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MUON CATALYZED FUSION — FISSION REACTOR DRIVEN BY A RECIRCULATING BEAM

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Abstract

The recent experimentally inferred value of multiplicity of fusion of deuterium and tritium catalyzed by muons has rekindled interest in its application to reactors. Since the main energy expended is in pion (and consequent muon) productions, we try to minimize the pion loss by magnetically confining pions where they are created. Although it appears at this moment not possible to achieve energy gain by pure fusion, it is possible to gain energy by combining catalyzed fusion with fission blankets. We present two new ideas that improve the muon fusion reactor concept. The first idea is to combine the target, the converter of pions into muons, and the synthesizer into one (the synergetic concept). This is accomplished by injecting a tritium or deuterium beam of 1 GeV/nucleon into DT fuel contained in a magnetic mirror. The confined pions slow down and decay into muons, which are confined in the fuel causing little muon loss. The necessary quantity of tritium to keep the reactor viable has been derived. The second idea is that the beam passing through the target is collected for reuse and recirculated, while the strongly interacted portion of the beam is directed to electronuclear blankets. The present concepts are based on known technologies and on known physical processes and data.

I. Introduction

The idea that negative muons might be able to catalyze proton-deuterium (p-d) fusion was first considered by Frank[1] in 1947 and it has been seen experimentally in 1956 by Alvarez et al.[2] Deuterium-tritium (d-t) fusion catalyzed by muons was discussed in 1948 by Sakharov and Zeldovich.[3] In 1957 Jackson[4] reconsidered these possibilities and concluded that although the d-t interaction has the highest probability, it is impossible to get energy gain by using muon catalyzed reactions. Although the theoretical and experimental picture has been significantly modified[5,6] since 1957, Jackson's conclusion about energy gain using "pure" fusion catalyzed muons still appears to be correct in 1985. However, as Petrov suggested[7–9] one can use muon catalyzed d-t fusion in a hybrid reactor with a fissile nuclide blanket (e.g., ^{238}U) in order to obtain a positive energy and further to breed a thermal fissile isotope (^{233}U, ^{239}Pu) gain. In this paper we propose an embodiment of such a scheme and discuss its advantages and difficulties.

Muon absorption in matter and the induced fusion of deuterium and tritium is a remarkable phenomenon since the negative muon is capable during its lifetime ($\tau_\mu = 2.2\times10^{-6}$ sec) of inducing[6] about 100 nuclear dt fusion reactions in a liquid density medium. This end result of nuclear fusion occurs after a chain of atomic and molecular processes.[10] The most crucial step in the physics of muon catalysis cycle is the resonant formation,[11,12] of the $dt\mu$ molecule which increases the probability of the end result by at least two orders of magnitude (relative to nonresonant $dt\mu$ formation). However, this high production rate of the $dt\mu$ molecule is disturbed by muons lost in the catalysis cycle. There seems to be a probability of about 1% that the helium ion created during the nuclear fusion interaction will capture the negative muon.[13–15] However, recent experiments at Los Alamos National Laboratories[16] suggest that the "sticking" problem is not yet completely understood. The present knowledge of the muon catalysis cycle for *dt* nuclear fusion is summarized in the first part (Sec. A) of Chapter II.

The "energy cost" for the creation of a muon is one of the most important practical parameters in analyzing the relevance of muon catalyzed fusion for energy production. The muons are produced during the decay of pions which can be created in nucleon-nucleon collisions. The energy threshold for pion generation is about 500 MeV of projectile kinetic energy and the process of negative muon generation seems to be most effective for nucleonic projectiles with a kinetic energy of 1 GeV per nucleon. An optimistic estimate[7] requires 5 GeV of energy to produce one negative muon. Therefore if one muon catalyzes about one hundred *dt* fusions the energy output is less than 2 GeV (one *dt* fusion gives 17.6 MeV) per muon, so that no energy gain seems to be possible from "pure" fusion nuclear reactions. Due to the crucial role of the negative pions and their energy cost, the second part (Sec. B) of Chapter II analyzes the nuclear physics and the energy balance for the relevant pion production.

Since pure fusion devices using muon catalyzed fusion seem to lose rather than to gain energy, Petrov[8] suggested using muon catalysis in a hybrid fusion-fission reactor. The first, and to our knowledge the only reactor concept so far was given by Petrov et al.[8,9] This reactor scheme includes <u>an accelerator</u> (of tritium or deuterium), <u>a target</u> (of tritium or beryllium) where the pions are created, <u>a converter</u> where the pions are confined (in vacuum) by strong magnetic fields, <u>a synthesizer</u> with the *d-t* fuel and <u>a blanket</u> where the fission and fissile materials are produced. The converter is a cylinder about 40 m long and 20 cm radius ($\approx 5\ m^3$ volume) having a longitudinal magnetic field of 11 to 16 Tesla and an applied d.c. electric field of 7.5×10^5 volt/m along the converter. Inside this vessel there is a cylinder 2 m long with a 2 cm radius target where the pions are created. The synthesizer is a second cylinder, about 20 m long with an average radius of 20 cm surrounded by a (longitudinal) magnetic field coil of 11 Tesla. The density of the *dt* fuel is 0.5 liquid hydrogen density with 30% tritium, so that the synthesizer has 80 kg of tritium. The main result of the converter-synthesizer is the conversion of about 75% of the created pions into muons that participate in the catalyzed *dt* fusion. This means that the energy cost of a stopped negative muon in the *dt* mixture is about 6 GeV (using 4.5 GeV to create one negative pion) for tritium projectiles and 8 GeV for a deuterium projectile beam. These results seem to be optimistic in this model reactor, mainly because pion and muon drifts to the wall due to collisions were neglected. Also, the influence of the magnetic field on the scattered proton beam was not considered. In fact, the proton Larmor radius is of the same order of magnitude as that of pions, therefore a significant portion of the proton beam is lost in the vessel in this concept reactor.[8,9] The fissions and the fissile material are produced in the blanket not only by the neutrons derived from *d-t* fusion but also from the fast nucleons of the incident beam which have about 80% of their initial energy after the creation of the pions. Taking into account the losses of the projectiles in the converter-synthesizer vessels due to the magnetic field will reduce the energy and the fissile material obtained from direct beam-blanket collisions. Moreover, by taking into account the diffusion due to collisions, the necessary quantity of tritium might increase significantly.

Following Petrov's concept of a muon catalyzed fusion-fission reactor, we suggest in Chapter III a reactor concept based on two new main ideas: (a) The high energy beam of tritium or deuterium ($\sim$ 1 GeV/nucleon) is injected into a target of tritium (or beryllium) with dimensions smaller than the mean free path for strong interactions. After passage through the target, the bulk of the beam is collected for reuse and only the small portion of the beam which suffered strong interactions is directed into an electronuclear blanket. (b) The pions created in the target are surrounded by the fuel of deuterium-tritium and are magnetically confined until they slow down and decay into muons which catalyze the fusion in situ. In this scheme the converter and synthesizer are combined into one vessel. The *d-t* fuel is the target and the produced pions are trapped in the fuel which slows down the pions, so that the necessary conditions for trapping the pions are sufficient conditions for stopping the muons. These muons cause the catalyzed chain reactions leading to nuclear *dt* fusion. In this way we have a very efficient trapping of muons (almost 100%) in a relatively small physical volume. We suggest a magnetic mirror configuration[17] in order to confine the pions and muons. The fuel is driven by a tritium (or deuterium) beam which is retrieved in part after passing through the target. The scattered portion of the beam hits the electronuclear blanket and the fusion created neutrons interact with the fissile blanket surrounding the fusion fuel. The fusion generated alpha particles have much smaller Larmor radii than the pions and therefore they are totally confined in the fuel (*dt*) vessel and heat the fuel to $\sim 500°$K where the *dt* molecular formation is most probable. Since the fuel is in the compressed gas state we recirculate the deuterium tritium fuel and cool it effectively with the help of coolers outside the fusion vessel.

The scheme of energy and fissile material production is described in Chapter IV. For tritium-tritium collisions we use about 20% of the beam energy for muon catalyzed fusion while the remaining 80% is used for the electronuclear direct beam-blanket interaction. It turns out that the fusion part (20% of the energy beam) contributes more effectively than the electronuclear interactions. Chapter V concludes this paper with a short summary and a comparison with other schemes.

II. μ^- Production and the Physics of Muon Catalyzed Fusion

A. *Muon Catalyzed Fusion*

The atomic, molecular and nuclear phenomena associated with processes occurring when a negative muon (μ^-) is absorbed in matter have been extensively investigated.[5] The electroneutrality and small dimensions of muon atoms makes the mesohydrogen atoms $\mu^- p(p =$ proton), $\mu^- d(d =$ deuterium), $\mu^- t(t =$ tritium) behave like neutrons which can penetrate freely through the electron shells of the atoms. When a mesohydrogen atom approaches a deuterium or a tritium molecule a mesomolecule can be created. During this process the mesic atom is attached to another hydrogen nucleus to form one of the centers of the electronic molecule. At this center of the molecule the *dd* or *dt* are kept close together by the negative muon and spontaneous fusion reactions occur in a very short time. The freed muon is then available to repeat the cycle. The size of the mesic atoms are of the order of the muon Bohr radius a_μ given by

$$a_\mu = a_0 m_e / m_\mu \simeq 2.6 \times 10^{-11} \mathrm{cm} \tag{2.1}$$

where a_0 is the Bohr radius for the electron atoms and $m_\mu / m_e \simeq 207$ is the ratio of the muon and electron masses. If the lifetime of the muon ($\tau_\mu \simeq 2.2 \times 10^{-6}$ sec) is long compared to the other processes, then many fusion reactions can occur during the lifetime of a muon so that in this case the muons have the role of a catalyzer.

When a negative muon enters a dense deuterium (D) - tritium (T) target it starts a chain of reactions: (we denote by capital letters D and T the deuterium and tritium atoms while the small letter denotes the nuclei).

(a) Stopping and capture of μ^- by d or t and a cascade to the ground state of the hydrogen-like atom,

$$\mu^- + D \xrightarrow{\lambda_a} (d\mu) + e^- \tag{2.2}$$

$$\mu^- + T \xrightarrow{\lambda_a} (t\mu) + e^- \tag{2.3}$$

where e^- denotes an electron and λ_a is the rate of the processes (2.2) and (2.3). In general, the rate $\lambda(\sec^{-1})$ is given by

$$\lambda = n\sigma v \left[\sec^{-1}\right] \tag{2.4}$$

where n $\left[\mathrm{cm}^{-3}\right]$ is the density of the target, σ $\left[\mathrm{cm}^2\right]$ is the cross-section describing the process under consideration and $v[\mathrm{cm}/\sec]$ is the velocity of the projectile. The rate of the muonic atomic formation[18] in Eqs. (2.2) and (2.3) for liquid hydrogen density, $n = 4.25 \times 10^{22}\mathrm{cm}^{-3}$, is $4 \times 10^{12} \sec^{-1}$.

(b) μ^- transfer from deuterium to tritium

$$(d\mu) + t \xrightarrow{\lambda_{dt}} (t\mu) + d. \tag{2.5}$$

The rate for this process at liquid density is estimated[5b,6] to be $2 \times 10^8 \sec^{-1}$. The 48 eV difference in the binding energies of $(d\mu)$ and $(t\mu)$ causes the transfer of μ in an irreversible process described by Eq. (2.5).

(c) At this stage, a $(dt\mu)$ molecular ion may be formed at the center of an "H_2 type" molecule. The formation of the relevant hydrogen type mesomolecules are described by

$$(d\mu) + D_2 \xrightarrow{\lambda_{dd\mu}} [((dd\mu)d2e] \tag{2.6}$$

$$(t\mu) + D_2 \xrightarrow{\lambda_{dt\mu-d}} [(dt\mu)d2e] \tag{2.7}$$

$$(t\mu) + DT \xrightarrow{\lambda_{dt\mu-t}} [(dt\mu)t2e] \tag{2.8}$$

$$(t\mu) + T_2 \xrightarrow{\lambda_{tt\mu}} [(tt\mu)t2e] \tag{2.9}$$

where in general the mesomolecule is in an excited state. The values of the rates describing Eqs. (2.6)-(2.9) were measured[6,16] to be $\lambda_{dd\mu} \simeq 3 \times 10^6\,\mathrm{sec}^{-1}$, $\lambda_{tt\mu} \simeq 3 \times 10^6\,\mathrm{sec}^{-1}$ and $\lambda_{dt\mu} \gtrsim 10^8\,\mathrm{sec}^{-1}$, where $\lambda_{dt\mu}$ is defined by

$$\lambda_{dt\mu} = \lambda_{dt\mu-d}c_d + \lambda_{dt\mu-t}c_t. \tag{2.10}$$

c_d and c_t are the concentrations of the deuterium and tritium nuclei, so that if the presence of He and other impurities is neglected, one has

$$c_d + c_t = 1. \tag{2.11}$$

The value of the $dt\mu$ rate is almost two orders of magnitude larger than the $dd\mu$ and $tt\mu$ rates. This phenomena can be explained by the resonant formation of these molecules.[11,12] A degeneracy in the excited state of the $dt\mu$ ion and the excited state of the electron molecular complex causes a strong resonance effect. The rate $\lambda_{dt\mu}$ is found to be dependent on the temperature[6] since the kinetic energy of the $t\mu$ atom, which forms the $dt\mu$ ion, is temperature dependent. This resonance condition can be described by energy conservation

$$\varepsilon_e = \varepsilon_T + \varepsilon_\mu \tag{2.12}$$

where $\varepsilon_\mu \simeq 0.7\ eV$ is the energy of $dt\mu$ ion in the quantum state $J = 1$, $\nu = 1$ (J is the rotational quantum number and ν is the vibrational quantum number of the dt nuclei forming the ion $(dt\mu)^+$), ε_e is the electron energy in the H_2-type molecule and $\varepsilon_T = \frac{3}{2}kT$ is the thermal energy of the $t\mu$ atom at a temperature T (for $T = 500°$ one has $\varepsilon_T \simeq 0.04$ eV). The energy released $(\varepsilon_T + \varepsilon_\mu)$ during the formation of the $J = 1, \nu = 1$ $dt\mu$ ion is absorbed resonantly by the electronic excited states of the mesomolecule $[(dt\mu)d2e]$. Since the thermal and the electronic molecular excitation have broad energy distributions, the resonance condition can be satisfied over a large range of temperatures. This effect has been observed experimentally.[6]

