DOE/ET-53088-222

IFSR #**222**

The Intrinsic Electromagnetic Solitary Vortices in Magnetized Plasma

J. Liu and *W. Horton*

Department of Physics
and
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712-1060

January 1986

The Intrinsic Electromagnetic Solitary Vortices in Magnetized Plasma

Jixing Liu and Wendell Horton

Department of Physics

and

The Institute for Fusion Studies

The University of Texas at Austin, Austin, Texas 78712

Abstract

Several Rossby type vortex solutions constructed for electromagnetic perturbations in magnetized plasma encounter the difficulty that the perturbed magnetic field and the parallel current are not continuous on the boundary between two regions. We find that fourth order differential equations must be solved to remove this discontinuity. Special solutions for two types of boundary value problems for the fourth order partial differential equations are presented. By applying these solutions to different nonlinear equations in magnetized plasma, the intrinsic electromagnetic solitary drift-Alfven vortex (along with solitary Alfven vortex) and the intrinsic electromagnetic solitary electron vortex (along with short-wavelength drift vortex) are constructed. While still keeping a localized dipole structure, these new vortices have more complicated radial structures in the inner and outer regions than the usual Rossby-wave vortex. The new type of vortices guarantees the continuity of the perturbed magnetic field $\delta\mathbf{B}_\perp$ and the parallel curent $j_{||}$ on the boundary between inner and outer regions of the vortex. The allowed regions of propagation speeds for these vortices are analyzed, and we find that the complementary relation between the vortex propagating speeds and the corresponding phase velocities of the linear modes no longer exists.

1. Introduction

Recently, the solitary vortex solution obtained by Larichev and Reznik (1976) in study of nonlinear Rossby waves in rotating fluids has been extensively applied to magnetized plasma. Along with the solitary vortex solutions for electrostatic perturbation the solitary vortex solutions for electromagnetic perturbations has been disscussed (Mikhailovskii et al. 1984_a;1985; Shukla et al. 1985_a; 1985_b; Hazeltine et al. 1985). In works by Mikhailovskii et al (1984_a) and by Shukla et al (1985_a) Alfven vortices have been constructed. In work by Shukla et al (1985_b) the drift-Alfven vortex has been analyzed. In work by Mikhailovskii et al (1985) two other nonlinear equations of electromagnetic pertubations in magnetized plasma have been derived, and the possibility of the existence of solitary vortex solutions for these equations was disscussed without presentation of explicit solutions. In work by Hazeltine et al (1985) a natural and simple electromagnetic generalization of drift-solitary wave for certain nonlinear equations has been suggested.

Although the above mentioned Rossby type vortex solutions obtained for electromagnetic perturbations exhibit attractive features such as localization and coherent structure, a close examination of the vortex solutions given by Mikhailovskii et al (1984_a) and by Shukla et al (1985_a) reveals that the perturbed magnetized field $\delta\mathbf{B}_\perp$ and the parallel current $j_{||}$ are discontinuous on the boundary between the inner and outer regions of the vortex (see **Appendix A**). These discontinuities do not appear in the solution given by Shukla et al (1985_b), but only because that solution is an approximate one. We can show that the exact Rossby type vortex solution of the same nonlinear equations used by Shukla et al (1985_b) also has the same difficulty (see **Appendix B**). Furthermore, for the nonlinear equations derived by Mikhailovskii et al (1985) we also can construct Rossby type vortex solutions (see section **4.1**), and they have the same difficulty. Based on these observations, it seems to us that the discontinuity difficulty related with the Rossby type vortex solutions of electromagnetic perturbations is a basic problem, and that the construction of a new type of solitary vortices without this difficulty for electromagnetic perturbations is necessary.

The purpose of this work is twofold. First, we construct a new type of solitary vortex solution for the same problems analyzed in works by Mikhailovskii et al (1984_a) and by Shukla et al (1985_a; 1985_b) which overcomes the discontinuity difficulty. Second, we give explicit solitary vortex solutions for two sets of unsolved nonlinear equations for electromagnetic perturbation derived by Mikhailovskii et al (1985).

The essential steps for deriving the Rossby type vortex of an appropriate nonlinear equation are first to find the corresponding linear differential equation, and then to solve the latter in separated regions requiring localization of the solution and continuity of the solution and its derivatives on the boundary between the two regions. It is well known that the original Rossby wave vortex solution in a rotating fluid corresponds to a second order linear differential equation. Our examination of the solutions given previously (Mikhailovskii et al. 1984_a; Shukla et al. 1985_a; 1985_b) shows that these solutions in fact also correspond to the second order differential equation obtained by forcing two coupled second order equations for two independent physical fields to be identical through a linear algebraic relation between the two fields. We believe that the discontinuity problem comes from this additional requirement. Abandoning this requirement leads to a linear equation which is fourth order. Hence, to construct the new type of vortex solutions for electromagnetic perturbations, we have to solve the characteristic fourth order linear differential equation corresponding to the nonlinear equation.

Limiting our study to dipolar vortex solutions, the characteristic fourth order differential equations corresponding to the nonlinear problems treated previously (Mikhailovskii et al. 1984_a, 1985; Shukla et al. 1985_a; 1985_b) with the requirements of localization and regularity for the solutions give rise to two types of boundary value problems in the inner and outer regions. We find that both these problems can be solved analytically. By using these solutions we can construct the new type of solitary vortices. To distinguish the new vortices from the Rossby wave vortices, henceforth we call them intrinsic electromagnetic vortices.

By using the solutions of the first boundary problem we can construct new solutions for both the Alfven vortices and the drift-Alfven vortices. Comparing these new solutions with the corresponding Rossby type solutions, we show that the new solutions have a more complicated structure in the inner region and decay more slowly in the outer region while guaranteeing the continuity of $\delta \mathbf{B}_\perp$ and $j_\parallel$ throughout the structure.

The nonlinear equations for the electromagnetic electron vortices and the electromagnetic short-wavelength drift vortices were derived by Mikhailovskii et al (1985) without giving explicit solutions. We give both the Rossby type and intrinsic electromagnetic vortices solutions for these equations. While the Rossby type vortex solutions still have the surface discontinuities, the intrinsic electromagnetic vortex solutions constructed by using the solutions of the second boundary problem are continuous.

The analysis of the corresponding nonlinear dispersion relations of the intrinsic electromagnetic vortices shows that the allowed regions for vortex propagation speeds, unlike the situation for drift vortices and drift-acoustic vortices (Meiss & Horton, 1983), are not complementary to the allowed regions for phase velocity of the corresponding linear modes.

The arrangement of this work is follows: In section **2** we give the special solutions of two boundary value problems for the fourth order partial differential equations. In section **3** we construct the new type of solution for both Alfven vortices and drift-Alfven vortices. In section **4** we solve the nonlinear equations for electromagnetic electron vortices and electromagnetic short-wavelength drift vortices. Both the Rossby type vortices and intrinsic electromagnetic vortices are given. Section **5** is a discussion and summary. In Appendix **A** we show that for the Alfven vortices constructed previously (Mikhailovskii et al. 1984_a; Shukla et al. 1985_a) the fields $\delta\mathbf{B}_\perp$ and $j_\parallel$ are discontinuous on the boundary between two regions. In Appendix **B** we show that the Rossby type solitary drift-Alfven vortex for the equations used by Shukla et al (1985_b) exhibits the same problem.

2. Special solutions for boundary value problems

In this section we give the special solutions for the two types of two-dimensional boundary value problems arising from the characteristic fourth order partial differential equations. In the following sections we apply these solutions to several nonlinear equations in magnetized plasma and construct the corresponding solitary vortex solutions.

Consider the following two types of boundary value problems for fourth order two-dimensional partial differential equations.

Problem 1:

$$\nabla_\perp^4 \phi(r,\theta) - \alpha_1 \nabla_\perp^2 \phi(r,\theta) = 0, \quad (r > a) \tag{1}$$

$$\nabla_\perp^4 \phi(r,\theta) + \alpha_2 \nabla_\perp^2 \phi(r,\theta) + \alpha_3[\phi(r,\theta) - \alpha_4 r\cos\theta] = 0, \quad (r < a) \tag{2}$$

where $\nabla_\perp^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$ and $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are constants determined by the properties of the system under consideration and a is a constant parameter. Find the localized, continuous solution of ϕ along with the conditions on α_i $(i = 1,2,3,4)$.

Problem 2:

$$\nabla_\perp^4 \phi - \beta_1 \nabla_\perp^2 \phi + \beta_2 \phi = 0 \quad (r > a) \tag{3}$$

$$\nabla_\perp^4 \phi + \beta_3 \nabla_\perp^2 \phi + \beta_4 \phi + \beta_5 r\cos\theta = 0 \quad (r < a) \tag{4}$$

where $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ are constants determined by the system considered and a is a constant parameter. Find the localized, continuous solution of ϕ along with the conditions on β_i (i=1,2,3,4,5).

We limit our study to solutions of the form

$$\phi(r,\theta) = \Phi(r)\cos\theta, \tag{5}$$

for type 1 and type 2 problems with the boundary conditions

$$\phi(r,\theta) \to 0 \text{ as } r \to \infty, \text{ and } \phi \text{ is regular when } r = 0. \tag{6}$$

In the following we give in relative detail the procedure for solving problem 1; while for problem 2, the procedure is similar and we only give the solutions.

