DOE-ET-53088-231-R IFSR#231-R

ANCHOR STABILIZATION OF TRAPPED PARTICLE MODES IN MIRROR MACHINES

H.L. Berk and *G.V. Roslyakov**
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712-1060

* Institute of Nuclear Physics
Soviet Academy of Sciences, Novosibirsk, USSR

July 1986

PLEASE DISCARD REPORT #231 ISSUED BACK IN APRIL, 1986. THE FOLLOWING REPORT IS A RE-ISSUE OF SAME WITH EXTENSIVE REVISIONS.

THANK YOU.

Anchor Stabilization of Trapped Particle Modes in Mirror Machines

H.L. Berk and *G.V. Roslyakov* *

Institute for Fusion Studies

The University of Texas at Austin

Austin, Texas 78712-1060

Abstract

It is shown that for trapped particle modes in tandem mirrors, the pressure of the passing particles in the anchor region introduces a stabilizing term proportional to the sum of the anchor's field line curvature and total diamagnetic pressure. The theory is applied to the proposed gas dynamic trap experiment.

Trapped particle modes in tokamaks[1] and tandem mirrors[2] arise from "electrostatic" perturbations that are localized to the region of unfavorable curvature. At low beta, these electrostatic modes do not produce any significant line bending, so that the instability can be driven by the unfavorable curvature. In tandem mirrors[3] and other mirror machine variants such as the gas dynamic trap,[4,5] this instability can have growth rates that are on MHD time scales if there is a sufficiently small number of confined particles sampling both the good and bad curvature regions (these are called the passing particles). If there are a different fraction of positively and negatively charged passing particles, then with a large enough fraction of passing particles the trapped particle modes can be stabilized by a charge uncovering mechanism.[2] However, this mechanism is somewhat fragile as the system is still susceptible to dissipative instabilities[6] or the removal of the charge uncovering mechanism at short wavelengths due to: (1) the ion finite Larmor radius charge

* Permanent address: Institute of Nuclear Physics, Soviet Academy of Sciences, Siberian Branch, Novosibirsk 630090 U.S.S.R.

uncovering mechanism cancelling the electrons' axial charge uncovering mechanism;[2] (2) the bounce frequency of ions in the stabilizing anchor region being less than the wave frequency.[7]

In this note we point out that it is in principle possible to stabilize a trapped particle mode with strong anchor stabilization. The mechanism of stabilization arises from the observation that the trapped particle mode's drive for destabilization is proportional to a weighted average of the drifts of both passing particles and particles trapped in the center cell. Past analyses tended to ignore the contribution to the instability drive from the passing particles as it was assumed that the passing particle partial pressure was too small to make a significant contribution to the term proportional to the curvature drive. However, with proper design, the passing term can be important, especially if one takes advantage of the following aspects:

(1) The unfavorable curvature drive from the trapped particles can be made proportional to P_c/L_cB_c where P_c is the central-cell pressure (sum of perpendicular plus parallel pressure) and L_c length of the central cell, and B_c the typical magnetic field of the central cell, while the favorable passing particle contribution at low beta is proportional to fP_c/L_AB_A, where fP_c is the partial pressure contribution of the passing particles in the anchor region, L_A the length of the anchor and B_A the typical magnetic field in the anchor. Thus, for $f > \dfrac{L_AB_c}{L_cB_A}$ one achieves a robust stabilization of trapped particle modes.

(2) With a strongly stabilized anchor region, the contribution of the passing particles to the curvature drive will also include a stabilizing diamagnetic contribution due to the enhancement of grad-B particle drifts arising from the pressure gradient. Thus, if the stable anchor region contains a plasma at finite beta, β_A, the stabilizing contribution of the passing particles will be proportional to:

$$\frac{fP_c}{B_A}\left[\frac{1}{L_A}+\frac{f_\perp\beta_A L_A}{fr_{pA}^2}\right]$$

where r_{pA} is the plasma radius in the anchor and $f_\perp P_c$ is the partial perpendicular pressure of passing particles in the anchor.

As high beta MHD anchor regions can be readily established, it would appear that tandem mirrors can be designed to be free of trapped particle modes. Similar stability considerations apply to the gas dynamic trap although the contribution from β_A in this case does not appear to be significant.

