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ISLAND BOOTSTRAP CURRENT MODIFICATION OF THE NONLINEAR DYNAMICS OF THE TEARING MODE

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Abstract

A kinetic theory for the nonlinear evolution of a magnetic island in a collisionless plasma confined in a toroidal magnetic system is presented. An asymptotic analysis of a Grad-Shafranov equation including neoclassical effects such as island bootstrap current defines an equation for the time dependence of the island width. Initially, the island bootstrap current strongly influences the island evolution. As the island surpasses a certain critical width the effect of the island bootstrap current diminishes and the island grows at the Rutherford rate. For current profiles such that $\Delta' < 0$ the island bootstrap current saturates the island.

Tokamak confinement devices of large aspect ratio are predicted to have parallel current that is not driven by an electric field. This current results from the interaction of the radial pressure gradient with the drifting motion of trapped particles off the magnetic surfaces. For a system in equilibrium this diffusion driven toroidal current makes possible the idea of a self-reliant tokamak operating in a steady state without externally induced toroidal electric field. Therefore, this current is called "bootstrap current."[1] Resonant magnetic fluctuations change the magnetic field topology of a tokamak plasma with finite resistivity; magnetic islands appear close to the resonant rational surface. The changing magnetic field induces both an Ohmic current and an island bootstrap current associated with the pressure gradient. Note that pressure gradients are modified by the presence of the island and flattened in the island interior. In this Letter we study the nonlinear evolution of a coherent magnetic island in a high temperature tokamak plasma. Specifically, we clarify the influence of the island bootstrap current in the island dynamics.

We consider a toroidal equilibrium with helical symmetry described by the flux coordinates χ, θ and α (χ = equilibrium poloidal flux or radial variable, θ = poloidal angle, α = helical angle = $\varsigma - q_s\theta$ with q_s = safety factor at the rational surface and ς = toroidal angle). We use large aspect ratio approximations in what follows. The total magnetic field in a low-β plasma can be expressed as $\vec{B} = I\vec{\nabla}\varsigma + \vec{\nabla}\varsigma \times \vec{\nabla}(\chi + \psi)$ where I measures the toroidal equilibrium field and ψ is the perturbed poloidal flux. We suppose the islands under consideration to be centered on some surface $\chi = \chi_s$ with $q = q_s$. These islands are caused by resonant magnetic perturbations of mode number $m \geq 2$. A perturbed magnetic flux function Ψ, is obtained as a solution of the equation $\vec{B} \cdot \vec{\nabla}\Psi = 0$. When one resonant harmonic dominates $\Psi = \psi_0 - \psi = \psi_0(\chi) - \psi_1(t)\cos\alpha$, where $\psi_0 = q_s'\tilde{\chi}^2/2q_s$ is the equilibrium helical flux, $q_s' = dq/d\chi(q = q_s)$, $\tilde{\chi} = \chi - \chi_s$ and we have considered thin islands. This form of Ψ describes a coherent magnetic island whose half-width is $w = 2[q_s\psi_1/q_s']^{1/2}$. It is convenient to define the normalized flux coordinate $\Omega = \Psi/\psi_1$ where $\Omega = 1$ corresponds to the island separatrix. Then we can write the flux surface average of a quantity F in the reconnected region as

$$\langle F \rangle_\Psi = \int_\Psi \frac{d\alpha \bar{F}}{[\Omega + \cos\alpha]^{1/2}} \bigg/ \int_\Psi \frac{d\alpha}{[\Omega + \cos\alpha]^{1/2}}$$

