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Reply to Comments of J. A. Krommes on Theory of Dissipative Density-Gradient Driven Turbulence in the Tokamak Edge

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Theory of Dissipative Density-Gradient Driven Turbulence in the Tokamak Edge

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We appreciate the interest of Krommes in our recent paper[1] and welcome the opportunity to discuss his comments and other related issues. In our opinion, most of the objections he has raised follow from a misunderstanding of the physics treated by clump and hole theory.[1–3] In particular, throughout his critique Krommes attempts to extrapolate results and intuition of homogeneous Navier-Stokes turbulence (HN-ST) to the more complicated case of dissipative drift-wave turbulence (DD-WT). Since these two cases are so dissimilar with regard to their fundamental constituents, drive, characteristic scales and interaction mechanisms, extrapolations from one case to the other are unwarranted and misleading. Moreover, the hypotheses and results of clump and hole theories have fared well in several tests using laboratory and simulation data which *is* relevant to the theoretical models analyzed.

To begin with, it may be instructive to contrast HN-ST and DD-WT. In the case of HN-ST, there are *two* characteristic scale lengths, viz., the system size L and the dissipation scale $\ell_d = (\mu^3/\varepsilon)^{1/4}$, and the turbulence is assumed to be forced only at long wavelengths ($kL \sim 1$). Fluid eddies are the basic excitations, and different scales (i.e., different eddy sizes) interact only via cascading i.e., local nonlinear mode-coupling in k space. In contrast, DD-WT has *three* basic scales: the density gradient scale length L_n, the gyroradius ρ_s, and the dissipation scale $\Delta_{r_d} = Rq\Delta\eta_c(D/D_{\|})^{1/2}$, where the notation is that of Ref. 1. Turbulence is driven at all wavenumbers k present in the fluctuation spectrum. The basic excitations of the system include density 'blobs', which are analogous to fluid eddies, as well as collective resonances, i.e., drift waves with $\omega = \omega_{*e}/(1+k_\perp^2\rho_s^2)$. In DD-WT, the intermediate scale ρ_s characterizes both density blobs and the wave spectrum. Finally, in DD-WT different scales can interact by the mechanisms of wave scattering and wave-blob scattering, as well as by mode-coupling, i.e., cascading.

Renormalized theories of both HN-ST and DD-WT ultimately arrive at an equation of the form

$$\left(\frac{\partial}{\partial t} + T(1,2) + \nu\right) C(1,2) = S(1,2), \tag{1}$$

where $C(1,2)$ is the relevant correlation function, for the velocity $\langle \hat{\mathrm{v}}^2 \rangle$ or density $\langle \hat{n}^2 \rangle$, respectively, $T(1,2) = T_{1,1} + T_{1,2}$ is the two-point evolution operator, ν is the dissipation, and $S(1,2)$ is the source function, representing driving. In both cases, $T(1,2) \to 0$ as $r_{1,2} \to 0$, where $r_{1,2}$ is the distance between points 1 and 2. Similarly, $T(1,2) \to T_{1,1}$ when $r_{1,2} > k_0^{-1}$, where k_0 is the average wavenumber of the turbulence.

For the case of HN-ST, $S(1,2) = 0$, except when $kL \sim 1$. Hence, the inertial range spectrum is determined by

$$T_{1,1} + T_{1,2} = 0, \tag{2}$$

which requires a delicate balance of $T_{1,1}$, the one-point operator which corresponds to coherent mode coupling, and $T_{1,2}$, the cross operator which corresponds to incoherent mode coupling. Such a balance naturally yields a self-similar solution in which small scale correlation is excited only by cascade from larger scales.

In contrast, DD-WT is driven over a broad range of scales, so that

$$S(1,2) = S_{1,2}[C(1,2), L_n] \tag{3}$$

i.e., the source is determined by the gradient scale length and the relevant spectral intensity. It is worth noting that Krommes' assertion that small scale correlation is parasitic does apply in the case of HN-ST where S=0. Here, however, density correlations are directly driven by free energy stored in the density gradient. This and the fact that possible interaction mechanisms include wave-blob scattering as well as cascading, indicate that small scales cannot be considered parasitic in DD-WT. Time stationarity implies that

$$C(1,2) = [T(1,2) + \nu]^{-1} S_{1,2}[C(1,2), L_n)]. \tag{4}$$

The clump theory *approximates* $[T(1,2)+\nu]^{-1}$ by a 'clump-lifetime' $\tau_{\rm cl}(r_{1,2})$, the expression for which is consistent with the $r_{1,2} \to 0$ and $r_{1,2} > k_0^{-1}$ limiting case requirements. Furthermore, the density gradient defines a coherent response $n^c(1) = R(1)\hat{\phi}(1)$, where $\hat{\phi}(1)$ is a field variable, and a dielectric medium response function ϵ. In view of the $r_{1,2} \to 0$ singularity of $[T(1,2)]^{-1}$, the fluctuation $\hat{n}$ may be written as $\hat{n} = n^c + \tilde{n}$, where $\epsilon(1)\hat{\phi}(1) \sim \tilde{n}$, the incoherent fluctuation. It then follows that

$$\tilde{C}(1,2) \approx (\tau_{\rm cl}(r_{1,2}) - \tau_c) S\left[\tilde{C}(1,2)/|\epsilon|^2, L_n\right]. \tag{5}$$

Hence, the fluctuation spectrum is determined by the processes of wave-blob scattering, i.e., shielding $\sim |\epsilon|^{-2}$ and wave-wave scattering, accounted for in ϵ, as well as mode coupling, which determines $\tau_{\rm cl}$. It is interesting to note that the dynamics of passive scalar convection,[4] an example frequently invoked by Krommes, can be described using mode-coupling only. Hence, it is not all surprising to find "contradictions" between the passive scalar convection theory and our model of DD-WT.