In the remainder of this note we will justify the above statements and apply our theory in detail to the gas dynamic trap.

We now consider a response to an eigenfunction that does not induce any bending energy. If the anchor region is strongly stabilizing, the mode amplitude in the anchor region will be very small compared to the amplitude in the central-cell region, and the response is flute-like. If the central cell's axial length is much larger than the anchor's axial length, and $\omega\tau_A \ll 1$, with ω the wave frequency and τ_A a passing particle's transit time through the anchor region, then the quadratic form governing the response of the system (neglecting bending energy and magnetic compressibility) is given by,[7,8]

$$
\begin{aligned}
\mathcal{L} = \int_0^\infty d\alpha \int_{-L_c}^{L_c} \frac{ds}{B} \Bigg\{ & \left[\frac{\ell^2 \kappa \hat{\phi}^2}{Br} \frac{D(P_\perp + P_\parallel + n_i m_i \bar{v}_\parallel^2)}{D\alpha} \right] \\
& + n_i m_i \omega^2 \left[r^2 B^2 \left(\frac{D}{D\alpha}\left(\frac{\hat{\phi}}{B}\right)\right)^2 + \frac{\ell^2 \hat{\phi}^2}{B^2 r^2} \right] \\
& + n_i m_i A \left[r^4 B^2 \left(\frac{D}{D\alpha}\left(\frac{\hat{\phi}}{rB}\right)\right)^2 + (\ell^2 - 1)\frac{\hat{\phi}^2}{B^2 r^2} \right] \Bigg\} \\
& - \sum_j \frac{8\pi q_j^2}{m_j^2} \int d\alpha\, \hat{\phi}^2 \int d\varepsilon d\mu \left(\omega \frac{\partial F_j}{\partial \varepsilon} + \frac{\ell}{q_j}\frac{\partial F_j}{\partial \alpha}\right) \int_{t_c}^{t_b} d\tau \Big(\omega - \omega_d(\tau) \Big) \\
& + \mathcal{O}(\kappa^2 r^2) = 0
\end{aligned}
\tag{1}
$$

where we have taken the perturbation to be finite only in the azimuthally symmetric regions of the plasma ($|s| < L_c$) so that we are justified in considering eigenfunctions of the form $\phi \propto \hat{\phi}(\alpha)\exp(i\ell\theta)$ where ℓ is the mode number and θ the azimuthally angles, the field line curvature is then only in the $\nabla\alpha$ direction and in the paraxial approximation it is given by $\boldsymbol{\kappa} = d^2r/ds^2$, the magnetic field $\mathbf{B}$ is given by $\mathbf{B} = \nabla\alpha \times \nabla\theta$ so that α is the magnetic flux with $d\alpha = Brdr$ (at low beta$=B(s)r^2/2$), s is the distance along a field line, $n_j = \int d^3vF_i$ is the density of species j and F_j the equilibrium distribution function, q_j the charge, $\varepsilon = \frac{1}{2}m_jv_\parallel^2 + \mu B(s,\alpha) + q_j\Phi(s,\alpha)$ is the particle energy, $\mu \equiv \frac{m_jv_\perp^2}{B}$ is the magnetic moment, $\tau(s) = \int_0^s \frac{ds}{(v_\parallel(\varepsilon,\mu,s))}$ $\tau_c = \tau(L_c)$, $\tau_b = \tau(L_b)$ with $v_\parallel(\varepsilon,\mu,L_b) = 0$ (note that the kinetic term only includes those particles whose turning point L_b is greater than L_c),

$$\omega_d = \frac{\mu\ell}{q_j}\frac{\partial B}{\partial\alpha} + \frac{m_jv_\parallel^2\ell\kappa}{q_jBr} \equiv \omega_B + \omega_\kappa$$

$$\frac{\partial B}{\partial\alpha} = \sigma\frac{\kappa B}{r} - \frac{1}{B}\frac{DP_\perp}{D\alpha} \quad , \quad \sigma = 1 + \frac{P_\perp - P_\parallel}{B^2},$$

