The Electromagnetic Solitary Vortices In Rotating Plasma

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(Revised version)

Abstract

The nonlinear equations describing drift-Alfven solitary vortices in a low β, rotating plasma are derived. Two types of solitary vortex solutions along with their corresponding nonlinear dispersion relations are obtained. Both solutions have the localized coherent dilopar structure. The first type of solution belongs to the family of the usual Rossby or drift wave vortex, while the second type of solution is intrinsic to the electromagnetic perturbation in a magnetized plasma and is a complicated structure. While the first type of vortex is a solution to a second order differential equation the second one is the solution of a fourth order differential equation intrinsic to the electromagnetic problem. The fourth order vortex solution has two intrinsic space scales in contrast to the single space scale of the previous drift vortex solution. With the second short scale length the parallel current density at the vortex interface becomes continuous. As special cases the rotational electron drift vortex and the rotational ballooning vortex also are given.

I. Introduction

In a recent work by Horton et al [1] the elecrostatic flute solitary vortex in a low β rotating plasma is analyzed. Their dipolar vortex solution belongs to the usual Rossby or drift wave solitary vortex family. [2,3] In another recent work [4] analyzing the the intrinsic electromagnetic solitary vortex problem we show that generally the solution involves solving fourth order partial differential equations. These new solutions are quite different from the Rossby or drift wave vortices. In this work we generalize these two earlier results [1,4] in the following manner: by including the parallel electron dynamics and the perturbation of magnetic fields we generalize Ref.1 to describe the electromagnetic perturbations and by including plasma rotation in the equations studied in Ref.4 we look for the new type of vortex from a more complicated equation.

When the nonlinear parallel electron dynamics and the perpendicular magnetic field perturbation are included, a set of nonlinear equations describing the drift-Alfven vortices in a rotating plasma is derived. The main difference of this set of equations with the corresponding equations derived in the rest plasma [5,6] is contained in the new terms arising from plasma rotation which adds complexity to solving the equations.

Our analysis shows that, like in Refs.5-7, these new equations also allow the existence of the usual Rossby or drift wave type vortex. The corresponding nonlinear dispersion relations for this type of vortex also is obtained. Close examination of the Rossby-drift type vortex solution [4] reveals that for the electromagnetic problem the drift vortex solutions in Refs.5-7 have a discontinuity of some physical quantities such as parallel current density $j_{||}$ and the magnetic field perturbarion $\delta\mathbf{B}_{\perp}$ across the border of the interior and external regions.

As a remedy to this flaw we obtain a new type of vortex solution which envolves solving a fourth order linear partial differential equation for perturbation functions. While still being dipole-like in structure, this new type of vortex solution has a more complicated radial structure in both the inner and outer regions. The new solution, however, overcomes the discontinuity difficulty of $j_{||}$ and $\delta\mathbf{B}_{\perp}$ across the boundary surface. The nonlinear dispersion relation for this type of vortex solution also is derived here for the first time. Unlike the first type of vortex, the second type of vortex appears to have no hydrodynamic analogy, whereupon we call it the intrinsic electromagnetic vortex to distinguish it from the previous one.

The arrangement of this work is as follows. In section **II** we derive the nonlinear equations. In section **III** we solve the derived equations and give the two types of vortex solutions explicitly. As special cases, we also give the rotational drift vortex and the rotational balloonining vortex. Finally, in section **IV** we discuss some properties of these solutions and give the conclusion.

II. Electromagnetic Drift-Alfven Vortex Equation

In this section we derive the nonlinear equations describing the coupling of the drift wave to the Alfven wave in a rotating plasma. The coupling of the compressional Alfven mode is neglected by using only the parallel component of the vector potential $A_\| = A_z(r, \theta, z, t)$ and the electrostatic potential $\phi(r, \theta, z, t)$.

The electromagnetic fields in the plasma are

$$\mathbf{E} = E_r(r)\hat{\mathbf{r}} - \nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}A_\|\hat{\mathbf{z}}, \tag{1}$$

$$\mathbf{B} = B_o\hat{\mathbf{z}} + \nabla \times (A_\|\hat{\mathbf{z}}). \tag{2}$$

where $E_r(r)$ is the equilibrium electric field which drives the plasma rotating with angular velocity

$$\boldsymbol{\Omega} = -\frac{cE_r}{rB_o}\hat{\theta},$$

and $\phi, A_\|$ are perturbation functions.

For low frequency $\omega \ll \omega_{ci} = eB_o/m_i c$ and large space scale $\lambda_\perp \gg \rho_i, \lambda_{De}$ motions the Maxwell field equations reduce to the quasineutrality condition

$$\nabla \cdot \mathbf{J} = \nabla_\perp \cdot \mathbf{J}_\perp + \mathbf{b} \cdot \nabla j_\| = 0, \tag{3}$$

and to the Ampere's law

$$\nabla_\perp^2 A_\| = -\frac{4\pi}{c}j_\|, \tag{4}$$

where $\rho_i = (T_i/m_i)^{1/2}/\omega_{ci}$, $\lambda_{De} = (T_e/4\pi n_o e^2)^{1/2}$, m_i-ion mass, T_i, T_e-ion and electron temperature, n_o-equilibrium number density of ions or electrons, e-electron charge, c- speed of light,and $\mathbf{J}$-electric current.