Substituting Eq.(5) into (1) and (2) gives

$$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2})[(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - \alpha_1)\Phi(r)] = 0 \ (r > a) \tag{7}$$

and

$$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2})[(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + \alpha_2)\Phi(r)] + \alpha_3[\Phi(r) - \alpha_4 r] = 0 \ (r < a) \tag{8}$$

Due to the property of the operator $\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}$, the general solution of equation (7) must satisfy the second order equation

$$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - \alpha_1)\Phi(r) - f(r) = 0 \tag{9}$$

where

$$f(r) = Ar + Br^{-1}; \quad A, B = constants. \tag{10}$$

The solution of Eq.(9) which satisfies the condition at infinity is

$$\Phi_{out}(r) = A_1 K_1(\sqrt{\alpha_1} r) + A_2 r^{-1}, \tag{11}$$

where K_1 is the first order McDonald function and A_1, A_2 are integration constants. From the reality requirement of $\sqrt{\alpha_1}$ we have

$$\alpha_1 > 0 \tag{12}$$

From the form of Eq.(8) and the boundary condition at the origin, the solution of Eq. (8) can be any one of the following three expressions, depending on the relations between the coefficients α_2 and α_3 in Eq.(2):

$$\Phi_{in}(r) = A_3 J_1(\lambda r) + A_4 I_1(\mu r) + \alpha_4 r, \tag{13.a}$$

$$\Phi_{in}(r) = A_3 J_1(\lambda^{(1)} r) + A_4 J_1(\lambda^{(2)} r) + \alpha_4 r, \tag{13.b}$$

$$\Phi_{in}(r) = A_3 I_1(\mu_1 r) + A_4 I_1(\mu_2 r) + \alpha_4 r, \tag{13.c}$$

where J_1, I_1 are the first order Bessel and modified Bessel functions, respectively, and A_3, A_4 are integration constants. Substituting Eqs. (13.a)-(13.c) into Eq. (8) yields the following conditions corresponding to the different solutions:

Solution (13.a) in which

$$\lambda^2 = \frac{1}{2}(\sqrt{\alpha_2^2 - 4\alpha_3} + \alpha_2), \tag{14.a}$$

$$\mu^2 = \frac{1}{2}(\sqrt{\alpha_2^2 - 4\alpha_3} - \alpha_2). \tag{14.b}$$

corresponds to

$$\alpha_3 < 0 \tag{15}$$

Solution (13.b) in which

$$(\lambda^{(1),(2)})^2 = \frac{1}{2}(\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_3}) \tag{16}$$

corresponds to

$$\alpha_2 > 0, \text{ and } \alpha_2^2 > 4\alpha_3 > 0. \tag{17}$$

Solution (13.c) in which

$$\mu_{1,2}^2 = \frac{1}{2}(-\alpha_2 \pm \sqrt{\alpha_2^2 - 4\alpha_3}) \tag{18}$$

corresponds to

$$\alpha_2 < 0, \text{and } \alpha_2^2 > 4\alpha_3 > 0. \tag{19}$$

It is easy to see that Eqs.(13.a)-(13.c) with corresponding conditions (15), (17), (19) all give inner solutions of Φ which are regular at the origin.

To further determine the solution, we need to impose some matching conditions to match Φ_{out} and Φ_{in} on the common boundary $r = a$. We will do this in Section **3** when we discuss the applications of the solutions.

By analogy, problem 2 can be solved in a similar way. The solution of Eqs. (3), (4) is

$$\Phi_{out} = B_1 K_1(\lambda_1 r) + B_2 K_1(\lambda_2 r), \quad (r > a) \tag{20}$$

and any one of the following three expressions according to the relations between the coefficients in Eq.(4)

$$\Phi_{in} = B_3 J_1(\lambda_3 r) + B_4 I_1(\lambda_4 r) - \frac{\beta_5}{\beta_4} r; \tag{21.a}$$

$$\Phi_{in} = B_3 J_1(\lambda_3^{(1)} r) + B_4 J_1(\lambda_3^{(2)} r) - \frac{\beta_5}{\beta_4} r; \tag{21.b}$$

$$\Phi_{in} = B_3 I_1(\lambda_4^{(1)} r) + B_4 I_1(\lambda_4^{(2)} r) - \frac{\beta_5}{\beta_4} r, \tag{21.c}$$

where

$$\lambda_{1,2}^2 = \frac{1}{2}(\beta_1 \pm \sqrt{\beta_1^2 - 4\beta_2}), \tag{22}$$

$$\lambda_{3,4}^2 = \frac{1}{2}(\sqrt{\beta_3^2 - 4\beta_4} \pm \beta_3), \tag{23.a}$$

$$(\lambda_3^{(1),(2)})^2 = \frac{1}{2}(\beta_3 \pm \sqrt{\beta_3^2 - 4\beta_4}), \tag{23.b}$$

$$(\lambda_4^{(1),(2)})^2 = \frac{1}{2}(-\beta_3 \pm \sqrt{\beta_3^2 - 4\beta_4}), \tag{23.c}$$

and $B_1,\ B_2,\ B_3,\ B_4$ are integration constants.

The corresponding conditions on coefficients β_i $(i = 1, 2, 3, 4, 5)$ are:

$$\beta_1 > 0, \text{ and } \beta_1^2 > 4\beta_2 > 0; \tag{24}$$

$$\beta_4 < 0, \quad \text{for solution (21.a);} \qquad (25.a)$$

$$\beta_3 > 0, \text{ and } \beta_3^2 > 4\beta_4 > 0, \quad \text{for solution (21.b);} \qquad (25.b)$$

$$\beta_3 < 0, \text{ and } \beta_3^2 > 4\beta_4 > 0, \quad \text{for solution (21.c).} \qquad (25.c)$$

As in problem 1, we leave the determination of the integration constants to the following sections where the continuity conditions for the relevant physical quantities are given.

Comparison between the solutions (11), (13) of problem 1 and solutions (20), (21) of problem 2 shows that the inner solutions of the two problems are quite similar, but the outside solutions are very different. Problem 1 has a long tail solution while the solution of problem 2 decreases to zero very rapidly when $r \to \infty$.

It is interesting to note that the solutions of these two problems are localized dipole solutions like the Rossby wave vortices (Larichev & Reznik, 1976) or drift wave vortices (Meiss & Horton, 1983) obtained from solving the corresponding one-potential equivalent linear equation of the nonlinear Rossby wave equation or nonlinear drift wave equation. However, the radial structure of solutions given here are very different from the Rossby or drift vortex solutions because the equations considered here are fourth order equations while the Rossby wave or drift wave problems correspond to second order equation,

The results presented above show that while the outer solutions for both problems 1 and 2 are uniquely given by Eqs.(11) and (20) the inner solutions for them can be either one of Eqs.(13.a)-(13.c) and (21.a)-(21.c) according to the different relations between the coefficients in Eq.(2) and in Eq.(4), respectively. In the following sections, we limit our analysis to solution (13.a) for problem 1 and to solution (21.a) for problem 2 with the corresponding conditions Eq.(15) and Eq.(25.a). The treatment of the other cases is similar.

3.The intrinsic Alfven and intrinsic drift-Alfven vortices

In this section we apply solutions of the first boundary problem given in the previous section to construct the intrinsic drift-Alfven vortices. The intrinsic Alfven vorices may be obtained from this solution in the limit of negligible plasma inhomogeneity.

We consider the plasma which is low β ($8\pi n_o T_e/B_o^2 \ll 1$), inhomogeneous ($n_o = n_o(x)$), isothermal ($T_i, T_e = const$), cold ($T_i = 0$), and immersed in a strong, uniform, constant magnetic field $\mathbf{B_o} = B_o\hat{\mathbf{z}}$. One can choose the three perturbed fields describing the system to be parallel component of vector potential $A_\parallel(x,y,z,t)$ which describes the magnetic field line perturbation $\delta\mathbf{B}_\perp(x,y,z,t)$ perpendicular to $\mathbf{B}_o$, the electrostatic potential pertubation $\phi(x,y,z,t)$, and the normalized density perturbation $\tilde{n}(x,y,z,t) \equiv \delta n(x,y,z,t)/n_o$. Using two fluid model, neglecting the electron mass effect and the parallel motion of the ions, one can use the quasineutrality condition with Ampere's law, the continuity equation and the parallel momentum balance equation of the electrons to describe the low-frequency dynamics of the plasma system. These equations are

$$\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}\nabla_\perp^2\phi + \frac{c_A^2}{c}\{\partial_z - \frac{1}{B_o}[A_\parallel,\ ]\}\nabla_\perp^2 A_\parallel = 0, \tag{26}$$

$$\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}\tilde{n} + \frac{c}{4\pi n_o e}\{\partial_z - \frac{1}{B_o}[A_\parallel,\ ]\}\nabla_\perp^2 A_\parallel + \frac{c\kappa_n}{B_o}\partial_y\phi = 0, \tag{27}$$

$$\frac{1}{c}\partial_t A_\parallel + \{\partial_z - \frac{1}{B_o}[A_\parallel,\ ]\}(\phi - \frac{T_e}{e}\tilde{n}) + \frac{T_e\kappa_n}{eB_o}\partial_y A_\parallel = 0 \tag{28}$$

where

$$\nabla_\perp^2 \equiv \partial_x^2 + \partial_y^2$$

$$[f,g] \equiv \partial_x f \partial_y g - \partial_y f \partial_x g, \ \textit{Poisson bracket}$$

$$c \equiv \textit{velocity of light}$$

$$c_A \equiv \sqrt{\frac{B_o^2}{4\pi n_o m_i}}, \ \textit{Alfven velocity}$$

$$m_i \equiv \textit{mass of ion}$$

$$n_o(x) \equiv \textit{equilibrium number density of ions and electrons}$$

$$e \equiv \textit{electron charge}$$

$$T_e \equiv \textit{electron temperature}$$

$$\kappa_n \equiv -\frac{d \ln n_o}{dx}.$$

Here in equations (26)-(28) we include the nonlinear terms that arise from the ion polarization nonlinearity, the $\mathbf{E} \times \mathbf{B}_o$ drift fluid convection, and the coupling of the perturbed magnetic field with the parallel electron flow. For the homogeneous plasma Eqs.(26)-(28) are equivalent to the nondissipative version of the inclusive nonlinear equations derived by Hazeltine (1983).