$\perp$ and $\parallel$ refer to perpendicular and parallel the magnetic field,

$$A = \frac{-\ell\omega}{n_iBq_i}\frac{D(P_{\perp i}B)}{D\alpha} + \frac{\ell^2}{2n_iq_i^2}\frac{1}{B}\frac{DB}{D\alpha}\frac{DH_i}{D\alpha}$$

$$H_i = m_i^2\int d^3v\frac{v_\perp^4}{4}F_i, \qquad \frac{D}{D\alpha} = \nabla\alpha\cdot\nabla.$$

The validity of Eq. (1) requires $L_A/L_c \ll 1$ and $\omega_{dA} \gg \omega_{dc}$ where ω_{dA} and ω_{dc} are ω_d in the anchor and central-cell regions respectively. The inclusion of ω_{dA} in the kinetic term of Eq. (1) gives rise to the principal new effect of this paper.

For simplicity we have neglected a finite bending energy term discussed in Ref. 8. We have included a somewhat unconventional term in the curvature that is due to the additional momentum flux arising from the axial outflow of the plasma.[9] This term is essential for establishing flute stability of the gas dynamic trap. Note that the integral in the kinetic term in Eq. (1) only includes those passing particles species that are confined.

We now analyze Eq. (1) for the stability of the $\ell = 1$ displacement-like mode at low beta (hence $\alpha \doteq \frac{1}{2}Br^2$) in the paraxial limit ($\kappa r \ll 1$). For systems with very long central cells, the higher ℓ-modes are assumed FLR stabilized, although more detailed of the higher ℓ-modes may also be appropriate.

In order to annihilate the response of the large FLR term, the perturbed eigenfunction in the central cell is taken to have the form $\hat{\phi} \propto \alpha^{1/2}$. After some manipulation of terms with the assumption that κ/r is independent of α, the dispersion relation becomes

$$\int_0^\infty d\alpha \left\{ \left[\int_{-L_c}^{L_c} \frac{ds}{B^2} \left[-\frac{\kappa}{r}\left(2P_c + n_i m_i \bar{v}_{||}^2\right) + n_i m_i \omega^2 \right] \right] \right.$$
$$+ 2\sum_j \int_{L_c}^{L_{\max}} ds \left[\frac{\omega^2 q_j^2 r^2}{T_j} n_{psj} + \frac{q_j r^2}{B} \omega n_{psj} \right.$$
$$\left. \left. - \left(P_{\perp pj} + P_{||psj}\right) \frac{\kappa}{B^2 r} - \frac{r^2}{2} \frac{DP_{psj}}{D\alpha} \frac{DP_{\perp A}}{D\alpha} \right] \right\} = 0 \tag{2}$$

where P_c is the central-cell pressure (where $P_\perp = P_{||}$), n_{psj}, $P_{\perp psj}$, $P_{||psj}$ are for passing particles of species j; the density, the partial perpendicular pressure and the partial parallel pressure, all in the anchor region and $L_{\max}$ represents the overall effective half-length of the system. We observe that we have the usual curvature drive in the central cell weighted by the central-cell pressure, as well as the stabilizing contribution from the anchor region, which includes both the favorable curvature from that region and the favorable diamagnetic well contribution from the pressure stored in the anchor. Because there is favorable curvature in the anchor region, one can support a local beta comparable to unity there and/or have a large favorable curvature there by providing a short anchor length. Thus, although the partial pressure of the passing particle is small, there is a possibility that the total favorable weighting from the anchor, surpasses the unfavorable weighting from the central cell. We can rewrite Eq. (2) as,

$$\omega^2(1+Q) - \omega\omega_{*i}\Delta Q + \gamma^2_{\mathrm{MHDC}} - \Omega^2_{\mathrm{MHDA}} = 0 \tag{3}$$

where

$$Q = -4\sum_j \int_{L_c}^{L_{\max}} \frac{ds q_j^2}{BT_j} \int_0^\infty d\alpha n_{ps}\alpha \Big/ K$$

$$\omega_{*i}\Delta Q = 4\sum_j q_j \frac{T_i}{T_j} \int_{L_c}^{L_{\max}} \frac{ds}{B} \int_0^\infty d\alpha n_{ps} \Big/ K$$