where $\bar{F} = \int_\alpha d\theta F/2\pi$. Our analysis is determined by the collisionality of the bounce motion around the major axis of the torus and by the collisionality of the particle motion around the island. Charged particles orbit the magnetic island with a frequency $\omega_\alpha \sim (w/a)\omega_T$ where $\omega_T = v_t/qR$ is the frequency of particle motion around the torus (a = minor radius, R = major radius, $v_t = (2T/m)^{1/2}$, T = electron temperature). We suppose that the motion of the electrons around the major axis is collisionless whereas the motion around the island is collisional. Then, defining $\Delta = \nu_e/\omega_T$ (ν_e = Coulomb collision frequency), $\Delta < \varepsilon^{3/2} < 1$ ($\varepsilon = a/R$) and $w/a \sim \Delta^2$. The particles move trying to follow the perturbed magnetic surfaces but they also drift on banana shaped orbits. The drift velocity of the electron guiding center can be written as

$$\vec{v}_D = c\frac{\vec{B}\times\vec{\nabla}\Phi}{B^2} - \frac{v_{\parallel}}{B}\vec{\nabla}\times\left(\frac{\vec{B}v_{\parallel}}{\Omega_e}\right),$$

where Φ is the electrostatic potential, $v_{\parallel} = \vec{B}\cdot\vec{v}/B$ and $\Omega_e = eB/mc$. The curl above is taken at fixed ω, μ (ω = particle kinetic energy, μ = particle magnetic moment).

The toroidal component of the perturbed Ampere's law can be written in terms of the perturbed poloidal flux as

$$\vec{\nabla}\cdot\left(\frac{1}{R^2}\vec{\nabla}\psi\right) = \frac{4\pi}{c}\vec{\nabla}\varsigma\cdot\vec{J}_1, \quad (1)$$

where J_1 is the perturbed current distribution. Considering thin islands, integrating across the tearing layer and isolating the $\cos\alpha$-Fourier component, we obtain

$$\frac{q_s'}{q_s}\frac{c\Delta'}{8\sqrt{2}R}|\vec{\nabla}\chi|_s^2 w = \int_{-1}^{\infty} d\Omega \oint \frac{d\alpha\cos\alpha}{[\Omega+\cos\alpha]^{1/2}}J_{\parallel}, \quad (2)$$

where Δ' measures the discontinuity in the vector potential amplitude across the tearing layer and $J_{\parallel}$ is the parallel island current. The helical angle integration has the effect of projecting out the dominant harmonic of the total island current. Equation (2) determines the island dynamics when coupled with the island Ohm's law. An explicit form of this equation is obtained here by solving the nonlinear drift kinetic equation for the electrons in the island region. Thus we write

$$\frac{\partial f}{\partial t} + v_{\parallel}\nabla_{\parallel}f + \vec{v}_D\cdot\vec{\nabla}f - \frac{e}{m}v_{\parallel}E_{\parallel}\frac{\partial f}{\partial\omega} = C(f,f),$$

where $v_{\parallel}\nabla_{\parallel}f = \frac{v_{\parallel}}{qR}\left(\frac{\partial f}{\partial\theta} + W_{\parallel}f\right)$, $W_{\parallel} = (q-q_s)\frac{\partial f}{\partial\alpha} - \left(\frac{\partial\psi}{\partial\theta} - q_s\frac{\partial\psi}{\partial\alpha}\right)\frac{\partial f}{\partial\chi}$,

$$\frac{e}{m}v_{\parallel}E_{\parallel} = \frac{e}{m}\frac{v_{\parallel}}{qR}\left[-\frac{\partial\tilde{\Phi}}{\partial\theta} - (q-q_s)\frac{\partial\Phi}{\partial\alpha} + \left(\frac{\partial\psi}{\partial\theta} - q_s\frac{\partial\psi}{\partial\alpha}\right)\frac{\partial\Phi}{\partial\chi} + \frac{q}{c}\frac{\partial\psi}{\partial t}\right]$$