(d) Next, in the chain of reactions, one has the de-excitation of $dt\mu$, $dd\mu$ and $tt\mu$ ions to their ground states and the occurrence of the nuclear fusion reactions

$$d + t \xrightarrow{\lambda_f} {}^4\mathrm{He} + n + 17.6\ \mathrm{MeV} \tag{2.13}$$

$$d + d \xrightarrow{\frac{1}{2}\lambda_{fd}} {}^3\mathrm{He} + n + 3.3\ \mathrm{MeV} \tag{2.14a}$$

$$d + d \xrightarrow{\frac{1}{2}\lambda_{fd}} t + p + 4\ \mathrm{MeV} \tag{2.14b}$$

$$t + t \xrightarrow{\lambda_{ft}} {}^4\mathrm{He} + 2n + 10\ \mathrm{MeV}. \tag{2.15}$$

The fusion rates are estimated[19] to be $\lambda_f \simeq 10^{12}\,\mathrm{sec}^{-1}$ for the dt nuclear reaction while $\lambda_{fd} \sim \lambda_{ft} \simeq 10^{11}\,\mathrm{sec}^{-1}$.

(e) During the fusion reaction there is a possibility that the muon sticks to a charged product. In particular, in reaction (2.13) the muon can stick to the ^{4}He by forming a munonic helium ion ($^4\mathrm{He}\mu$). In this case, if the muon stays bound to the He particle it is lost for the chain of reactions described ((a) to (d)). The sticking probabilities for the reactions given by Eqs. (2.13)-(2.15) were estimated to be[13,16] $\omega_s \simeq 0.9 \times 10^{-2}$, $\omega_d \simeq 0.13$ (for Eq. (2.14a)), $\omega_{d'} \simeq 0.003$ (for Eq. (2.14b)) and $\omega_t \simeq 0.05$. For the important dt case, the value of ω_s implies that the muon sticks to the He particle and is lost after catalyzing $1/\omega_s \simeq 110$ fusions, no matter how fast the other processes leading to the mesomoleculer formation and fusion are. The sticking process may be the bottleneck of the muon catalyzed fusion cycle.

A set of rate equations can be written to describe the kinetics[10] of the chain reactions described above from (a) to (e). This chain of reactions is described schematically in Fig. 1. The rates for these reactions and the sticking probabilities are summarized in Table 1 for liquid density of the hydrogen isotopes ($n = 4.25 \times 10^{22}\mathrm{cm}^{-3}$). The solution of the rate equations results in an expression for the fusion neutron yield — X_μ, namely the average number of dt fusions catalyzed by one muon. For high density mixtures of d and t and neglecting small effects, one obtains that the value of X_μ^{-1} is given by the sum of probabilities of muon decay during a catalysis cycle and the muon capture by ^{3}He or ^{4}He (sticking)

$$X_\mu^{-1} = \frac{\lambda_\mu}{\lambda_c} + W \tag{2.16}$$

where $\lambda_\mu = 1/\tau_\mu = 0.45 \times 10^6\,\mathrm{sec}^{-1}$, λ_c is the muon cycling rate, estimated to be,

$$\frac{1}{\lambda_c} = \frac{c_d}{\lambda_{dt} c_t} + \frac{1}{\lambda_{dt\mu} c_d} \tag{2.17}$$

and W is the probability of muon capture by He, and can be approximated for the chain of reactions given in Fig. 1 by

$$W = \omega_s + \frac{0.5\lambda_{dd\mu}\omega_d c_d}{\lambda_{dd\mu} c_d + \lambda_{dt} c_t} + \frac{\lambda_{tt\mu}\omega_t c_t}{\lambda_{dt\mu} c_d}. \tag{2.18}$$

The rate equations described by Fig. 1 are more complex if one takes into account the existence of ^{3}He and ^{4}He concentrations in the deuterium-tritium target. In this case Eq. (2.11) is replaced by

$$c_d + c_t + c_{\mathrm{He}} = 1 \tag{2.19}$$

where $c_{\mathrm{He}} = c_{e_{\mathrm{He}}} + c_{4_{\mathrm{He}}}$. The role of the He-sink and its complexity is described schematically in Fig. 2.

We end this section by citing the preliminary experimental observation[20] of 170 fusions per muon (i.e., $X_\mu = 170$) at $T = 800^\circ K$ and densities less than liquid hydrogen density. These recent results imply that some of the theoretical estimates, such as the sticking probability ω_s, are inconsistent with the recent experiments.[16] For example, from Eqs. (2.16)-(2.18) one gets

$$X_\mu \leq \frac{1}{\omega_s} \approx 110 \tag{2.20}$$

in contradiction with $X_{\mathrm{exp}} \simeq 170$. At this stage one can conclude only that better theories and more experimental evidence are necessary to complete our understanding of the physics of muon catalyzed fusion.

B. The Production of μ^-

The μ^- particles are produced by the decay of the negative pion, $\pi^- \to \mu^- + \bar{\nu}_\mu$. The π^- mesons are generated by using proton, deuterium or tritium accelerators. For kinetic energies per nucleon T_0 smaller than 0.7 GeV/nucleon, the probability of inelastic nucleon-nucleon collision, $V_{ab}(T_0)$, ($a, b = p$ (proton) or n (neutron)) is very small so that a π^- cannot effectively be produced. V_{ab} is usually defined by the ratio of inelastic cross-section, σ_{ab}^{in}, to the total cross-section, $\sigma_{ab}^{\mathrm{tot}}$, for strong interactions ($\sigma_{ab}^{\mathrm{tot}} = \sigma_{ab}^{\mathrm{in}} + \sigma_{ab}^{\mathrm{el}}$ where σ^{el} is the strong elastic cross-section)

$$V_{ab}(T_0) = \sigma_{ab}^{\mathrm{in}}(T_0)/\sigma_{ab}^{\mathrm{tot}}(T_0), \quad \left.\begin{matrix} a \\ b \end{matrix}\right\} = p, n. \tag{2.21}$$

We do not consider initial energies larger than 2 GeV per nucleon since in this case the number of undesirable particles (e.g., neutral pions) that are created increases. Therefore, in the energy domain under consideration, $0.7 \le T_0 \le 2$ GeV, the negative pions are mainly created in the following processes,

$$p + p \to p + p + \pi^+ + \pi^- \tag{2.22}$$

$$p + n \to p + p + \pi^- \tag{2.23}$$

$$n + n \to n + p + \pi^- \tag{2.24}$$

with cross-sections $\sigma_{pp}^{\pi^-}$ (Eq. (2.22)), $\sigma_{pn}^{\pi^-}$ (Eq. (2.23)) and $\sigma_{nn}^{\pi^-}$ (Eq. 2.24)). The probabilities W_{ab} of producing a π^- once the inelastic collision occurs are given by

$$W_{ab}(T_0) = \sigma_{ab}^{\pi^-}(T_0)/\sigma_{ab}^{\mathrm{in}}(T_0), \quad \left.\begin{matrix} a \\ b \end{matrix}\right\} = p, n. \tag{2.25}$$

In this work we are interested in collisions of protons, or deuteriums (d) or tritium (t) with light nuclei targets such as d, t, Be, etc. In particular we estimate the multiplicity $y^{\pi}(x)$ of the negative pions ($(y^{\pi})^{-1}$ is equal to the number of nucleonic projectiles necessary to create one pion) for the case where the projectiles are passing through x cm of longitudinal target dimensions. For x smaller than the mean free path for strong interactions, $x << \ell_s$, we can calculate $y^{\pi}(x)$ by using the approximation

$$y^{\pi}_{AB}(x) = Y^{\pi}_{AB}\frac{x}{\ell_{s,AB}} \quad \left.\begin{matrix} A \\ B \end{matrix}\right\} = p, d, t, \text{etc.} \tag{2.26}$$

where Y^{π} is the appropriate multiplicity for projectiles passing through a target with infinite dimensions, $y^{\pi}(x \to \infty) = Y^{\pi}$. The multiplicities Y^{π}_{AB} were calculated by Petrov and Shabelskii[7] to be*

$$Y^{\pi}_{AB} = C^{AB}_{pp}V_{pp}W_{pp} + C^{AB}_{pn}V_{pn}W_{pn} + C^{AB}_{nn}V_{nn}W_{nn} \tag{2.27}$$

where V_{ab} and W_{ab} are defined in Eqs. (2.21) and (2.25) and C^{AB}_{ab} are given by

$$C^{AB}_{pp} = Z_A Z_B \sigma^{\text{tot}}_{pp}/\sigma^{\text{tot}}_{AB} \tag{2.28a}$$

$$C^{AB}_{pn} = [Z_A(A_B - Z_B) + Z_B(A_A - Z_A)]\,\sigma^{\text{tot}}_{pn}/\sigma^{\text{tot}}_{AB} \tag{2.28b}$$

$$C^{AB}_{nn} = (A_A - Z_A)(A_B - Z_B)\sigma^{\text{tot}}_{nn}/\sigma^{\text{tot}}_{AB} \tag{2.28c}$$

$$\begin{aligned}\sigma^{\text{tot}}_{AB} &= Z_A Z_B \sigma^{\text{tot}}_{pp} + (A_A - Z_A)(A_B - Z_B)\sigma^{\text{tot}}_{nn} \\ &\quad + [Z_A(A_B - Z_B) + Z_B(A_A - Z_A)]\,\sigma^{\text{tot}}_{pn}\end{aligned} \tag{2.29}$$

where C^{AB} give the fraction of pp, pn and nn collisions for a projectile with A_A nucleons and Z_A protons and a target with A_B nucleons and Z_B protons. The following normalization is satisfied for Eqs. (2.28)-(2.29)

$$\sum_{a,b} C^{AB}_{ab} = 1 \quad \left.\begin{matrix} a \\ b \end{matrix}\right\} = p, n \quad \left.\begin{matrix} A \\ B \end{matrix}\right\} = p, d, t, Be, \text{etc.}. \tag{2.30}$$

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* The yields Y_{π} in Petrov-Shabelskii's paper are equal to Y^{π}/T_0 in our paper, where T_0 is the kinetic energy of the projectile.

In deriving Eq. (2.27) it is assumed that the interaction between the projectile and the target is described by free nucleon-nucleon collisions, neglecting multiple scattering and shadow corrections.

Using Eqs. (2.27)-(2.29), the definitions of V_{ab} and W_{ab} in (2.21) and (2.25), and the values of mean free path $\ell_{S,AB}$

$$\ell_{S,AB}^{-1} = n\sigma_{AB}^{\rm tot} \equiv \phi n_0 \sigma_{AB}^{\rm tot}, \tag{2.31}$$

where $\sigma_{AB}^{\rm tot}$ is given in Eq: (2.29), (n_0 is the density of liquid hydrogen density (ℓ.h.d.) defined to be $n_0 = 4.25 \times 10^{22}$ atoms/cm^3 so that ϕ is the density in ℓ.h.d. units, $\phi = n/n_0$) one gets for $y_{AB}^{\pi}(x)$ in Eq. (2.26) the following expression

$$\begin{aligned} y_{AB}^{\pi}(x) = n_0\phi x\Big\{ & Z_A Z_B \sigma_{pp}^{\pi^-} + [Z_A(A_B - Z_B) + Z_B(A_A - Z_A)]\,\sigma_{pn}^{\pi^-} \\ & + (A_A - Z_A)(A_B - Z_B)\sigma_{nn}^{\pi^-}\Big\}. \end{aligned} \tag{2.32}$$

$\left(y_{AB}^{\pi}(x)\right)^{-1}$ is the number of nucleonic projectiles necessary to create one pion while the projectile A passes through x cm of target B, and $(y_{AB}^{\pi})^{-1}T_0$ is the energy yielded for this type of π^- creation. Using $\sigma_{pn}^{\rm tot} \simeq 0.8\sigma_{pp}^{\rm tot}$ (which is satisfied for our range of energies) and $\sigma_{pp}^{\rm tot} = \sigma_{nn}^{\rm tot}$, the values of C_{ab}^{AB}, $\sigma_{AB}^{\rm tot}$ (Eq. (2.29)) and $\ell_{S,AB}$ (Eq. (2.31)) are calculated in Table 2 for the hydrogen isotope collisions. The cross-section is estimated as $\sigma_{pp}^{\rm tot}(1\text{ GeV}) \simeq 45$ mb(1 mb $= 10^{-27}$ cm^2) and the mean free path for strong interaction is calculated for a liquid hydrogen density target defined by the density $n = n_0 = 4.25 \times 10^{22}$ atoms/cm^3 (i.e., $\phi = 1$). Moreover, following Petrov and Shabelskii,[7] we assume

$$\begin{aligned} & V_{pp} \simeq V_{pn} \simeq V_{nn} \simeq 0.5 \\ T_0 = 1\text{ GeV}: \quad & W_{pp} \simeq 0 \;,\quad W_{pn} \simeq 0.23 \;,\quad W_{nn} \simeq 0.80 \\ T_0 = 1.5\text{ GeV}: \quad & W_{pp} \simeq 0.05 \;,\quad W_{pn} \simeq 0.4 \;,\quad W_{nn} \simeq 0.80 \end{aligned} \tag{2.33}$$

in order to calculate (see Table 3) Y^{π}_{AB} (Eq. (2.27)), y^{π}_{AB}/x (Eq. (2.26) or Eq. (2.32)) and $(y^{\pi}_{AB}/x)T_0$. For dd and tt collisions the isotopic spin invariance relations are used, namely $\sum_{a,b} C^{AB}_{ab} W_{ab}$ equals 1/3 for the dd collision and 0.5 for the tt collision. One can see for example from Table 3, that in tritium-tritium collisions along 64 cm, one π^- is created by four nucleons having a total kinetic energy of 4 GeV.