For convenience we introduce a scalar potential function ψ which describes the axial perturbed electric field such that

$$E_z(x,y,z,t) = -\partial_z \psi.$$

Then we find the relation between $A_\|$ and ϕ, ψ to be

$$\frac{1}{c}\partial_t A_\| = -\partial_z(\phi - \psi) \tag{29}$$

Suppose that the perturbation functions ϕ, ψ, and $\tilde{n}$ are in the form of a travelling disturbance

$$\begin{aligned} \phi(x,y,z,t) &= \phi(x,\eta) \\ \psi(x,y,z,t) &= \psi(x,\eta) \\ \tilde{n}(x,y,z,t) &= \tilde{n}(x,\eta) \end{aligned} \tag{30}$$

where

$$\eta = y + \alpha z - ut, \quad u, \alpha = consts.$$

Substituting (29),(30) into (26)-(28) yields

$$\hat{L}_1 \nabla_\perp^2 \phi - \frac{c_A^2 \alpha^2}{u^2} \hat{L}_2 \nabla_\perp^2 (\phi - \psi) = 0, \tag{31}$$

$$\hat{L}_1 \tilde{n} - \frac{e}{T_e}\frac{\alpha^2 c_A^2}{u^2} \rho_s^2 \hat{L}_2 \nabla_\perp^2 (\phi - \psi) - \frac{e}{T_e}\frac{v_{de}}{u}\partial_\eta \phi = 0, \tag{32}$$

$$\hat{L}_2(\psi - \frac{T_e}{e}\tilde{n}) + \frac{v_{de}}{u}\partial_\eta(\phi - \psi) = 0, \tag{33}$$

where

$$\hat{L}_1 = \partial_\eta - \frac{c}{B_o u}[\phi, \ ]$$

$$\hat{L}_2 = \partial_\eta - \frac{c}{B_o u}[\phi - \psi, \ ]$$

$$v_{de} \equiv \frac{cT_e\kappa_n}{eB_o}, \textit{diamagnetic drift velocity of electrons}$$

$$\rho_s^2 \equiv \frac{T_e}{m_i \omega_{ci}^2}$$

$$\omega_{ci} \equiv \frac{eB_o}{m_i c}, \; \textit{ion gyrofrequency.}$$

Eq.(33) has the exact solution

$$\frac{T_e}{e}\tilde{n} = \psi + \frac{v_{de}}{u}(\phi - \psi). \tag{34}$$

Substituting (34) into (32) and eliminating $\tilde{n}$ gives

$$\hat{L}_2\{\psi - \rho_s^2 \frac{\alpha^2 c_A^2}{u(u - v_{de})} \nabla_\perp^2(\phi - \psi)\} = 0 \tag{35}$$

Using the identity $\hat{L}_1\psi = \hat{L}_2\psi$ and substituting (35) into (31) yields

$$\hat{L}_1\{\psi - \rho_s^2(1 - \frac{v_{de}}{u})^{-1}\nabla_\perp^2\phi\} = 0. \tag{36}$$

By using the property of the Poisson bracket we can write Eqs. (35),(36) as

$$[\phi - \psi - \frac{B_o u}{c}x, \; \psi - \rho_s^2 \frac{\alpha^2 c_A^2}{u(u - v_{de})} \nabla_\perp^2(\phi - \psi)] = 0. \tag{37}$$

$$[\phi - \frac{B_o u}{c}x, \; \psi - \rho_s^2(1 - \frac{v_{de}}{u})^{-1}\nabla_\perp^2\phi] = 0, \tag{38}$$

The general solutions of nonlinear equations (37), (38) leads to

$$\psi - \frac{\alpha^2 c_A^2}{u(u - v_{de})}\rho_s^2 \nabla_\perp^2(\phi - \psi) = f_1(\phi - \psi - \frac{B_o u}{c}x), \tag{39}$$

$$\psi - \rho_s^2 (1 - \frac{v_{de}}{u})^{-1} \nabla_\perp^2 \phi = f_2(\phi - \frac{B_o u}{c} x), \tag{40}$$

where $f_1(w), f_2(w)$ are arbitrary nonsingular functions of their arguments. In a way similar to the construction of the Rossby wave vortices (Larichev & Reznik, 1976; Meiss & Horton, 1983), we only consider the case where f_1, f_2 are linear functions of their arguments

$$f_1(w) = C_1 w; \quad f_2(w) = C_2 w. \tag{41}$$

Following the standard procedure (Larichev & Reznik, 1976; Meiss & Horton, 1983) we use the polar coordinates defined by

$$r = \sqrt{x^2 + \eta^2}, \qquad \theta = \tan^{-1} \frac{\eta}{x}$$

and look for the solutions of Eqs.(39),(40) in two regions divided by a circle $r = a$ in the $r - \theta$ plane: the inner region ($r < a$) and the outer region ($r > a$).

From the requirement of locality, we impose the conditions

$$\phi \to 0, \ \psi \to 0, \quad \text{as } r \to \infty \tag{42}$$

to the solutions in the outside region. From the requirement of regularity we impose the conditions

$$\phi, \ \psi \text{ finite when } r = 0, \tag{43}$$

to the solution in the inner region.

Condition (42) implies that for the outer region, after substituting Eq.(41) into equations (39), (40),C_1 and C_2 must be zero. Therefore, we have

$$\psi - (1 - \frac{v_{de}}{u})^{-1}(\frac{\alpha c_A}{u})^2 \rho_s^2 \nabla_\perp^2 (\phi - \psi) = 0 \tag{44}$$

$$\psi - (1 - \frac{v_{de}}{u})^{-1} \rho_s^2 \nabla_\perp^2 \phi = 0 \tag{45}$$

$$(r > a)$$

For the inner region, the only physical requirement imposed on the solution is (43), which does not imply an explicit requirement on the choice of integation constants C_1 and C_2. For simplicity we choose

$$C_1 = 0, \qquad C_2 \neq 0. \tag{46}$$

in the inner region.

For this choice Eqs.(39), (40) become

$$\psi - (1 - \frac{v_{de}}{u})^{-1}(\frac{\alpha c_A}{u})^2 \rho_s^2 \nabla_\perp^2 (\phi - \psi) = 0 \tag{47}$$

$$\psi - (1 - \frac{v_{de}}{u})^{-1} \rho_s^2 \nabla_\perp^2 \phi = C_2(\phi - \frac{B_o u}{c} r \cos\theta) \tag{48}$$

$$(r < a).$$

Eliminating ψ from Eqs. (44),(45) and (47),(48), we have fourth order equations for ϕ in both the inner and outer regions:

$$\nabla_\perp^4 \phi - \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})(1 - (\frac{u}{\alpha c_A})^2) \nabla_\perp^2 \phi = 0, \ (r > a) \tag{49}$$

$$\nabla_\perp^4\phi + \frac{1}{\rho_s^2}(1-\frac{v_{de}}{u})[(\frac{u}{\alpha c_A})^2 - 1 + C_2]\nabla_\perp^2\phi +$$

$$+\frac{1}{\rho_s^4}C_2(1-\frac{v_{de}}{u})^2(\frac{u}{\alpha c_A})^2(\phi - \frac{B_o u}{c} r\cos\theta) = 0. \ (r < a) \qquad (50)$$

Equations (49), (50) with boundary conditions (42), (43) form a boundary value problem which belongs to the type 1 problem we solved in section **2**. As we mentioned at the end of previous section, here we use (13.a) with the corresponding condition (15) as the inner solution. Substituting the corresponding coefficients of (49), (50) into (11), (13.a), (14.a)- (14.b) and using Eqs. (45),(48) gives the special solutions of the nonlinear equations (35), (36) under assumptions (5), (15),(41), (46) as

$$\begin{aligned} \phi_{out} &= \{A_1 K_1(\nu r) + A_2 \frac{1}{r}\}\frac{B_o u}{c} a\cos\theta, \quad (r > a) \\ \phi_{in} &= \{A_3 J_1(\lambda r) + A_4 I_1(\mu r) + \frac{r}{a}\}\frac{B_o u}{c} a\cos\theta, \quad (r < a) \end{aligned} \qquad (51)$$

and

$$\begin{aligned} \psi_{out} &= A_1[1 - (\frac{u}{\alpha c_A}^2)]K_1(\nu r)\frac{B_o u}{c} a\cos\theta, \quad (r > a) \\ \psi_{in} &= \{(C_2 - \frac{\rho_s^2\lambda^2}{1 - v_{de}/u})A_3 J_1(\lambda r) + (C_2 + \frac{\rho_s^2\mu^2}{1 - v_{de}/u})A_4 I_1(\mu r)\}\frac{B_o u}{c}\cos\theta, \ (r < a) \end{aligned} \qquad (52)$$

where

$$\nu^2 = \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})[1 - (\frac{u}{\alpha c_A})^2], \qquad (53)$$

$$\lambda^2 = \frac{1 - v_{de}/u}{2\rho_s^2}\Big\{\{[C_2 - 1 + (\frac{u}{\alpha c_A})^2]^2 - 4(\frac{u}{\alpha c_A})^2\}^{\frac{1}{2}} + [C_2 - 1 + (\frac{u}{\alpha c_A})^2]\Big\}, \quad (54)$$

$$\mu^2 = \frac{1 - v_{de}/u}{2\rho_s^2}\Big\{\{[C_2 - 1 + (\frac{u}{\alpha c_A})^2]^2 - 4(\frac{u}{\alpha c_A})^2\}^{\frac{1}{2}} - [C_2 - 1 + (\frac{u}{\alpha c_A})^2]\Big\}. \quad (55)$$