It seems to us that the form of Eq. (5), referred to as the "spectrum balance equation", is generic and, indeed, inevitable. The real issue broached by Krommes centers on the extent to which $[T(1,2)+\nu]^{-1}$ is accurately represented by the approximation $(\tau_{\rm cl} - \tau_c)$ in Eq. (5). The evolution operator $T(1,2)$ is, in reality, an extremely complicated nonlinear operator which is intractable from any practical point of view. One is thus powerfully motivated to make approximations! An obvious approximation is simply to replace $T(1,2)$ with a relative diffusion operator. Taking $\nu \to 0$ and expanding around $r_{1,2} = 0$, the leading term is proportional to $r_{1,2}^2$ and leads to the exponential "orbit diffusion" of the clump theory. In spite of the crudeness of this approximation, we have found that in the case of 3D resistivity-gradient-driven i.e., rippling mode turbulence[5] – a relevant case-study where numerical simulations were available for comparison – the theory successfully predicted the spectral intensity shape, the correlation lengths and the rms fluctuation amplitude with great accuracy.[6] The success of the theory in addressing this detailed problem (and also in treating the electron-ion plasma studied by Dupree, et al.[7]), as well as the relatively simple and intuitive physical picture it provides, establish the clump theory as a powerful and practical tool.

The preceding discussion has summarized our differences with the views of Krommes. We will now briefly reply to specific points he has raised which our general discussion may not have adequately addressed.

(i) Paragraph 1: The "well-established facts about the small scale behavior of turbulent fluids" used by Krommes for comparison are drawn from the inappropriate analogy of HN-ST. Since this model differs strongly with DD-WT, agreement is not to be expected.

(ii) Paragraph 2: Krommes restricts his discussions to the high Reynolds number regime but infers that the low Reynolds number regime has "related difficulties." Even if his assertions regarding high Reynolds number turbulence are assumed to have merit, they cannot possibly apply to the low Reynolds number case. This is because in the low Reynolds number limit, the evolution operator is inverted without any assumptions concerning small scale size or any expansion about small scales, i.e., the approximations leading to the spectrum balance equation in the small Reynolds number limit hold for all scales.

(iii) Paragraph 4: While calculations of "macroscopic quantities" such as the diffusivities are indeed rather insensitive to the physics of exponential orbit divergence, the wavenumber and frequency spectra most certainly are not. Moreover, the spectra, which provide one of the best opportunities for detailed comparison with laboratory and simulation measurements, cannot be obtained through dimensional analysis and scaling theory.

(iv) Paragraph 7: It is incorrect to claim that the spectral density equation is valid only at asymptotically small wavelengths, but dominated by large wavelengths. As pointed out, in DD-WT the intermediate scale ρ_s characterizes both the energy containing large scales ($k_\theta \rho_s \sim 1/3$) and the scales of density blobs. Thus the approximations are not inconsistent with the scales that dominate the balance.

(v) Paragraph 9: The analogy to discrete particles is misleading. A better one is to associate k_D with Δ_{r_d}, not k_0 ($k_0 \rho_s \sim 1/3$). Also, exponential divergence is a result of the competition between correlated ($D_{1,2}$) and uncorrelated ($D_{1,1}$) diffusion. Hence, analogies between uncorrelated particles and exponential divergence are incorrect.

In conclusion, we offer a few comments on the approach advocated by Krommes in his paragraph 12. The ultimate test of any theory is how it fares in detailed comparison with either numerical or laboratory experiments. The clump theory is clearly approximate. Nevertheless, it has yielded predictions of spectra, correlation lengths, diffusivities, and fluctuation levels; the accuracy of which has been established by comparison with pertinent numerical and laboratory data. Indeed, this remarkable agreement is what originally motivated us to write our paper on DD-WT. We doubt that a better answer would be obtained if the complete TFM equations were used. Such equations would require numerical solution (a ludicrous thought, since the *exact* equations could be so solved) and would be relatively devoid of physical insight. Therefore, we feel that Krommes must forsake generalities and shoulder the burden of proof. He should actually implement the solution of the TFM equations for a relevant problem, compare them with the results from simulations and with those of the clump theory, and pinpoint and explain any discrepancies. We anxiously await the outcome of this undertaking.

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1. P.W. Terry and P.H. Diamond, Phys. Fluids **28**, 1419 (1985).
2. T.H. Dupree, Phys. Fluids **15**, 334 (1972).
3. T.H. Dupree, Phys. Fluids **25**, 277 (1982).
4. G.K. Batchelor, J. Fluid Mech. **5**, 113 (1959).
5. L. Garcia, P.H. Diamond, B.A. Carreras, and J.D. Callen, Phys. Fluids **28**, 2147 (1985).
6. P.W. Terry, P.H. Diamond, K.C. Shaing, L. Garcia, and B.A. Carreras, Institute for Fusion Studies Report no. #174 (1986); submitted to Phys. Fluids.
7. R.H. Berman, D.J. Tetreault and T.H. Dupree, Phys. Rev. Lett. **48**, 1249 (1982).