typical CPU times for N particles with truncation error ϵ_T tolerance of $10^{-6} \leq \epsilon_T \leq 10^{-2}$ is $T_{\text{cpu}} = 8 \times 10^{-4} N \log_{10}(1/\epsilon_T)$ per $2\pi/\omega_{be}$.

The effect of classical collisional diffusion $D_0 = \nu_e \rho_e^2$ of the guiding center is studied by adding Gaussian random perturbations δx, δy each bounce period with a variance $\langle \delta x^2 \rangle = \langle \delta y^2 \rangle = 2D_0 (2\pi/\omega_b)$, which in dimensionless units is $\langle \delta x^2 \rangle = (\beta_e/2)(2\pi\nu_e/\omega_{be})$. With a view toward reducing the cost of the collisionless calculation, we compare the effect of the external stochasticity from D_0 with the effect of the truncation error ϵ_T in the numerical integration. We find that the effects are comparable with the correspondence given roughly by $D_0 = \pi\beta_e\nu_e/\omega_{be} \simeq \epsilon_T^2/10$.

The effect of D_0 or the truncation error ϵ_T is two-fold. First, it produces a residual background diffusion of the guiding centers, and secondly, it modifies the anomalous transport coefficient so that $D = D_0 + D_A(\alpha, D_0)$ where α are system parameters. For $D_0 \ll D_A$, the second effect dominates and becomes important near or below the onset of stochastic diffusion.

Below and near the onset of stochasticity the effect of D_0 is to produce a hybrid diffusion given by $D_{\text{eff}} = \left[D_0 \omega_b (c/\omega_{pe})^2 \tilde{\varphi}\right]^{1/2}$ where $\tilde{\varphi}$ is the mean amplitude of the integrable convective motion. The square root arises from the boundary layer transport across the separatrices of the flow (see Fig. 1).

Above the threshold regime the stochastic components δx, δy eliminate all islands and KAM torii structures of scale ϵ and smaller, both of which block the particle transport to some extent. The effect may be viewed as introducing finite lifetime τ_{cl} of the microscopic correlations of the particles in a given $H(x, y, t, \alpha)$. In the limit $\epsilon_T \to 0$, the finite time orbits are reversible and, strictly speaking, not describable by an irreversible diffusion. For finite ϵ_T, the lifetime τ_{cl} becomes the time beyond which the motion is truly irreversible. This microscopic lifetime or correlation time is essentially the so-called clump time and is approximately given by

$$\tau_{\rm cl} = \frac{\ell_c^2}{D} \ln\left(\frac{\ell_c}{\epsilon}\right),$$

where D is the diffusion coefficient and ℓ_c the macroscopic scale of the convective structures $\ell_c \sim 1/\Delta k_\perp$. For orbit times greater than $\tau_{\rm cl}(\ell_c/\epsilon)$, the motion is intrinsically irreversible and well described by diffusion. An example of the ϵ dependence of the diffusion coefficient is shown in Fig. 2. The large variations with the parameters α characteristic of the collisionless diffusion coefficient $D_A(\alpha)$ are substantially reduced by the weak background diffusion D_0.

For typical tokamak parameters, the residual diffusion even deep into the banana regime exceeds $D_0 \simeq (\pi\beta_e\nu_e/\omega_{be}) > 10^{-6} - 10^{-5}$ which implies that a truncation error of order $\epsilon_T \sim 10^{-2} - 10^{-3}$ per bounce period $2\pi/\omega_{be}$ is adequate to calculate the nearly collisionless anomalous transport. One must be aware, however, that there is an order unity, $\ln(\ell_c/\epsilon_T)$, variation of the diffusion coefficient in changing the truncation error of the integration used in Eqs. (29) as shown in Fig. 2. We have studied variations of ϵ from 10^{-2} to 10^{-6}.

A. Dependence on $A_\parallel$

In Fig. 3 we show the variation of the diffusion coefficient with the strength of the electromagnetic coupling $v_\parallel \delta A_\parallel / c\delta\phi$ effect as measured by γ_1 defined in Eq. (27). For trapped electrons $\gamma_0 = 0$ and γ_1 ranges from $0 \leq \gamma_1 \leq 15$. The reference parameters are $\phi_1 = 1.0$, $\gamma_0 = 0$ (trapped electrons), $k_1 = 0.3$ ($k_{\perp\rm min} \simeq 0.3\omega_{pe}/c = 0.3\,(m_i\beta_e/2m_e)^{1/2}\rho_s^{-1}$), and $u = v_d = v_s = 0$ for a 5×5 randomly-phased spectrum. The electrostatic model of Ref. 2 corresponds to $\gamma_0 = \gamma_1 = 0$ where $D = 0$ in the present model since no phase velocity dispersion ($\omega = k_y u$) is accounted for here.

The onset of global stochasticity occurs at $\gamma_1 \cong 0.2$ and the dimensionless diffusion coefficient reaches a maximum value of $D \simeq 2.0$ for $\gamma_1 \simeq 1 - 6$, which is somewhat larger than reported in Ref. 3. For $\gamma_1 \to \infty$, the Hamiltonian reduces to $h = \gamma_1\phi_1 h_{em}(x,y)\sin t$ which is essentially integrable and the diffusion coefficient decreases.

B. Dependence on amplitude and spectral index

The specification of the amplitude and the shape of the fluctuation spectrum is given by ϕ_1 and p. The parameter ϕ_1 is the amplitude of the spectrum at the $k = k_\perp c/\omega_{pe} = 1$. For $p = 0$, the decay of the spectrum $\phi_k = \phi_1/k^2$ is such that the nonlinear $\mathbf{E}\times\mathbf{B}$ trapping or mixing frequency is constant throughout the spectrum. The $p = 0$ reference spectrum is also an estimate from mode-coupling theory for the dissipative drift wave turbulence. Figure 4 shows the dependence of the diffusion coefficient on $\tilde{\varphi}$ and p.

Comparing Figs. 1 and 4 we see that the results are consistent with the large-scale structures producing larger diffusion $D \propto \omega_{be}\ell_c^2$ provided $\tilde{\varphi}$ is large enough ($\tilde{\varphi} > \tilde{\varphi}_{\rm crit} \sim 0.2$) to make the large-scale $\mathbf{E}\times\mathbf{B}$ circulation frequency $\bar{k}^2\tilde{\varphi}$ comparable with the electron bounce frequency ω_{be}. For the smaller-scale spectrum $p = -1$, the comparable frequency condition is first met in the short wavelength $k > 1$ part of the spectrum, while for the $p = +1$, it is first met in the long wavelength $k < 1$ part of the spectrum.