In the energy balance it is necessary to count the number of elastic strong collisions and the number of inelastic strong collisions during the creation of one negative pion. Defining the yields for elastic collisions along a mean free path by y^{el}_{AB} and the appropriate yield for inelastic collisions by y^{in}_{AB}, one gets

$$y^{\mathrm{el}}_{AB} = C^{AB}_{pp}(1 - V_{pp}) + C^{AB}_{pn}(1 - V_{pn}) + C^{AB}_{nn}(1 - V_{nn}) \tag{2.34}$$

$$y^{\mathrm{in}}_{AB} = C^{AB}_{pp} V_{pp} + C^{AB}_{pn} V_{pn} + C^{AB}_{nn} V_{nn}. \tag{2.35}$$

Using the approximation of Eq. (2.33) and the normalization condition Eq. (2.30) we estimate

$$y^{\mathrm{el}}_{AB} \simeq 0.5 \quad , \quad y^{\mathrm{in}}_{AB} \simeq 0.5. \tag{2.36}$$

The average energy loss in elastic collisions, $\varepsilon_{\mathrm{el}}$, is about 100 MeV, while it is reasonable to assume that the energy loss in an inelastic collision, ε_{in}, is on the average 300 MeV (this includes the energy required to create a negative pion). Using Eq. (2.36) we estimate that the energy loss in strong interactions during the creation of one negative pion along a mean free path is

$$E_{\pi} = \frac{1}{y_{\pi}} \left(\frac{\varepsilon_{\mathrm{el}}}{y_{\mathrm{el}}} + \frac{\varepsilon_{\mathrm{in}}}{y_{\mathrm{in}}} \right) \simeq \frac{1}{2y_{\pi}} (\varepsilon_{\mathrm{el}} + \varepsilon_{\mathrm{in}}) \tag{2.37}$$

where the indices AB were omitted for simplicity and the values of y_{π}^{-1} are given in Table 3, $\varepsilon_{\mathrm{el}} \simeq 100$ MeV and $\varepsilon_{in} \simeq 300$ MeV.

It is important to emphasize at this stage that the calculations related to Eq. (2.37) refer to the nucleons which have only one strong interaction, elastic or inelastic. This situation can be achieved if the projectiles are interacting with targets of small longitudinal dimensions, i.e., target dimension is much smaller than the mean free path for strong interactions. For large target dimensions, the nucleon projectile has multiple collisions in the target. Following the estimate of Eq. (2.36), about 50% of the incident nucleons have an elastic scattering in the first collision. These nucleons can have in their second or third scattering an inelastic collision if the transverse and longitudinal dimensions are large enough. However, since the probability of a strong inelastic interaction (where a π^- can be created) is very small for energies less than 0.6 GeV, all the nucleons with energies below this value are ineffective and their energy is actually lost as far as the creation of negative pions is concerned. In our scheme, suggested in the next section, the π^- mesons are created inside the *d-t* fuel. Therefore, pions created by multiple collisions are not lost into an undesirable surrounding material. The multiplication factor is calculated by adding the probabilities of the first collision being inelastic and the appropriate probabilities that the possible second and third collisions are inelastic. The contributions of interactions of higher multiplicity are small since V_{AB} (Eq. (2.21)) is negligible for $T_0 \lesssim 600$ MeV and a nucleon loses about 100 MeV per elastic collision. The multiplication factors K_{AB} are found[7] by using the energy spectra of secondary nucleons in inelastic NN collisions[24] to be

$$Y_{AB}^{\pi,\mathrm{eff}} = K_{AB}^{-1} Y_{AB}^{\pi}$$

$$K_{tt} \simeq 2 \quad , \quad K_{dt} \simeq K_{dd} \approx 1.3 \tag{2.38}$$

where Y_{AB}^{π} is given by Eq. (2.27). [Note: Some people[7] define the multiplication factor $K_{AB}^{*} = K_{AB} V_{AB}$, where V_{AB} is given by (2.21), so that in this case $K_{tt}^{*} \simeq 1$, $K_{dt}^{*} \simeq K_{dd}^{*} \simeq$

0.67. In Table 3 we give the necessary energy to create a π^- in *d-d*, *d-t* and *t-t* collisions. We will mainly consider the case where the target is small, i.e., single collisions, in order to restrict reactor dimensions.

When a charged particle passes through matter it also makes electromagnetic interactions and its energy is reduced. A projectile more massive than an electron having a velocity $\beta c (\beta = v/c$, v is the particle velocity and c is the light velocity), where $\beta >> 1/137$, dissipates its energy into the medium via interactions with the electrons of the medium. The incident particle loses its kinetic energy by collisions until it is brought to a stop. For calculating the mean rate of this energy loss per unit path length x, we use the Bethe-Bloch formula[21]

$$-\frac{dT}{dx} = \frac{4\pi Z_A^2 e^4 n Z_B}{mc^2\beta^2}\left\{\ell n\left[\frac{2mc^2\beta^2\gamma^2}{I}\right] - \beta^2\right\} \tag{2.39}$$

where T is the kinetic energy of the projectile, which has a mass M and a charge Z_A, and a velocity βc. γ is related to β by

$$\beta = \frac{v}{c} \ ; \quad \gamma \equiv 1 + \frac{T}{Mc^2} \ ; \quad \gamma^2 = 1/(1-\beta^2), \tag{2.40}$$

e and m are the electron charge and mass respectively, Z_B is the charge of the medium atoms, n is the density of the target in atoms/cm^3 and I is a phenomenological constant which characterizes the binding of the electrons of the medium. For hydrogen (H_2 isotopes) at liquid density we take $I = 20$eV. The quantity $(dT/dx)\delta x \equiv \delta\varepsilon_I$ is the mean total energy loss via ionization and other inelastic electromagnetic interactions of the projectile with electrons of the medium in a layer of thickness δx. For a hydrogen isotope, Eq. (2.39) yields

$$\frac{\delta\varepsilon_I}{\delta x} \simeq \begin{cases} 0.66\cdot 10^6 \text{ eV/cm} & \text{for} \quad T = 1 \text{ GeV} \\ 0.62\cdot 10^6 \text{ eV/cm} & \text{for} \quad T = 1.5 \text{ GeV} \end{cases} \tag{2.41}$$

We denote by ε_I the energy loss when the beam passes through an effective layer of thickness ℓ_S, the mean free path for strong interactions ($\ell_S = \sum(\delta x)$). In general the values for the range and energy loss are taken from the "Review of Particle Properties Data".[22] However, for our model reactor driven by an accelerator with recirculating beams, the values of β (and γ) do not change along an effective path of thickness ℓ_S, so that we can approximate ε_I by

$$\varepsilon_I = \left(\frac{\delta\varepsilon_I}{\delta x}\right) \cdot \ell_S \tag{2.42}$$

where $(\delta\varepsilon_I/\delta x)$ is given by Eq. (2.39). The values of (2.42) are equal for proton, deuteriums or tritiums, therefore the energy loss per a nucleon projectile is given by ε_I/A, where $A = 1, 2, 3$ for p, d and t accordingly.

Using Eqs. (2.37) and (2.42), we derive the energy loss E_π during the creation of one negative pion as

$$E_\pi = \frac{1}{y_\pi}\left(\frac{\varepsilon_{\rm el}}{y_{\rm el}} + \frac{\varepsilon_{in}}{y_{in}} + \frac{\varepsilon_I}{A}\right) \simeq \frac{1}{2y_\pi}\left(\varepsilon_{\rm el} + \varepsilon_{in} + \frac{2\varepsilon_I}{A}\right) \tag{2.43}$$

where $A = 1$ for p, 2 for d and 3 for a t target. Numerical estimates for E_π can be obtained by using $\varepsilon_{\rm el} \simeq 100$ MeV, $\varepsilon_{in} \simeq 300$ MeV and $\varepsilon_I \simeq 0.66\ \ell_S$ MeV for projectiles with kinetic energies of $T_0 = 1$ GeV. One finds that the energy losses E_π in *d-d*, *d-t* and *t-t* collisions are smaller than the fusion energy produced by one muon with 100 catalyses (which releases 1.76 GeV of energy). However, one has to remember that the necessary projectile energy to create a pion, E_0, is larger than E_π in (2.43), where E_0 is

$$E_0 = Y_\pi^{-1} T_0 \simeq \begin{cases} 4.5 \text{ GeV} & \text{for d} - \text{d collisions} \\ 3.7 \text{ GeV} & \text{for t} - \text{d collisions} \\ 2.0 \text{ GeV} & \text{for t} - \text{t collisions} \end{cases} \tag{2.44}$$

and $y_\pi \equiv Y_{AB}^{\pi,\rm eff}$ is given in (2.38). The difference between E_0 and E_π represents energy remaining in the projectile after π^- production, which we will use in a spallation electro-nuclear (EN) target.

III. Synergetic Reactor Concept — The Mirror

In a muon-catalyzed fusion reaction the main energy expended is in the production of muons. It is therefore of paramount interest to minimize the necessary energy for muon creation and, once created, to minimize the loss of muons, in order to introduce a reactor concept utilizing muon fusion catalysis. We have to identify the most efficient way to create muons and to optimize the efficiency of an accelerator system. We must devise (an) effective configuration(s) to utilize created muons as efficiently as possible with minimal loss. Following Petrov's concept[8] of a muon catalyzed fusion-fission reactor, one may break down the above required tasks for a reactor concept into (i) an accelerator, (ii) a target, (iii) a converter, and (iv) a synthesizer for the catalyzed fusion. In addition we need (v) a blanket for fission and breeding driven by fusion created neutrons. One notes that these five components involve different levels of physics: (i) involves accelerator physics, (ii) nuclear physics, (iii) dynamics of charged particles (accelerator physics and plasma physics), (iv) atomic and molecular physics, and (v) nuclear physics. It may be fair to say that any reactor concept must optimize some or all of these five areas. In the present article, however, no attempt will be made to further the efficiency of the atomic and molecular physics processes in item (iv). We note that there are not particularly difficult physical requirements for the confinement and ignition of the fuel as posed in conventional magnetic or inertial fusion reactor concepts. Because the fuel is not ignited, there is no requirement for α-particle confinement, either.

In this chapter the general idea of the reactor is described (Sec. A) followed by the beam injection into and pion creation within the *d-t* fuel (Sec. B). In Sec. C, the π^- capture by strong interaction is estimated and the efficiency of π^- to μ^- conversion is derived. In Sec. D we describe beam recirculation. The confinement of pions by the mirror is considered in Sec. E, and the size of the vessel and the amount of the fuel is estimated. Finally, in Sec. F we list some necessary technical considerations for future engineering studies.

A. General Ideas

Our reactor concept is based on three ideas: (a) The high energy beam of tritium or deuterium is injected into the deuterium-tritium (d-t) fuel and, after passage through the fuel, part of the beam is collected for reuse while the portion of the beam which suffered a strong interaction is directed into an electronuclear blanket, (see Fig. 3). (b) The pions created in the target are surrounded by the fuel of deuterium and tritium and are magnetically confined until they slow down and decay into muons which catalyze the fusion in situ; (c) The fusion created neutrons are absorbed by blankets to breed fissile matter for energy production. In comparison to Petrov's concept the highlights of our concept are (a) and (b). Instead of a separate target, converter and synthesizer we combine these three functions into one. The D-T fuel is the target and the produced pions are trapped in the fuel which slows them down before they decay into muons. The muons are created in the fuel and trapped there, catalyzing the D-T fusion through the atomic and molecular processes reviewed in the previous section until they in turn decay. In this way we solve one of the most difficult problems of muon catalyzed fusion, i.e., efficient trapping of muons in a relatively small physical volume. The present concept solves this problem by creating mesons in the fuel and by confining them magnetically. We may call this a synergistic approach.

Figure 3 sketches a version of our reactor concept. We drive the fuel by a tritium (or deuterium) beam which is retrieved in part after passing through the target. The significantly scattered portion of the beam feeds into the electronuclear blanket. The fusion created neutrons are captured in the fissile blanket surrounding the fusion fuel. Figure 3 shows a magnetic mirror configuration. The mirror is filled with a pressurized gas mixture of deuterium and tritium gas. The gas is circulated through the mirror, with a cooling section between traversals. The mirror is enclosed by ^{238}U or ^{232}Th blankets with admixed lithium for breeding of Pu, ^{232}U and tritium. Magnets provide a magnetic field configuration with a mirror ratio R_m. The field at the mirror throats is typically $10T$.