Due to the bending of the magnetic field the parallel direction is given by

$$\hat{\mathbf{b}} = \hat{\mathbf{z}} + \frac{1}{B_o}\nabla_\perp A_\parallel \times \hat{\mathbf{z}}$$

and the parallel gradient by

$$\nabla_\parallel = \hat{\mathbf{b}} \cdot \nabla = \partial_z - \frac{1}{B_o}[A_\parallel, \ ], \tag{5}$$

where $[f, g] = \hat{\mathbf{z}} \cdot \nabla f \times \nabla g$ is the Poisson bracket in the plane perpendicular to the equilibrium magnetic field.

The plasma dynamics is described by the two component fluid equations

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{V}_j) = o \tag{6}$$

$$m_j n_j \frac{d\mathbf{V}_j}{dt} = e_j n_j (\mathbf{E} + \frac{\mathbf{V}_j \times \mathbf{B}}{c}) - \nabla p_j + m_j n_j \mathbf{g} \tag{7}$$

$$(j = i, e)$$

with $T_j = const$ and $g = g(r) = -\partial_r U(r)$ is gravity expressing the curvature of the equilibrium magnetic field line. In a previous study[1] the vortex solutions for the finite Larmor radius (FLR) electrostatic flute modes in the rotating plasma are analyzed. The FLR terms may be included here by using the results of that work.

In this work we simplify the analysis by taking $T_i = 0$ which eliminates the FLR effects. We also neglect the coupling to the ion-acoustic mode which is given in Ref.3.

Solving Eq.(7) for the low frequency perturbed ($\omega \ll \omega_{ci}$) perpendicular plasma current gives

$$\mathbf{J}_\perp = \frac{m_i n_i c^2}{B_o^2}\frac{d\mathbf{E}_\perp}{dt} + \frac{c\hat{\mathbf{z}} \times \nabla p_e}{B_o} + \frac{m_i n_i c\mathbf{g} \times \hat{\mathbf{z}}}{B_o} \tag{8}$$

where the first term is the ion polarization current and the third term is the ion gravity drift current. The currents due to $\mathbf{E} \times \mathbf{B}$ drift motion of ions and electrons canceal each other and only electron diamagnetic drift contributes to the perpendicular current by the second term.

Assume that the plasma rotates uniformly, i.e, $\Omega = -\frac{cE_r}{rB_o} = const.$ which implies that $E_r(r) = \frac{B_o\Omega}{c}r$, and let $n = n_i = n_e = n_o(r) + \delta n(r, \theta, z, t)$. In the rotating frame comoving with the bulk plasma where the equilibrium electric field is transformed out and only the perturbation fields are left, we can write the quasineutrality condition (3), electron continuity equation and electron parallel momentum balance equation as

$$\frac{m_i c^2}{B_o^2}\nabla\cdot\{n\frac{\partial}{\partial t}\nabla_\perp\phi + \frac{c}{B}n[\phi, \nabla_\perp\phi]\} - \frac{2c^2 m_i\Omega}{B_o^2}[n, \phi] - \frac{m_i c}{B_o}[n,\ U - \frac{r^2\Omega^2}{2}] - \nabla_\| j_\| = 0 \tag{9}$$

$$\frac{\partial\delta n}{\partial t} - \frac{c}{B_o}\frac{dn_o}{dr}\frac{\partial\phi}{r\partial\theta} + \frac{c}{B_o}[\phi, \delta n] - \frac{1}{e}\nabla_\| j_\| = 0, \tag{10}$$

$$m_e n(\frac{\partial}{\partial t} + \frac{c}{B_o}[\phi,\ ])v_{\|e} = -enE_{\|} - T_e \nabla_{\|} n. \qquad (11)$$

where we neglect the ion contribution to the parallel current, hence $j_{\|} = -env_{\|e}$. Suppose that the space scale of the pertubations $\phi, A_{\|}$ and δn is much smaller than the length scale of the equilibrium quantity such as

$$r_n = \mid \frac{1}{n_o}\frac{dn_o}{dr} \mid^{-1} \gg \mid \frac{\nabla \delta n}{\delta n} \mid^{-1}, \mid \frac{\nabla \phi}{\phi} \mid^{-1}, \mid \frac{\nabla A_{\|}}{A_{\|}} \mid^{-1},$$

then equation (9) can be written as

$$\frac{m_i n_o c^2}{B_o^2}\{\frac{\partial}{\partial t} + \frac{c}{B_o}[\phi,\ ]\}\nabla_{\perp}^2 \phi - \frac{2m_i c^2 \Omega}{B_o^2}\frac{dn_o}{dr}\frac{\partial \phi}{r\partial \theta} + \frac{m_i c}{B_o}(\Omega^2 r + g(r))\frac{\partial \delta n}{r \partial \theta} - \nabla_{\|} j_{\|} = 0. \qquad (12)$$

Since we are interested in the study of localized perturbations in the region far away from the center of the rotating plasma column which is the usual case in some magnetized rotating plasma experiments, we can use local Cartesian coordinates in the region of vortex instead of the cylindrical coordinates. By using the coordinate transformation

$$r - r_o \to x, \quad r(\theta - \theta_o) \to y, \quad z \to z$$

where $(r_o, \theta_o, 0)$ are the cylinderical coordinates of a reference point inside the localized region under study, we can rewrite Eqs.(10)-(12) as

$$\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}\tilde{n} + \frac{c\kappa_n}{B_o}\partial_y \phi + \frac{c}{4\pi n_o e}\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\nabla_{\perp}^2 A_{\|} = 0, \qquad (13)$$

$$m_e\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}v_{\|e} = e\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\phi + \frac{e}{c}\partial_t A_{\|} - T_e\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\tilde{n} + \frac{T_e \kappa_n}{B_o}\partial_y A_{\|}, \qquad (14)$$