Once the stochastic motion in x, y sets in, the particles stay phase-locked in a mode k for $\tau_c(k) < 1/k^2D$. The resulting diffusion is now limited by further increase of $\tilde{\varphi}$. As a consequence, the diffusion coefficient has a broad maximum only weakly dependent on the value of $\tilde{\varphi}$ for $5 < \tilde{\varphi} < 20$.

C. Dependence on the diamagnetic drift

Figure 5 shows the dependence of the diffusion coefficient on the dimensionless phase velocity u for the three wavenumber spectra at a given mean fluctuation amplitude. The diffusion decreases sharply for $u = 0.5$ and then remains at approximately constant until $|u| \geq \bar{k}_x\tilde{\varphi}(1+\gamma_1)$. For $|u|$ above this value few electrons are $\mathbf{E}\times\mathbf{B}$ trapped and the diffusion is negligible.

D. Dependence on the toroidal drift velocity

Increasing the toroidal drift velocity increases the radial excursion in x but also produces a rapid phase mixing in the y motion. The phase mixing reduces the diffusion rather strongly for $v_d \gtrsim 1.0$. Figure 6 shows the effect of increasing v_d. We observe that for $v_d \neq 0$, the passing electrons $\gamma_0 \gg \gamma_1$ also diffuse.

E. Effect of magnetic shear

Figure 7 shows the dependence of the diffusion coefficient on the shear parameters $v_s^{(1)}$ and $v_s^{(2)}$. For the two models of magnetic shear, we have $D\left(v_s^{(1)}\right)$ for $k_{\|} \sim 1/qR$ and $D\left(v_s^{(2)}\right)$ for $k_{\|} \sim sx/qR$. The two choices represent opposite limits of the toroidal eigenmode structure problem. The diffusion coefficient decreases rapidly with increasing shear parameter $v_s^{(j)}$ in both cases. The constant $k_{\|}$ or $v_s^{(1)}$ model has the stronger effect on D for the same value of $v_s^{(1)}$ and $v_s^{(2)}$.

F. Effect of ballooning mode dependence

As a simple test of the sensitivity of the diffusion coefficient to ballooning of the wave amplitude toward the outside of the torus, we introduce a ballooning model parameterized by α for the θ dependent mode amplitude

$$\phi(t) = \phi_1 \exp\left[\alpha_0(\cos\theta(t) - 1)\right].$$

Figure 8 shows the results of varying α_0 from $\alpha = 0$ to 1.5. For $\alpha_0 \leq 0.5$ the previous results are retrieved. For $\alpha_0 \geq 0.5$ there is a significant decrease in the diffusion coefficient for $\tilde{\varphi} = 5.0$. The diffusion is approximately equivalent to a reduction of ϕ_1 by $\phi_1 \to \phi_1 \exp\left(-\alpha_0 \left\langle\theta^2\right\rangle/2\right)$ where $\left\langle\theta^2\right\rangle$ is the amplitude of the electron motion in $\theta(t) = \theta_0\left(v_{\|}/v_{\perp}\right) sn\left(\omega_b t\right)$ given in Eqs. (7)–(9).

From these studies we see that for $\tilde{\varphi}$ well into the fully nonlinear regime and for a spectrum of $\vec{k}$ modes, there is some degree of insensitivity of the diffusion coefficient to the details of the spectrum. The strongest mechanism in reducing the diffusion arises from the $v_s^{(1)}$-type of magnetic shear effect. The extent to which this mechanism is operative depends on the details of the toroidal mode structure and requires further study.

V. Conclusions

The motion of sample electrons in a low β, axisymmetric toroidal trap are integrated over many bounce or transit periods while taking into account the presence of electromagnetic drift wave type fluctuations. Due to the low frequency and long wavelength of the drift modes, the electron motion is given accurately by the guiding center equations of motion. Furthermore, since the electron acceleration along the magnetic field is dominated by the equilibrium mirror force $-\mu\nabla_{\|}B \gg eE_{\|}$, the cross-field motion of the guiding center reduces to a $1\frac{1}{2}$-D Hamiltonian flow. The Hamiltonian $H(x,y,t)$ describes the cross-field motion due to the $\mathbf{E}\times\mathbf{B}$ drift and the $v_{\|}\delta\mathbf{B}_{\perp}/B$ guiding center drift arising from electromagnetic drift wave type fluctuations.

We show that the nonlinear Ohm's law has an exact large amplitude solution for vortical plasma flows of the form $\phi(x, y+\alpha z-ut)$ that is valid for large drift velocities $k_{\perp}\tilde{v}_{\perp} > \omega$ corresponding to the regime of broadband spectral lines observed in the experiments. Here $\tilde{v}_{\perp}$ is the total guiding center speed across the ambient confining magnetic field. The nonlinear Ohm's law $j_{\|} = \sigma E_{\|}$ taken with Ampere's law $\nabla_{\perp}^2 A_{\|} = -4\pi j_{\|}/c$ determines the self-consistent relationship $\delta A_{\|}/\delta\varphi$ between the vector and scalar electromagnetic potentials given by Eq. (26). The cross-field Hamiltonian $h(x,y,t)$ is now expressed as a low-order sum of drift wave-like fluctuation fields $\varphi_{\mathbf{k}}(x,y,t)$ with randomly-phased components. The isotropic spectral distribution is specified by an amplitude ϕ_1, spectral index p, and minimum and maximum wavenumbers k_1 and $k_m = Nk_1$.

The cross-field Hamiltonian is a function of the dimensionless parameter set $\{\alpha\}$ given by γ_0, γ_1, u, v_d, v_s and the spectral parameters ϕ_1, k_1, p, N. The condition for the onset of global stochasticity is that the $\mathbf{E}\times\mathbf{B}$ circulation frequency $\Omega_k = ck_xk_y\phi_{\mathbf{k}}/B$ in the time-frozen Hamiltonian be comparable with either (1) the dispersion of the drift-wave frequencies[2] $\Delta\omega$ in the wave spectrum or (2) the bounce frequency[3] of the trapped electrons $\omega_b = v_{\|}/qR \sim \varepsilon^{1/2}v_e/qR$. When either of these conditions are met there is a statistically well-defined diffusion coefficient for the trapped electrons.

