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Destabilization of Alfvén-Resonant Modes by Resistivity and Diamagnetic Drifts

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Abstract

The existence of unstable resistive drift-Alfvén modes is shown in a cylindrical geometry. The modes, driven by nonuniformities in the diamagnetic drift frequency, are unstable even when the associated purely resistive mode is stable. The growth rate has a weak, $\eta^{1/4}$, dependence on resistivity. Because of their large growth rates, they may play an important role in tokamak confinement.

The low frequency magnetic fluctuations associated with drift and drift-Alfvén waves are thought to play an important role in tokamak confinement degradation; in particular, they are regarded as a possible mechanism for anomalous electron thermal transport[1]. Both drift waves and shear-Alfvén waves can be unstable in the local approximation[2].However, they are stabilized by magnetic shear in the collisionless[3,4,5], and collisional[6,7] limits in a sheared-slab geometry. The collisionless drift-Alfvén waves can be destabilized by trapped-electron effects[4,8], and temperature gradients[8], whereas the ion diamagnetic drift and finite-β effects were found to be destabilizing for the resistive modes[6]. However, the resistive drift-Alfvén mode of Ref. 6 was later shown to be stable[9].

In this Letter, we report a new resistive drift-Alfvén mode. The salient features of this mode are as follows:1) The mode is resonant, not at the mode rational surface where the parallel wave-vector $k_{\|}$ vanishes, but at the drift-Alfvén resonance where $\omega^2 \simeq \omega_{*e}^2 \simeq k_{\|}^2 \ v_A^2/(1+T_i/T_e)$. Here ω_{*e} is the electron diamagnetic drift frequency, and v_A is the Alfvén speed. 2) The mode is destabilized by gradients in the drift frequency ω_{*e} ; it is neutral for uniform ω_{*e}. 3) The parallel electric field $E_{\|} = -i(\omega\psi - k_{\|}\phi)$ vanishes outside the resonance layer; thus, the flux function vanishes at the rational surface ($k_{\|} = 0$), distinguishing this mode from drift-tearing modes. 4) The growth rate γ of the mode has only a weak dependence on resistivity η: $\gamma \sim \eta^{1/4}$.

The mode is studied using conventional[3−8] linearized field equations. For convenience we use "reduced" normalizations, measuring speeds with respect to the Alfvén speed, v_A, and times with respect to the (perpendicular) Alfvén time τ_A[10]. Ignoring compressibility,temperature gradients, and curvature effects for simplicity, and assuming perturbations of the form $\psi = \psi(r)e^{-i\omega+i(m\theta-nz)}$, we can write the relevant equations as

$$\omega\psi - k_{\|}\phi + \frac{1}{i\sigma_*}\nabla_{\perp}^2\psi = 0 \tag{1}$$

$$-(\omega-\omega_{*i})\nabla_{\perp}^2\phi + k_{\|}\nabla_{\perp}^2\psi + k_{\perp}J_0'\psi = 0 \tag{2}$$

where ψ and ϕ are the poloidal flux function and the electrostatic potential, respectively. Equation. (1) is the parallel Ohm's law. Equation (2), in which J_0' is the gradient of the equilibrium current density, is the vorticity equation modified by ion gyroviscosity. The "ac" conductivity is given by $\sigma_* = (\omega - \omega_{*e})/(\eta\omega)$, where η is the resistivity. The electron and ion diamagnetic drift frequencies are $\omega_{*e} = -\delta k_\perp p_0'$, and $\omega_{*i} = -(T_i/T_e)\omega_{*e}$, respectively, where $k_\perp = m/r$. The gyroradius parameter[10] δ is given by $\delta = (2\Omega_i\tau_A)^{-1}$, where $\Omega_i = eB_T/(m_ic)$ is the ion gyrofrequency. Finally, the parallel wave-vector $k_\parallel = m/q - n$, where $(1/q) = -(1/r)(\partial\psi_o/\partial r)$, and m, n are the poloidal and toroidal mode numbers. Note that in our normalized units, the Alfvén speed v_A is dropped from expressions of the form $k_\parallel v_A$.