Eqs. (51)-(52) represent the localized perturbations with dipole structure. As we mentioned in the end of previous section,we still need to determine the integration constants $A_1,\ A_2,\ A_3,\ A_4$ by matching the solutions in both regions on the border $r = a$. For solutions with continuous density, flow velocity, magnetic field, vorticity and current density, we impose following matching conditions:

$$\phi_{in}\ |_{r=a} = \phi_{out}\ |_{r=a}, \quad (56.a)$$

$$\frac{\partial \phi_{in}}{\partial r}\ |_{r=a} = \frac{\partial \phi_{out}}{\partial r}\ |_{r=a}, \quad (56.b)$$

$$\nabla_\perp^2 \phi_{in}\ |_{r=a} = \nabla_\perp^2 \phi_{out}\ |_{r=a}, \quad (56.c)$$

and

$$\psi_{in}\ |_{r=a} = \psi_{out}\ |_{r=a}, \quad (57.a)$$

$$\frac{\partial \psi_{in}}{\partial r}\ |_{r=a} = \frac{\partial \psi_{out}}{\partial r}\ |_{r=a}, \quad (57.b)$$

$$\nabla_\perp^2 \psi_{in}\ |_{r=a} = \nabla_\perp^2 \psi_{out}\ |_{r=a}\ . \quad (57.c)$$

Substituting (51)-(52) to (56.a)-(57,a) gives

$$A_1 = 2\frac{\lambda^2+\mu^2}{K_1(\nu a)\Delta}, \tag{58}$$

$$A_2 = (1-2\frac{\lambda^2+\mu^2}{K_1(\nu a)\Delta})a, \tag{59}$$

$$A_3 = -2\frac{\nu^2}{J_1(\lambda a)\Delta}, \tag{60}$$

$$A_4 = 2\frac{\nu^2}{I_1(\mu a)\Delta}, \tag{61}$$

where

$$\Delta = (\lambda^2+\mu^2)[2-\frac{\nu a K_2(\nu a)}{K_1(\nu a)}] - \nu^2[\frac{\lambda a J_2(\lambda a)}{J_1(\lambda a)} + \frac{\mu a I_2(\mu a)}{I_1(\mu a)}]. \tag{62}$$

According to the choice of the integration constants C_1, C_2 in (46), from Eqs.(44), (47) we can see that the condition (57.c) is satisfied automatically after conditions (56.a)-(57.a) are satisfied.

The matching condition (57.b) gives a relation between parameters ν, λ, μ

$$(\mu^2+\lambda^2)\nu\frac{K_2(\nu a)}{K_1(\nu a)} + (\nu^2-\mu^2)\lambda\frac{J_2(\lambda a)}{J_1(\lambda a)} + (\nu^2+\lambda^2)\mu\frac{I_2(\mu a)}{I_1(\mu a)} = 0. \tag{63}$$

Eq.(63) can be considered as the generalization of the similar relation found in the Rossby vortex solution (e.g, Eq.(B.26) in Appendix **B**), and, in fact, it is a condition to determine C_2 in terms of u, a, α.

At this stage the special solutions of nonlinear equation (35), (36) are given, they contain three free parameters u, a, α. All other parameters A_i, μ, λ, ν are now determined.

For this new solitary vortex solution ϕ and ψ are continuous on the boundary up to their second derivatives, therefore both the perturbed perpendicular magnetic field

$$\delta\mathbf{B}_\perp = \nabla_\perp A_\| \times \hat{\mathbf{z}} = -\frac{\alpha c}{u}\hat{\mathbf{z}} \times \nabla_\perp(\phi - \psi)$$

and the parallel current

$$j_\| = -\frac{c}{4\pi}\nabla_\perp^2 A_\| = -\frac{c^2\alpha}{4\pi u}\nabla_\perp^2(\phi - \psi)$$

are continuous cross the boundary $r = a$. The discontinuity encountered in the Rossby type vortices is eliminated in these newly constructed vortices. We call them the intrinsic drift-Alfven vortices.

The constant parameter u which describes the propagation speed of the vortices is constrained by the the nonlinear dispersion relation of the vortices

$$\nu^2 = (1 - \frac{v_{de}}{u})[1 - (\frac{u}{\alpha c_A})^2]/\rho_s^2 > 0, \tag{64}$$

and further restricted by the condition

$$C_2(1 - \frac{v_{de}}{u})(\frac{u}{\alpha c_A})^2 < 0 \tag{65}$$

which arises from the choice of the inner solution form.

At the limit $\kappa_n = 0$ the equations describing drift-Alfven vortex reduce to the equations describe Alfven vortex, so after proper change of the coefficients, (51)-(52) also represent the intrinsic Alfven vortices. Accordingly, from (64) the nonlinear dispersion relation for Alfven vortices becomes

$$\alpha^2 c_A^2 - u^2 > 0. \tag{66}$$

which is exactly the same condition given for Rossby type Alfven vortices (Mikhailovskii et al. 1984_a; Shukla et al. 1985_a). The further restriction corresponding to Eq. (65) becomes

$$C_2 (\frac{u}{\alpha C_A})^2 < 0. \tag{67}$$

4. Electromagnetic electron vortices and electromagnetic short-wavelength drift vortices

In this section we analyze two sets of nonlinear equations in magnetized plasma derived by Mikhailovskii et al (1985). Unlike the equations we discussed in section **3**, no explicit electromagnetic vortex solutions for these equations have been presented except for their electrostatic limit (Aburdzaniya et al. 1984; Mikhailovskii et al. 1984_b; 1985). Therefore here we construct the two types of vortex solutions for them: the Rossby type vortices and the intrinsic electromagnetic vortices. As we show later, the Rossby type vortices of these equations also suffer from the discontinuity problem. For the intrinsic electromagnetic vortices constructed by using the solutions of the second boundary value problem given in section **2** the discontinuity is removed.

By considering very different physics, Mikhailovskii et al (1985) derived two sets of nonlinear equations which are remarkably similar. For the details of the derivation and validity condition of the equations we refer to Mikhailovskii et al (1985). In terms of the perturbation fields ϕ and ψ we defined in section **3** the first set of equations which describes the electromagnetic electron vortices is

$$\hat{L}_1 \nabla_\perp^2 \phi - (\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u}) \partial_\eta \phi - (\frac{\alpha c_{Ae}}{u})^2 \hat{L}_2 \nabla_\perp^2 (\phi - \psi) = 0, \tag{68}$$

$$\hat{L}_1 [\nabla_\perp^2 (\phi - \psi) + (\frac{\omega_{pe}}{c})^2 \psi] = 0. \tag{69}$$

where

$$\omega_{pe} \equiv \sqrt{\frac{4\pi n_o e^2}{m_e}}, \quad electron\ plasma\ frequency$$

$$\omega_{ce} \equiv \frac{eB_o}{m_e c}, \quad electron\ gyrofrequency$$

and

$$c_{Ae} \equiv \sqrt{\frac{B_o^2}{4\pi m_e n_o}}, \quad Alfven\ velocity\ calculated\ by\ electron\ mass.$$

Other notations are the same as in section **3**.

The set of equations which describes the electromagnetic short-wavelength drift vortices is

$$\hat{L}_1 \nabla_\perp^2 \phi - \frac{1}{\rho_{ei}^2}(1 - \frac{v_{di}}{u})\frac{\partial \phi}{\partial \eta} - (\frac{\alpha c_A}{u})^2 \hat{L}_2 \nabla_\perp^2 (\phi - \psi) = 0, \tag{70}$$

$$\hat{L}_1 \{\nabla_\perp^2 (\phi - \psi) + (\frac{\omega_{pe}}{c})^2 \psi\} = 0, \tag{71}$$

where

$$v_{di} \equiv -\frac{cT_i \kappa_n}{eB_o}, ion\ diamagnetic\ drift\ velocity,$$

$$\rho_{ei}^2 = \frac{T_i}{m_e \omega_{ce}^2}$$

The similarity between these two sets of equations is obvious: (69) and (71) are the same; (68) and (70) have the same structures, only the constant coefficients of the second and third terms for them are different. In the following we only analyze the first set of equations, the solutions to the second being obtained simply by changing the coefficients.

Following the same procedure given in the section **3**, we can get the equivalent linear equations for (68), (69) as

$$\nabla_\perp^2(\phi - \psi) + \frac{\omega_{pe}^2}{c^2}\psi - C_1(\phi - \frac{B_o u}{c}x) = 0, \tag{72}$$

$$\nabla_\perp^2\phi - (\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n\omega_{ce}}{u})\phi + (\frac{\alpha c_{Ae}}{u})^2(\frac{\omega_{pe}^2}{c^2} - C_1)\psi + C_2(\phi - \frac{B_o u}{c}x) = 0, \tag{73}$$

where C_1, C_2 are integration constants.

In the following we give two types of solitary vortices solutions for Eqs.(72) and (73) with the localization and regularity conditions (42)-(43).