and $\tilde{\Phi} = \Phi - \bar{\Phi}$. We use a Lorentz collision operator $C^\ell(f) = \nu(\xi/B)(\partial/\partial\lambda)(\lambda\xi\partial/\partial\lambda)f$ with $\nu = 2\nu_e(v_t/v)^3$, $\xi = \sigma[1-\lambda B]^{1/2}$, $\sigma = \mathrm{sgn} v_\parallel$ and $\lambda = \mu/\omega$. In order to solve the electron kinetic problem analytically we consider two independent small parameters $\delta = \rho_{pe}/w$ and $\Delta = \nu_e/\omega_T$ (ρ_{pe} = poloidal electron gyroradius). In our perturbation procedure we need to define four basic ratios: $\omega_T^{-1}\ \partial/\partial t \sim \gamma$, $e\Phi/T \sim e\bar{\Phi}/T \sim A$, $\tilde{\Phi}/\Phi \sim B$ and $W_\parallel \sim w/a \sim S$. We have assumed that the motion of the electrons around the island is collisional, so that $S \sim \Delta^2$. We also assume that the electrons complete many orbits around the island in the time characteristic for the magnetic topology to change, so that $\gamma \sim \delta\Delta^2$. In addition, we consider the electrostatic and electromagnetic contributions to the parallel electric field near the island to be comparable, implying $A \sim \Delta^2$. Finally, for quasineutrality considerations we take $B \sim \Delta$. This ordering is consistent with experimental island sizes (greater than the ion gyroradius) and then island potential effects are important. Finally, we expand the distribution function as $f = \sum_{m,n} \Delta^m \delta^n f_n^m$.

To $O(\delta^0)$ we obtain the equation

$$v_\parallel \nabla_\parallel f_0 - \frac{e}{m} v_\parallel E_\parallel \frac{\partial f_0}{\partial \omega} = C(f_0, f_0).$$

From $O(\delta^0\Delta^0)$ and $O(\delta^0\Delta^1)$ we find $f_0^0 = f_M$ and $f_0^1 = \bar{f}_0^1$ (θ-independent) where f_M is the lowest order local Maxwellian solution. We ignore the complications due to equilibrium temperature gradients to emphasize the essentials of the banana regime dynamics. Applying the operators $\oint \frac{qRd\theta}{|v_\parallel|}(--)$ and $\langle \int d^3v f_0^1/f_M(--)\rangle_\Psi$ in the equation to $O(\delta^0\Delta^2)$ and using the Boltzmann-H theorem we find that $f_M = f_M(\Psi)$, $f_0^1 = f_M^1$ (perturbed Maxwellian) and $f_0^2 = \bar{f}_0^2$. Similarly, from the equation to $O(\delta^0\Delta^3)$ we find $f_M^1 = f_M^1(\Psi)$, $f_0^2 = f_M^2$ and $f_0^3 = (e\tilde{\Phi}/T)f_M + g_0^3$ with $g_0^3 = \bar{g}_0^3$. After bounce averaging the $O(\delta^0\Delta^4)$ equation we find

$$g_0^3 = \sigma \frac{\pi v[2\varepsilon]^{1/2}}{\nu q R_0} \left[\frac{e}{T}\left(\bar{W}_\parallel \bar{\Phi} - \frac{q}{c}\frac{\partial\psi}{\partial t}\right) f_M - W_\parallel f_M^2\right] I(\lambda)\theta(\lambda_c - \lambda)$$

where R_0 is the major radius at the axis, $I(\lambda)$ is an integral function of λ, θ is a step function, $\lambda_c = [B_0(1+\varepsilon)]^{-1}$ and B_0 is the magnetic field at the axis. Integrating this result in velocity space we obtain the lowest order island Ohm's law in the form

$$J_{\rm is} = 2.3(1. - 2.1\sqrt{\varepsilon})\sigma_s \bar{E}_{||},$$

where $\sigma_s = 1.97 n_0 e^2 \tau_e/m$ is the classical parallel Spitzer conductivity (n_0 = density at the island center), $J_{\rm is}$ corresponds to the island Spitzer current and $\bar{E}_{||}$ is the θ-averaged island electric field. Note that the factor 2.3 above becomes identically unity if we take $\sigma_s = n_0 e^2 \tau_e/m$ and $\nu = \nu(v = v_t)$. This result includes trapped particle corrections (trapped electrons do not respond to the island electric field). This component of the island current is driven by the island inductive electric field. In the nonlinear regime of interest when the island width exceeds the linear tearing layer width an electrostatic potential builds up in the island which then relaxes the island current so that approximately $J_{||}(x, \theta, \alpha) \simeq J_{||}(\Psi)$ (inertia and interchange effects are negligible here). The island potential is determined as the solution of the equation $\nabla_{||} J_{||} = 0$ or $J_{||} = \langle J_{||} \rangle_\Psi$. Thus, the lowest order island Ohm's law can be written as