$$\{\partial_t+\frac{c}{B_o}[\phi,\ ]\}\nabla_\perp^2\phi+2\Omega\kappa_n\partial_y\phi+\frac{B_o}{c}(\Omega^2 r_o+g(r_o))\partial_y\tilde{n}+\frac{c_A^2}{c}\{\partial_z-\frac{1}{B_o}[A_\parallel,\ ]\}\nabla_\perp^2 A_\parallel=0, \tag{15}$$

where

$$\tilde{n}\equiv\frac{\delta n}{n_o},\qquad \kappa_n\equiv\frac{1}{r_n}=-\frac{1}{n_o}\frac{dn_0(r)}{dr},\qquad c_A^2\equiv\frac{B_o^2}{4\pi n_o m_i},$$

and in third term of Eq.(15) we have used the localized value for the centrifugal force and gravity as an approximation which can be justified by the smallness of the scale length of the perturbations.

We look for the solutions in the form of a translating helical vortex with

$$\begin{aligned}\phi(x,y,z,t)&=\phi(x,\eta)\\ A_\parallel(x,y,z,t)&=A_\parallel(x,\eta)\\ \tilde{n}(x,y,z,t)&=\tilde{n}(x,\eta)\\ \eta&=y+\alpha z-ut,\end{aligned} \tag{16}$$

and for convenience we introduce a new potential function $\psi(x,y,z,t)=\psi(x,\eta)$ such that

$$\partial_z\psi=\partial_z\phi+\frac{1}{c}\partial_t A_\parallel \tag{17}$$

from which

$$E_z=-\partial_z\psi.$$

Substituting (16) into (17) we find that

$$A_\parallel=\frac{c\alpha}{u}(\phi-\psi). \tag{18}$$

Substituting (16),(18) in Eqs.(13)-(15) and neglecting the electron mass effect in Eq.(14) yields

$$\hat{L}_1\tilde{n} - \frac{ec_A^2\alpha^2}{T_e u^2}\rho_s^2\hat{L}_2\nabla_\perp^2(\phi-\psi) - \frac{e}{T_e}\frac{v_{de}}{u}\partial_\eta\phi = 0, \tag{19}$$

$$\hat{L}_2(\psi - \frac{T_e}{e}\tilde{n}) + \frac{v_{de}}{u}\partial_\eta(\phi-\psi) = 0, \tag{20}$$

$$\hat{L}_1\nabla_\perp^2\phi - \frac{2\Omega\kappa_n}{u}\partial_\eta\phi - \frac{B_o}{cu}(\Omega^2 r_o + g(r_o))\partial_\eta\tilde{n} - \frac{c_A^2\alpha^2}{u^2}\hat{L}_2\nabla_\perp^2(\phi-\psi) = 0, \tag{21}$$

where

$$\begin{aligned}\hat{L}_1 &= \partial_\eta - \frac{c}{B_o u}[\phi,\ ]\\ \hat{L}_2 &= \partial_\eta - \frac{c}{B_o u}[\phi-\psi,\ ]\end{aligned} \tag{22}$$

and

$$v_{de} = \frac{cT_e\kappa_n}{B_o e}, \quad v_s^2 = \frac{T_e}{m_i}, \quad \rho_s = \frac{v_s}{\omega_{ci}}$$

It is interesting to note that Eqs. (19)-(21) are a quite general set of equations. Special cases of these equations have been discussed by different authors. As examples we list the following: the $\Omega = 0, g = 0$ case was analyzed in Refs.4,7; the $\Omega = 0, g = 0, \kappa_n = 0$ case was treated in Refs.5,6; the $\Omega = 0, g \neq 0, \alpha = 0, \phi = \psi$ case was discussed in Ref.8; the $\Omega = 0, g \neq 0, \alpha \neq 0, \psi = 0,\ \rho_s^2 = v_s^2/\omega_{ci}^2 \rightarrow 0$ case was discussed in Ref.9; and finally, the problem solved in Ref.1 corresponds to the case where $\Omega \neq 0, g \neq 0, \phi = \psi$ and $\alpha = 0$.

In this work we intend to analyze this more general set of equations. To further reduce the basic equations, we solve equation (20) first. From the property of

Poisson bracket it is easy to see that the relation

$$\frac{T_e}{e}\tilde{n} = \frac{v_{de}}{u}\phi + (1 - \frac{v_{de}}{u})\psi \tag{23}$$

satisfies Eq.(20).

Substituting (23) into (19) yields

$$\hat{L}_1(1 - \frac{v_{de}}{u})\psi = \frac{c_A^2\alpha^2}{u^2}\rho_s^2\hat{L}_2\nabla_\perp^2(\phi - \psi) \tag{24}$$

Due to the fact that $\hat{L}_1\psi = \hat{L}_2\psi$ Eq.(24) reduces to

$$\hat{L}_2\{\psi - \frac{c_A^2\alpha^2}{u(u - v_{de})}\rho_s^2\nabla_\perp^2(\phi - \psi)\} = 0. \tag{25}$$

Substituting (23),(24) into Eq.(21) gives

$$\hat{L}_1\{\nabla_\perp^2\phi - \rho_s^{-2}(1 - \frac{v_{de}}{u})\psi\} = [\frac{2\Omega\kappa_n}{u} + \frac{\omega_{ci}v_{de}}{v_s^2u^2}(\Omega^2r_o + g(r_o))]\partial_\eta\phi$$

$$+\frac{\omega_{ci}}{v_s^2u}(1 - \frac{v_{de}}{u})(\Omega^2r_o + g(r_o))\partial_\eta\psi. \tag{26}$$

Eqs. (25)-(26) compose the electromagnetic vortex equations of the rotating plasma, in the next section we will give the solutions of these two equations.