$$\gamma^2_{\mathrm{MHDC}} = -\int_0^\infty d\alpha \int_{-L_c}^{L_c} \frac{ds\kappa}{B^2 r}\left[2P_c + n_i m_i \bar{v}_\parallel^2\right] \Big/ K$$

$$\Omega^2_{\mathrm{MHDA}} = \frac{1}{K}\Bigg[-2\sum_j \int_0^\infty d\alpha \int_{L_c}^{L_{\max}} \frac{ds\kappa}{B^2 r}\left(P_{\perp psj} + P_{\parallel psj}\right)$$
$$+ 2\sum_j \int_{L_c}^{L_{\max}} ds \int_0^\infty \frac{d\alpha\alpha}{B}\frac{DP_{psj}}{D\alpha}\frac{D}{D\alpha}P_{\perp A}\Bigg]$$

$$K = \int_{-L_c}^{L_c} ds \int_0^\infty \frac{d\alpha}{B^2} n_i m_i .$$

Stability is then always guaranteed if $\Omega^2_{\mathrm{MHDA}} > \gamma^2_{\mathrm{MHDC}}$. The precise stability condition is

$$\omega^2_{*i} > 4(\gamma^2_{\mathrm{MHDC}} - \Omega^2_{\mathrm{MHDA}})\frac{(1+Q)}{\Delta Q^2}. \tag{4}$$

Now let us apply this trapped particle mode theory to the gas dynamic trap experiment being planned.[4] The gas dynamic trap is a symmetric mirror machine that is stabilized by momentum flux arising from the outflow of the plasma. A schematic diagram of the experiment is shown in Fig. 1 and planned plasma parameters is shown in Table 1. There is a long central cell of length L_c, shaped in such a manner as to minimize the unfavorable curvature drive. At the ends there is a high mirror ratio, R, (e.g., ~ 80 as in Table 1). The collisionality of the plasma is such that $\nu_i > v_{\mathrm{thi}} \ln R/L_c R$, where ν_i is the ion collision time, and v_{thi} the ion thermal speed. In such a range of collisionality, the plasma distribution will be near a Maxwellian in the central cell, with the density nearly constant until one approaches the mirror throat. In the throat region the plasma escapes with an outward particle flux given by

$$\frac{n\bar{v}_\parallel}{B} \equiv \text{const.} \tag{5}$$

At the mirror throat the ions have an exit velocity comparable to their thermal speed. The density is somewhat reduced from the central-cell density by roughly a factor of two. Beyond the throat all the ions are lost, but most of the electrons are reflected by an ambipolar potential that is present to maintain charge neutrality. These reflected electrons, which have primarily parallel pressure (the perpendicular motion is converted to parallel motion by the expanding magnetic field), will play the role of passing particles in the trapped particle theory.

As the magnetic field expands, the ion thermal speed reaches an exit speed $\approx 3v_{\text{thi}}$ at $L{=}L_{\text{max}}$.[10] Over most of the expander this exit speed has only a small variation compared with the change of density and magnetic field which vary nearly together to maintain the equality of Eq. (5). At the end of the expander, the magnetic field is at a low value, and the major contribution to the stabilizing term is,

$$\mathcal{L}_{\text{flow}} = \int_0^\infty d\alpha \int_{-\infty}^{\infty} \frac{ds\kappa}{B^2 r} \hat{\phi} \frac{D}{D\alpha} n_i m_i \bar{v}_\parallel^2. \tag{6}$$

In the detailed design for the gas dynamic trap based on the figures in Table 1, the contribution from $\mathcal{L}_{\text{flow}}$ stabilizes the flute mode, whose eigenmode structure is $\hat{\phi} = \phi_0 \alpha^{1/2}$ and is independent of s. For the parameters in Table 1, it is larger than the destabilizing flute contribution,

$$\mathcal{L}_{\text{MHDF}} = -\int_0^\infty d\alpha \int_{-\infty}^{\infty} \frac{ds\kappa\phi_0^2}{B^2 r} \left[P_\perp + P_\parallel\right] \tag{7}$$

by roughly a factor of three. Ballooning mode analysis of this configuration predicts a large central MHD critical beta limit, ~ 0.5.[11]