In the "exterior region" where resistivity can be neglected, the vorticity equation can be written as

$$-\left[\omega(\omega - \omega_{*i}) - k_\parallel^2\right]\nabla_\perp^2\phi + (k_\parallel^2)'\left[\frac{\partial\phi}{\partial r} - \frac{\phi}{r}\right] = 0, \tag{3}$$

where we expressed J_0' in terms of $k_\parallel'$, and used $\psi = \frac{k_\parallel}{\omega}\phi$. For a mode with finite frequency ω, Eq. (3) is singular at $\omega(\omega - \omega_{*i}) \simeq k_\parallel^2$. In addition, in Eq. (1), $E_\parallel$ becomes significant at the $\omega \simeq \omega_{*e}$ resonance where $\sigma_* \to 0$. These two relations determine the drift-Alfvén resonance surface,$r = r_{DA}$, where the new Alfvén-resonant mode is localized.

The equations relevant to this singular layer are derived by letting $x = r - r_s$, $k_\parallel = k_\parallel' x$, and assuming $\frac{\partial}{\partial x} \gg k_\perp$, and $J_0' \to 0$, where $r = r_s$ is the rational surface where $k_\parallel$ vanishes. Then we obtain

$$\omega\psi - k_\parallel' x\phi + \frac{1}{i\sigma_*}\psi'' = 0, \tag{4}$$

$$-(\omega - \omega_{*i})\phi'' + k_\parallel' x\psi'' = 0, \tag{5}$$

where primes denote x-derivatives.

Before giving a variational solution to the above eigenvalue problem, we estimate the growth rate of the mode for $\omega_{*i} = 0$ as follows. Let $\omega_{*e} = \omega_* + \omega_*' x$, and define

$x_\omega = (\omega - \omega_*)/\omega'_*$, $x_A = \omega/k'_\parallel$, and $x_{DA} = \omega_*/[k'_\parallel(1+\zeta)]$, where $\zeta = -\omega'_*/k'_\parallel$. Noting that $\sigma_* \to 0$ as $x \to x_\omega$, we consider a mode centered at $x_{\omega_r} = \Re(x_\omega)$. Defining $\hat{x} = x - x_{\omega_r}$, we have $\omega - \omega_{*e} \to i\gamma - \omega'_*\hat{x}$, where γ is the growth rate ($\gamma \ll \omega_r$). Thus, the resistivity will be important only in the region $|\hat{x}| \leq w$, where the layer width is given by $w \simeq |\gamma/\omega'_*|$. It is clear from this relation that variations in ω_{*e} will be able to localize the mode away from the rational surface only if $w/x_A \simeq \gamma/(\zeta\omega) \ll 1$. For $\hat{x} \leq w$ the resistive term in Ohm's law becomes

$$\frac{\psi''}{\sigma_*} \simeq \frac{\eta\omega\psi}{\gamma w^2}. \tag{6}$$

This term is balanced by $E_\parallel \simeq \omega\psi - k_\parallel\phi$. $E_\parallel$ can be estimated by setting $x \simeq x_\omega \simeq x_A$ in Eq. (5) to obtain $\phi \sim \psi[1 + O(w/x_A)]$. Using this we find

$$E_\parallel \simeq \psi(\omega - k_\parallel) \simeq \psi k'_\parallel(x_A - x) \sim \psi k'_\parallel w. \tag{7}$$

Combining Eqs. (4),(6), and (7) yields

$$\gamma^4 \simeq \frac{\omega_*\omega'^3_*\eta}{k'_\parallel}. \tag{8}$$

Note that the growth rate scales linearly with ω_* and has only $\eta^{1/4}$ dependence on resistivity.