B. *Beam Injection and π^- Creation*

An accelerated tritium beam of approximately one GeV per nucleon and a current of about 0.1 Ampere is injected through a small tube perpendicular to the axial magnetic field into the fuel. When the high energy beam (1 GeV/nucleon) strongly interacts with the target, in effect proton-proton, proton-neutron, neutron-neutron scatterings take place. Bugg et al.[24] experimentally studied this in detail at the energy of 0.97 GeV. Figure 17(c) of Ref. 24 exhibits the angular distribution $g(\theta^*)$ of the number of events in the center-of-mass frame, where θ^* is the angle of a pion created with respect to the incoming proton (or neutron) in the center-of-mass frame. This distribution can be approximated by

$$g(\mu^*)d\mu^* = \frac{1}{4}(1 + 3\mu^{*2})d\mu^*, \tag{3.1}$$

where $\mu^* \equiv \cos\theta^*$. The pion creation angle θ_π in the laboratory frame is related to θ^* by

$$\tan\theta_\pi = \frac{P_{\text{c.m.}}\sin\theta^*}{\bar{\gamma}P_{\text{c.m.}}\cos\theta^* + \bar{\beta}\bar{\gamma}E_{\text{c.m.}}/c} \tag{3.2}$$

where $P_{\text{c.m.}}$ and $E_{\text{c.m.}}$ are the momentum and energy of the pion at the center of the mass system of the two colliding nucleons and $\bar{\gamma} = (1-\bar{\beta}^2)^{-1/2}$ is associated with the change from the laboratory frame to the center of mass frame,

$$\bar{\gamma} = \frac{E^N_{\text{lab}}E^N_{\text{c.m.}} - P^N_{\text{lab}}P^N_{\text{c.m.}}c^2}{(M_N c^2)^2} \tag{3.3}$$

where M_N is the nucleon (projectile) mass and P^N and E^N are the momentum and energy of the incoming nucleon projectile in the reference frame as indicated by the subscript in (3.3). For relativistic pions, Eq. (3.2) can be approximated by

$$\tan\theta_\pi = \frac{\sin\theta^*}{\bar{\gamma}(\cos\theta^* + \bar{\beta})} \tag{3.4}$$

which implies

$$\cos\theta_\pi \equiv \mu = \frac{\mu^* + \bar{\beta}}{1 + \bar{\beta}\mu^*} \quad ; \quad \cos\theta^* \equiv \mu^* = \frac{\mu - \bar{\beta}}{1 - \bar{\beta}\mu}. \tag{3.5}$$

The relativistic approach is satisfied since the nonrelativistic particles are stopped in a short range and thus are not able to leave the mirror. In this case the pion distribution (3.5) can be written in the laboratory frame by

$$g(\mu^*)d\mu^* = g(\mu)d\mu = \frac{1}{4}\frac{(1-\bar{\beta}^2)}{(1-\bar{\beta}\mu)^2}\left[1 + \frac{3(\mu-\bar{\beta})^2}{(1-\bar{\beta}\mu)^2}\right]d\mu. \tag{3.6}$$

For perpendicular injection of the nucleonic projectiles, as described by Fig. 3, the angle θ' between the π^- and the magnetic field of the mirror is given by

$$\mu' = \sqrt{1-\mu^2}\cos\phi \tag{3.7}$$

where $\mu' = \cos\theta'$ and ϕ is the azimuthal angle and changes between zero to π on each side of the loss cone. The loss cone is defined by

$$\mu' = \frac{v_\parallel}{v} \geq \left(1 - \frac{1}{R_m}\right)^{1/2} \tag{3.8}$$

where $R_m = \dfrac{B_{\max}}{B_{\min}}$ is the ratio of reflecting magnetic field at the mirror throat to field in the injection (target) region. This implies that ϕ is limited by

$$\phi \geq \cos^{-1}\left[\left(1 - \frac{1}{R_m}\right)^{1/2} \Big/ (1-\mu)^{1/2}\right], \tag{3.9}$$

so that the fraction of π^-'s in the loss cone is given by

$$G(\mu) = \frac{2}{\pi} \cos^{-1} \left[\left(1 - \frac{1}{R_m}\right)^{1/2} \Big/ (1 - \mu^2)^{1/2} \right]. \tag{3.10}$$

Therefore the fraction of pions created by the strong interactions in the loss cone is given by

$$f_0 = \int_0^{1/\sqrt{R}} G(\mu) g(\mu) d\mu, \tag{3.11}$$

where $g(\mu)$ and $G(\mu)$ are given by (3.6) and (3.11). The evaluation of the integral (3.11) for $\bar{\beta} \geq 0.55$ gives $f_0 \leqq 15\%$ for a mirror ratio $R_m = 1.5$, $f_0 \leqq 10\%$ for $R_m = 2$, and $f_0 \lesssim 5\%$ for $R_m \geq 3$. The muon loss cone is included in the pion loss cone since the nonrelativistic μ^- are absorbed in a short range while the relativistic μ^- are created mainly in the direction of the π^-.

C. π^- Capture

Another factor in considering this scheme is the fact that the pions which come to rest in the high density hydrogen fuel are absorbed by the protons and therefore lost by strong interactions before converting into a muon. The fraction of pions lost by nucleon capture is given by

$$f = \int_0^\infty \frac{dN}{dt} e^{-t/\tau_\pi} dt \Big/ \int_0^\infty \frac{dN}{dt} dt. \tag{3.12}$$

Since the initial distribution of pions is given as a function of their kinetic energy T, we express dN/dt by

$$\frac{dN}{dt} = \left(\frac{dN}{dT}\right) \left(\frac{dT}{dx}\right) \beta c \tag{3.13}$$

where $\frac{dT}{dx}$ is given by the Bethe-Bloch formula, Eq. (2.39), and $\frac{dN}{dT}$ is the experimental data for the initial pion distribution. The appropriate time, t, in Eq. (3.12) is derived from

$$\frac{t}{\tau_\pi} = \int_0^t \frac{dt}{\tau_\pi(\beta)} = \int_0^T \frac{dT}{\beta c \left(\frac{dT}{dx}\right) \tau_\pi(\beta)} \tag{3.14}$$

where again, $\tau_\pi(\beta) = \left[\tau_\pi/(1-\beta^2)^{1/2}\right]$, $\tau_\pi = 2.6\times10^{-8}$ sec, and $\frac{dT}{dx}$ is given by (2.39). Using the dimensionless quantity

$$\varsigma = T/m_\pi c^2 \tag{3.15}$$

we obtain

$$\begin{aligned}\frac{t}{\tau_\pi} &= \frac{1}{10}\left(\frac{mm_\pi c^3}{4\pi n e^4 \tau_\pi}\right)\int_0^\varsigma d\varsigma \frac{\sqrt{2\varsigma+\varsigma^2}}{(1+\varsigma)^2} \\ &= -\frac{k}{\phi}\cdot\left\{\frac{\sqrt{\varsigma^2+2\varsigma}}{1+\varsigma} - \ell n(1+\varsigma+\sqrt{\varsigma^2+2\varsigma})\right\}\end{aligned} \tag{3.16}$$

where $k \approx 1.0$ and the factor $\frac{1}{10}$ approximates the logarithm term in Eq. (2.39). The fraction of lost pions in Eq. (3.12) can be expressed conveniently in the following form

$$\begin{aligned} f \equiv \int_0^{T_{\max}} f(T)dT &= \int_0^{T_{\max}} \left(\frac{dN}{dT}\right) \\ &\times \frac{dT}{m_\pi c^2}(1+\varsigma+\sqrt{\varsigma^2+2\varsigma})^{-k/\phi}\exp\left\{\frac{k}{\phi}\left[\frac{\sqrt{\varsigma^2+2\varsigma}}{1+\varsigma}\right]\right\} \Big/ \int \frac{dN}{dT}dT \end{aligned} \tag{3.17}$$

where $\left(\frac{dN}{dT}\right)$ is the experimental data of the initial pion distribution and $\phi = n/n_0$ is the d-t fuel density in liquid hydrogen units, and f the probability for π^- capture. This probability f is plotted in Fig. 5, as a function of ϕ. In Fig. 5(a) we plot the relative number of events of π^- creation as a function of kinetic energy in the laboratory frame experimentally measured by Bugg et al.[24] In Fig. 5(b) we show the slow-down time (π^- capture time) normalized by the lifetime of π^- [Eq. (3.16)]. Figure 5(c) summarizes our result (3.17). Equation (3.17) yields $f \cong 0.32$, 0.48, 0.68, and 0.77 for $\phi = 0.3$, 0.5, 1.0 and 1.5 accordingly. Taking into account that the other losses were small, the conversion efficiency y_μ between π^- to μ^- seems to be

$$y_\mu \simeq 0.5 \tag{3.18}$$

for half liquid hydrogen density.

D. Beam Recirculation

A portion of the beam that suffers a strong interaction with fuel nuclei and a deflection with a substantial angle ($\sim 10°$) will be absorbed by the electronuclear blanket, which breeds the fissile material by bombarding neutrons, protons, or any other debris of tritium nuclei. The other portion of the beam that has not suffered a strong interaction but merely soft Coulomb collisions and ionization interactions does not get significantly scattered, although this portion of the beam experiences reduction in energy (see Sec. II and later in this section). This nearly undeflected portion of the beam goes through the exist tube and is collected. This portion has suffered soft electromagnetic collisions which give rise to small spreads of momentum $\Delta p_\perp/p$ per mean free path. The spread of momentum may be estimated by noting that the energy loss ΔE is incurred mainly by heating electrons $\Delta E \simeq (\Delta p_\perp)^2/2m$, while the original energy is roughly $E \sim p^2/2m_t$ where m_t is the projectile (tritium) mass. Therefore $\Delta p_\perp/p \sim (\Delta E/E)^{1/2}(m/m_t)^{1/2} \sim 10^{-3}$, since $\Delta E/E \sim (0.6/3)\ell_s$ MeV/1 GeV and $\ell_s \sim 60$cm for t-t collisions at liquid hydrogen density. The admittance of a warm beam into an accelerator is determined by the condition that the maximum phase space spread be less than the accelerator aperture. This condition may be written as[23]

$$\beta_a \varepsilon < \pi r_a^2 \tag{3.19}$$

where β_a is the so-called amplitude function of the betatron oscillations in the accelerator, ε is the emittance

$$\varepsilon = \pi \left(\frac{\Delta p_\perp}{p} \right) r_b \tag{3.20}$$

with r_b the radius of the beam, and r_a is the aperture of the accelerator. The β_a for the present accelerator would be typically of the order of one meter. Thus as long as the aperture of the accelerator satisfies

$$r_a \geq \left(\frac{\Delta Em}{Em_t}\right)^{1/4} (\beta_a r_b)^{1/2} \tag{3.21}$$

recirculation of the portion of the beam is possible. Even for the existing accelerators such as LAMPF lineac this condition is satisfied. LAMPF parameters are: $\beta_a \simeq 10\ m$, $r_a \simeq 1 - 3$cm, $\varepsilon \simeq 10^{-3}$ cm rad. Thus as long as r_b is less than 1 cm, the beam is admitted. Since the beam is hitting a small volume, the target is heating primarily by electromagnetic interactions. One has to consider appropriate cooling of this volume. As will be shown in Sec. F, the *d-t* fuel is moving with a velocity of about 10 m/sec. Therefore, for a beam radius of 1 cm and a longitudinal (the transverse dimension of the mirror) dimension of about 40 cm - 50 cm, the cooling of the interaction region seems not to be a difficulty. We mention that the recirculating beam must also be tailored, then accelerated back to the original energy of 1 GeV/nucleon.

E. π^- Confinement

As we stated earlier, the most important consideration is how not to lose pions and muons that are created by the high energy tritium (or deuterium) beam. Here we analyze the confinement of pions and subsequently muons in the mirror magnetic fields. The Bethe and Bloch formula or its derivative[21] gives the differential energy loss of a charged particle per unit length in a condensed matter as in Eq. (2.39). The mean free path or the stopping length is obtained by

$$\ell_{mfp} = \left(\frac{d}{dx}\ell n\ T\right)^{-1}, \tag{3.22}$$

which is a function of the energy. The mean free path is longer for higher energy particles. The collision frequency is defined as

$$\nu = \frac{\beta c}{\ell_{mfp}}. \tag{3.23}$$

For typical energies of pions created by the injected tritium (or deuterium) beam colliding the target, the collision frequency is approximately $\sim 10^{+8}\,\text{sec}^{-1}$. A more exact concept is the range, numerical values of which may be found in Ref. 22. The range for pions under consideration typically varies from less than $1m$ to 50 m for 0.5 liquid hydrogen density of the *d-t* fuel. It is clear that if we do not mirror particles (pions), we need a fairly long containment system. In order to confine pions up to energies of 500 MeV ($\sim$ 95%), the length of the converter/fuel vessel should at least be 45 m. Within this length most ($\sim$ 95%) of pions decay into muons and muon neutrinos. We further have to confine muons. If we apply the longitudinal magnetic field, the transverse size (the radius) of the fuel container would be at least $2\rho_\pi^{\text{max}}$, where ρ_π^{max} is the maximum (within 95% population) of the pion Larmor radius

$$\rho_\pi = \frac{\gamma\beta_\perp m_\pi c^2}{eB}, \tag{3.24}$$

where m_π is the pion rest mass, B is the applied longitudinal magnetic field, and $\beta_\perp$ is $v_\perp/c$ with $v_\perp$ being the velocity perpendicular to the magnetic field. With B of the order of 10 Tesla, $\rho_{\pi,\text{max}} \simeq 10$ cm so that $a \geq 20$ cm would suffice for $T_\pi \leq 500$ MeV. The mirror fields can reduce the system length by a substantial amount. Such a reduction of the system length will cut down the volume of the fuel container and thus the amount of the fuel tritium.