In the first regime the condition for global stochasticity leads to the spectral scaling

$$\frac{e\phi_{\mathbf{k}}}{T_e} \sim \frac{1}{k_x r_n} \sim \frac{\rho_s}{r_n}, \tag{32}$$

and the proper[2] dimensionless x, y, t variables for the guiding center motion are x/ρ_s, y/ρ_s, $\Delta\omega t = c_s t/r_n$. The diffusion coefficient from the trapped electrons is

$$D_\perp = \varepsilon^{1/2} \frac{\rho_s}{r_n} \left(\frac{cT_e}{eB} \right) \hat{D}_{es}(\alpha), \tag{33}$$

where $\hat{D}_{es}(\alpha)$ is the dimensionless diffusion coefficient, and $\{\alpha\}$ is dimensionless parameter set of the effective Hamiltonian. For the long wavelength part of the spectrum $k_\| v_e > k_\perp \tilde{v}_\perp$ so that only trapped or plateau regime electrons contribute to $D_\perp$. Numerical experiments and theoretical formulas for $\hat{D}_{es}(\alpha)$ are reported in Ref. 2. The diffusion coefficient $\hat{D}_{es}(\alpha)$ is of order unity and has a maximum in its dependence on the amplitude parameter for typical α.

In the second electromagnetic regime, the stochasticity[3] occurs for $\Omega_k \simeq \omega_{be}$ with the dominant spectral components for diffusion occurring for $k_\perp \simeq k_s \equiv \omega_{pe}/c$. The global stochasticity condition leads to $c\phi_{k_s}/B \simeq \left(c^2/\omega_{pe}^2\right)\left(\varepsilon^{1/2} v_e/qR\right)$, or equivalently,

$$\frac{e\phi_{k_s}}{T_e} \simeq \frac{cB\varepsilon^{1/2}}{4\pi n_0 e v_e qR} \simeq \frac{cB_\theta \varepsilon^{1/2}}{4\pi n_0 e a v_e} \simeq \frac{u_{\|e}}{v_e} \varepsilon^{1/2} \tag{34}$$

and

$$\frac{\delta B_\perp}{B} \lesssim \frac{ck_\|}{uB} \phi_k \lesssim \frac{\varepsilon^{3/2}}{q} \cdot \frac{e\phi_k}{T_e}. \tag{35}$$

An alternative way of writing the level (34) is

$$\frac{e\phi_{k_s}}{T_e} \simeq \left(\frac{2}{\beta_e} \right) \left(\frac{\omega_{be}}{\omega_{ce}} \right)$$

which is convenient for a numerical estimate of the required fluctuation level ($e\phi/T_e \sim \left(2/5 \times 10^{-3}\right)\left(10^6/10^{11}\right) \sim 4 \times 10^{-3}$). The fluctuation level (34) is equivalent to the dimensionless parameter $\phi_1 = 1$ in Eq. (28). The relation between the fluctuation levels (32) and (34) is given by

$$\left(\frac{e\phi_{k_s}}{T_e} \right) \Big/ \left(\frac{\rho_s}{r_n} \right) \sim \frac{2\varepsilon^{3/2}}{q\beta_e} \left(\frac{m_e}{m_i} \right)^{1/2}, \tag{36}$$

where $\beta_e = 8\pi n_e T_e/B^2$. For $\beta_e \sim (2\varepsilon/q)\left(m_e/m_i\right)^{1/2}$ the mixing length level (32) corresponds to $\phi_1 \sim 1$.

Now, in general, both parts of the fluctuation spectrum $k_\perp \sim \omega_{ci}/c_s$ and $k_\perp \sim \omega_{pe}/c$ are present with sufficient amplitude to produce stochastic electron diffusion. The total electron diffusion rate is given by

$$\begin{aligned} D_\perp &= D_{em} + D_{es} \\ &= \varepsilon^{1/2}\frac{c^2\omega_{be}}{\omega_{pe}^2}\hat{D}_{em}(\alpha) + \varepsilon^{1/2}\frac{\rho_s}{r_n}\frac{cT_e}{eB}\hat{D}_{es}(\alpha), \end{aligned} \tag{37}$$

where $\hat{D}_{em}(\alpha)$ and $\hat{D}_{es}(\alpha)$ are slowly varying functions in the stochastic plateau.

One of the important parametric dependences in α is the speed of the electron v/v_e relative to the thermal speed. A precise calculation of the particle flux Γ and thermal flux q requires the density and kinetic energy moments of $D_\perp(v)$ over the distribution function $f_e(v)$. Here we assume $\int_0^\infty v^2 D_\perp f_e v^2\, dv \sim v_e^2 \int_0^\infty D_\perp f_e v^2\, dv$ so that $\chi_e \sim D_\perp$ to within a numerical constant.

Thus, we find that for the purpose of interpreting the thermal transport properties of toroidal traps, the thermal diffusivity is

$$\chi_e = \hat{D}_{em}\frac{\varepsilon v_e c^2}{qR\omega_{pe}^2}\left(1 + A\beta_\theta\right), \tag{38}$$

where

$$A = \frac{1}{2}\left(\frac{\hat{D}_{es}}{\hat{D}_{em}}\right)\left(\frac{\varepsilon^{1/2}}{q}\right)\left(\frac{m_i}{m_e}\right)^{1/2} \simeq 1 - 3 \tag{39}$$

and $\beta_\theta = \beta_e q^2/\varepsilon^2 = 8\pi n_e T_e/B_\theta^2$. In applying formula (38) to experiments, the coefficients $\hat{D}_{em}$ and $\hat{D}_{es}/\hat{D}_{em}$ are adjustable constants of order unity corresponding to details of the fluctuations present in the range c/ω_{pe} and c_s/ω_{ci}, respectively.

Formula (38) for χ_e predicts the neo-Alcator and Merezhkin-Mukhovatov scaling laws

$$\tau_E \simeq \frac{qn_e aR^2}{T_e^{1/2}} \tag{40}$$

empirically deduced from tokamak power balance studies for $\beta_\theta < A^{-1} \simeq \mathrm{const}\left(2q/\varepsilon^{1/2}\right)\left(m_e/m_i\right)^{1/2}$. For $\beta_\theta \gtrsim A^{-1}$ the theory predicts a saturation of the energy confinement time

$$\tau_E \simeq \frac{a^{5/2}R^{1/2}B^2}{T_e^{3/2}} \tag{41}$$

with increasing β_θ or plasma density n_e.

The effects of Coulomb collisions on the electron orbits may weakly increase the diffusion over the stochastic value given in Eq. (38). Collisional studies will be reported in a future work along with more work on the effect of the nonlinear ballooning mode structure.

Acknowledgements

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Figure Captions

Fig. 1 Contours of constant $\varphi(x, y, t = 0)$ for the model spectrum used in the particle orbit calculations. The parameters $\varphi_1 = 1.0$, $\gamma_0 = 0$, $\gamma_1 = 1.0$, $k_1 = 0.3$, and $N = M = 5$. The spectral index is (a) $p = -1$, (b) $p = 0$, and in (c) $p = +1.0$.