Detailed analysis of the eigenvalue problem begins with the observation that Eqs.(4) and (5) can be combined to yield a second order equation for the radial electric field, $E \equiv \phi'$:

$$\frac{ix^2}{\omega}\left[\frac{E'}{\sigma_* x^2}\right]' = [1 - \frac{x^2}{x^2_{A*}}]E, \tag{9}$$

where $x^2_{A*} = \omega(\omega - \omega_{*i})/k'^2_\parallel$. Equation (9) possesses the well known variational principle[11] $\delta H = 0$, with

$$H \equiv \int dx \left[i\frac{E'^2}{\omega\sigma_* x^2} + (\frac{1}{x^2} - \frac{1}{x^2_{A*}})E^2\right]. \tag{10}$$

Furthermore, a variationally accurate dispersion relation is given by $H = 0$.

We expect the eigenfunction to be centered at some point, $x_o \neq 0$; on the other hand, Eq. (9) shows that E' must vanish at $x = 0$. Thus, a reasonable trial function has the form $E' = x \exp[-(\alpha/2)(x - x_0)^2]$, where x_0 and α are variational parameters. After substitution into Eq. (10) we find that

$$H = Z(\alpha^{1/2} x_*) - i\frac{2\omega'_*}{\eta\alpha^{3/2}}\left[1 - \frac{x_0^2}{x_{A*}^2} + \frac{1}{2\alpha x_{A*}^2}\right], \tag{11}$$

where Z is the plasma dispersion function and $x_* \equiv -x_0 + x_\omega$. The variational principle, $\delta H = 0$ (i.e., $\partial H/\partial\alpha = 0 = \partial H/\partial x_0$), and the dispersion relation $H = 0$ now provide three equations for α, x_0, and ω. We omit the detailed analysis of these equations and restrict attention to the narrow mode-width limit, $w \ll x_{A*}$. Setting $T_i = 0$ temporarily, one finds that

$$-(\omega - \omega_*)/(\zeta\omega) - 1 = (4/3)y(1 - 2y^2/3)^{-1/4}[i\eta/(2\omega'_*)]^{1/4}, \tag{12}$$

where $y \equiv \alpha^{1/2} x_*$ satisfies $Z(y) = (2/3)y/[1 - (2/3)y^2]$. Solution to this transcendantal equation reveals a double sequence of eigenvalues having the form $y_{n\pm} = \pm r_n - i\nu_n$, with, typically, $\nu_n \simeq \mathrm{r}_n$. The sequence, which is also observed numerically, is presumably a discretized remnant of the Alfvén continuum. We choose the smallest $y_n = -1.9435 - 0.76663i$, corresponding to the broadest radial eigenmode structure with $x_0 > 0$. After substitution into Eqs. (14) and (15), we obtain the growth rate

$$\frac{\gamma}{\omega_*} = 0.92(1 + \zeta)^{-5/4}\left[\frac{\zeta\gamma_c}{\omega_*}\right]^{3/4}, \tag{13}$$

where $\gamma_c \equiv (k'_\parallel)^{2/3}\eta^{1/3}$ is the growth rate of the classical $m = 1$ tearing mode, and $\zeta = -\omega'_*/k'_\parallel$.

The mode width is given by $w \simeq [\gamma_c^3/(\zeta\omega_*^3)]^{1/4}(\omega_*/k'_\parallel)$. Note that narrow mode-width condition requires the first factor here to be small: $\gamma_c^3/\omega_*^3 \ll \zeta$, an ordering which is very sensitive to electron temperature, and magnetic shear length, L_s, since $\gamma_c^3/(\zeta\omega_*^3) \sim L_s^{-3} T_e^{-11/2}$. It is not clearly satisfied in present tokamaks. We emphasize, however, that this is a constraint only for the asymptotic analysis; the numerical results show that the instability persists even when it is violated. Two other consistency conditions, $Re(\alpha) > 0$, and $Re(\alpha^{1/2}) > 0$ are easily seen to be satisfied.