Unfortunately, some of the pions in the mirror fields are untrapped and will be lost through the loss cone in velocity space

$$\frac{v_\perp}{v_\parallel} \leqq \frac{1}{(R_m - 1)^{1/2}}, \tag{3.25}$$

as discussed in Sec. IIIB. Given angle θ_π before electromagnetic scattering of pions in the fuel takes place, one can calculate the rate of pitch angle diffusion, which is small for the same reasons given in Sec. IIID. The results are shown in Fig. 4. The typical scattering angle $\overline{\langle\theta^2\rangle}^{1/2}$ is of the order of 2°. Therefore, the electromagnetic scattering does not much deflect the beam. The pion diffusion due to strong interactions in the d-t fuel is negligible since the rate of strong interaction is $n\sigma v \sim 10^5 - 10^6\,\mathrm{sec}^{-1}$ ($\sigma \approx 2.10^{-27}$ cm^2 and $n \approx 4.10^{22}$ cm^{-3}) while the pion lifetime is only $\tau_\pi \simeq 2.6 \cdot 10^{-8}$ sec. Since we have shown the angular scattering to be small, one can see that collisional radial diffusion is much less than ρ_π and thus negligible. Although to a zeroth order approximation, the magnetic moment μ of pions in the mirror is conserved as an adiabatic invariant, there are nonadiabatic changes to the magnetic moment as a particle bounces back and forth in the mirror field. Since the pitch angle scattering of pions by the DT fuel is relatively minor, one should also consider the nonadiabatic change of μ as a primary concern. Let $\Delta\mu$ be the jump in the magnetic moment per one passage of the midplane (i.e., per a half bounce of the trapped oscillations) in the mirror. A typical number of bounces in the mirror is given by

$$n = \frac{R}{2L\cos\theta'} \simeq \frac{R}{2L\sqrt{1-\frac{1}{R_m}}}, \tag{3.26}$$

where R is the range of the pion in the mirror. n is 43 for $\varepsilon_m = 1$, ($\varepsilon_m = a/L$ is the the inverse aspect ratio), R=5 m case while n=430 for $\varepsilon_m = 1$, R=50 m case. For these long ranged pions, it may be more appropriate to use the stochastic approximation to evaluate the time of flight for the pion to diffuse in velocity space, which yields the condition

$$\gamma\tau_\pi < \left(\frac{\mu}{\Delta\mu}\right)^2 \frac{2L}{v_{||}}, \tag{3.27}$$

where $v_{||}$ is the parallel velocity of the pion. The jump $\Delta\mu$ due to the nonadiabatic effect has been given[27] as

$$\frac{\Delta\mu}{\mu} = A\exp\left(-\frac{3L}{2\rho_\pi}\right), \tag{3.28}$$

with A being a numerical constant, approximately 5. Solving Eqs. (3.27) and (3.28) for

$v_{\|} = c\cos\theta'$, c being the velocity of light and $\cos\theta' = [1-(1/R_m)]^{1/2}$, we get the inequality

$$L \geq \frac{\rho_\pi}{3}\ell n\left[A^2\left(1-\frac{1}{R_m}\right)^{1/2}\frac{\gamma\tau_\pi c}{L}\right]. \tag{3.29}$$

Equation (3.29) yields for L the following estimate for a mirror ratio $R_m = 1.5$,

$$L \simeq 2a = 4\rho_{\pi,\mathrm{max}} \tag{3.30}$$

where $\rho_{\pi,\mathrm{max}} \simeq 10$ cm is the maximum pion Larmor radius for a magnetic field of 10 Tesla and pion kinetic energies of 500 MeV. At this stage the mirror volume and the tritium mass can be estimated. Since we have shown that an aspect ratio $L/a \simeq 2$ can be chosen with the accepted mirror ratio $R_m \simeq 1.5$, the mirror volume is

$$V \simeq \pi a^2 L \simeq 16\pi(\rho_{\pi,\mathrm{max}})^3 \simeq 5\cdot 10^4\ \mathrm{cm}^3 \tag{3.31}$$

where the maximum Larmor radius is 10 cm for the most energetic pions. The density of the *d-t* fuel is 0.5 liquid hydrogen density with about 30% tritium (apparently the muon catalysis is most efficient for 30% tritium concentration[16]) implying a tritium mass M_t in the vessel of

$$M_t \sim 1.5\ kg. \tag{3.32}$$

A total tritium inventory including the cooling and recirculating sections is perhaps twice as much.

F. Some Comments

Once muons are trapped in the fuel, they slow down until p_μ becomes $\alpha m_\mu c$ over the range of less than a centimeter and then they will quickly interact with the fuel through the atomic and molecular processes reviewed in Sec. II. These muons will catalyze the fusion reactions as described there. The fusion generated alpha particles have a range of about 0.1mm and, therefore, they are totally confined in and heat the fuel. The μ^- which are released from the $dt\mu$ molecule have even a shorter range than the α particles and therefore they do not diffuse appreciably during the catalytic cycle. The rate of heating by alpha particles is related to the amount of muons created in the fuel $\varepsilon_\alpha X_\mu N_\mu$, where ε_α is the α-particle energy of 3.6 MeV and N_μ is the number of muons created in the reactor per unit time, which is proportional to the power supplied to the accelerator. Therefore, we equate the temperature rise to the power input as

$$nc_p \frac{dT}{dt} = \varepsilon_\alpha X_\mu N_\mu / V \tag{3.33}$$

where n is the average density of deuterium and tritium, c_p is the specific heat under constant pressure, and T is the temperature of the fuel. By recirculating the fuel through gas flow, the heat generation can be carried away so that the fuel vessel stays at the optimal temperature, say $500^\circ\ K$. This condition is expressed approximately as

$$nTv_\ell L^{-1} = \varepsilon_\alpha X_\mu N_\mu / V, \tag{3.34}$$

where v_ℓ is the gas flow velocity of the fuel in the vessel. A typical estimate of the flow v_ℓ is that with $n \sim 2 \times 10^{22}$ cm^{-3}, $T \sim 0.1\ eV$, $V \sim 10^5$ cm^3, $L \sim 100$ cm, $\varepsilon_\alpha = 4 \times 10^6\ eV$, $X = 100$, and $N \approx 10^{18}\,\mathrm{sec}^{-1}$, one gets v_ℓ equals few m/sec, which is not unreasonable. The d-t fuel is cooled outside the mirror vessel. In this case one has to cool the fuel very efficiently and very fast, so that the tritium inventory is not significantly increased. One has to cool an amount of power as in a standard nuclear reactor, but with a volume about one order of magnitude smaller.

This paper does not discuss the materials problems related to the design of the mirror vessel in particular, and the whole reactor in general. Particular consideration should be given to the material strength and endurance of the mirror vessel, taking into account a vessel with about 1500 atmosphere pressure ($p \sim nkT$ for $T = 500^\circ$ and $\boldsymbol{n} \simeq 2 \cdot 10^{22}\ \mathrm{cm}^{-3}$, i.e., $\phi \simeq 0.5$), and a typical internal curvature of about 20 to 30 cm. In particular, the injection of the beam, the stresses on the vessel, the heat removal and the cooling technologies need serious engineering considerations and seem to be at the edge of technological possibilities.

IV. Energy Gain

The energy and particle flow for the muon catalyzed fusion-fission reactor is described schematically in Fig. 6. A particle accelerator of tritium (or deuterium) delivers (about 1 GeV/nucleon) a power P_A with an efficiency η_A into a μ^- generating catalysis chamber which contains the high density D-T fusion fuel. In this chamber the muon catalysis fusion takes place, releasing $\varepsilon_{\rm fus} = 17.6$ MeV/fusion. The configuration of this chamber is given in the previous section. The beam power P_A is used for μ^- production as well as for electronuclear (EN) direct beam-blanket interaction where r is the fraction of P_A used for the fusion processes

$$r = \frac{P_A(\text{to create } \mu^-)}{P_A} = \frac{E_\pi}{y_\mu^{-1} E_0} \tag{4.1}$$

where E_π is the energy loss during the creation of one negative pion and E_0 is the required energy to create one π^-. E_π is given by Eq. (2.43), E_0 by Eq. (2.44) and the π^- to μ^- efficiency conversion y_μ is estimated to be about 0.5 for $\phi = 0.5$ liquid hydrogen density. For the tritium-tritium interaction $r \simeq 20\%$. Then $(1-r)P_A$ is used for the electronuclear portion where the direct beam-blanket interaction is used for spallation breeding of the fissile fuel (e.g., ^{239}Pu or ^{233}U for a blanket of ^{238}U or ^{232}Th respectively). The gain value for fusion in our schematic diagram, Fig. 6, is defined by

$$Q = \frac{X_\mu \cdot \varepsilon_{\rm fus}}{r E_0}, \tag{4.2}$$

X_μ is the number of fusion reactions catalyzed by one muon ($X_\mu \simeq 100$, see Sec. II), $\varepsilon_{\rm fus} = 17.6$ MeV is the released fusion energy in one dt reaction and

$$E_0 = y_\pi^{-1} T_0, \tag{4.3}$$

where y_π^{-1} is the number of projectiles required to produce a π^- with an initial kinetic energy per nucleon $T_0 (T_0 \simeq 1$ GeV). y_π^{-1}, y_μ^{-1} and X_μ are the most important input (physical) parameters for the fusion-fission process catalyzed by the negative muons. Values of

y_π^{-1} for different relevant projectile-target interactions in our scheme are given in Table 3 of Sec. II.

The total gain factor is (see Fig. 6)

$$K \equiv \frac{E_{\text{blanket,out}}}{E_{\text{target,in}}} = \frac{E_{\text{blanket,out}}(EN) + E_{\text{blanket,out}}(\mu c)}{y_\mu^{-1} y_\pi^{-1} T_0}. \tag{4.4}$$

EN denotes the energy gain from "direct" beam-blanket interaction (see previous section) and μc denotes the energy gain from muon catalytic processes with input energy on target $(1-r)y_\pi^{-1}y_\mu^{-1}T_0$ and $ry_\pi^{-1}y_\mu^{-1}T_0$ for the EN and μc processes respectively. The value of K is given in terms of the physical qualities X_μ, y_μ^{-1} and y_π^{-1} by the formula

$$K = \frac{y_\mu^{-1} y_\pi^{-1} Z_e T_{of} \varepsilon_{\text{fis}} + X_\mu \delta_f \varepsilon_{\text{fis}}}{y_\mu^{-1} y_\pi^{-1} T_0} = \frac{Z_e T_{of} \varepsilon_{\text{fis}}}{T_0} + \frac{y_\mu y_\pi X_\mu \delta_f \varepsilon_{\text{fis}}}{T_0} \equiv K_{EN} + K_{\mu c} \tag{4.5}$$

where the EN contribution is $K_{EN} \equiv Z_e T_{of} \varepsilon_{\text{fis}} / T_0$ and the muon catalyzed gain is given by the second term. $\varepsilon_{\text{fis}} \simeq 0.2$ GeV is the uranium fission energy[28] (for an ^{238}U blanket), $Z_e \simeq 20$ fissions/GeV in the "direct" beam-blanket interaction[8] (i.e., the EN process), T_{of} is the beam kinetic energy before colliding with the blanket while T_0 is their initial kinetic energy before hitting the target (for pion $\rightarrow$ muon creation). $\delta_f \simeq 1$ is the number of fissions in the blanket[28] caused by one 14 MeV neutron from the fusion process. X_μ was measured experimentally to be about 100 for liquid hydrogen *d-t* targets and y_π was estimated in Sec. II and y_μ was estimated in Sec. III to be about 0.5. The above reactor produces a considerable power on its own. However, the fissile material produced in the blanket (e.g., ^{239}Pu) can be used to run between 3 to 6 satellite fission plants of equal output.[29] X_R in Fig. 6 denotes the number of these satellite fission plants. The value of X_R, of course, depends on the design and breeding ratio of those plants and is thus somewhat arbitrary. However, assuming a value allows an intercomparison between this scheme and others — pure EN, fusion hybrid, etc. We have also estimated that the number of fissile atoms bred per fission in the blanket is about the same for the EN and the fusion neutrons (taking account of T breeding). This assumption may be a reasonable one. The breeding factor X_{R_1} for the EN spallation case is proportional to the number of fissions triggered by the high energy projectiles. This yields $X_{R_1} \propto 20 \times 0.8$, where 0.8 is the fraction of projectile energy directed to the spallation blanket. On the other hand, the breeding factor X_{R_2} for the fusion-fission blanket is proportional to the number of neutrons by fusion times the effective fission number (~ 1), which yields $X_{R_2} \propto 1 \times X_\mu \times 0.2$, where 0.2 is the fraction of projectile energy spent in the fusion fuel. This estimate[8,28] yields $X_{R_1} \sim X_{R_2}$.