$$J_{\rm is} = 2.3(1. - 2.1\sqrt{\varepsilon})\sigma_s \langle \bar{E}_{||} \rangle_\Psi. \tag{3}$$

Substituting this result in Eq. (2) we obtain:

$$\frac{dw}{dt} = \frac{0.27}{1. - 2.1\sqrt{\varepsilon}} \frac{\Delta' \eta_s c^2}{4\pi} |\vec{\nabla}\chi|_s^2, \tag{4}$$

where $\eta_s = 1/\sigma_s$ and $|\vec{\nabla}\chi|_s$ is $|\vec{\nabla}\chi|$ evaluated at $\chi = \chi_s$. Therefore, the island grows algebraically on the resistive time scale.[2,3]

The $\mathcal{O}(\delta)$ neoclassical corrections are obtained from the kinetic equation

$$\frac{\partial f_0}{\partial t} + v_{||}\nabla_{||} f_1 + \vec{v}_D \cdot \vec{\nabla} f_0 - \frac{e}{m} v_{||} E_{||} \frac{\partial f_1}{\partial \omega} = C(f_1, f_0) + C(f_0, f_1).$$

To lowest order in Δ we obtain $f_1^0 = v_{||} B_\alpha (\partial f_M/\partial \chi)/\Omega_e + g_1^0$, where B_α is the α covariant component of $\vec{B}$ and $g_1^0 = \bar{g}_1^0$. After bounce averaging the next order equation we find

$$f_1^0 = \sigma \frac{B_\alpha}{\Omega_e} v \frac{\partial f_M}{\partial \chi} \left[|\xi| - \theta(\lambda_c - \lambda) \int_\lambda^{\lambda_c} \frac{B_0 d\lambda}{2|\bar{\xi}|} \right].$$

The perturbed flux surface average of the current due to f_1^0 has the form

$$J_{ib} = -1.46\sqrt{\varepsilon}\,\frac{cIT}{B}\,\frac{q_s' w}{q_s I_2}\,\frac{\partial n}{\partial \Psi}$$

where $I_2(\Omega) = \oint_\Omega d\alpha[2/(\Omega + \cos\alpha)]^{1/2}/2\pi$. This is the island bootstrap current. Using this expression and Eq. (1) we can write a perturbed neoclassical Grad-Shafranov equation for the resonant field as

$$\Delta^*\psi = \left(4.5\frac{1. - 2.1\sqrt{\varepsilon}}{c^2\eta_s/4\pi}\right)\left\langle\frac{\partial\psi}{\partial t}\right\rangle_\Psi - 2.92\sqrt{\varepsilon}\,\frac{4\pi}{I_2 B_p^2} \cdot \left(\frac{q_s'\psi_1}{q_s}\right)^{1/2}\frac{\partial p}{\partial\Psi}\,|\vec{\nabla}\chi|_s^2, \qquad (5)$$

where $\Delta^*\psi = R^2\vec{\nabla}\cdot(\vec{\nabla}\psi/R^2)$, B_p is the poloidal magnetic field and p is the plasma pressure. In the long mean-free path regime appropriate to thermonuclear temperatures, there are two crucial effects of the field variation in a tokamak: particle trapping and $\vec{\nabla}B$ and curvature drifts of particles across field lines. These effects are included in Eq. (5). The island dynamics equation is obtained by an asymptotic analysis of this equation.