The finite-T_i analysis is not significantly different, although the results are superficially more complicated. It is convenient to take $\zeta < 1$, and neglect $O(\zeta^2)$. Then the real frequency is given by $\omega_r = \omega_* - \zeta[\omega_*(\omega_* - \omega_{*i})]^{1/2} + O(\zeta^2)$, while Eq. (13) becomes

$$\gamma = 0.92[\omega_*(\omega_* - \omega_{*i})]^{1/2}[\zeta\gamma_c/\omega_*]^{3/4}. \tag{14}$$

The Alfvén-resonant instability was first seen in numerical simulations, using the four-field model[10], of the effects of diamagnetic drifts on the m=1 resistive tearing mode in tokamaks[12]. These effects were examined earlier; because the spatial variation of ω_* was ignored, they were found merely to reduce the growth rate of the mode, without introducing a real frequency[13]. This branch of the dispersion relation (referred to as the Waddell branch) has a growth rate $\gamma_W \simeq \gamma_c^3/\omega_{*e}\omega_{*i}$ for $\omega_{*e} \gg \gamma_c$. For comparison, in Fig. 1, numerically calculated growth rates of the Waddell branch, and the Alfvén-resonant branch are plotted as a function of ω_{*e} for a typical equilibrium safety factor profile with $q_o = 0.95$, and $q_l = 3.0$. The pressure profile is $p_o \sim \psi^2$, and the Lundquist number $S = \eta^{-1}$ has a value of 5×10^8. For $\omega_{*e} \ll \gamma_c$, $\gamma_W \simeq \gamma_c = 8.51 \times 10^{-4}$, while γ_{AR} is negligible. For neutral beam heated TFTR discharges[14] ($B_T = 47kG, n_e = 4.7 \times 10^{13}/cm^3, T_e = 4.9keV$), we estimate $\omega_{*e} = 2.9 \times 10^{-3}$, which gives $\gamma_{AR}/\gamma_W \sim 1/2.5$. However, since $\gamma_{AR}/\gamma_W \sim T_e^{33/8}$, for $T_e > 6.1keV$, the Alfvén-resonant branch becomes dominant.

Figures 2 a,b show the eigenfunction ϕ for the purely resistive ($\omega_{*e} = 0$) $m = 1$ mode, and the Alfvén-resonant mode ($\omega_{*e} = 4.5 \times 10^{-3}$), respectively. As expected, both modes have $\phi \sim r$ in the exterior region ($\phi = re^{im\theta}$ is a solution to Eq. (3) for $m = 1$). For the resistive mode, $\phi \to 0$ in a boundary layer around the rational surface at $r = 0.259$. However, the Alfvén-resonant mode "ignores" the $k_\parallel = 0$ surface and exhibits a resonance structure near the $\omega \simeq \omega_{*e}$ surface at $r = 0.285$. A more significant deviation is in the temporal behavior of this mode: the resistive mode, and the Waddell branch are purely growing modes, whereas the Alfvén-resonant mode rotates with $\omega_r \simeq \omega_{*e}$.

The Alfvén-resonant modes exist for $m > 1$ also, which is not surprising since these are essentially MHD continuum modes destabilized by local effects at the resonance surface. In fact, unstable modes exist even when the associated purely resistive mode is stable ($\Delta' < 0$). For example, for $m/n = 15/10$ and the TFTR parameters quoted earlier, the Alfvén-resonant mode has a growth rate $\gamma = 1.85 \times 10^{-4}$, which is larger than that of the corresponding $m = 1$ mode.

In conclusion, we have shown, in a cylindrical geometry, the existence of unstable drift-Alfvén modes driven by radial gradients in the diamagnetic drift-frequency. The modes are localized to the drift-Alfvén resonance surfaces where $\omega(\omega - \omega_{*i}) \simeq k_{\|}^2$, and rotate at the electron drift frequency, ω_{*e}. In this letter, we have ignored effects of parallel compressibility, and toroidal curvature, which are important in present-day large tokamaks. These effects, and the nonlinear evolution of these modes are being investigated and will be reported on in the near future.