III. The Solitary Vortex Solutions

The basic equations (19)-(21) or their reduced form allow a wide group of solitary vortex solutions. Limiting consideration to dipole-like vortex solutions, we find two types of solutions. The first type of solution corresponds to the usual Rossby or drift wave vortex. The second type of solution is intrinsic to the electromagnetic perturbation and there is no analogy of it in hydrodynamics or the analysis of an electrostatic perturbation in a magnetized plasma. In this section we give these two types of the solutions.

A. Rossby or drift wave vortex solutions

For simplifying the notations we rewrite Eqs. (25)-(26) as

$$\hat{L}_2\{\nabla_\perp^2(\phi - \psi) - \beta_1\psi\} = 0, \tag{27}$$

$$\hat{L}_1\{\nabla_\perp^2\phi - \beta_2\psi\} - \beta_3\partial_\eta\phi - \beta_4\partial_\eta\psi = 0, \tag{28}$$

where

$$\beta_1 = \rho_s^{-2}\frac{u(u - v_{de})}{c_A^2\alpha^2}$$

$$\beta_2 = \rho_s^{-2}(1 - \frac{v_{de}}{u})$$

$$\beta_3 = \frac{2\Omega\kappa_n}{u} + \frac{\omega_{ci}v_{de}}{v_s^2u^2}[\Omega^2 r_o + g(r_o)]$$

$$\beta_4 = \frac{\omega_{ci}}{v_s^2u}(1 - \frac{v_{de}}{u})[\Omega^2 r_o + g(r_o)].$$

Considering the nonlinear third order differential structures of Eqs. (27)-(28) and noticing that the essence of the technique for seeking a Rossby vortex type solution is to reduce the nonlinear equation to linear second order differential equations in two regions separated by a circle with radius a as a characteric space scale of the vortex in the $x-\eta$ plane leads us to take the following steps.

We define the polar coordinates in $x-\eta$ plane as

$$r^2 = x^2 + \eta^2, \quad \theta = \tan^{-1}\frac{\eta}{x}$$

and suppose that the two perturbation potentials ϕ and ψ have a linear algebraic relation in both $r > a$ and $r < a$ regions as

$$\psi(r,\theta) = \alpha_1\phi(r,\theta) \quad (r > a) \tag{29}$$

$$\psi(r,\theta) = \alpha_2\phi(r,\theta) + \alpha_3\frac{B_o u}{c} r\cos\theta \quad (r < a), \tag{30}$$

where α_1, α_2 and α_3 are some real constant parameters.

Since we are looking for regular localized solutions, the potential functions must meet following requirements:

$$\phi, \psi \to 0 \quad \text{as} \quad r \to \infty \tag{31}$$

and

$$\phi, \psi = finite \quad \text{when} \quad r = 0. \tag{32}$$

Substituting Eqs.(29)-(30) into (27)-(28) and considering the conditions (31)-(32), along with proper choice of integration constants yields the linear equations satisfied by ϕ in both regions:

$$\nabla_\perp^2 \phi - k^2 \phi = 0, \quad (r > a) \tag{33}$$

$$\nabla_\perp^2 \phi + p^2 \phi + q \frac{B_o u}{c} r \cos\theta = 0 \quad (r < a) \tag{34}$$

where

$$k^2 = \frac{\alpha_1 \beta_1}{1 - \alpha_1} > 0 \tag{35}$$

$$p^2 = \frac{\alpha_2 \beta_1}{1 - \alpha_2} + \frac{(\alpha_2 + \alpha_3)[(\beta_2 + \beta_4)(\alpha_2 - 1) + \beta_1] + (\alpha_2 - 1)}{2 - \alpha_2 + \alpha_3} > 0 \tag{36}$$

$$q = \frac{\alpha_3 \beta_1}{\alpha_2 - 1} + \frac{(1 + \alpha_3)(\alpha_2 - \alpha_3)[(\beta_2 + \beta_3) + \beta_1/(\alpha_2 - 1)] + \beta_2(1 + \alpha_3)}{2 - \alpha_2 + \alpha_3} \tag{37}$$

and the parameter α_1 now is determined as one of the roots of the quadratic equation

$$\alpha_1^2 - [1 - (\beta_1 + \beta_3)/(\beta_2 + \beta_4)]\alpha_1 - \beta_3/(\beta_2 + \beta_4) = 0 \tag{38}$$

which satisfies the condition (35) while α_2, α_3 still are free.

Following the standard procedure[2,3] to solve Eqs.(33)-(34) under conditions (31)-(32) and matching the inner and outer solutions on the border $r = a$ for continuity of the function ϕ up to second derivatives gives

$$\begin{aligned} \phi_{out}(r,\theta) &= -\frac{qa}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c}\cos\theta \quad (r>a) \\ \phi_{in}(r,\theta) &= \{\frac{k^2}{p^2+k^2}\frac{a}{r}\frac{J_1(pr)}{J_1(pa)} - 1\}\frac{q}{p^2}\frac{B_o u}{c} r\cos\theta \quad (r<a) \end{aligned} \tag{39}$$

The relation between k and p is given by

$$\frac{1}{pa}\frac{J_2(pa)}{J_1(pa)} + \frac{1}{ka}\frac{K_2(ka)}{K_1(ka)} = 0, \tag{40}$$

Among the seven parameters $u, a, \alpha, p, k, \alpha_2$ and α_3 introduced to obtain the solutions (39) only three of them, say $u, a\ \alpha$, are free, the other four can be determined when the three parameters are given. For given $u, a\ \alpha$, the parameter ka is determined by Eq.(35), and then we can obtain pa by solving (40). Once p, k were determined we can use Eqs. (36),(37) and the continuity condition of ψ on the border $r = a$

$$\alpha_1\phi_{out}\mid_{r=a} = \alpha_2\phi_{in}\mid_{r=a} + \alpha_3\frac{B_o u}{c}a\cos\theta \tag{41}$$

to fix the parameters α_2 and α_3. After completing this procedure the dipole vortex solution $\phi(r,\theta)$ with three free parameters a, u, α is obtained.