Fig. 2 Effect of truncation error ϵ_T or collisions on the diffusion coefficient for the $p = +1$ spectrum. Here $N = 128$ and $\omega_{be} t = 2\pi \times 100$.

Fig. 3 Dependence of the diffusion coefficient on γ_1 for $\varphi_1 = 1.0$, $\gamma_0 = 0$, $k_1 = 0.3$, $N = M = 5$. Here $N = 128$, $\omega_{be} t = 2\pi \times 100$ and $\epsilon_T = 10^{-5}$.

Fig. 4 Dependence of the diffusion coefficient on $\tilde{\varphi}$ for $\gamma_1 = 1$ and the same reference parameters in Fig. 3.

Fig. 5 Dependence of the diffusion coefficient on u for $\gamma_1 = 1$ and $\tilde{\varphi} = 5$.

Fig. 6 Dependence of the diffusion coefficient on v_d for $\gamma_1 = 1$ and $\tilde{\varphi} = 5$. Here, $\gamma_0 = 0$ for trapped electrons and $\gamma_0 = 0.5$ for passing electrons.

Fig. 7 Dependence of the diffusion coefficient on $v_s^{(1)}$ and $v_s^{(2)}$.

Fig. 8 Dependence of the diffusion coefficient on α_0.

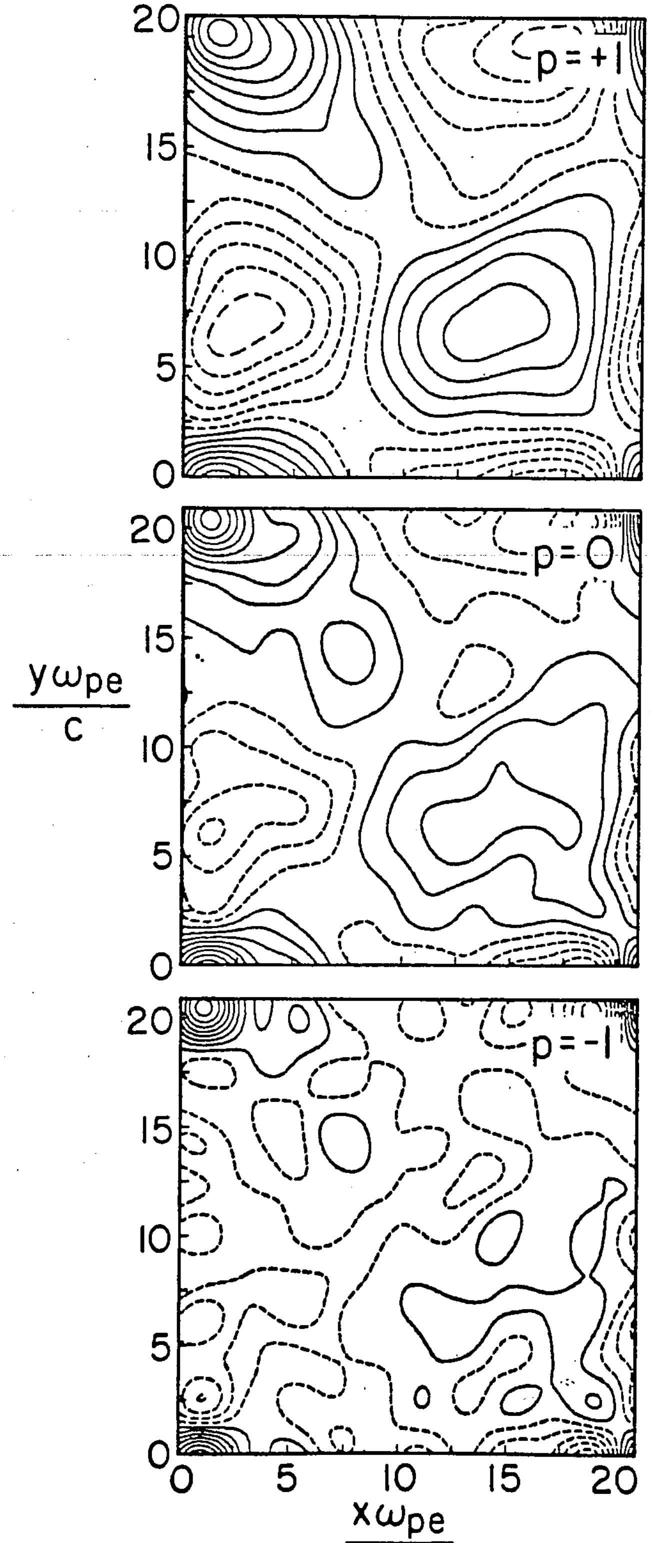
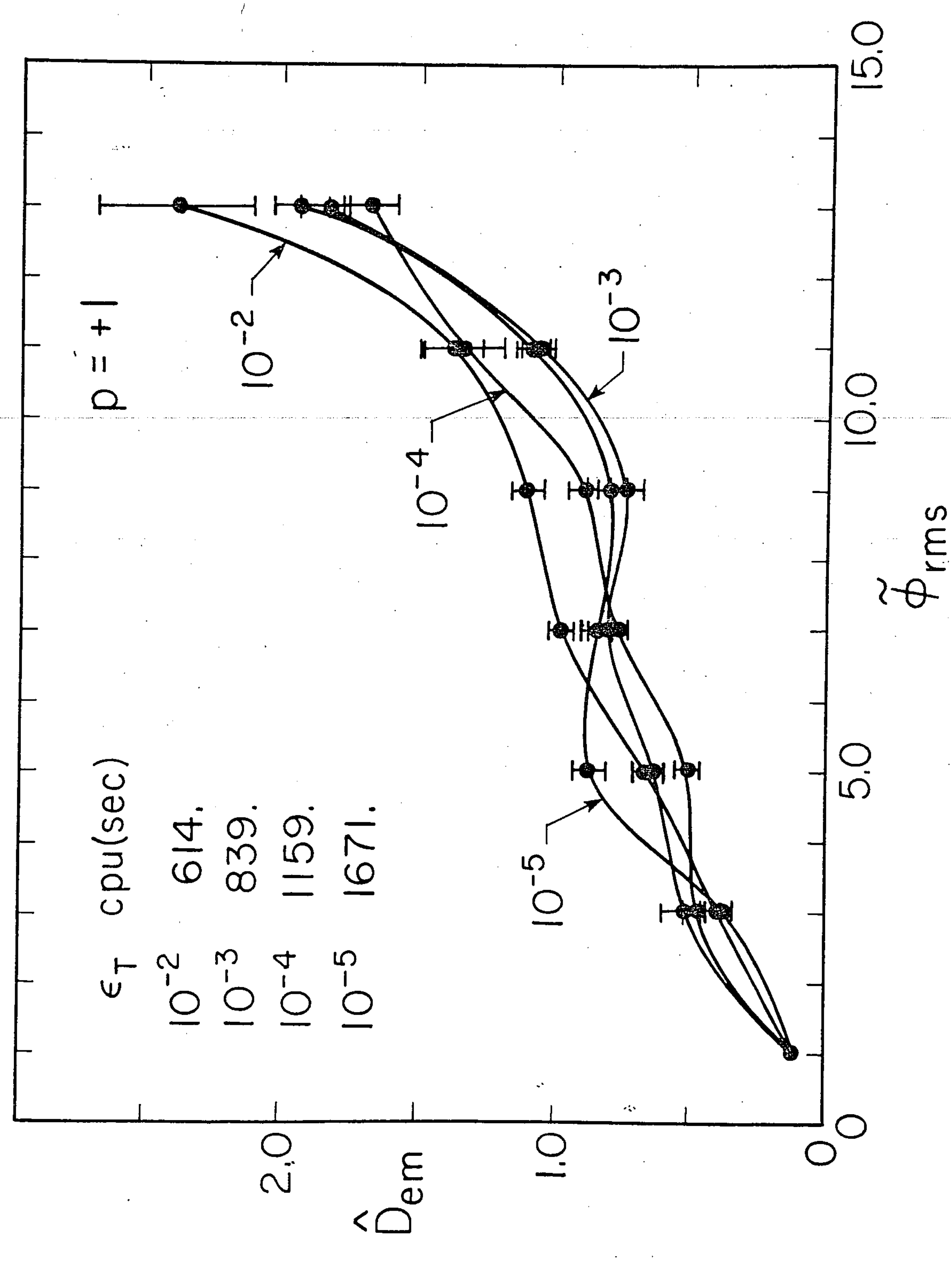