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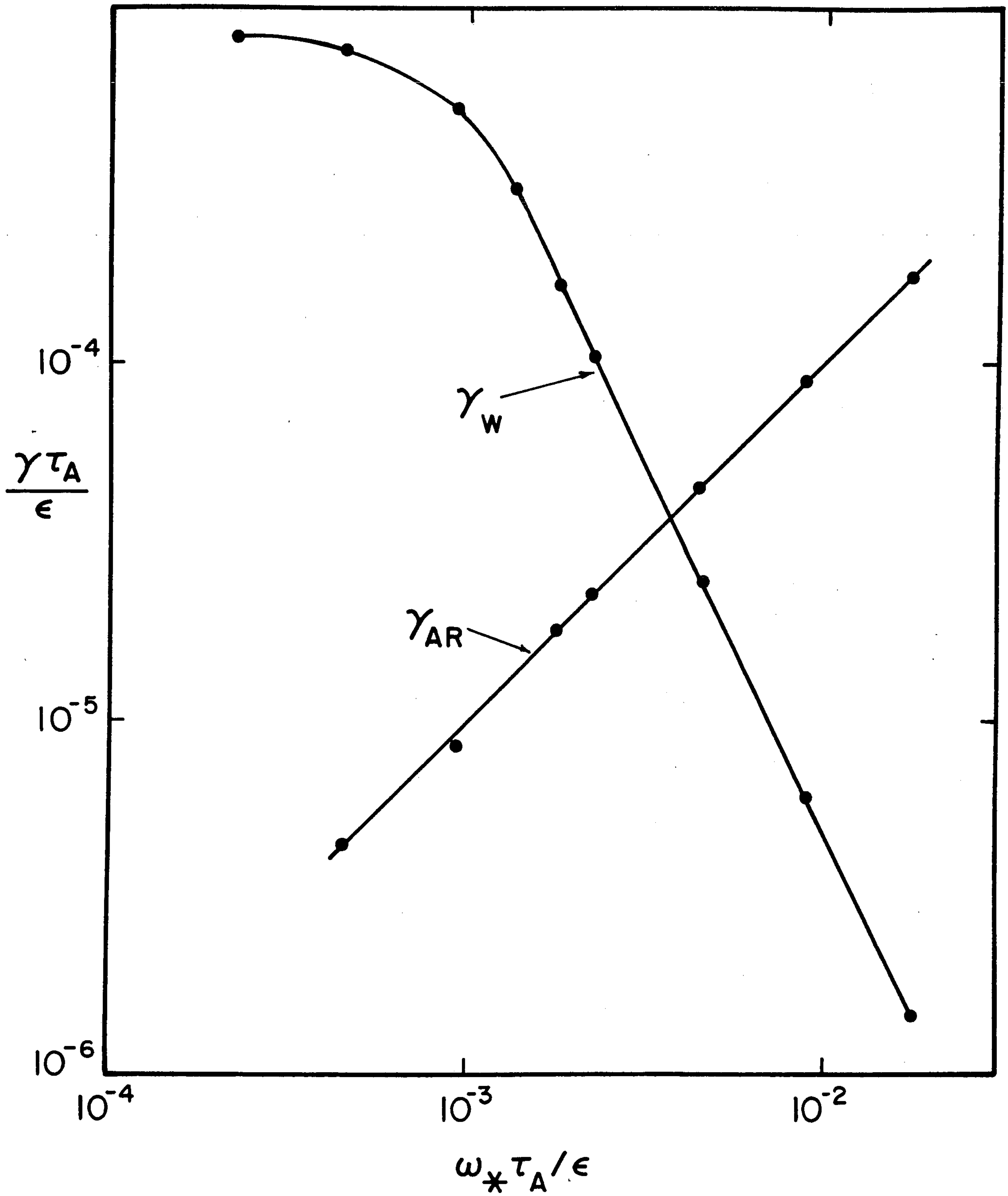