Substituting (39) into (29)-(30) gives the potential function ψ in both regions as

$$\begin{aligned} \psi_{out}(r,\theta) &= -\alpha_1\frac{qa}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c}\cos\theta \quad (r>a) \\ \psi_{in}(r,\theta) &= \{\alpha_2[\frac{k^2}{p^2+k^2}\frac{a}{r}\frac{J_1(pr)}{J_1(pa)} - 1]\frac{q}{p^2} + \alpha_3\}\frac{B_o u}{c} r\cos\theta \quad (r<0) \end{aligned} \tag{42}$$

Substituting (39) and (42) into (24) the corresponding density perturbation is obtained as

$$
\begin{aligned}
\tilde{n}_{out}(r,\theta) &= -\frac{T}{e}[\frac{v_{de}}{u}(1-\alpha_1)]\frac{q}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c}a\cos\theta \quad (r>a) \\
\tilde{n}_{in}(r,\theta) &= \frac{T_e}{e}\{[\frac{v_{de}}{u}(1-\alpha_2)+\alpha_2][\frac{k^2}{p^2+k^2}\frac{a}{r}\frac{J_(pr)}{J_(pa)}-1]\frac{q}{p^2} \\
&+\alpha_3(1-\frac{v_{de}}{u})\}\frac{B_o u}{c}r\cos\theta \quad (r<0)
\end{aligned}
\tag{43}
$$

Eqs. (39),(42) and (43) represent three perturbation functions with the same dipole vortex structure as the Rossby vortex in a rotating fluid[2]. This vortex moves with the constant speed u in the $x-z$ plane without changing its shape. The velocity vector forms an angle $\gamma = \tan^{-1}(1/\alpha)$ with the direction of the equilibrium magnetic field direction. Henceforth we call this solution the Rossby type vortex solution.

The allowed region of propagating speeds u for these vortices is determined by condition (35) hence we call it the nonlinear dispersion relation of the vortices. After solving equation (38) this nonlinear dispersion relation can be written explicitly in terms of the coefficients of Eqs. (27)-(28) as

$$k^2 = \frac{1}{2}[\beta_2+\beta_3+\beta_4-\beta_1 \pm \sqrt{(\beta_2+\beta_3+\beta_4-\beta_1)^2+4\beta_1\beta_3}] > 0. \tag{44}$$

When there is no rotation and gravity, i.e. $\beta_3=\beta_4=0$, (44) degenerates to

$$-k^2 = (1-\frac{v_{de}}{u})(1-\frac{\alpha^2 c_A^2}{u^2}) < 0 \tag{45}$$

which is the nonlinear dispersion relation of usual drift-Alfven vortex. [4,7] The full relation (44) is considerable more complicated than (45) and we leave further discussion of this nolinear dispersion relation to Section **IV**.

B. The intrinsic electromagnetic vortex solution

In subsection **III.A** we obtained the Rossby type vortex solutions of Eqs. (27)-(28) and the corresponding nonlinear dispersion relation. Although this type of solution shares many nice properties with the Rossby-drift wave vortex, it has the unsatisfactory property shared with the other electromagnetic vortex solutions obtained in previous studies[5,6,7] that some physical quantities related with higher derivatives of the perturbation potential such as the perturbed parallel current $j_{\|}$ and the perpendicular magnetic perturbation $\delta \mathbf{B}_{\perp}$ are discontinuous across the boundary between the interior and external regions. The reason for this discontinuity is related with the fact that the method for seeking the Rossby-drift vortex solution lowers the order of the basic differential equations and which, in turn, limits the freedom for choosing enough parameters to meet the requirement of continuity for higher derivatives of the perturbation functions on the boundary.

As an effort to eliminate the discontinuity, in this subsection we try to construct another type of vortex solution for Eqs. (27)-(28) which envolves solving the full fourth order linear differential equation. [4]

In first looking for the new solutions we suppose that the rotation of plasma is sufficiently slow and the gravity is sufficiently weak that we can neglect the terms connected with centrifugal force and gravity. Under this condition we drop the last term of Eq.(27) and only keep the Coriolis effect in coefficient β_3, hence Eq.(28)

becomes

$$\hat{L}_1\{\nabla_\perp^2 \phi - \beta_2 \psi - \beta_3 \phi\} = 0, \tag{46}$$

where

$$\beta_3 = \frac{2\Omega\kappa_n}{u}.$$

Integrating Eqs. (27) and (46) yields

$$\nabla_\perp^2(\phi - \psi) - \beta_1 \psi = C_1(\phi - \psi - \frac{B_o u}{c} x), \tag{47}$$

$$\nabla_\perp^2 \phi - \beta_2 \psi - \beta_3 \phi = C_2(\phi - \frac{B_o u}{c} x), \tag{48}$$

where C_1, C_2 are integration constants.