Fig. 1



p = +1
ϵ_T cpu(sec)
10^{-2} 614.
10^{-3} 839.
10^{-4} 1159.
10^{-5} 1671.
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$\hat{D}_{em}$
2.0
1.0
0.0
5.0
10.0
15.0
$\tilde{\phi}_{rms}$

Fig. 2

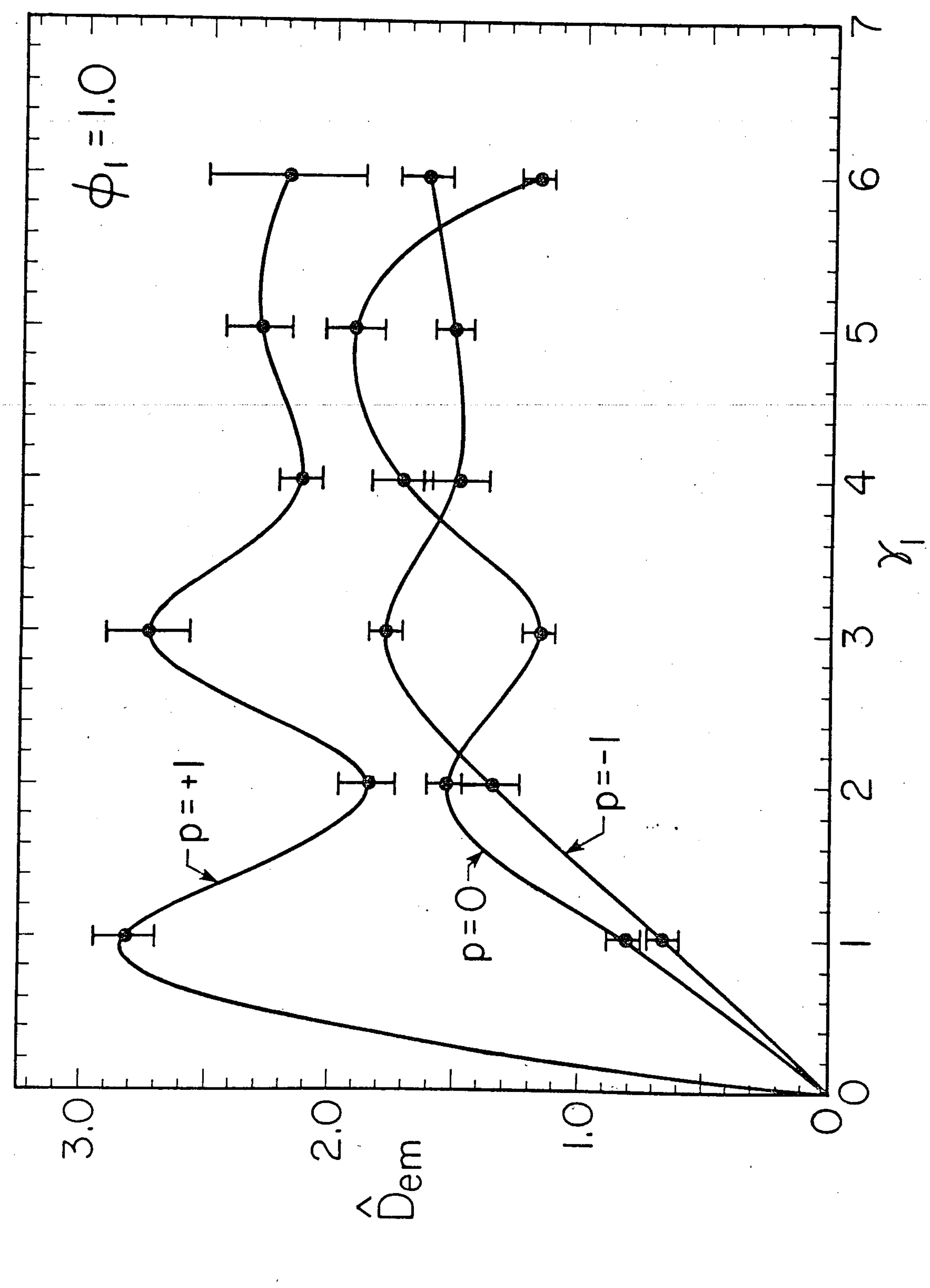


Fig. 3

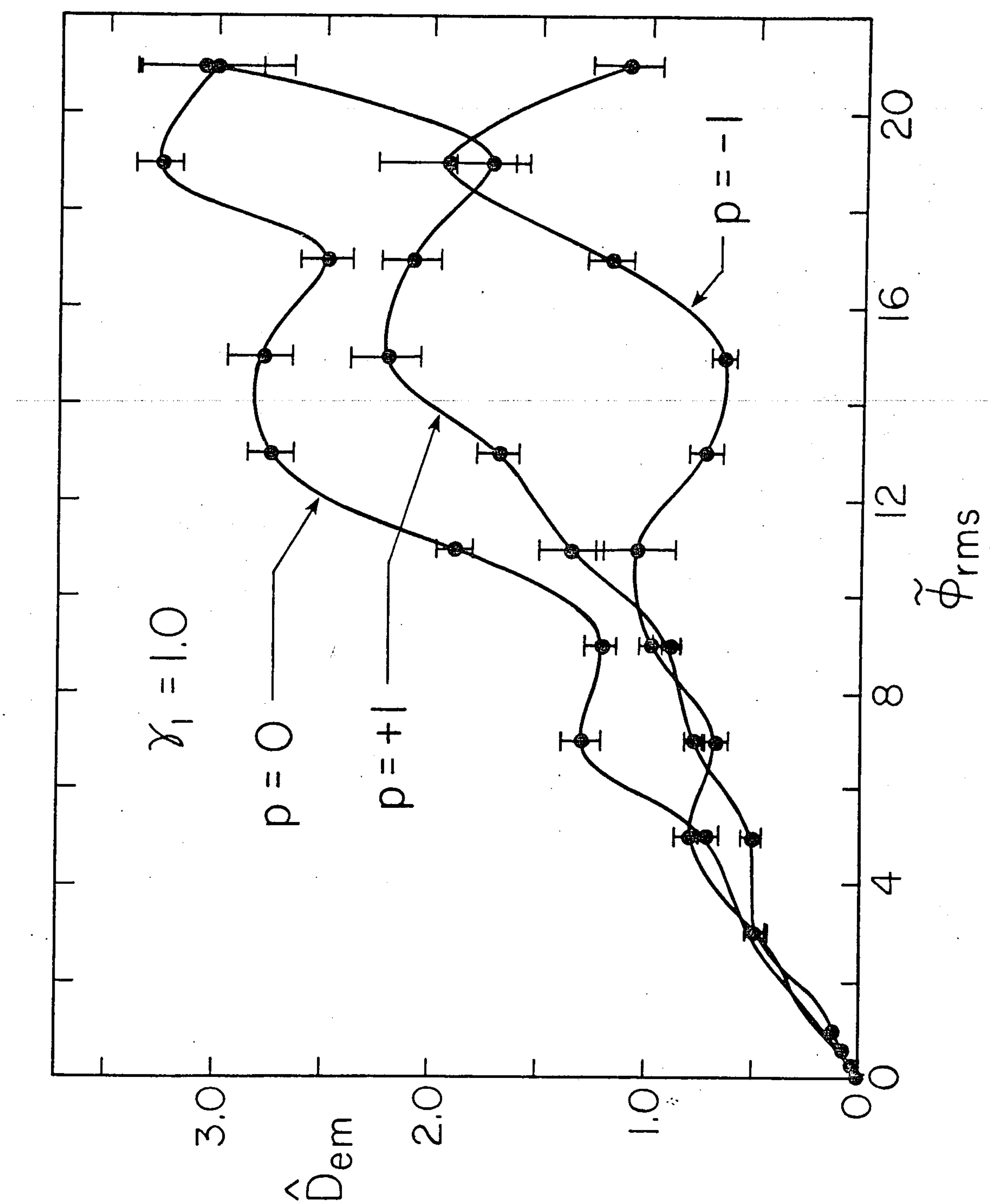