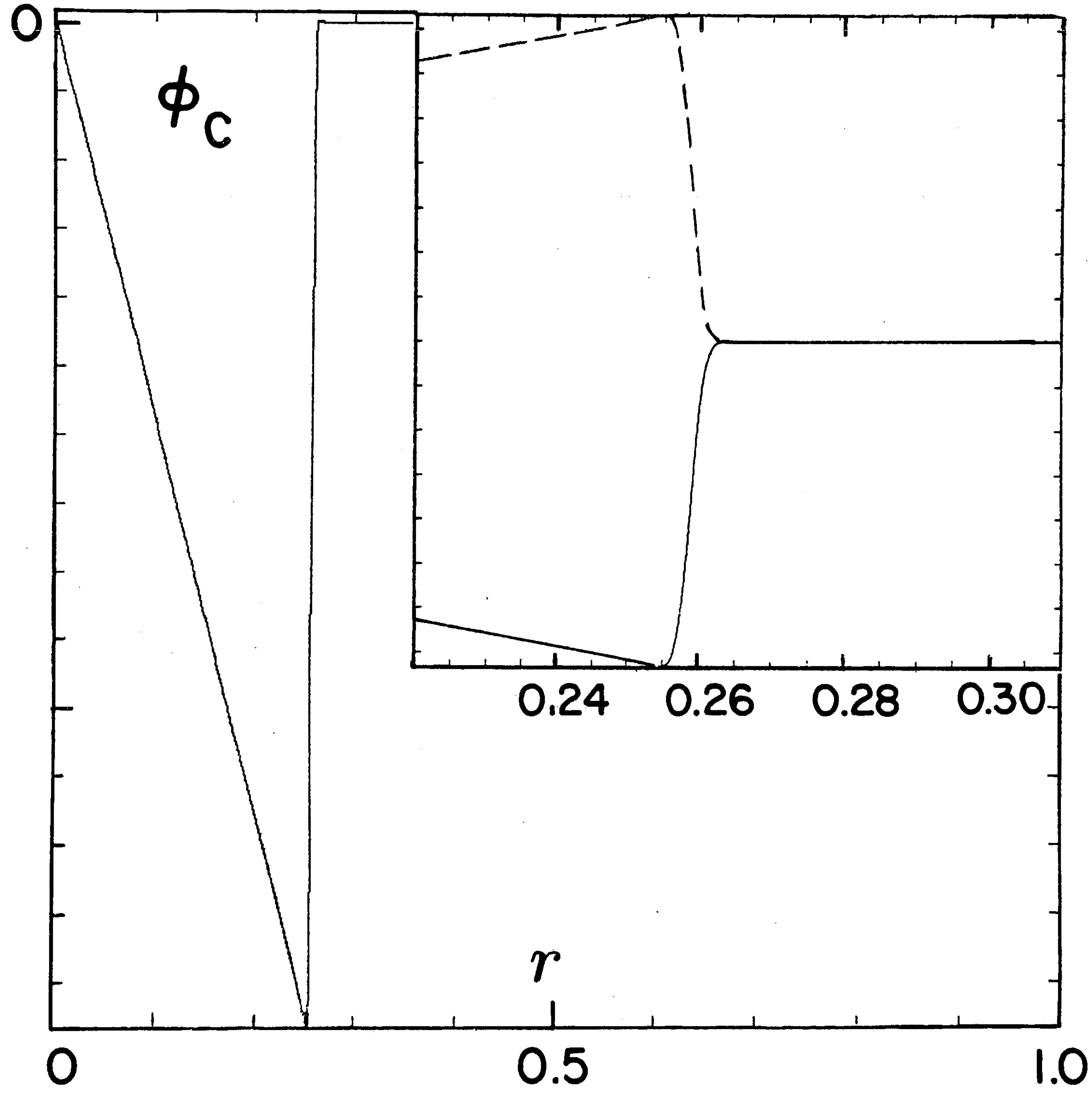
Figure Captions