4.1. Rossby type vortices

Similar to the method used in **Appendix B**, in this subsection we give the Rossby type vortex solution of Eqs. (72)-(73). We suppose linear algebraic relations between the two functions ϕ and ψ in both the inner and outer regions as

$$\psi(r,\theta) = a_1\phi(r,\theta) \quad (r > a) \tag{74}$$

$$\psi(r,\theta) = a_2\phi(r,\theta) + a_3\frac{B_o u}{c} r\cos\theta \quad (r < a) \tag{75}$$

Substituting (74)-(75) into (72)-(73) with the requirement that both (72) and (73) give identical equation in the corresponding region yields

$$\nabla_\perp^2\phi - k^2\phi = 0 \quad (r > a) \tag{76}$$

$$\nabla_\perp^2\phi + p^2\phi + q\frac{B_o u}{c} r\cos\theta \quad (r < a) \tag{77}$$

where a_1 is the real root of quadratic equation

$$a_1^2 - \left(1 + \frac{c^2}{\omega_{ce}^2}\left(\frac{u}{\alpha c_A}\right)^2\frac{\kappa_n\omega_{ce}}{u}\right)a_1 - \left(\frac{u}{\alpha c_{Ae}}\right)^2\left(1 - \frac{c^2}{\omega_{ce}^2}\frac{\kappa_n\omega_{ce}}{u}\right) = 0 \tag{78}$$

and

$$k^2 = \frac{\omega_{pe}^2}{c^2}\{1+\frac{1}{2}[(\frac{\alpha c_{Ae}}{u})^2 - \frac{c^2}{\omega_{pe}^2}\frac{\kappa_n\omega_{ce}}{u} \pm \sqrt{((\frac{\alpha c_{Ae}}{u})^2 - \frac{c^2}{\omega_{pe}^2}\frac{\kappa_n\omega_{ce}}{u})^2 + 4(\frac{\alpha c_{Ae}}{u})^2}]\} > 0 \tag{79}$$

$$p^2 = \frac{\omega_{pe}^2}{c^2}\{(\frac{u}{\alpha c_{Ae}})^2\frac{a_2 + a_3 + (1-a_2)(1 - \frac{c^2}{\omega_{pe}^2}\frac{\kappa_n\omega_{ce}}{u})}{(a_2+a_3)(1-a_2)^2} - 1\} > 0 \tag{80}$$

$$q = \frac{\omega_{pe}^2}{c^2}\{\frac{a_3+1}{1-a_2} - (\frac{u}{\alpha c_{Ae}})^2 \frac{a_2+a_3+(1-a_2)(1-\frac{c^2}{\omega_{pe}^2}\frac{\kappa_n \omega_{ce}}{u})}{(a_2+a_3)(1-a_2)^2}\} \tag{81}$$

Following the standard procedure (Larichev & Reznik, 1976; Meiss & Horton, 1983), the solution of (74),(75) is given by

$$\begin{aligned} \phi_{out} &= A_1 \frac{K_1(kr)}{K_1(ka)} \frac{B_o u}{c} a \cos\theta \quad (r > a) \\ \phi_{in} &= \{A_2 \frac{J_1(pr)}{J_1(pa)} \frac{a}{r} - \frac{q}{p^2}\} \frac{B_o u}{c} r \cos\theta \quad (r < a) \end{aligned} \tag{82}$$

Inserting (82) into (74)-(75) gives the solution of ψ as

$$\begin{aligned} \psi_{out} &= a_1 A_1 \frac{K_1(kr)}{K_1(ka)} \frac{B_o u}{c} a \cos\theta \quad (r > a) \\ \psi_{in} &= \{a_2 A_2 \frac{J_1(pr)}{J_1(pa)} \frac{a}{r} + (-a_2 \frac{q}{p^2} + a_3)\} \frac{B_o u}{c} r \cos\theta \quad (r < a) \end{aligned} \tag{83}$$

Matching the inner and outer solutions on the boundary $r = a$ by requiring ϕ to be continuous up to its second derivatives gives

$$A_1 = -\frac{q}{p^2+k^2} \tag{84}$$

$$A_2 = \frac{k^2}{p^2+k^2} \frac{q}{p^2} \tag{85}$$

and the relation

$$\frac{1}{ka} \frac{K_2(ka)}{K_1(ka)} + \frac{1}{pa} \frac{J_2(pa)}{J_1(pa)} = 0 \tag{86}$$

Eqs.(82)-(83) with the coefficients given by (84)-(85) give the usual Rossby type solitary vortex solutions for Eqs.(72)-(73), but with five parameters a_2, a_3, u, a, α. In fact a_2, a_3 are determined by u, a, α in the following way: given u, a, α one determines k, p from Eq.(79),(86); then solving Eq.(80) and the continuity condition of ψ on the boundary

$$a_1 \phi_{out} |_a = a_2 \phi_{in} |_a + a_3 \frac{B_o u}{c} a \cos\theta \tag{87}$$

Taking consideration of equation (81), finally determines a_2, a_3.

Apparently, due to the assumption (74)-(75), fields $\delta\mathbf{B}_\perp$ and $j_{||}$ are not continuous across the boundary, as was the case for the Rossby type Alfven vortices and drift-Alfven vortices solutions (see **Appendices A,B**).

4.2 Intrinsic electromagnetic vortices

By imposing the same boundary conditions as (42), (43) on ϕ, ψ, and choosing the integration constants $C_1 = 0$, $C_2 \neq 0$ in the inner region ($r < a$), after eliminating ψ from Eqs.(72)-(73) we find the fourth order equations for ϕ in two regions

$$\nabla_\perp^4 \phi - (\frac{2\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u} + \frac{\omega_{pe}^2}{c^2}\frac{\alpha^2 c_A^2}{u^2})\nabla_\perp^2 \phi + \frac{\omega_{pe}^2}{c^2}(\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u})\phi = 0, \ (r > a) \quad (88)$$

$$\nabla_\perp^4 \phi + (C_2 - \frac{2\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u} - \frac{\omega_{pe}^2}{c^2}\frac{\alpha^2 c_{Ae}^2}{u^2})\nabla_\perp^2 \phi + \frac{\omega_{pe}^2}{c^2}(\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u} - C_2)\phi$$
$$+\frac{\omega_{pe}^2}{c^2}\frac{B_o u}{c} C_2 r \cos\theta = 0. \ (r < a) \quad (89)$$

Eqs.(88), (89) and boundary conditions (42), (43) form the type 2 boundary value problem we solved in section **2**. Hence the special solutions of nonlinear equations (68), (69), under assumptions (5), (25.a),(41),(46), can be given by substituting the corresponding coefficients of Eqs. (88), (89) into Eqs.(20),(21.a),(22),(23.a). This yields

$$\begin{aligned} \phi(r,\theta)_{out} &= \{B_1 K_1(\lambda_1 r) + B_2 K_1(\lambda_2 r)\}\frac{B_o u}{c} a\cos\theta, \quad (r > a) \\ \phi(r,\theta)_{in} &= \{B_3 J_1(\lambda_3 r) + B_4 I_1(\lambda_4 r) + B_5 \frac{r}{a}\}\frac{B_o u}{c} a\cos\theta \quad (r < a) \end{aligned} \quad (90)$$

and

$$\begin{aligned} \psi(r,\theta)_{out} &= \{D_1 K_1(\lambda_1 r) + D_2 K_1(\lambda_2 r)\}\frac{B_o u}{c} a\cos\theta, \quad (r > a) \\ \psi(r,\theta)_{in} &= \{D_3 J_1(\lambda_3 r) + D_4 I_1(\lambda_4 r)\}\frac{B_o u}{c} a\cos\theta \quad (r < a) \end{aligned} \quad (91)$$

where

$$B_5 = C_2/(C_2 - \frac{\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u})$$

$$D_1 = -(\frac{c}{\omega_{pe}} \frac{u}{\alpha c_{Ae}})^2 (\lambda_1^2 - \frac{\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u}) B_1 \tag{92}$$

$$D_2 = -(\frac{c}{\omega pe} \frac{u}{\alpha c_{Ae}})^2 (\lambda_2^2 - \frac{\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u}) B_2 \tag{93}$$

$$D_3 = -(\frac{c}{\omega_{pe}} \frac{u}{\alpha c_{Ae}})^2 (C_2 - \frac{\omega_{pe}}{c^2} + \frac{\kappa_n \omega_{ce}}{u} - \lambda_3^2) B_3 \tag{94}$$

$$D_4 = -(\frac{c}{\omega_{pe}} \frac{u}{\alpha c_{Ae}})^2 (C_2 - \frac{\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u} + \lambda_4^2) B_4 \tag{95}$$

$$\lambda_{1,2}^2 = \frac{1}{2}\{(\frac{2\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u} + \frac{\omega_{pe}^2}{c^2} \frac{\alpha^2 c_{Ae}^2}{u^2})$$

$$\pm[(\frac{2\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u} + \frac{\omega_{pe}^2}{c^2} \frac{\alpha^2 c_{Ae}^2}{u^2})^2 - 4\frac{\omega_{pe}^2}{c^2}(\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u})]^{1/2}\}, \tag{96}$$

$$\lambda_{3,4}^2 = \frac{1}{2}\{[(C_2 - \frac{2\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u} - \frac{\omega_{pe}^2}{c^2} \frac{\alpha^2 c_{Ae}^2}{u^2})^2 - 4\frac{\omega_{pe}^2}{c^2}(\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n \omega_{ce}}{u} - C_2)]^{1/2}$$

$$\pm(C_2 - \frac{2\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u} - \frac{\omega_{pe}^2}{c^2} \frac{\alpha^2 c_{Ae}^2}{u^2})\}, \tag{97}$$

Taking into account the continuity of the relevant physical quantities as we did for the intrinsic drift-Alfven vortex solution, we impose the matching conditions (56.a)-(57.c) to solutions (90)-(91) to determine the integration constants B_1, B_2, B_3, B_4. From condition (57.b) we get a relation between the parameters entering the inner and outside solutions. As long as we keep the choice of integration constants as $C_1 = 0, C_2 \neq 0$ in the inner region, the satisfaction of conditions (56.a)-(57.a) makes the condition (57.c) satisfied automatically.