Fig. 4

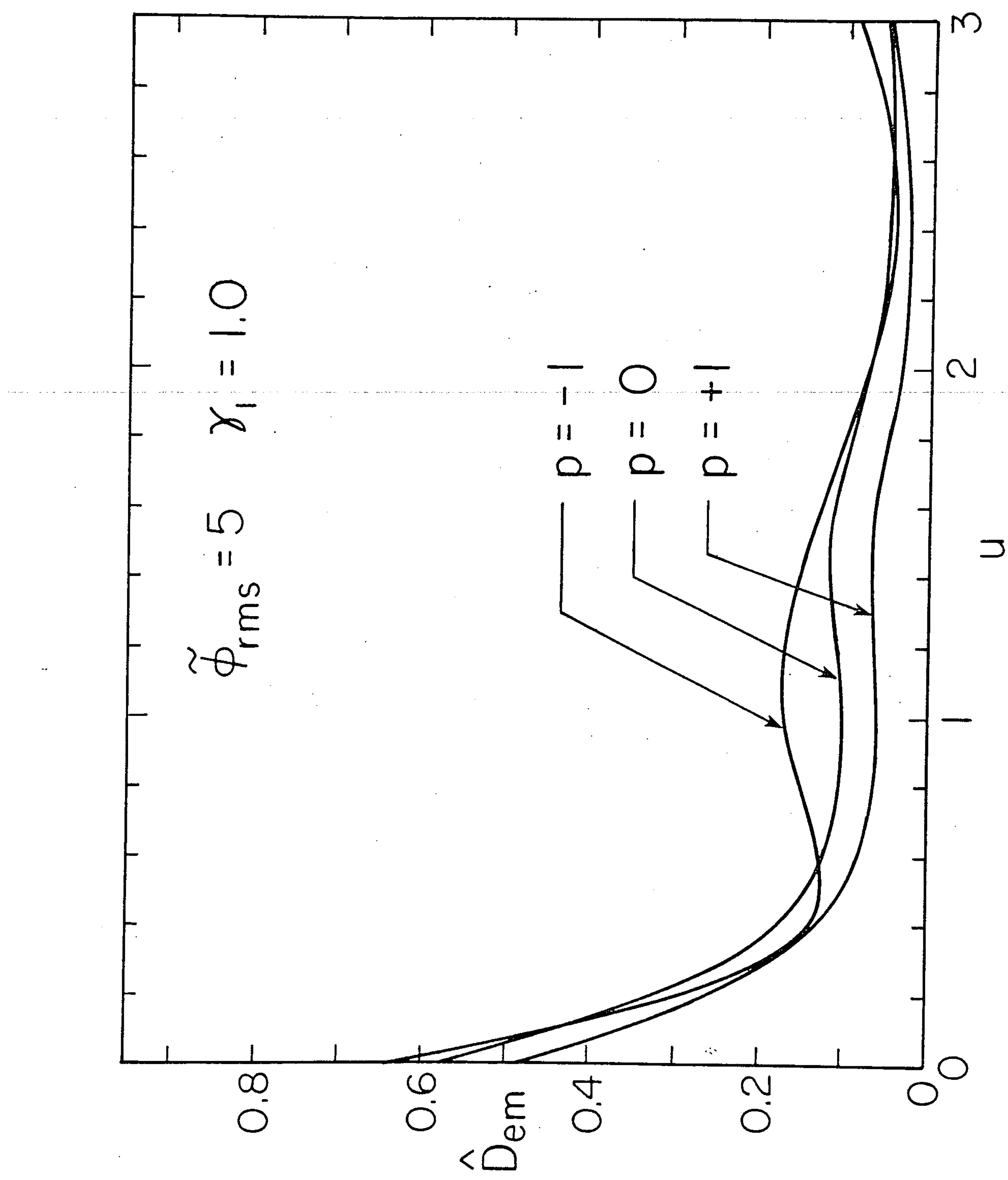
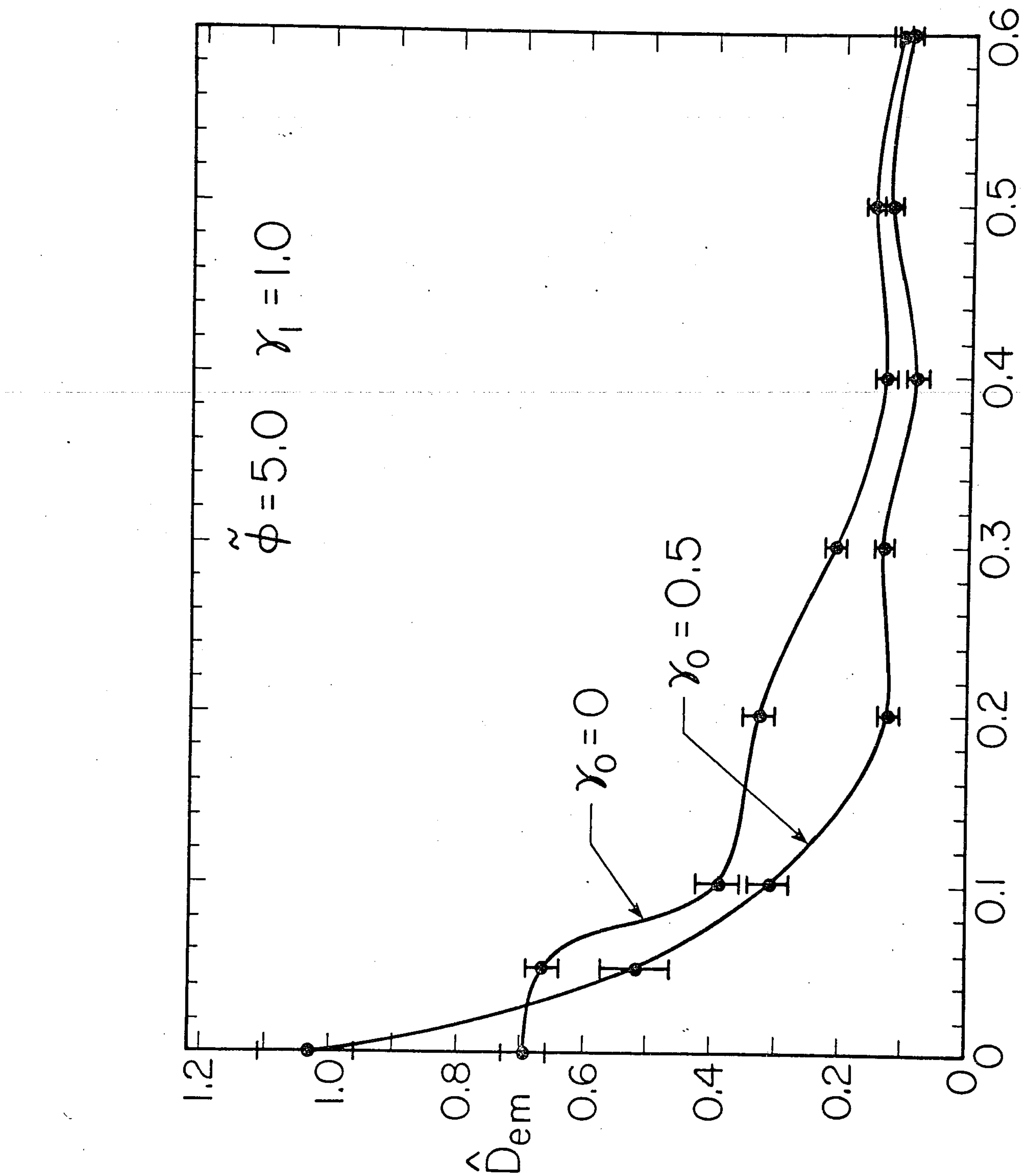
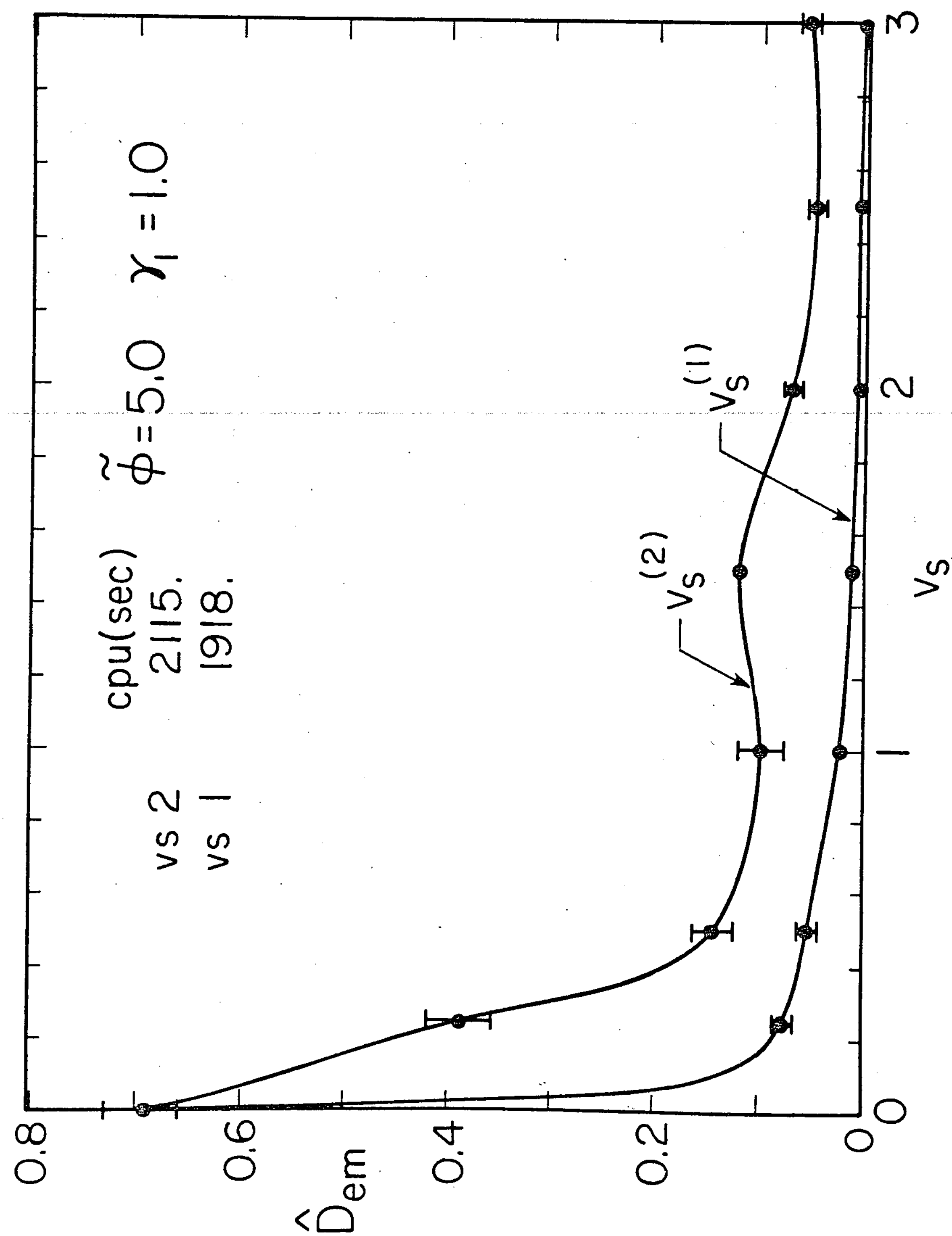