In the trapped particle theory, where there is zero perturbation in the end-cells, the direct ion momentum term does not contribute as the flowing ions do not return to the central cell. However, the electron pressure term does produce a stabilizing contribution to the dispersion relation. As $\bar{v}_\parallel^2 \approx 10 v_{\text{thi}}^2$, $P_{\parallel pse} \approx \frac{1}{10} n_i m_i \bar{v}_\parallel^2$, and for the parameters shown in Table 1, one finds that

$$\mathcal{L}_{\text{anchor}} = -2\phi_0 \int_0^\infty d\alpha \int_{L_c}^{L_{\max}} \frac{ds\kappa}{B^2 r} P_{\parallel pse} \approx .1\mathcal{L}_{\text{flow}} \approx .3\mathcal{L}_{\text{MHDF}}. \tag{8}$$

At low beta, the quantity $\mathcal{L}_{\text{anchor}}$ is the additional stabilizing term that enters trapped particle mode theory. For the present design of the gas dynamic trap experiment $\mathcal{L}_{\text{anchor}}$ does not appear large enough to guarantee robust stability to the trapped particle interchange mode. However, if one can modify the design by making the ratio of expander length to central cell smaller by a factor of three, or lower the mirror ratio (roughly $\mathcal{L}_{\text{anchor}} \propto 1/B_{\text{max}}$) to 25, one can obtain robust stability to the curvature driven trapped particle mode. Even if the robust stabilization condition ($\Omega^2_{\text{MHDA}} > \gamma^2_{\text{MHDC}}$) is not satisfied, stability can still be achieved by satisfying Eq. (4). For the gas dynamic trap, $\Delta Q/Q = -1$, and we choose $T_e = T_i = 150\text{eV}$, and the scale lengths given in Table 1. Then

$$Q = \frac{n_{ps}}{n_p}\frac{2L_{ex}}{L_c}\frac{1}{R}\frac{r_p^2}{a_H^2} \approx 1$$

where $n_{ps}/n_0 = 1/2$, $\dfrac{2L_{ex}}{L_c} = 3.6/7 = .5$, r_p=11cm. is the plasma radius, where a_H is the ion Larmor radius in a 2kg central-cell magnetic field, and n_0 the central-cell density. We estimate $\gamma^2_{\text{MHDC}}/\omega^2_{*i}$ as

$$\frac{\gamma^2_{\text{MHDC}}}{\omega^2_{*i}} \approx \frac{2r^4}{L_c^2 a_H^2} = \frac{2 \times (.11)^4}{7^2 \cdot (6 \times 10^{-3})^2} = 1.7 \times 10^{-1}.$$

Consequently, from Eq. (4) the stability condition for the base case is satisfied but close to marginal stability.

$$\frac{4\gamma^2_{\text{MHDC}}}{\omega^2_{*i}}\left(1 - \frac{\Omega^2_{\text{MHDA}}}{\gamma^2_{\text{MHDC}}}\right)\frac{(1+Q)}{\Delta Q^2} \approx .68\left(\frac{2}{3}\right)\frac{2}{1} \approx .9 < 1.$$

We now consider the stability criterion, as the parameters $\tau \equiv T_e \dfrac{B_{co}^2}{B_c^2}$ (B_{co} is the nominal central-cell magnetic field of 2kg and B_c is a variable central-cell magnetic field) and R are varied. In terms of these parameters we have

$$\frac{\gamma^2_{\text{MHDC}}}{\omega^2_{*i}} = 1.7 \times 10^{-1}\left(\frac{150}{\tau}\right)$$
$$Q = -\Delta Q = \left(\frac{150}{\tau}\right)\frac{80}{R}$$
$$\frac{\Omega^2_{\text{MHDA}}}{\gamma^2_{\text{MHDC}}} = .3\left(\frac{80}{R}\right).$$

Then, using the same scale parameters as before, the stability condition becomes,

$$\tau < 150 \left(\frac{80}{R}\right) \left[\frac{1.5\left(\frac{80}{R}\right)}{1 - 25/R} - 1 \right]$$

where the units of τ is in electron volts. Without the anchor stabilization term (the term "$25/R$") stability requires for R=80, $\tau <$ 75eV, while with this term $\tau <$ 177eV is predicted for stability. Thus there is a substantial increase in the region predicted to be stable. In Table II we list the threshold of the normalized temperature for the onset of the trapped particle mode, as a function of the mirror ratio R. Observe, that the stable regime widens quite significantly at lower mirror ratios.