Substituting Eqs. (90)-(91) into (56.a)-(57.a), after some algebra, we have

$$B_1 = \frac{a_{11}A + a_{12}B}{\Gamma}, \tag{98}$$

$$B_2 = \frac{a_{21}A + a_{22}B}{\Gamma}, \tag{99}$$

$$B_3 = \frac{a_{31}A + a_{32}B}{\Gamma}, \tag{100}$$

$$B_4 = \frac{a_{41}A + a_{42}B}{\Gamma}, \tag{101}$$

where

$$\Gamma = (\lambda_1^2 - \lambda_2^2)[(\lambda_3 J_2(\lambda_3 a) I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a) J_1(\lambda_3 a)] K_1(\lambda_1 a) K_1(\lambda_2 a)$$

$$+(\lambda_3^2 + \lambda_4^2)[(\lambda_1 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a) K_1(\lambda_1 a))] J_1(\lambda_3 a) I_1(\lambda_4 a),$$

$$A = 1,$$

$$B = 1 - B_5,$$

and

$$a_{11} = -\{(\lambda_3^2 + \lambda_4^2)\lambda_2 K_2(\lambda_2 a) J_1(\lambda_3 a) I_1(\lambda_4 a)$$

$$+\lambda_2^2[\lambda_3 J_2(\lambda_3 a) I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a) J_1(\lambda_3 a)] K_1(\lambda_2 a)\},$$

$$a_{12} = \lambda_3\lambda_4[\lambda_4 J_2(\lambda_3 a) I_1(\lambda_4 a) - \lambda_3 I_2(\lambda_4 a) J_1(\lambda_3 a)] K_1(\lambda_2 a);$$

$$a_{21} = \lambda_1^2[\lambda_3 J_2(\lambda_3 a) I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a) J_1(\lambda_3 a)] K_1(\lambda_1 a)$$

$$+(\lambda_3^2 + \lambda_4^2)\lambda_1 K_2(\lambda_1 a) J_1(\lambda_3 a) I_1(\lambda_4 a),$$

$$a_{22} = \lambda_3\lambda_4[\lambda_3 I_2(\lambda_4 a) J_1(\lambda_3 a) - \lambda_4 J_2(\lambda_3 a) I_1(\lambda_4 a)] K_1(\lambda_1 a);$$

$$a_{31} = \lambda_1\lambda_2[\lambda_1 K_2(\lambda_2 a) K_1(\lambda_1 a) - \lambda_2 K_2(\lambda_1 a) K_1(\lambda_2 a)] I_1(\lambda_4 a),$$

$$a_{32} = (\lambda_1^2 - \lambda_2^2)\lambda_4 I_2(\lambda_4 a) K_1(\lambda_1 a) K_1(\lambda_2 a)$$

$$+\lambda_4^2[\lambda_1 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a) K_1(\lambda_1 a)];$$

$$a_{41} = \lambda_1\lambda_2[\lambda_2 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_1 K_2(\lambda_2 a) K_1(\lambda_1 a)] J_1(\lambda_3 a),$$

$$a_{42} = (\lambda_1^2 - \lambda_2^2)\lambda_3 J_2(\lambda_3 a) K_1(\lambda_1 a) K_1(\lambda_2 a)$$

$$+\lambda_3^2[\lambda_1 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a) K_1(\lambda_1 a)] J_1(\lambda_3 a).$$

Condition (57.b) gives the relation

$$\lambda_1^3 K_2(\lambda_1 a) B_1 + \lambda_2^3 K_2(\lambda_2 a) B_2 + (\lambda_3^2 - C_2)\lambda_3 J_2(\lambda_3 a) B_3 + (\lambda_4^2 + C_2)\lambda_4 I_2(\lambda_4 a) = 0 \tag{102}$$

which, in fact, is a condition to determine C_2 in terms of parameters u, a, α.

Eqs.(90)-(91), with the constant coefficients given in Eqs.(92)-(95),(97)-(101), are the intrinsic electromagnetic electron vortices described by nonlinear equations (68)-(69) which guarantee the continuity of $\delta\mathbf{B}_\perp$ and $j_\parallel$ at boundary $r = a$. Changing coefficients $\frac{\omega_{pe}^2}{c^2} - \frac{\kappa_n\omega_{ce}}{u} \to \frac{1}{\rho_{ei}^2}(1 - \frac{v_{di}}{u})$; $c_{Ae} \to c_A$ in Eqs.(90)-(95), (97)-(101) gives the intrinsic electromagnetic short-wavelength drift vortices.

The existence regions for these vortices are determined by the conditions (24) and (25.a) which we will discuss in the next section.

5. Discussion and summary

In the previous sections we constructed the intrinsic Alfven vortices, intrinsic drift-Alfven vortices, two types of electromagnetic electron vortices and electromagnetic short-wavelength drift vortices. In this section we discuss some properties of these vortices, and finally, give the summary.

(1). Common features

From the expressions in Eqs.(51)-(52),(90)-(91) we see that the intrinsic electromagnetic solitary vortices share the basic feature of the Rossby wave vortices: both vortices have localized dipole structure and move in the $y-x$ plane with constant speed u at an angle $\gamma = \tan^{-1}\frac{1}{\alpha}$ with respect to the equilibrium magnetic field direction without changing their shapes. The localized structures decay to zero as $r \to \infty$. For the electromagnetic electron vortices (and electromagnetic short-wavelenth vortices) the asymptotic form as $r \to \infty$ for both Rossby and intrinsic vortices is $e^{-\lambda r}/\sqrt{r}$, where for Rossby type vortices $\lambda = k$, whereas for intrinsic vortices $\lambda = min\{\lambda_1, \lambda_2\}$ with λ_1, λ_2 given by Eq.(96). The intrinsic Alfven and intrinsic drift-Alfven vortices, in contrast to the exponentialy decaying Rossby type vortices (Mikhailovskii et al. 1984_a; Shukla et al. 1985_a; 1985_b), slowly decay to zero as $1/r$ for $r \to \infty$.

(2). Differences in Structure

Comparing the intrinsic vortices with the corresponding Rossby vortices, we find that the intrinsic vortices have a more complicated radial structure than the Rossby type vortices in both inner and outer regions. In the inner region, with our choice of the inner solutions, the amplitude of the intrinsic vortices is a linear combination of both regular and modified Bessel functions J_1 and I_1. In the outer region of the intrinsic electron vortices (or intrinsic short-wavelength drift vortices) the amplitude is a linear combination of two first order McDonald functions, while for the intrinsic drift-Alfven (or intrinsic Alfven) vortices the amplitude is a combination of the inverse power law radial decay and the McDonald function. In contrast, the amplitude of the Rossby type vortices is simply $J_1(pr)$ in the inner region and $K_1(kr)$ in the outer region.

(3). Different allowed regions of the vortex propagation speed

Another remarkable difference between intrinsic electromagnetic vortices and their corresponding Rossby vortices is that they have different allowed regions of vortex propagation. For Rossby vortices, the allowed region of vortex propagation is determined by the single nonlinear dispersion relation condition

$$k^2 > 0. \tag{103}$$

The explicit expressions for Eq.(103) for Alfven and drift-Alfen vortices, given first in works by Mikailovskii et al (1984_a) and by Shukla et al (1985_a; 1985_b), are given by Eqs.(66) and (64) in Sec. **3**. For electromagnetic electron vortices (or electromagnetic short-wavelength drift vortices), k^2 in Eq.(103) is defined by Eq.(77) or, equivalently, by Eq.(96) in Sec. 4. Unlike Rossby type vortices, for intrinsic electromagnetic vortices, in addition to the restriction from condition (103), the allowed region of vortex propagation speed is further constrained by additional conditions. With our choices of the inner solutions, for the intrinsic Alfven and intrinsic drift-Alfven vortices, the additional conditions are Eq.(67) and Eq.(65), respectively; for intrinsic electromagnetic electron vortices (or electromagnetic short-wavelength vortices), the additional condition is Eq.(25.a) which has the explicit form

$$C_2 - \frac{\omega_{pe}^2}{c^2} + \frac{\kappa_n \omega_{ce}}{u} > 0. \tag{104}$$

Therefore we can see that in general the allowed regions for intrinsic electromagnetic vortices are narrower than the allowed region for the corresponding Rossby vortices.

In summary, in this work we show that the existing Rossby type vortices for electromagnetic perturbations in magnetized plasma exhibit discontinuities of $\delta \mathbf{B}_\perp$ and $j_\parallel$ on the boundary between the inner and outer regions. In an effort to overcome this difficulty, we construct a new type of solitary vortices solution based on solutions of the fourth order differential equation. This new type of solitary vortex seems to have no hydrodynamic analogue; hence we call them intrinsic electromagnetic vortices. Comparing them with the corresponding Rossby type vortices, intrinsic vortices have more complicated radial structure in both inner and outer regions and narrower allowed regions of existence. For two sets of unsolved nonlinear equations, we constructed both Rossby and intrinsic vortices solutions. While the

Rossby vortices for these equations also show the discontinuity, the intrinsic ones are free of it. In light of the four examples we give in this work and one we treated in a recent work on electromagnetic solitary vortices in rotating plasma (Liu & Horton, 1985), we believe that the discontinuity difficulty encountered by constructing Rossby type vortices for electromagnetic perturbations in magnetized plasma is a generic problem, and the intrinsic solitary vortices we construct here may be the natural physical vortical flow states of the electromagnetic collective modes of the magnetized plasma.