Similar to the procedure of seeking the Rossby type vortex, we solve Eqs. (47)-(48) in two regions under constraints (31)-(32). Imposing the condition (31) on Eqs. (47)-(48) on the external region ($r > a$) gives

$$C_1 = C_2 = 0. \tag{49}$$

Substituting (49) into (47)-(48) and eliminating ψ from them yeilds

$$\nabla_\perp^4 \phi + \gamma_1 \nabla_\perp^2 \phi + \gamma_2 \phi = 0 \quad (r > a), \tag{50}$$

where

$$\begin{aligned} \gamma_1 &= \beta_1 - \beta_2 - \beta_3 \\ \gamma_2 &= -\beta_1 \beta_3 \end{aligned} \tag{51}$$

In the inner region where $r < a$, for simplicity we choose

$$C_1 = 0, \; C_2 \neq 0. \tag{52}$$

Then (47)-(48) can be reduced to

$$\nabla_\perp^4 \phi + \gamma_3 \nabla_\perp^2 \phi + \gamma_4 \phi + \gamma_5 \frac{B_o u}{c} r \cos\theta = 0 \quad (r < a), \tag{53}$$

where

$$\begin{aligned} \gamma_3 &= \beta_1 - \beta_2 - (\beta_3 + C_2) \\ \gamma_4 &= -\beta_1(\beta_3 + C_2) \\ \gamma_5 &= C_2 \beta_1 \end{aligned} \tag{54}$$

Following the method given in Ref.4, solving Eqs.(50) and (53) in both regions with constraints (31)-(32) and assuming the solution with form of $\phi(r,\theta) = \Phi(r)\cos\theta$ gives

$$\begin{aligned} \phi_{out}(r,\theta) &= \{A_1 \frac{K_1(\lambda_1 r)}{K_1(\lambda_1 a)} + A_2 \frac{K_1(\lambda_2 r)}{K_1(\lambda_2 a)}\} \cos\theta \quad (r > a) \\ \phi_{in}(r,\theta) &= \{A_3 \frac{J_1(\lambda_3 r)}{J_1(\lambda_3 a)} + A_4 \frac{I_1(\lambda_4 r)}{I_1(\lambda_4 a)} - \frac{\gamma_5}{\gamma_4} \frac{B_o u}{c} r\} \cos\theta \quad (r < a) \end{aligned} \tag{55}$$

where I_1 is modified Bessel function, A_1, A_2, A_3, A_4 are intgration constants, and

$$\lambda_{1,2}^2 = \frac{1}{2}[-\gamma_1 \pm \sqrt{\gamma_1^2 - 4\gamma_2}] \tag{56}$$

$$\lambda_{3,4}^2 = \frac{1}{2}[\sqrt{\gamma_3^2 - 4\gamma_4} \pm \gamma_3] \tag{57}$$

To keep $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ real, we need the following conditions:

$$\gamma_1 < 0; \qquad \gamma_1^2 > 4\gamma_2 > 0 \tag{58}$$

and

$$\gamma_4 < 0. \tag{59}$$

Substituting (55) into (48), considering the choice of C_1, C_2 in both regions gives

$$\begin{aligned}
\psi(r,\theta)_{out} &= \frac{1}{\beta_2}\{(\lambda_1^2 - \beta_3)A_1\frac{K_1(\lambda_1 r)}{K_1(\lambda_1 a)} + (\lambda_2^2 - \beta_3)A_2\frac{K_1(\lambda_2 r)}{K_1(\lambda_2 a)}\}\cos\theta, \quad (r > a) \\
\psi(r,\theta)_{in} &= \frac{1}{\beta_2}\{-(\lambda_3^2 + C_2 + \beta_3)A_3\frac{J_1(\lambda_3 r)}{J_1(\lambda_3 a)} + (\lambda_4^2 - C_2 - \beta_3)A_4\frac{I_1(\lambda_4 r)}{I_1(\lambda_4 a)}\}\times \\
&\quad \times \cos\theta. \ (r < a)
\end{aligned} \tag{60}$$

For determining the integration constants $C_2,\ A_1,\ A_2,\ A_3,\ A_4$ we impose the following matching conditions on the boundary for the solutions (55) and (60):

$$\phi_{in}\,|_a = \phi_{out}\,|_a \tag{61a}$$

$$\frac{\partial\phi_{in}}{\partial r}\,|_a = \frac{\partial\phi_{out}}{\partial r}\,|_a \tag{61b}$$

$$\nabla_\perp^2\phi_{in}\,|_a = \nabla_\perp^2\phi_{out}\,|_a \tag{61c}$$

$$\psi_{in}\,|_a = \psi_{out}\,|_a \tag{62a}$$

$$\frac{\partial\psi_{in}}{\partial r}\,|_a = \frac{\partial\psi_{out}}{\partial r}\,|_a \tag{62b}$$

$$\nabla_\perp^2 \psi_{in} \,|_a = \nabla_\perp^2 \psi_{out} \,|_a \,. \tag{62c}$$