In summary this study shows that the stability for the curvature driven trapped particle mode is close to the planned experimental parameters. By a suitable variation of parameters it may be possible to identify the trapped particle mode and to determine whether a more quiescent plasma is obtained as the trapped particle stabilization conditions are more strongly fulfilled. More generally, we have shown that it is in principle possible to stabilize trapped particle modes in a robust manner in tandem mirrors, and it may be worthwhile to attempt to design configurations to satisfy the condition $\Omega^2_{\mathrm{MHDA}} > \gamma^2_{\mathrm{MHDC}}$.

References

1. B.B. Kadomtsev and O.P. Pogutse, Sov. Phys. JETP **24**, 1172 (1967).
2. H.L. Berk, M.N. Rosenbluth, H.V. Wong, T.M. Antonsen, and D.E. Baldwin, Fiz. Plasmy **9**, 176 (1983) [Sov. J. Plasma Phys. **9**, 108 (1983)].
3. "Physics Basis for an Axicell Design for the End Plugs of MFTF-B", Eds. N.E. Baldwin and B.G. Logan, Lawrence Livermore Laboratory Report, UCID 19359, April 21, 1982.
4. D.D. Ryutov "Physics of Open Traps", presented at *XII European Conference on Controlled Fusion and Plasma Physics*, Budapest, September 1985), preprint: Institute of Nuclear Physics, Novosibirsk, 1985, Report #85-127.
5. V.P. Nagornyj, D.D. Ryutov, G.V. Stupakov, Nucl. Fusion **24**, 1421 (1984).
6. H.L. Berk, M.N. Rosenbluth, R. H. Cohen and W.M. Nevins, Phys. Fluids **28**, 2824 (985).
7. H.L. Berk and B.G. Lane, Phys. Fluids **29**, 1076 (1986).
8. H.L. Berk and B.G. Lane "Variational Quadratic Form for Low Frequency Electromagnetic Perturbations; (II) Applications to Tandem Mirrors Report, Plasma Fusion Center, Massachusetts Institute of Technology (submitted to The Physics of Fluids).
9. V.V. Mirnov and D.D. Ryutov, Pis'ma JTF (JETP Lett.) **5**, 678 (1979).
10. V.V. Mirnov and O.A. Tkachenko, "The Electrostatic Potential Distribution in Gas Dynamic Trap", preprint, Institute of Nuclear Physics, 1986, Report #86-28.
11. O.A. Bushkova and V.V. Mirnov, "The Effect of the Magnetic Field Profile on MHD Stability in Gas Dynamic Trap", preprint: Institute of Nuclear Physics, Novosibirsk, 1985, Report #85-103.

Table I

PARAMETERS OF THE EXPERIMENT

Initial plasma density	$4 \times 10^{13} \mathrm{cm}^{-3}$
Temperature of heated plasma	150 eV
Mirror-to-mirror length	7 m
Plasma radius	11 cm
Expander length	1.8 m
Mirror field	16 T
Mirror ratio	80
Injection energy	20 keV
Trapped injection current	250 A
Duration of the injection pulse	0.25 ms
Density decay time	4 ms
Plasma β in the solenoid	10%