Acknowledgement

The authors thank J.D. Meiss and R.D. Hazeltine for encouraging discussions.

This work was supported by the Department of Energy Contract No.DE-FG05-80ET-53088.

Appendix A

The Rossby type Alfven vortex solution given by Mikhailovskii et al (1984_a) probably is the first solitary vortex solution of this type constructed for electromagnetic perturbations in magnetized plasma. Later Shukla et al (1985_a), retaining more nonlinear terms, also gave the Rossby type solitary Alfven vortices. The work by Shukla et al (1985_a) contain more physics through the additional nonlinear terms, the solution given there is exact,in contrast to the approximate solution given by Mikhailovskii et al (1984_a)

However, both Alfven vortices constructed in above mentioned works have discontinuities of the perturbed magnetic field $\delta \mathbf{B}_\perp$ and parallel current density $j_{||}$ on the boundary between the inner and outer regions. In this appendix we explicitly show this fact. For convenience in referring to the original works, we keep their original notations, respectively.

For the solution given by Mikhailovskii et al (1984_a)**:**

By using Eqs.(8),(11),(13),(14) in work by Mikhailovskii et al (1984_a) and continuity conditions of ϕ, $\nabla\phi$,$\nabla^2\phi$ on the boundary, from their Eqs.(1), (3) we can calculate $\delta \mathbf{B}_\perp^{(out)} |_a$, $\delta \mathbf{B}_\perp^{(in)} |_a$; $j_{||}^{(out)} |_a$, $j_{||}^{(in)} |_a$; $\Delta\delta\mathbf{B}_\perp$ and $\Delta j_{||}$, where superscripts "out" and "in" represent corresponding quantities calculated from the solutions for the outer and the inner regions, "$|_a$" represents the value on the boundary, and "Δ" represents the jump of the same quantity across the boundary.

$$\delta \mathbf{B}_{\perp}^{(out)} |_a = -\frac{\alpha c}{u}(1 - \frac{\alpha^2 c_A^2}{u^2}\frac{\beta^2\rho^2}{a^2})\hat{\mathbf{z}} \times \nabla\phi |_a, \qquad (A.1)$$

$$\delta \mathbf{B}_{\perp}^{(in)} |_a = -\frac{\alpha c}{u}(1 + \frac{\alpha^2 c_A^2}{u^2}\frac{\gamma^2\rho^2}{a^2})\hat{\mathbf{z}} \times \nabla\phi |_a - \xi\frac{\alpha^2 c}{u}(\frac{\alpha c_A}{u})^2\frac{(\beta^2+\gamma^2)\rho^2}{a^4}\hat{\mathbf{y}}, \qquad (A.2)$$

$$j_{\parallel}^{(out)} |_a = -\frac{\alpha c^2}{4\pi u}(1 - \alpha^2 c_A^2\frac{\beta^2\rho^2}{a^2})\nabla_{\perp}^2\phi |_a, \qquad (A.3)$$

$$j_{\parallel}^{(in)} |_a = -\frac{\alpha c^2}{4\pi u}(1 + \frac{\alpha^2 c_A^2}{u^2}\frac{\gamma^2\rho^2}{a^2})\nabla_{\perp}^2\phi |_a . \qquad (A.4)$$

From Eqs.(A.1)-(A.4), obviously, there are jumps of $\delta\mathbf{B}_{\perp}$ and $j_{\parallel}$ across the boundary:

$$\Delta\delta\mathbf{B}_{\perp} = \frac{\alpha c}{u}(\frac{\alpha c_A}{u})^2\frac{(\beta^2+\gamma^2)\rho^2}{a^4}[\hat{\mathbf{z}} \times \nabla_{\perp}\phi |_a + \frac{\xi\alpha}{a^2}\hat{\mathbf{y}}], \qquad (A.5)$$

$$\Delta j_{\parallel} = \frac{\alpha c^2}{4\pi u}(\frac{\alpha c_A}{u})^2\frac{(\beta^2+\gamma^2)\rho^2}{a^2}\nabla_{\perp}^2\phi |_a . \qquad (A.6)$$

For the solution given by Shukla et al (1985_a):

From their Eqs.(10),(13)-(16) and the definitions for constants d_o, d_i, e_o, e_i, according to the convention used in Ref.3 we have

$$\delta \mathbf{B}_{\perp}^{(out)} |_a = -d_o\hat{\mathbf{z}} \times \nabla_{\perp}\phi^{(out)} |_a, \qquad (A.7)$$

$$\delta \mathbf{B}_{\perp}^{(in)} = -d_i\hat{\mathbf{z}} \times \nabla_{\perp}\phi^{(in)} |_a - e_i\hat{\mathbf{y}}. \qquad (A.8)$$

$$j_{\|}^{(out)} \mid_a = \frac{c}{4\pi} d_o \nabla_\perp^2 \phi^{(out)} \mid_a, \qquad (A.9)$$

$$j_{\|}^{(in)} \mid_a = \frac{c}{4\pi} d_i \nabla_\perp^2 \phi^{(in)} \mid_a . \qquad (A.10)$$

Since $\nabla_\perp \phi^{(out)} \mid_a = \nabla_\perp \phi^{(in)} \mid_a$, $\nabla_\perp^2 \phi^{(out)} \mid_a = \nabla_\perp^2 \phi^{(in)} \mid_a$ but $d_i \neq d_o$, accordingly, there are jumps of $\delta \mathbf{B}_\perp$ and $j_{\|}$ across the boundary:

$$\Delta \delta \mathbf{B} = -(d_o - d_i) \hat{\mathbf{z}} \times \nabla_\perp \phi \mid_a + e_i \hat{\mathbf{y}} \neq 0, \qquad (A.11)$$

$$\Delta j_{\|} = \frac{c}{4\pi} (d_o - d_i) \nabla_\perp^2 \phi \mid_a \neq 0. \qquad (A.12)$$

Appendix B

For the Rossby type drift-Alfven vortices constructed by Shukla et al (1985_b) the discontinuity problem for the Alfven vortices seems not to appear. The reason for this apparent discrepancy is that the solution given there is an approximate iterative one (see their Eqs.(8),(10)). In this appendix we show that the set of nonlinear equations used by Shukla et al (1985_b) in fact can be exactly solved without using iteration, and the exact Rossby type solitary vortices solution suffers the same discontinuity problem as the Alfven vortices discussed in **Appendix A.**

The basic equations used by Shukla et al (1985_b), i.e. Eqs. (2)-(4) there, are almost identical to equations (26)-(28) in this work. The only difference is that in their Eq.(2) there is a linear ion drift convection term which we dropped with the assumption $T_i = 0$ in our equation (26). By using our notations, assuming a travelling disturbance form for the perturbations, their equations (2)-(4) can be written as

$$\hat{L}_1 \nabla_\perp^2 \phi - \frac{\alpha^2 c_A^2}{u^2} \hat{L}_2 \nabla_\perp^2 (\phi - \psi) - \frac{v_{di}}{u} \partial_\eta \nabla_\perp^2 \phi = 0, \qquad (B.1)$$

$$\hat{L}_2 (\psi - \frac{T_e}{e} \tilde{n}) + \frac{v_{de}}{u} \partial_\eta (\phi - \psi) = 0, \qquad (B.2)$$

$$\hat{L}_1 (\frac{T_e}{e} \tilde{n}) - \frac{\alpha^2 c_A^2}{u^2} \rho_s^2 \hat{L}_2 \nabla_\perp^2 (\phi - \psi) - \frac{v_{de}}{u} \partial_\eta \phi = 0, \qquad (B.3)$$

where

$$v_{di} \equiv -\frac{c}{B_o} \frac{T_i}{e} \kappa_n, \quad \textit{the ion diamagnetic drift velocity}$$

Solving (B.2) by

$$\frac{T_e}{e}\tilde{n} = \psi + \frac{v_{de}}{u}(\phi - \psi), \tag{B.4}$$

and substituting (B.4) into (B.1),(B.3) yields

$$\hat{L}_2\{(1 - \frac{v_{de}}{u})\psi - \frac{\alpha^2 c_A^2}{u^2}\rho_s^2\nabla_\perp^2(\phi - \psi)\} = 0, \tag{B.5}$$

$$\hat{L}_1\{\nabla_\perp^2\phi - (1 - \frac{v_{de}}{u})\rho_s^{-2}\psi\} - \frac{v_{di}}{u}\partial_\eta\nabla_\perp^2\phi = 0. \tag{B.6}$$

Introducing polar coordinates, we solve Eqs.(B.5)-(B.6) in regions $r > a$ and $r < a$.