1. The growth rates of the $m = 1$ resistive drift-tearing mode (γ_W), and the Alfvén-resonant mode (γ_{AR}) as a function of ω_{*e}, for $S = 5 \times 10^8$.
2. The eigenfunction ϕ as a function of the minor radius in a "cylindrical tokamak" geometry: a)the classical ($\omega_* = 0$) $m = 1$ resistive tearing mode, b) The Alfvén-resonant mode. The larger plots show only $\Re(\phi)$, whereas the insets show both the real and imaginary parts in a magnified region in the resonance layer.



10^{-4}
10^{-5}
10^{-6}
$\frac{\gamma \tau_A}{\epsilon}$
γ_W
γ_{AR}
10^{-4}
10^{-3}
10^{-2}
$\omega_* \tau_A / \epsilon$



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r
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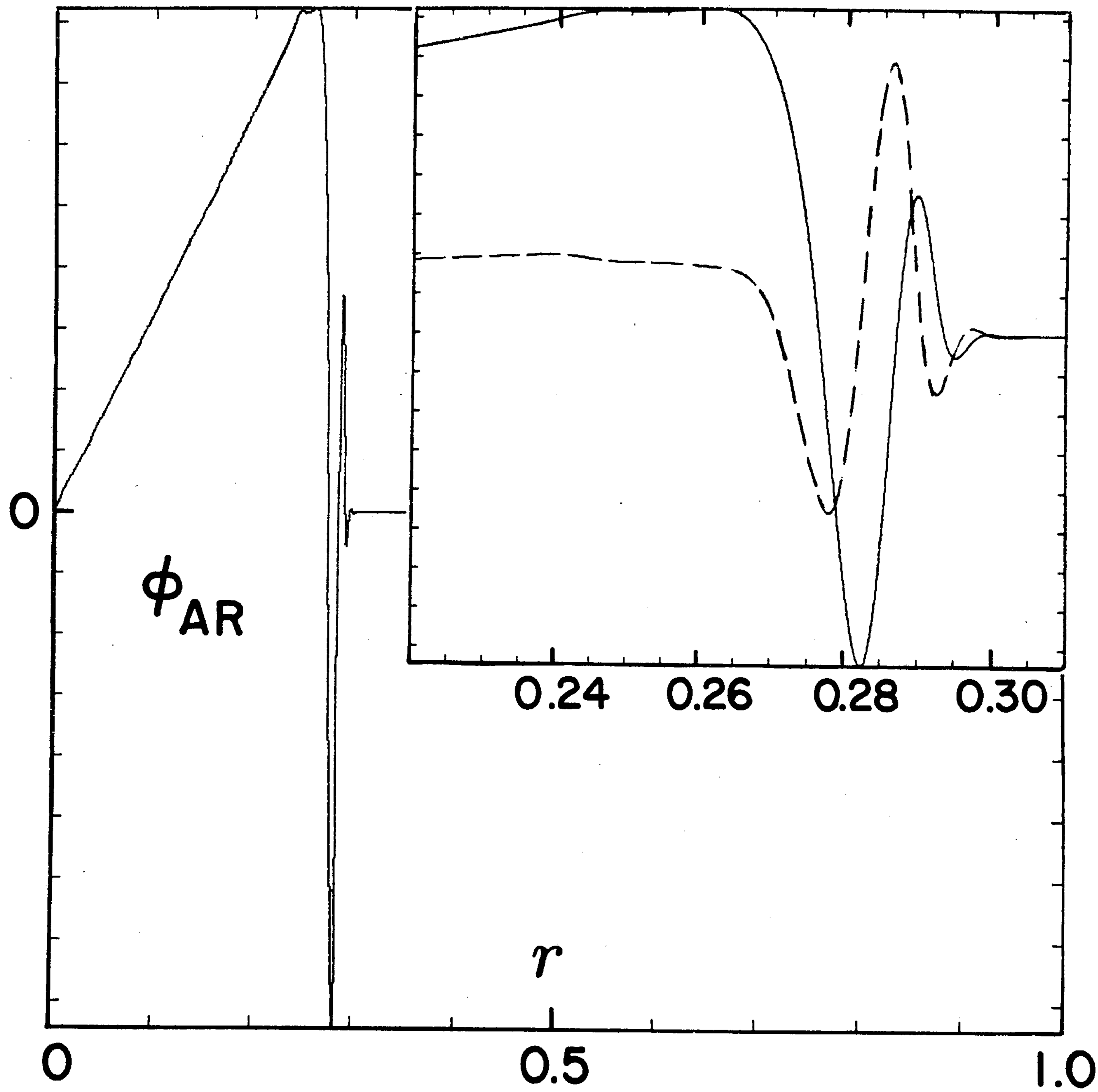
0.5
1.0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r
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0.5
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