Substituting Eqs. (55),(60) into (61a)-(62a), after some algebra, we have determined A_1, A_2, A_3, A_4 as

$$\begin{aligned}
A_1 &= \frac{a_{11}a_1 + a_{12}a_2}{\Gamma}\frac{B_o ua}{c}K_1(\lambda_1 a) \\
A_2 &= \frac{a_{21}a_1 + a_{22}a_2}{\Gamma}\frac{B_o ua}{c}K_1(\lambda_2 a) \\
A_3 &= \frac{a_{31}a_1 + a_{32}a_2}{\Gamma}\frac{B_o ua}{c}J_1(\lambda_3 a) \\
A_4 &= \frac{a_{41}a_1 + a_{42}a_2}{\Gamma}\frac{B_o ua}{c}I_1(\lambda_4 a)
\end{aligned} \tag{63}$$

where

$$\Gamma = (\lambda_1^2 - \lambda_2^2)[(\lambda_3 J_2(\lambda_3 a)I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_1 a)K_1(\lambda_2 a)$$

$$+(\lambda_3^2 + \lambda_4^2)[(\lambda_1 K_2(\lambda_1 a)K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a)K_1(\lambda_1 a))]J_1(\lambda_3 a)I_1(\lambda_4 a), \tag{64}$$

$$a_1 = 1 \tag{65}$$

$$a_2 = \frac{\beta_3}{\beta_3 + C_2} \tag{66}$$

$$a_{11} = -\{(\lambda_3^2 + \lambda_4^2)\lambda_2 K_2(\lambda_2 a)J_1(\lambda_3 a)I_1(\lambda_4 a)$$

$$+\lambda_2^2[\lambda_3 J_2(\lambda_3 a)I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_2 a)\} \tag{67}$$

$$a_{12} = \lambda_3\lambda_4[\lambda_4 J_2(\lambda_3 a)I_1(\lambda_4 a) - \lambda_3 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_2 a), \tag{68}$$

$$a_{21} = \lambda_1^2[\lambda_3 J_2(\lambda_3 a) I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a) J_1(\lambda_3 a)] K_1(\lambda_1 a)$$

$$+(\lambda_3^2 + \lambda_4^2)\lambda_1 K_2(\lambda_1 a) J_1(\lambda_3 a) I_1(\lambda_4 a), \tag{69}$$

$$a_{22} = \lambda_3\lambda_4[\lambda_3 I_2(\lambda_4 a) J_1(\lambda_3 a) - \lambda_4 J_2(\lambda_3 a) I_1(\lambda_4 a)] K_1(\lambda_1 a), \tag{70}$$

$$a_{31} = \lambda_1\lambda_2[\lambda_1 K_2(\lambda_2 a) K_1(\lambda_1 a) - \lambda_2 K_2(\lambda_1 a) K_1(\lambda_2 a)] I_1(\lambda_4 a), \tag{71}$$

$$a_{32} = (\lambda_1^2 - \lambda_2^2)\lambda_4 I_2(\lambda_4 a) K_1(\lambda_1 a) K_1(\lambda_2 a)$$

$$+\lambda_4^2[\lambda_1 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a) K_1(\lambda_1 a)] I_1(\lambda_4 a), \tag{72}$$

$$a_{41} = \lambda_1\lambda_2[\lambda_2 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_1 K_2(\lambda_2 a) K_1(\lambda_1 a)] J_1(\lambda_3 a), \tag{73}$$

$$a_{42} = (\lambda_1^2 - \lambda_2^2)\lambda_3 J_2(\lambda_3 a) K_1(\lambda_1 a) K_1(\lambda_2 a)$$

$$+\lambda_3^2[\lambda_1 K_2(\lambda_1 a) K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a) K_1(\lambda_1 a)] J_1(\lambda_3 a). \tag{74}$$

Notice that with the choice of constants $C_1 = 0$ Eq. (47) for both inner and external regions becomes the same equation as

$$\nabla_\perp^2 \psi = \nabla_\perp^2 \phi - \beta_1 \psi, \tag{75}$$

so condition (62c) is automatically satisfied.

Condition (62b) gives a relation

$$\lambda_1^2 \frac{\lambda_1 K_2(\lambda_1 a)}{K_1(\lambda_1 a)} A_1 + \lambda_2^2 \frac{\lambda_2 K_2(\lambda_2 a)}{K_1(\lambda_2 a)} A_2 + (\lambda_3^2 + C_2) \frac{\lambda_3 J_2(\lambda_3 a)}{J_1(\lambda_3 a)} A_3$$

$$+(\lambda_4^2 - C_2) \frac{\lambda_4 I_2(\lambda_4 a)}{I_1(\lambda_4 a)} A_4 = 0 \qquad (76)$$

through which we can determine constant C_2 for given free parameters u, a, α.

Obviously, substituting Eqs. (55) and (60) into (24) we can give the explicit expression for corresponding density pertubation function $\tilde{n}(r, \theta)$.

Eqs. (55) and (60) give the intrinsic electromagnetic solitary vortex solutions in the rotating plasma under slow rotation assumption we made in the begining of this subsection. After all integration constants were determined by the coefficients of Eqs. (27) and (46), the constraint conditions (58)-(59) play the role of nonlinear dispersion relation for this type of vortex, which will determine the allowed region of the vortices. We leave the discussion of this relation in the next section.

C. Special cases

As we mentioned in section II, the Eqs. (19)-(21) are a quite general set of nonlinear equations describing the dynamics of a magnetized plasma. Beside the special cases analyzed previously which we listed there, here we present two interesting special cases which have not been studied yet. These two special cases correspond to the electron drift vortex and the ballooning vortex in a rotating plasma.