Next we assume that there is a diffusion process operating in the equilibrium and also that pressure sources exist in the plasma interior. Then the density gradient in the island region is found by the condition that the flux of particles be a constant[4] requiring $\partial n/\partial\Psi = q_s(\partial n_\infty/\partial\chi)/wq_s' I_0$ where $\partial n_\infty/\partial\chi$ is the equilibrium density gradient and $I_0(\Omega) = \oint_\Omega d\alpha[(\Omega + \cos\alpha)/2]^{1/2}/2\pi$. Thus, the island bootstrap current takes the form

$$J_{ib} = -0.73\sqrt{\varepsilon}\,\frac{\rho_{pe}}{\bar{a}}\,en_0 v_t \theta(\Omega - 1), \qquad (6)$$

where $(\bar{a})^{-1} = (|\vec{\nabla}\chi|_s/n_0 I_0 I_2)(\partial n_\infty/\partial\chi)$ and J_{ib} vanishes inside the separatrix. A boundary layer integration in velocity space shows that this current is mostly carried by the boundary layer untrapped electrons. The diamagnetic current originates in the trapped electrons. Because the friction between the trapped and untrapped electrons is very intense it causes this small diamagnetic current to be amplified according to the following mechanism: the untrapped electrons start drifting in the same direction as the trapped electrons due to collisions between them and the drift becomes steady state due to collisions with ions. The island Ohm's law including trapped particle effects takes the form

$$J_{\|} = 2.3(1. - 2.1\sqrt{\varepsilon})\sigma_s \langle \bar{E}_{\|} \rangle_{\Psi} - 0.73\sqrt{\varepsilon}\frac{\rho_{pe}}{\bar{a}}\theta(\Omega - 1). \tag{7}$$

Substituting Eq. (7) into Eq. (2) we obtain

$$\frac{dw}{dt} = \frac{0.27}{1. - 2.1\sqrt{\varepsilon}} \frac{\Delta' \eta_s c^2}{4\pi} |\vec{\nabla}\chi|_s^2 - 1.23\sqrt{\varepsilon} \left(\frac{\eta_s c^2 \ell_s}{B_p} \right) \left(\frac{r_s}{q_s} \right) \frac{p'}{w} |\vec{\nabla}\chi|_s^2, \tag{8}$$

where $\ell_s = Rq_s^2/r_s q_s' |\vec{\nabla}\chi|_s$ is the shear length, $r = r_s$ is the radius at the rational surface, and $p' = \partial p_\infty / \partial \chi$ is the equilibrium pressure gradient. Equation (8) is our main result. It is an equation for nonlinear island evolution in a collisionless tokamak plasma including conductivity decrease due to trapping in the toroidal magnetic field and bootstrap current effects. When toroidicity is neglected, this result coincides with Rutherford.[2,3] In the general case the island dynamics is driven by the magnetic free energy and by the island bootstrap current. Initially, the effects of the island bootstrap current are dominant. When the island grows there is a certain island width (Δw_c) such that once it is surpassed, the effect of the island bootstrap current diminishes and the island growth rate approaches the Rutherford's rate[2] with growth dominated by Δ'. The critical island width for a $n = 1$, $m = 2$ island with $\Delta' a \sim 4$, $r_s \sim a/2$ is typically $\Delta w_c / a \sim \sqrt{\varepsilon}\beta_{pe}$. Then for TFTR, $\Delta w_c / a \sim 9\%$ and for a reactor $\Delta w_c / a \sim 26\%$. We note that interchange effects[4] in the nonlinear dynamics of magnetic islands in tokamaks are of opposite sign (and of $\mathcal{O}(\sqrt{\varepsilon})$ in magnitude) with respect to the effects considered here. In plasmas with $\Delta' > 0$ the island growth saturates due to geometry considerations ($w \sim a$) as shown in Ref. 5. For current distributions such that $\Delta' < 0$ the island bootstrap current causes the island to grow until it saturates at width Δw_c. The particle diffusion in high temperature tokamak plasmas might be understood in terms of magnetic field stochasticity induced when the bootstrap-saturated islands overlap.

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