The total electrical gain K_{tot} is given by

$$K_{\mathrm{tot}} = \frac{\eta_{\mathrm{th}} P_{\mathrm{out}}}{P_A} = \eta_{\mathrm{th}}(1 + X_R)K \tag{4.6}$$

where $\eta_{\mathrm{th}} \simeq 0.35$ is the thermal to electric power conversion efficiency and K is given in Eq. (4.5). Defining α as the fraction of the total electric power required to recirculate in order to operate the accelerator and the auxiliary facilities, then the electric net output is

$$P_E = (1 - \alpha)\eta_{\mathrm{th}}(1 + X_R)KP_A. \tag{4.7}$$

One gets the following relations

$$\frac{P_A}{\eta_A} = \alpha \eta_{\mathrm{th}} P_{\mathrm{out}} \tag{4.8}$$

implying

$$\alpha = \frac{1}{\eta_A \eta_{\mathrm{th}} K(1 + X_R)} = \frac{1}{\eta_A K_{\mathrm{tot}}}. \tag{4.9}$$

Substituting in Eqs. (4.5) and (4.9) the following parameters[8,28,29] for a fuel containing 30% tritium and 70% deuterium at 0.5 liquid hydrogen density,

$$\left\{ \begin{array}{l} Z_e = 20\ GeV^{-1},\ \varepsilon_{\mathrm{fis}} = 0.2\ GeV,\ \delta_f = 1,\ T_0 = 1\ GeV,\ X_R = 5 \\ \eta_A = 0.6,\ \eta_{\mathrm{th}} = 0.35,\ y_\mu = 0.5,\ y_\pi = 0.31,\ T_{of} = 0.8\ GeV,\ X_\mu = 100 \end{array} \right\} \tag{4.10}$$

we get

$$K_{EN} = 3.2,\ K_{\mu c} = 3.1,\ K = 6.3,\ K_{\rm tot} = 13.2,\ \alpha = 0.13. \tag{4.11}$$

Note that although the μc has an input energy smaller by a factor of four than the EN input energy, the output gain from μc is about the same as the EN gain. An important result of this section is the estimate of the parameter α, which can serve as a figure of merit in a "driven" nuclear reactor. As usual, assumed output electricity for the system of $P_E = (1 + X_R) \times 10^9 W$ and for $\alpha = 0.13$ and $X_R = 5$, we need an accelerator power of

$$P_A \simeq 8.5 \times 10^8 W \tag{4.12}$$

which for 1 GeV/nucleon projectiles implies an average current of 0.85 Ampere.

V. Summary and Discussion

The observed values of multiplicity[6] of deuterium-tritium fusion catalyzed by muons have increased the interest in muon catalyzed fusion reactors.[9] At present, it seems impossible to achieve energy gain by a pure fusion process. Therefore, it was suggested[8] to use muon catalysis in a hybrid fusion-fission reactor. In this scheme, the fusion reactor vessel is surrounded by a blanket of ^{238}U or ^{232}Th for a positive energy gain and further to breed a thermal fissile isotope (^{233}U,^{239}Pu) in order to use these materials in satellite fission reactors.[32]

In the present paper we suggest two new ideas that improve the muon fusion reactor concept. (i) Since the main energy expended is in the pion and consequent muon production, we try to minimize the pion loss by magnetically confining pions in the deuterium-tritium fuel. For this purpose, the target which creates pions, the converter of pions into muons and the synthesizer with pressurized gas of the fusion fuel are combined into one vessel. A tritium or a deuterium beam of 1 GeV/nucleon is injected in the (above) synergetic vessel which has the structure of a magnetic mirror. The magnetic configuration has been chosen for the best possible pion confinement, with a minimum quantity of tritium. The necessary quantity of tritium in these reactors is estimated to be of the order of several kilograms, and the fusion-reactor vessel has a volume of about 0.1 m^3. It might be possible to have the DT fuel only near the center of the vessel where the pions decay and leave the freshly created pions in the surrounding vacuum. In this case the necessary quantity of tritium would be reduced. (ii) In order to minimize beam energy losses, we suggest recirculating the beam that passes through the target so that we use the maximum probability for pion creation with a 1 GeV/nucleon base. For kinetic energies smaller than 0.6 GeV/nucleon, the probability of inelastic nucleon-nucleon collisions is very small so that the π^- cannot efficiently be produced. Therefore, part of the beam which suffers only electromagnetic energy losses (ionization, etc.) in the target is collected for reuse and it is recirculated. The other part of the beam, which has strong interactions with the target has a deflection with a substantial angle ($\sim 10°$) and therefore is directed and absorbed by an electronuclear blanket which breeds fissile material.

The process of direct beam-blanket interaction is less efficient than the fusion induced part. On the other hand, it seems that one has to use a substantial ($\sim 80\%$) part of the beam energy for electronuclear fission, since only a limited energy range is useful for π production. If we want to use all of the beam for muon catalyzed fusion, we have to collect also the strongly scattered beam ($\sim 10°$), to tailor it, to cool and to accelerate it back to the original 1 GeV/nucleon energy. This process is difficult and probably impossible due to the Liouville theorem.

A comparison of the present reactor concept with other driven reactor concepts is considered in Table 4. Since the reactor discussed in this paper can not achieve ignition, the comparison with self-ignited reactors is difficult. The drivers considered in the table are: lasers and particle beam accelerators for inertial confinement fusion (ICF), accelerators for electronuclear (EN) breeders and for muon catalyzed fusion-fission (μcFF) reactors, and radio frequency (RF) wave generators and/or accelerators for magnetic fusion (MF) reactors. We also show the ICFF and MCFF which are fusion-fission hybrid schemes for the respective processes. The efficiency of the drivers are optimistically taken. Numbers of merit for reactors may be the total energy gain K_{tot} (Eq. (4.6)) and the fraction of driving energy α (Eq. (4.9)). It is difficult to predict the science and technologies necessary for the different schemes. We thus indicate in Table 4 equivalent energy gain Q necessary to achieve $\alpha = 0.13$, which is the value we have estimated for the present scheme. Order of magnitude for the fuel temperature (T in eV), reactor volume including the blankets (V in m^3), and the necessary tritium mass in the reactor at a given time (M_t in gr) are also given in Table 4.

The reactor concept discussed in this paper may merit further serious investigations based on our preliminary study. It is worth pointing out that the present scheme is based on mostly readily available technologies and available data from experiments.

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Figure Captions

Fig. 1 The kinetic of muon catalyzed fusion in a mixture of deuterium and tritium. The muon can decay at any stage with a lifetime $\tau_\mu = 2.2 \times 10^{-6}$ sec. The values for λ and ω are given in Table 1.

Fig. 2 The He sink. He denotes ^{3}He, ^{4}He, ^{3}Heμ, and ^{4}Heμ. The concentrations c_d, c_t and c_{He} satisfy the relation $c_d + c_t + c_{\mathrm{He}} = 1$.

Fig. 3 The mirror reactor concept. The gas deuterium-tritium fuel at the liquid hydrogen density in 400°K temperature range is contained in the mirror. The target (tritium) is held in the middle of the mirror. The tritium or deuterium beam is injected along the mirror axis and an undeflected portion is collected for recirculation and a deflected portion is absorbed by electronuclear blankets. Pions are confined by the mirror field. Fusion neutrons are absorbed by the surrounding fission blankets. The heated fuel is cooled by circulating flow of the fuel.

Fig. 4 The average deflection angle of a pion in matter as a function of the distance travelled by a pion.

Fig. 5 The fraction of stopped pions before decaying into μ^- inside the high density hydrogen fuel.

(a) Experimental number of events of π^- creation as a function of the Lab frame kinetic energy (Ref. 24).

(b) The slow-down time (the π^- capture time) as a function of the Lab frame kinetic energy for various ϕ ($\phi = 0.3, 0.5, 1.0$, and 1.5 from the above to below).

(c) The π^- nuclear capture fraction as a function of the normalized density ϕ.

Fig. 6 Scheme of energy production in a hybrid muon catalyzed fusion-fission electronuclear reactor.

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Table 1. Estimates of the rate coefficients at hydrogen liquid density ($n = 4.25 \times 10^{22}$ cm^{-3}) normalized to the rate of muon decay ($\tau_\mu = 2.2 \times 10^{-6}$ sec). The sticking probabilities are given by their inverse ($1/\omega$).

Process	$\lambda\tau_\mu$	$1/\omega$
$\mu^- + D \rightarrow (d\mu) + e^-$ $\mu^- + T \rightarrow (t\mu) + e^-$	$\lambda_a \cdot \tau_\mu = 8.8 \times 10^6$	--
$d\mu + t \rightarrow t\mu + d$	$\lambda_{dt} \cdot \tau_\mu = 4.4 \times 10^2$	--
$(d\mu) + D_2 \rightarrow (dd\mu)d2e$	$\lambda_{dd\mu} \cdot \tau_\mu = 6.6$	--
$(t\mu) + D_2 \rightarrow (dt\mu)d2e$ + $(t\mu) + DT \rightarrow (dt\mu)t2e$	$\lambda_{dt\mu}\tau_\mu \geq 2.2 \times 10^2$	--
$(t\mu) + T_2 \rightarrow (tt\mu)t2e$	$\lambda_{tt\mu}\tau_\mu = 6.6$	--
$d + t \rightarrow {}^4He + n$	$\lambda_f \cdot \tau_\mu = 2.2 \times 10^6$	$\frac{1}{\omega_s} = 110$
$d + d \begin{cases} \xrightarrow{50\%} {}^3He + n \\ \xrightarrow{50\%} t + p \end{cases}$	$\lambda_{fd}\tau_\mu = 2.2 \times 10^5$	$\frac{1}{\omega_d} = 7.7$
$t + t \rightarrow {}^4He + 2n$	$\lambda_{ft}\tau_\mu = 2.2 \times 10^5$	$\frac{1}{\omega_t} = 20$

Table 2. The values of C_{ab}^{AB} (a,b = p or n) from eqs. (2.28) for hydrogen isotopes collisions, the total cross section σ_{AB} from eq. (2.29) and an estimate of the mean free path $\ell_{S,AB}$ for strong interaction in a liquid density target ($n_0 = 4.25 \times 10^{22}$ atoms/cm^3).

target →	d					t				
projectile	$\sigma_{AB}^{tot} \equiv \sigma$ * [mb] = 10^{-27} cm^2	ℓ_S [cm]	c_{pp}	c_{pn}	c_{nn}	$\sigma_{AB}^{tot} \equiv \sigma$ [mb] = 10^{-27} cm^2	ℓ_S [cm]	c_{pp}	c_{pn}	c_{nn}
p	$\sigma_{pp}^{tot} + \sigma_{pn}^{tot}$ $\simeq 81$	290	σ_{pp}^{tot}/σ $\simeq 0.56$	σ_{pn}^{tot}/σ $\simeq 0.44$	0	$\sigma_{pp}^{tot} + 2\sigma_{pn}^{tot}$ $\simeq 117$	201	σ_{pp}^{tot}/σ $\simeq 0.38$	σ_{pn}^{tot}/σ $\simeq 0.62$	0
d	$\sigma_{pp}^{tot} + \sigma_{nn}^{tot} + 2\sigma_{pn}^{tot}$ $\simeq 162$	145	σ_{pp}^{tot}/σ $\simeq 0.28$	σ_{pn}^{tot}/σ $\simeq 0.44$	σ_{nn}^{tot}/σ $\simeq 0.28$	$\sigma_{pp}^{tot} + 2\sigma_{nn}^{tot} + 3\sigma_{pn}^{tot}$ $\simeq 243$	97	σ_{pp}^{tot}/σ $\simeq 0.19$	$3\sigma_{pn}^{tot}/\sigma$ $\simeq 0.44$	$2\sigma_{nn}^{tot}/\sigma$ $\simeq 0.37$
t	identical to d projectile on t target					$\sigma_{pp}^{tot} + 4\sigma_{nn}^{tot} + 4\sigma_{pn}^{tot}$ $\simeq 369$	64	σ_{pp}^{tot}/σ $\simeq 0.12$	$4\sigma_{pn}^{tot}/\sigma$ $\simeq 0.39$	$4\sigma_{nn}^{tot}/\sigma$ $\simeq 0.49$

*We assume σ_{pp} (1 GeV) = 45 mb.

Table 3. Values of y_π and the energy $y_\pi^{-1}T_0$ required to create one negative pion. In column a we give the results taking multiple scattering into account while in b one single collision is considered.