Outer Region $(r > a)$

Integrating (B.5) and considering $\phi, \psi \to 0$ as $r \to \infty$ gives

$$(1 - \frac{v_{de}}{u})\psi - \frac{\alpha^2 c_A^2}{u^2}\rho_s^2\nabla_\perp^2(\phi - \psi) = 0 \qquad (B.7).$$

Assuming a linear algebraic relation between ϕ and ψ as

$$\psi = a_1\phi \tag{B.8}$$

and substituting it into (B.7) gives

$$\nabla_\perp^2\phi - k_1^2\phi = 0, \tag{B.9}$$

where

$$k_1^2 = \frac{1}{\rho_s^2}\frac{a_1}{1 - a_1}\frac{u(u - v_{de})}{\alpha^2 c_A^2}. \tag{B.10}$$

Substituting (B.8)-(B.9) into Eq.(B.6) gives

$$[x - \frac{c}{B_o u}\phi,\ \ \nabla_\perp^2\phi - \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})a_1\phi - \frac{v_{di}}{u}k_1^2\phi] = 0 \tag{B.11}$$

With the localization condition, integrating (B.11) gives

$$\nabla_\perp^2\phi - k_2^2\phi = 0, \tag{B.12}$$

where

$$k_2^2 = \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})a_1 + \frac{v_{di}}{u}k_1^2 \tag{B.13}$$

The requirement of identity of (B.9) and (B.12) leads to

$$k_1^2 = k_2^2 = k^2 = \frac{1}{\rho_s^2}\frac{(u - v_{de})[1 - u(u - v_{di})/\alpha^2 c_A^2]}{u - v_{di}} \tag{B.14}$$

which implies that

$$a_1 = 1 - \frac{u(u - v_{di})}{\alpha^2 c_A^2} \tag{B.15}$$

Inner Region ($r < a$):

Integrating (B.5) gives

$$(1 - \frac{v_{de}}{u})\psi - \frac{\alpha^2 c_A^2}{u^2}\rho_s^2\nabla_\perp^2(\phi - \psi) = f_1(\phi - \psi - \frac{B_o u}{c}x) \tag{B.16}$$

where $f_1(w)$ is an arbitrary function of the argument w. As usual we only consider the case

$$f_1(w) = C_1 w, \tag{B.17}$$

where C_1 is an integration constant. Assuming a linear algebraic relation between ϕ and ψ

$$\psi = a_2\phi + a_3\frac{B_o u}{c} r\cos\theta \qquad (B.18)$$

then Eq.(B.16) becomes

$$\nabla_\perp^2\phi + p_1^2\phi + q_1\frac{B_o u}{c} r\cos\theta = 0, \qquad (B.19)$$

where

$$p_1^2 = \frac{1}{\rho_s^2}\frac{u^2}{\alpha^2 c_A^2}[C_1 - (1 - \frac{v_{de}}{u})\frac{a_2}{1-a_2}]$$

$$q_1 = -\frac{1}{\rho_s^2}\frac{u^2}{\alpha^2 c_A^2}[(1 - \frac{v_{de}}{u})\frac{a_3}{1-a_2} + C_1\frac{1+a_3}{1-a_2}].$$

Substituting (B.18)-(B.19) into (B.6) gives

$$[x - \frac{c}{B_o u}\phi \, , \, \nabla_\perp^2\phi - \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})(a_2\phi + a_3\frac{B_o u}{c}x) + \frac{v_{di}}{u}p_1^2\phi] = 0 \qquad (B.20)$$

Integrating (B.20) yields

$$\nabla_\perp^2\phi + p_2^2\phi + q_2\frac{B_o u}{c} r\cos\theta = 0, \qquad (B.21)$$

where

$$p_2^2 = \frac{1}{\rho_s^2}\{\frac{v_{di}}{u}\frac{u^2}{\alpha^2 c_A^2}[C_1 - (1 - \frac{v_{de}}{u})\frac{a_2}{1-a_2}] - (1 - \frac{v_{de}}{u})a_2\} - C_2$$

$$q_2 = C_2 - \frac{1}{\rho_s^2}(1 - \frac{v_{de}}{u})a_3$$

and C_2 is another undetermined integration constant introduced by integrating Eq.(B.20). The requirement for identity of Eq.(B.19) and (B.21) leads to

$$p_1^2 = p_2^2 = p^2, \quad q_1 = q_2 = q \tag{B.22}$$

Solving (B.22) by eliminating two integration constants C_1, C_2 we find that

$$p^2 = \frac{1}{\rho_s^2}\frac{u^2}{\alpha^2 c_A^2}(1 - \frac{v_{de}}{u})\{\frac{(1 - v_{di}/u)a_2 + a_3 - \alpha^2 c_A^2/u^2(1 - a_2)(a_2 + a_3)}{(1 - v_{di}/u)(1 - a_2) - (1 + a_3)} - \frac{a_2}{1 - a_2}\} \tag{B.23}$$

$$q = -\frac{1}{\rho_s^2}\frac{u^2}{\alpha^2 c_A^2}\frac{1 - v_{de}/u}{1 - a_2}\{a_3 +$$

$$+\frac{(1 + a_3)[(1 - v_{di}/u)a_2 + a_3 - \frac{\alpha^2 c_A^2}{u^2}(a_2 + a_3)(1 - a_2)]}{(1 - v_{di}/u)(1 - a_2) - (1 + a_3)}\} \tag{B.24}$$

where the two undetermined constant parameters a_2, a_3 introduced in Eq.(B.18) will be fixed from the matching conditions of the solutions on the boundary.

Obviously, Eq.(B.9) and Eq.(B.19) (or Eq.(B.12) and Eq.(B.21)), with k^2 determined by Eq.(B.14) and p^2, q determined by Eqs.(B.23)-(B.24), have a Rossby (or drift) solitary vortex solution (Larichev & Reznik, 1976; Meiss & Horton, 1983) which is continuous on the boundary up to second derivatives

$$\begin{aligned} \phi_{out} &= -\frac{q}{p^2 + k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c} a\cos\theta \quad (r > a) \\ \phi_{in} &= \{\frac{k^2}{p^2 + k^2}\frac{a}{r}\frac{J_1(pr)}{J_1(pa)} - 1\}\frac{q}{p^2}\frac{B_o u}{c} r\cos\theta \quad (r < a) \end{aligned} \tag{B.25}$$

where parameters p and k are related by the relation

$$\frac{1}{ka}\frac{K_2(ka)}{K_1(ka)} + \frac{1}{pa}\frac{J_2(pa)}{J_1(pa)} = 0. \tag{B.26}$$

For the eight parameters $a, u, \alpha, k, p, q, a_2, a_3$ we have introduced into the solution, there are four constraint relations,e.g. (B.14),(B.23)-(B.24) and (B.26). In fact we can introduce the fifth relation, the continuity of ψ on the boundary:

$$a_2\phi_{in} \mid_a + a_3\frac{B_o u}{c} a\cos\theta = a_1\phi_{out} \mid_a . \qquad (B.27)$$

Therefore the solution (B.25) only contain three free parameters a, u, α as in Ref.8.

As the procedure shows, the solution we give here is an exact solitary solution for the nonlinear equations used by Shukla et al (1985_b). The nonlinear dispersion relation for this vortex solution is the inequality

$$k^2 = \frac{(u - v_{de})[1 - u(u - v_{di})/\alpha^2 c_A^2]}{u - v_{di}} > 0 \qquad (B.28)$$

which is identical with the relation

$$\eta^2 = -Pa^2/Q > 0$$

in work by Shukla et al (1985_b). But unlike their solution, here we do not need the further constraint relation (their Eq.(22)) to restrict the values of the free parameters u, a, α.

Since, for the solutions of ϕ and ψ given here only ϕ, $\nabla\phi$, $\nabla_\perp^2\phi$ and ψ are continuous, the perturbed perpendicular magnetic field

$$\delta\mathbf{B}_\perp = -\frac{\alpha c}{u}\hat{\mathbf{z}} \times \nabla(\phi - \psi)$$

and parallel current density

$$j_{\|} = -\frac{\alpha c^2}{4\pi u}\nabla_{\perp}^2(\phi - \psi)$$

have discontinuities on the boundary between the two regions in the vortex. The jumps of these two fields across the boundary are

$$\Delta\delta\mathbf{B}_{\perp} = -\frac{\alpha c}{u}[(a_1 - a_2)\hat{\mathbf{z}} \times \nabla\phi \mid_a - a_3\frac{B_o u}{c}a\hat{\mathbf{y}}, \qquad (B.29)$$

and

$$\Delta j_{\|} = -\frac{\alpha c^2}{4\pi u}(a_1 - a_2)\nabla_{\perp}^2\phi \mid_a, \qquad (B.30)$$

respectively.

References

Aburdzhaniya, G.D., Kamenets, F.F., Lakhin, V.P., Mikhailovskii, A.B.,& Onishchenko, O.G. 1984 *Phys.Lett.* **105A** 48.

Hazeltine, R.D. 1983 *Phys.Fluids* **26** 3242.

Hazeltine, R.D., Holm, D.D., & Morrison, P.J. 1985 *J.Plasma Physics* **34** 103.

Larichev, V.D. & Reznik, G.M. 1976 *Dokl.Akad.Nauk USSR* **231** 1077.

Liu, J. & Horton, W. 1985 " *Electromagnetic solitary vortices in rotating plasma*" (Submitted to *The Physics of Fluids*) **IFSR #214**.

Meiss, J.D. & Horton, W. 1983 *Phys.Fluids* **26** 990.

Mikhailovskii, A.B., Aburdzhaniya, G.D., Onishchenko, O.G., & Churikov, A.P. 1984_a *Phys.Lett.* **101A** 263.

Mikhailovskii, A.B., Aburdzhaniya, G.D., Onishchenko, O.G., & Sharapov, S.E. 1984_b *Phys.Lett.* **104A** 94.

Mikhailovskii, A.B., Lakhin, V.P., Mikhailovskaya, L.A., & Onishchenko, O.G. 1985 *Sov.Phys.JETP* **59** 1198.

Shukla, P.K., Yu, M.Y., & Varma, R.K. 1985_a *Phys.Lett.* **109A** 322.

Shukla, P.K., Yu, M.Y., & Varma, R.K. 1985_b *Phys.Fluids* **28** 1719.