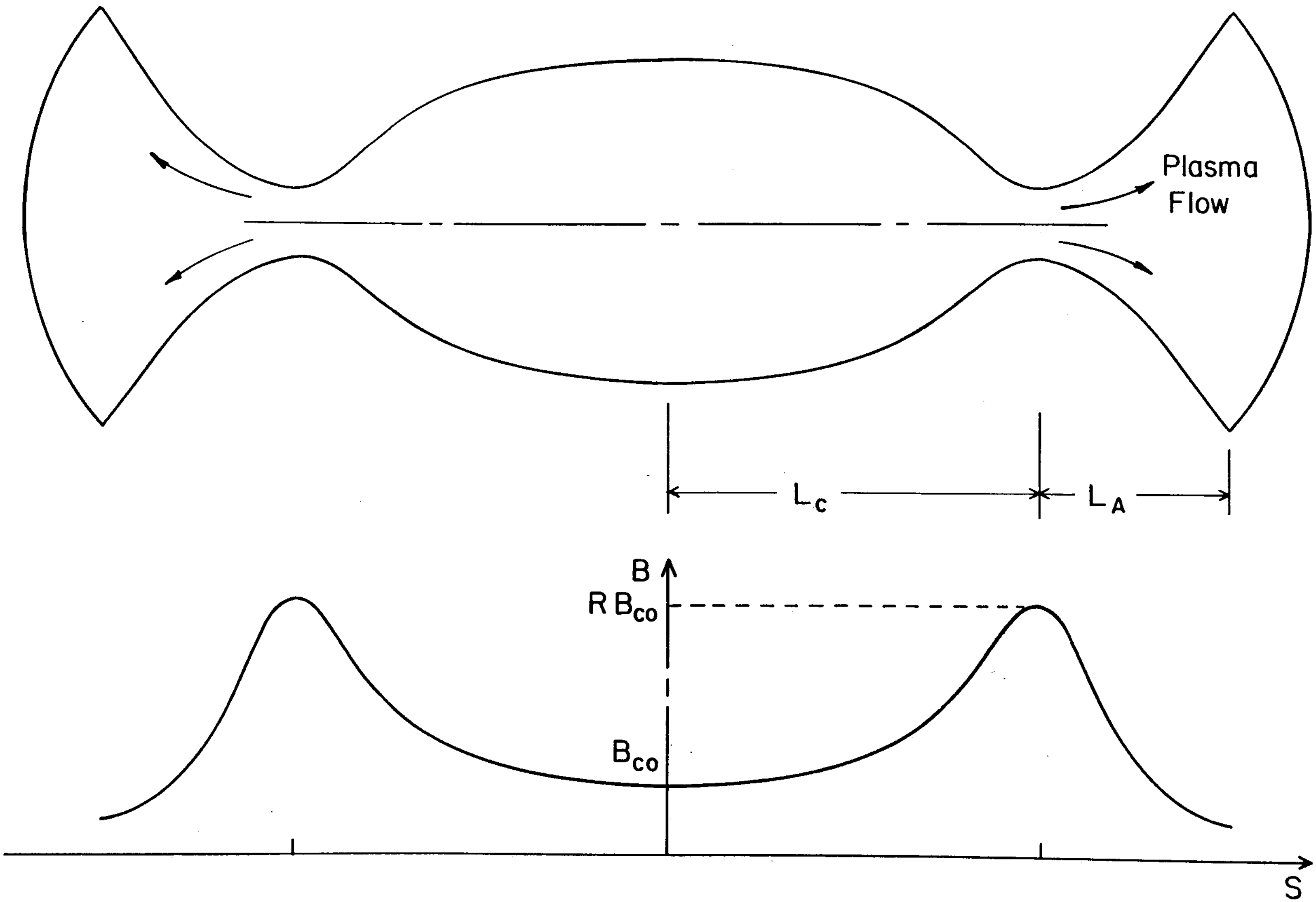
Table II

STABLE "TEMPERATURE" RANGE FOR GIVEN MIRROR RATIO

Mirror Ratio R	Maximum Stable "Temperature" (eV)
100	$\tau > 72$
90	$\tau > 112$
80	$\tau > 177$
70	$\tau > 285$
60	$\tau > 486$
50	$\tau > 912$
40	$\tau > 2100$
30	$\tau > 9200$
25	No Instability

Figure Caption

Schematic figure of gas dynamic trap. Top figure is the radial and axial shape of the plasma flux tube. Bottom figure is the variation of magnetic field along the axis. Central region has a half-length of L_c, an expander length of L_A, and a midplane magnetic field value of B_{c0} and a peak magnetic field of RB_{c0}.


Plasma
Flow
L_c
L_A
B
RB_{co}
B_{co}
S