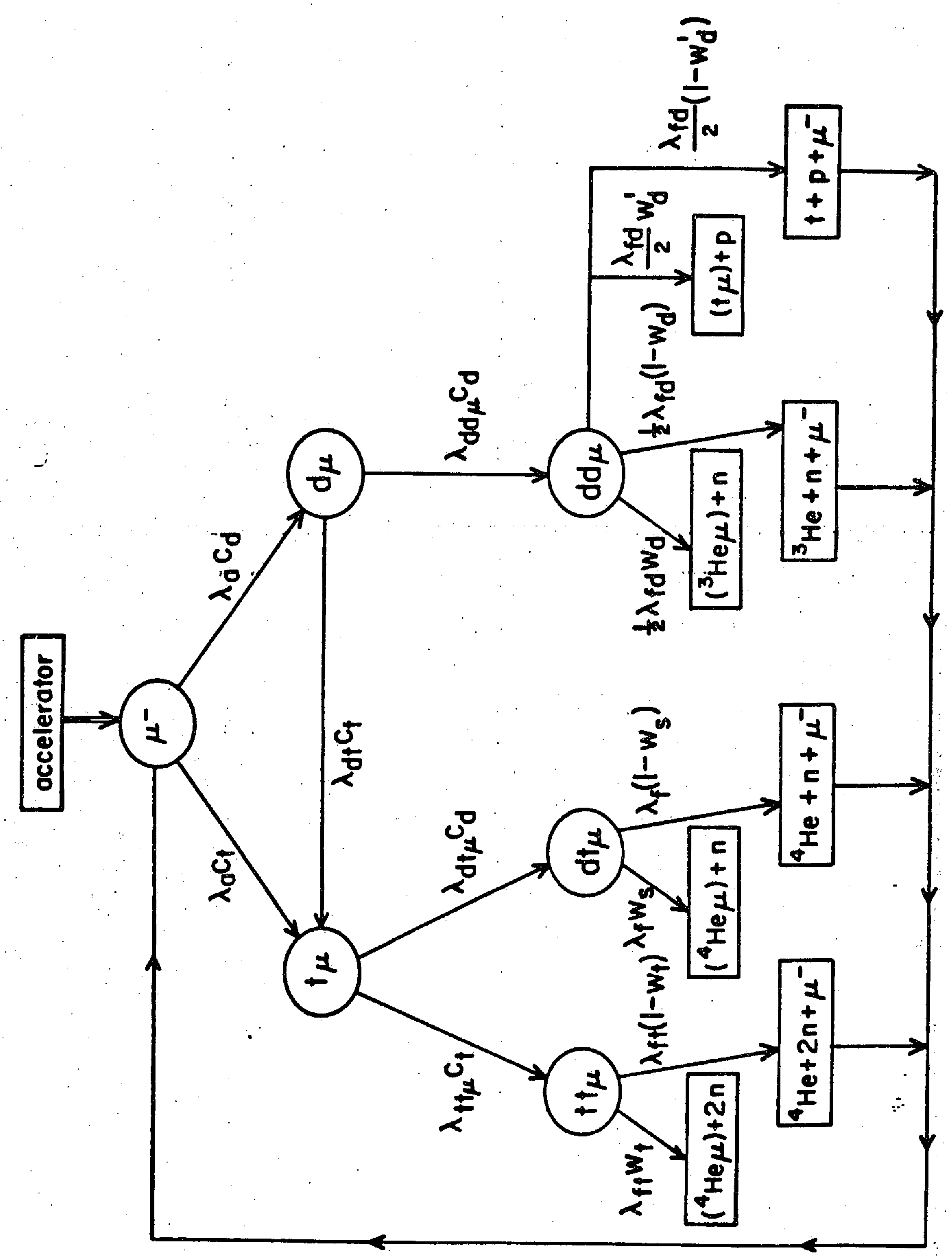
T_0 = energy per nucleon	y_π		$E_0 = y_\pi^{-1}T_0$	
	a	b	a	b
d(1 GeV) - d	0.22	0.17	4.5	5.9
t(1 GeV) - d	0.27	0.21	3.7	4.8
t(1 GeV) - t	0.50	0.25	2	4
t(1 - GeV) - (70% d + 30% t)	0.31	0.22	3.2	4.6

TABLE 4

A comparison of the present reactor concept (μFF) with other driven reactor concepts, Inertial confinement fusion (ICF) and fusion-fission (ICFF) with laser and ion beams as drivers, the electronuclear (EN) breeder and the magnetic fusion (MF) and fusion-fission (MFF) reactors. Order of magnitude for the fuel temperature (T), the reactor volume including blanket (V), and the tritium mass (M_t) in the reactor at a given time. α is the fraction of the total electrical output which must be recirculated for the driver.

driver	η(driver)	scheme	K_{tot} (energy gain)†	α	T (eV)	V (m^3)	M_t (gr)
accelerator	0.6	μcFF	13.2	0.13	0.1	1	10^3
accelerator	0.6	EN	8.4	0.2	--	1	0
laser	0.05	ICF / ICFF	$Q \simeq 430$ / $Q \simeq 75$	0.13	10^4	1-10	10^{-2}
accelerator	0.6	ICF / ICFF	$Q \simeq 37$ / $Q \simeq 6.5$	0.13	10^4	1-10	10^{-2}
accelerator, RF, etc.	0.6	MF / MFF	$Q \simeq 37$ / $Q \simeq 6.5$	0.13	10^4	10^2	10^{-2}

† The thermal efficiency included in K_{tot} is assumed to be $\eta_{th} = 0.35$. The number of satellite nuclear plants fueled by the blanket breeder, is taken $X_R = 5$. The value of Q is taken in such a way that $\alpha = 0.13$ in the equation $Q = \frac{1}{\eta_{th}\,\eta_A\,\alpha}$ for pure fusion and $Q = \frac{1}{\eta_{th}\eta_A(1+X_R)\alpha}$ for the fusion-fission schemes.



accelerator
μ^-
$\lambda_a C_d$
$\lambda_a C_t$
$d\mu$
$t\mu$
$\lambda_{dt} C_t$
$\lambda_{dd\mu} C_d$
$\lambda_{dt\mu} C_d$
$\lambda_{tt\mu} C_t$
$dd\mu$
$dt\mu$
$tt\mu$
$\frac{1}{2}\lambda_{fd} W_d$
$\frac{1}{2}\lambda_{fd}(1-W_d)$
$\frac{\lambda_{fd}}{2} W_d'$
$\frac{\lambda_{fd}}{2}(1-W_d')$
$(^3He\mu)+n$
$^3He+n+\mu^-$
$(t\mu)+p$
$t+p+\mu^-$
$\lambda_f W_s$
$\lambda_f(1-W_s)$
$(^4He\mu)+n$
$^4He+n+\mu^-$
$\lambda_{ft} W_t$
$\lambda_{ft}(1-W_t)$
$(^4He\mu)+2n$
$^4He+2n+\mu^-$

Fig. 1

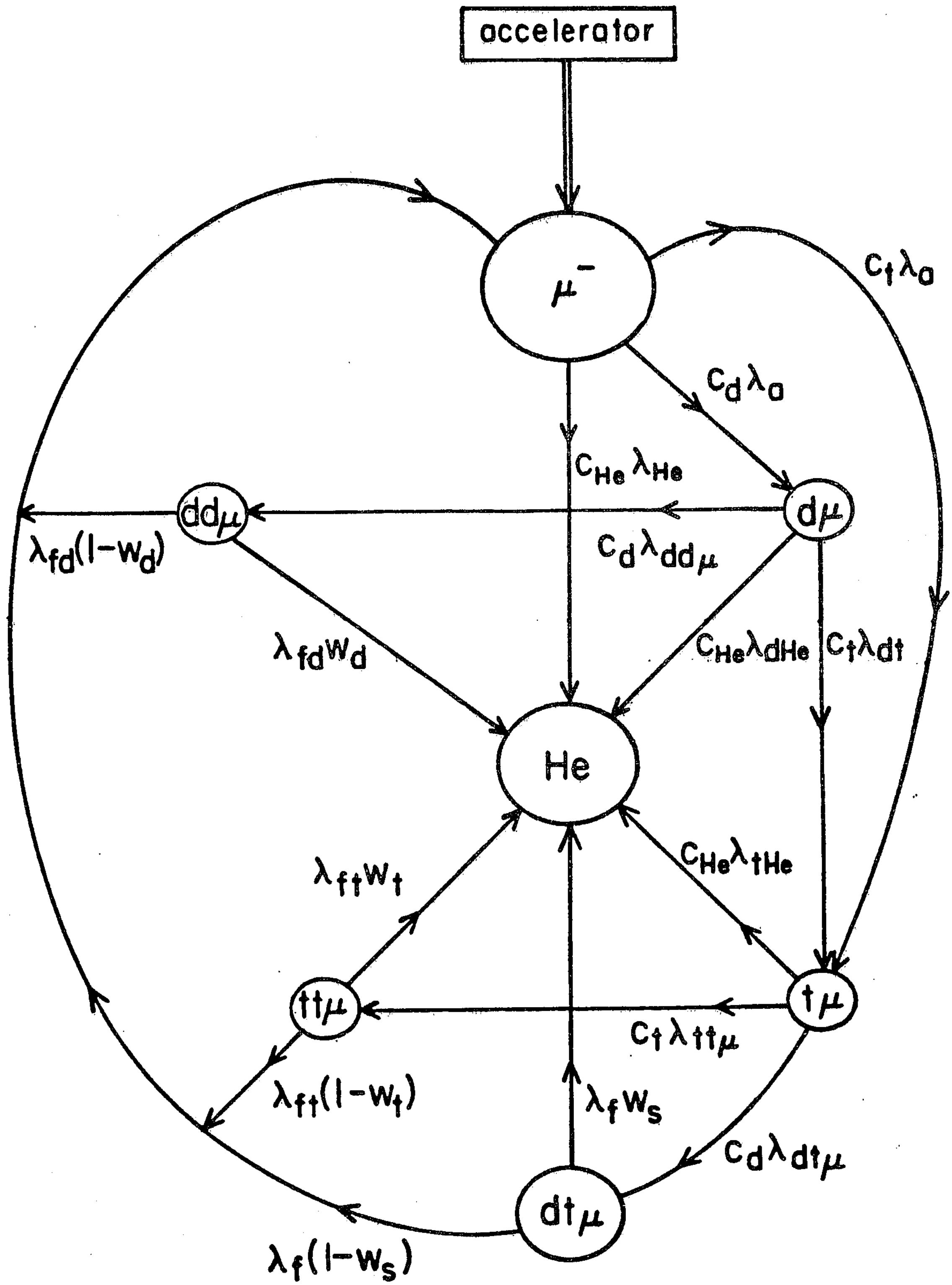

accelerator
μ^-
$c_t\lambda_a$
$c_d\lambda_a$
$c_{He}\lambda_{He}$
$dd\mu$
$d\mu$
$\lambda_{fd}(1-w_d)$
$c_d\lambda_{dd\mu}$
$\lambda_{fd}w_d$
$c_{He}\lambda_{dHe}$
$c_t\lambda_{dt}$
He
$\lambda_{ft}w_t$
$c_{He}\lambda_{tHe}$
$tt\mu$
$t\mu$
$c_t\lambda_{tt\mu}$
$\lambda_{ft}(1-w_t)$
$\lambda_f w_s$
$c_d\lambda_{dt\mu}$
$dt\mu$
$\lambda_f(1-w_s)$

Fig. 2

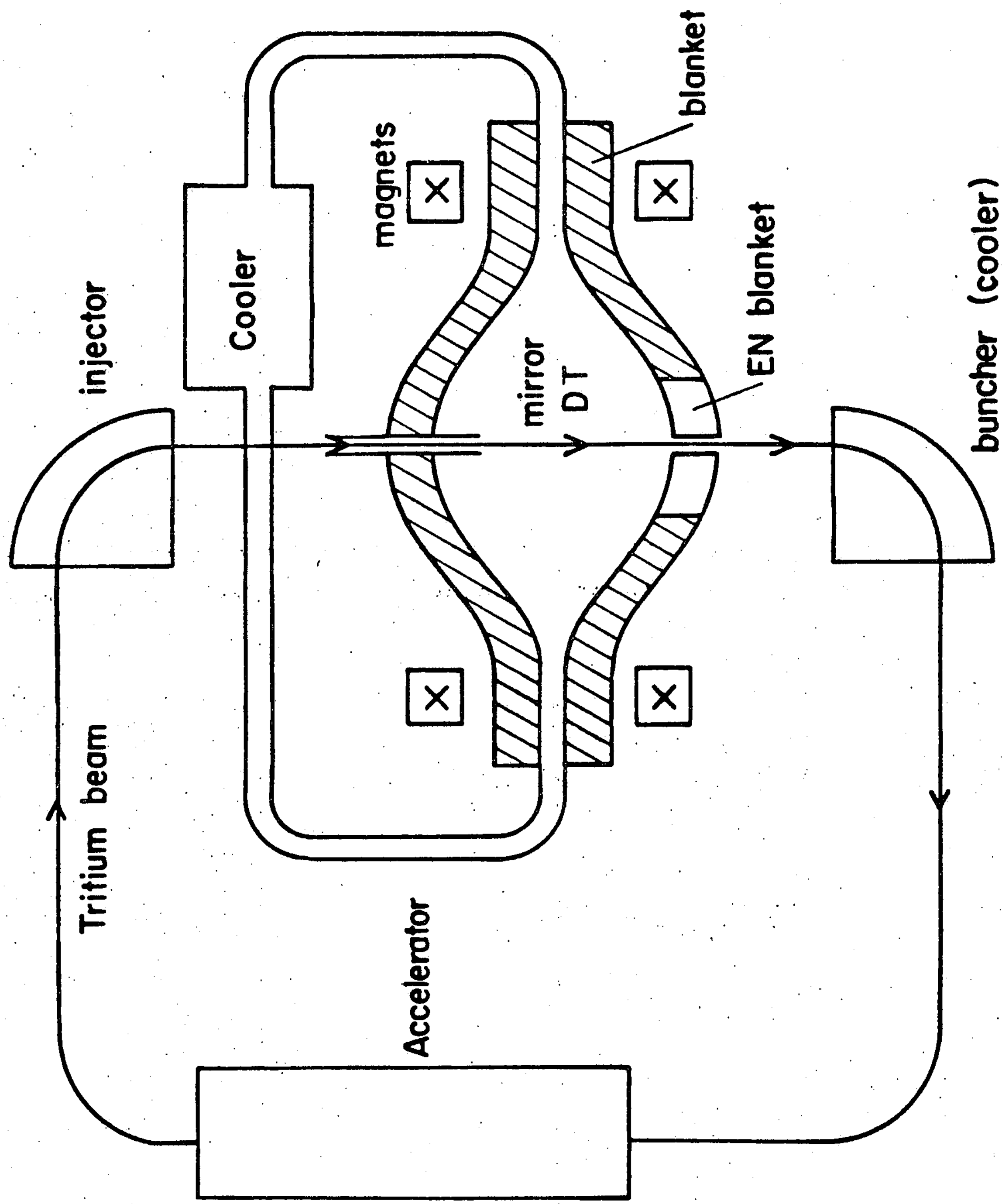

injector
Cooler
magnets
blanket
mirror
DT
EN blanket
buncher (cooler)
Tritium beam
Accelerator