Fig. 5



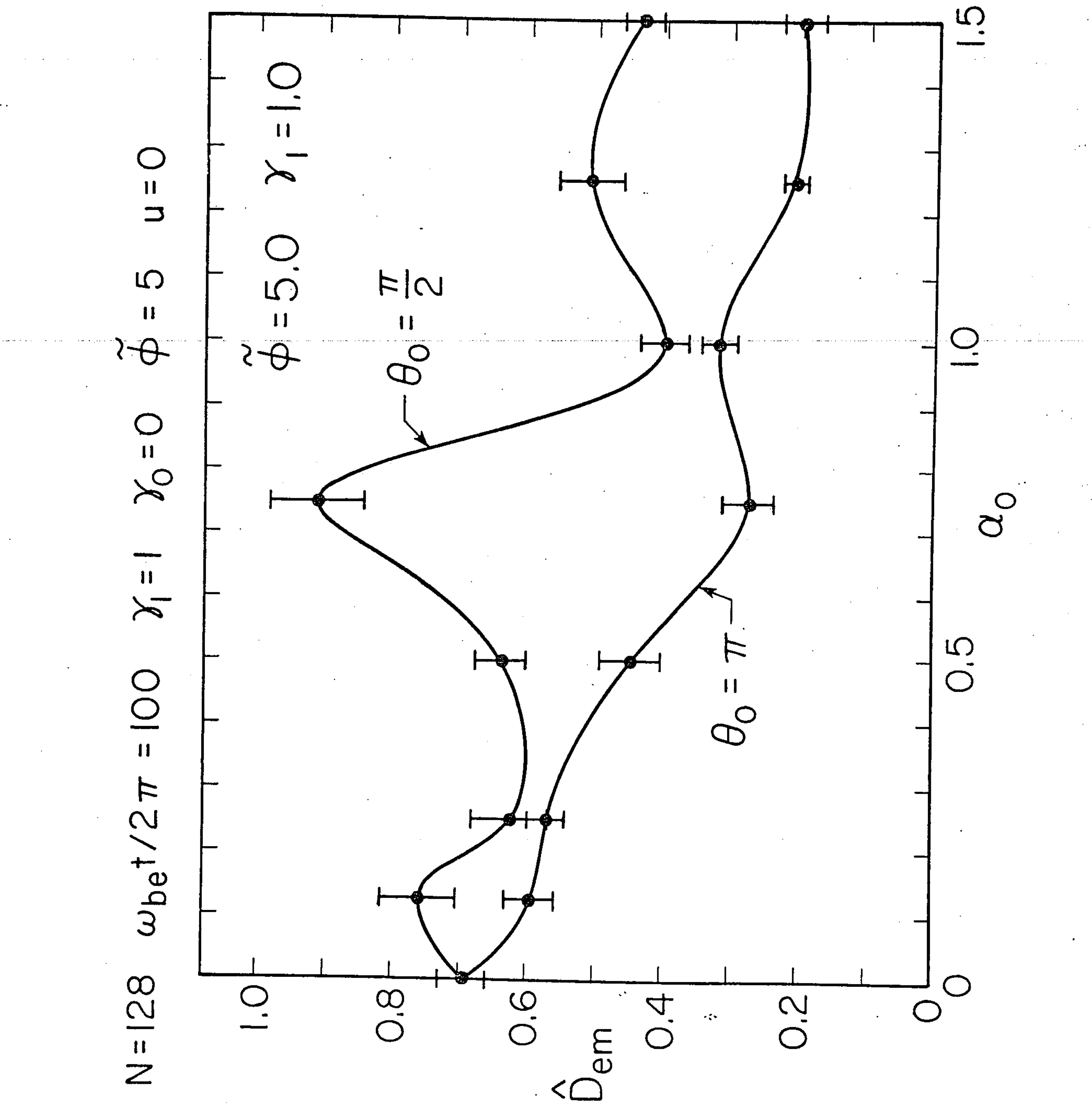
$\tilde{\phi} = 5.0$
$\gamma_1 = 1.0$
$\gamma_0 = 0$
$\gamma_0 = 0.5$
$\hat{D}_{em}$
1.2
1.0
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
v_d

Fig. 6



$\tilde{\phi} = 5.0 \quad \gamma_1 = 1.0$
cpu(sec)
vs 2 2115.
vs 1 1918.
$v_s^{(2)}$
$v_s^{(1)}$
$\hat{D}_{em}$
0.8
0.6
0.4
0.2
0
0
1
2
3
vs

Fig. 7



N=128 $\omega_{be}t/2\pi$ = 100 γ_1 = 1 γ_0 = 0 $\tilde{\phi}$ = 5 u = 0
$\tilde{\phi}$ = 5.0 γ_1 = 1.0
$\theta_0 = \frac{\pi}{2}$
$\theta_0 = \pi$
$\hat{D}_{em}$
1.0
0.8
0.6
0.4
0.2
0
0
0.5
1.0
1.5
α_0

Fig. 8