Fig. 3

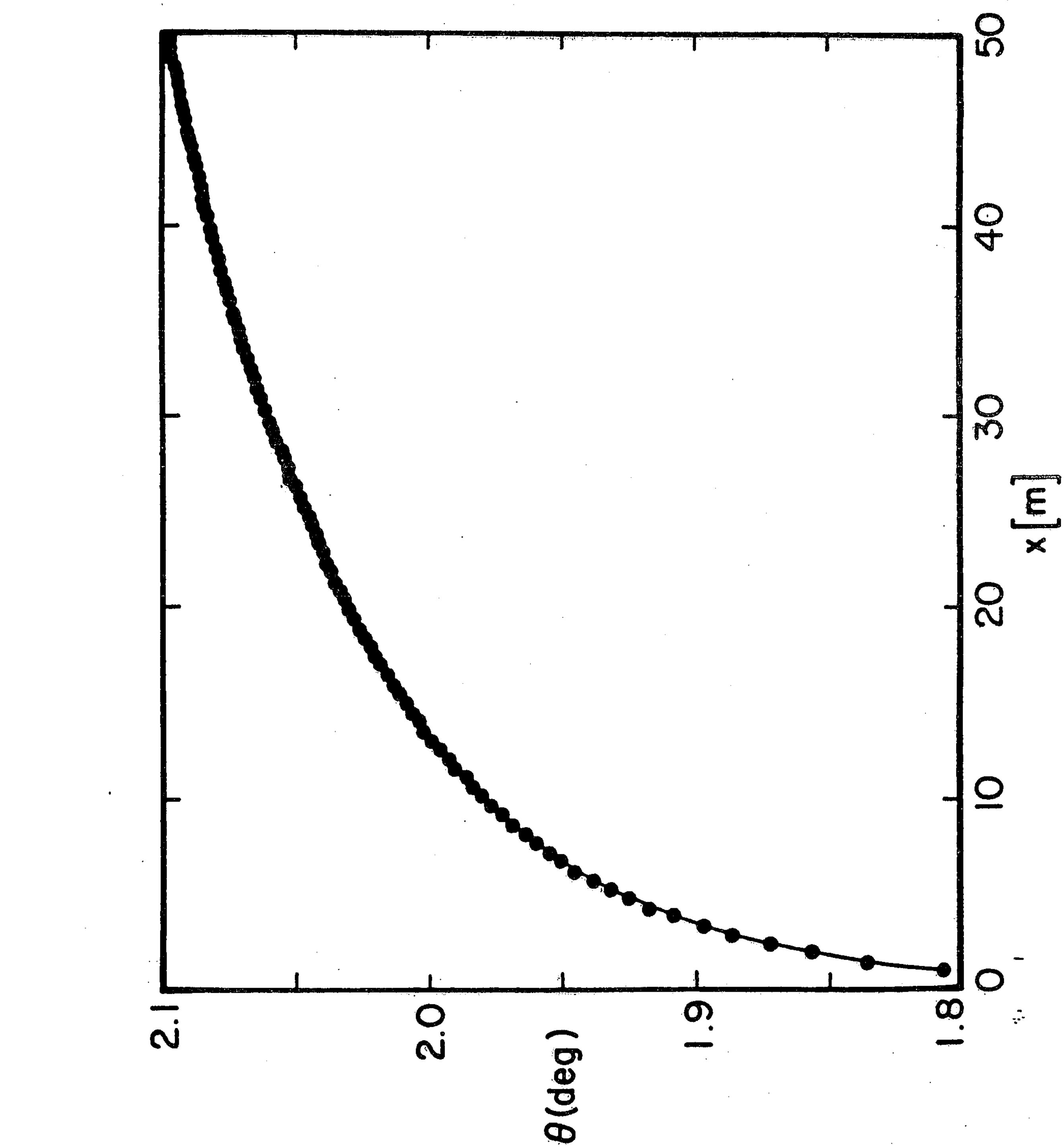
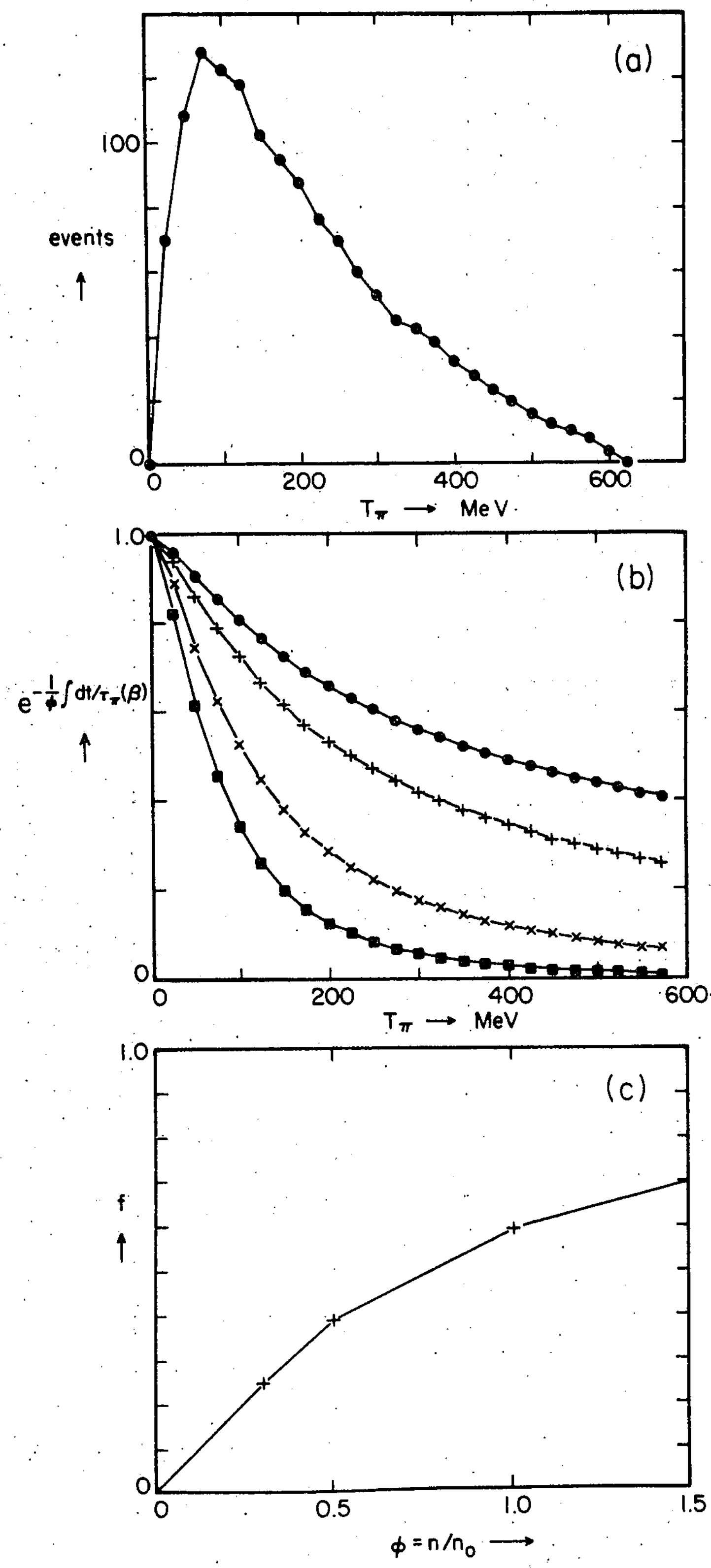


Fig. 4


(a)
100
0
events
0
200
400
600
T_π → MeV
1.0
(b)
0
0
200
400
600
T_π → MeV
(c)
1.0
f
0
0
0.5
1.0
1.5
φ = n/n_0 →

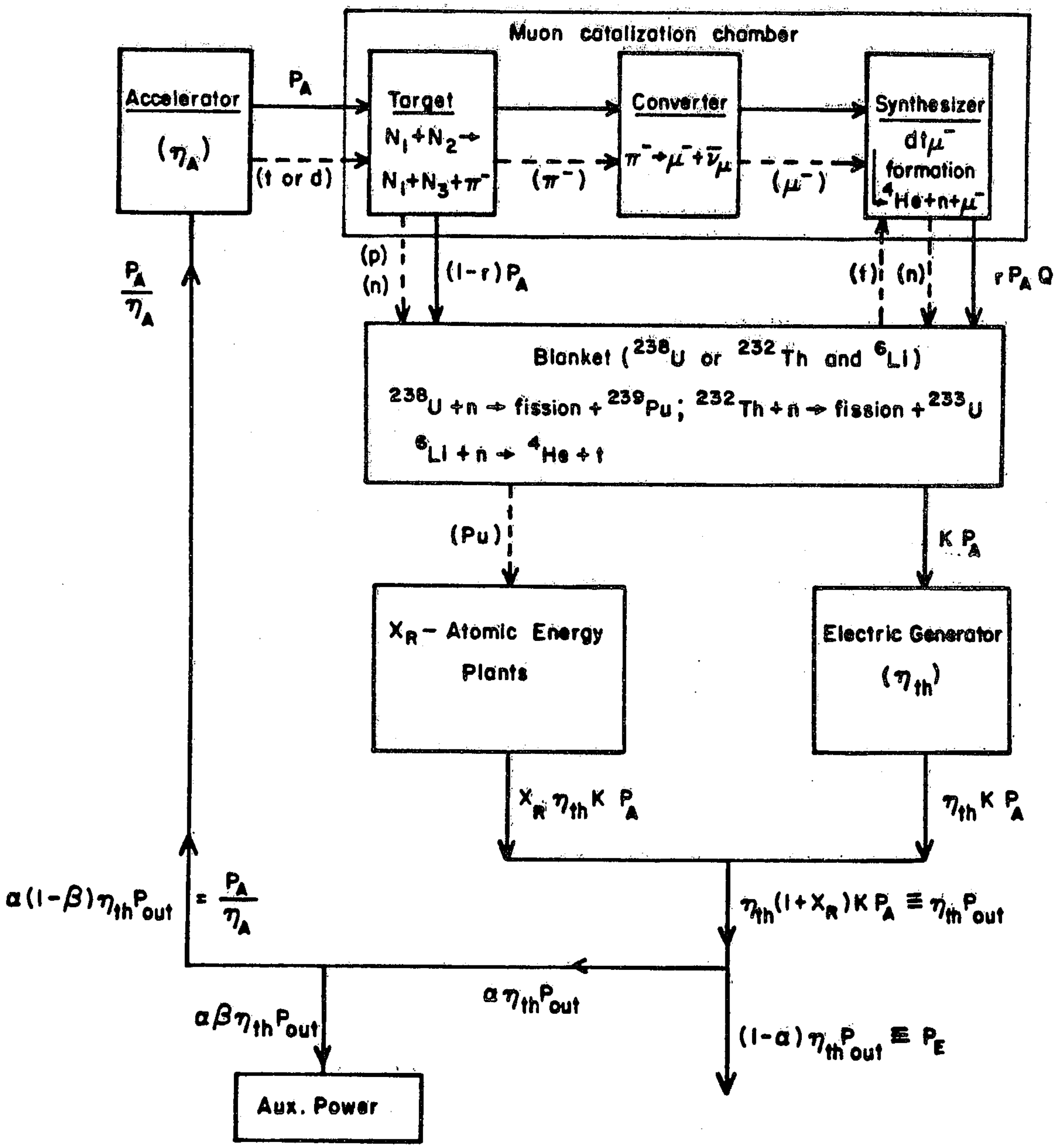

Muon catalization chamber
Accelerator
(η_A)
P_A
(t or d)
Target
$N_1+N_2 \rightarrow$
$N_1+N_3+\pi^-$
(π^-)
Converter
$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
(μ^-)
Synthesizer
$dt\mu^-$
formation
$^4He+n+\mu^-$
(p)
(n)
$(1-r)P_A$
(t)
(n)
rP_AQ
$\frac{P_A}{\eta_A}$
Blanket (^{238}U or ^{232}Th and ^6Li)
$^{238}U + n \rightarrow$ fission $+ ^{239}Pu$; $^{232}Th + n \rightarrow$ fission $+ ^{233}U$
$^6Li + n \rightarrow {}^4He + t$
(Pu)
KP_A
X_R – Atomic Energy Plants
Electric Generator
(η_{th})
$X_R \eta_{th} K P_A$
$\eta_{th} K P_A$
$\eta_{th}(1+X_R)KP_A \equiv \eta_{th}P_{out}$
$\alpha(1-\beta)\eta_{th}P_{out} = \frac{P_A}{\eta_A}$
$\alpha\eta_{th}P_{out}$
$\alpha\beta\eta_{th}P_{out}$
$(1-\alpha)\eta_{th}P_{out} \equiv P_E$
Aux. Power

Fig. 6