1. The rotational electron drift vortex

When $A_{\|} = 0$, but $\Omega \neq 0$, $g \neq 0$ and $\alpha \neq 0$, we are treating the nonflute case of electrostatic perturbation in a rotating plasma. For this case it is convenient to start from Eqs.(10)-(12). Neglecting electron mass, from Eq.(11) we get the adiabatic electron density

$$\tilde{n} = \frac{e\phi}{T_e}. \tag{77}$$

Substututing (77) into (10) and (12), eliminating $\nabla_{\|} j_{\|}$ yields the single equation for ϕ

$$\partial_t \phi - \rho_s^2 \{\partial_t + \frac{c}{B_o}[\phi,\ ]\} \nabla_{\perp}^2 \phi + \{v_{de} - \rho_s^2 [2\Omega\kappa_n + \frac{eB_o}{c}(\Omega^2 r_o + g(r_o))]\} \partial_y \phi = 0. \tag{78}$$

Comparing Eq.(78) with the Eq.(10) of Ref. 3, we see that (78) is the nonlinear electron drift wave equation modified by the plasma rotation.

Following the procedure of Ref.3 the localized solution of the perturbation potential ϕ in form of (16) is given as

$$\phi_{out}(r,\theta) = \frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c} a\cos\theta, \quad (r > a)$$
$$\phi_{in}(r,\theta) = \{\frac{k^2+p^2}{p^2}\frac{r}{a} - \frac{k^2}{p^2}\frac{J_1(pr)}{J_1(pa)}\}\frac{B_o u}{c} a\cos\theta, \quad (r < a) \tag{79}$$

where p, k are related by the relation

$$\frac{1}{ka}\frac{K_2(ka)}{K_1(ka)} + \frac{1}{pa}\frac{J_2(pa)}{J_1(pa)} = 0.$$

The nonlinear dispersion relation is given by

$$k^2 = \frac{1}{\rho_s^2}\{1 - \frac{v_{de}}{u} + \frac{2\kappa_n \rho_s^2 \Omega + (\Omega^2 r_o + g(r_o))/\omega_{ci}}{u}\} > 0 \tag{80}$$

From Eq.(80) we see that the allowed region of the usual electron drift vortex propagating speeds is restrained by the plasma rotation through the last term. So we call this solution a rotational electron drift vortex.

2. The rotational ballooning vortex

If we follow Ref. 10, take the limit

$$\psi = 0, \quad \rho_s^2 \to 0,$$

then from Eq. (20) we have

$$\tilde{n} = \frac{v_{de}}{u}\frac{e\phi}{T_e}. \tag{81}$$

Substituting (81) into Eq.(21) gives

$$(u^2 - \alpha^2 c_A^2)\hat{L}_1 \nabla_\perp^2 \phi = \Gamma^2 \partial_\eta \phi \tag{82}$$

where

$$\Gamma^2 = \kappa_n[g(r_o) + (\Omega^2 r_o + 2\Omega u)] \tag{83}$$

Comparing Eqs. (82)-(83) with Eq. (9) in Ref.9, we see that the Eq.(82) describes the so-called ballooning vortex in rotating plasma with the same potential function as (79). And the nonlinear dispersion is given by

$$\begin{aligned} k^2 &= \frac{\Gamma^2}{u^2 - \alpha^2 c_A^2} \\ &= \kappa_n[g(r_o) + \Omega^2 r_o + 2\Omega u]/(u^2 - \alpha^2 c_A^2) > 0 \end{aligned} \tag{84}$$

Since the analogy of this vortex with the one analyzed in Ref.9 for the nonrotating plasma, we call it the rotational ballooning vortex.

IV. Discussion and Conclusion

In previous section we give two types of solitary vortex solutions in a rotating plasma. In this section we discuss some properties of these solutions.

(1). Common features of two types of vortices

From direct observation of expressions (39),(42) for the Rossby type vortex and expressions (55),(60) for the intrinsic electromagnetic vortex we can see that both solutions have the common features of a localized dipole structure; moving in the $y - z$ plane with constant speed u in the direction at an angle $\gamma = \tan^{-1} \frac{1}{\alpha}$ respect to the equilibrium magnetic field direction without changing their shape. The localized structures decay to zero as $r \to \infty$ with the asymptotic form $e^{-\lambda r}/\sqrt{r}$, where for Rossby-drift vortex $\lambda = k$ while for the intrinsic electromagnetic one $\lambda = min\{\lambda_1, \lambda_2\}$, which can be seen from Eqs.(39) and (60).

(2). Differences between the two types of vortices

Comparing (55), (60) with (39) and (42) we find that the intrinsic electromagnetic vortex is a more complicated structure than the Rossby-drift vortex in both its inner and outer regions. The amplitude of the intrinsic electromagnetic vortex is a linear combination of both regular and modified Bessel functions J_1 and I_1 in the interior region and the combination of two first order Mcdonald functions with different decay lengths in external region. In contrast, the amplitude of the Rossby-drift vortex is single $J_1(pr)$ in the interior region and $K_1(kr)$ in the external region. Related with this difference, most significantly, these two types of vortex are different physically. From Eqs.(2), (4) and (18) we have

$$\delta \mathbf{B}_{\perp} = \frac{c\alpha}{u} \nabla(\phi - \psi) \times \hat{\mathbf{z}} \tag{85}$$

and

$$j_{\|} = -\frac{c^2 \alpha}{4\pi u} \nabla_{\perp}^2 (\phi - \psi). \tag{86}$$

Due to the fact that for the intrinsic electromagnetic vortex $\nabla_{\perp}\psi$ and $\nabla_{\perp}^2\psi$ are continuous on the border $r = a$, hence from Eqs. (85)-(86) we see that this type of vortex keeps $j_{\|}$ and $\delta \mathbf{B}_{\perp}$ continuous across the boundary. In contrast, for the Rossby type vortex both $\delta \mathbf{B}_{\perp}$ and $j_{\|}$ have jumps on the boundary because for it only ψ but not it's derivatives is continuous across the border. The jumps of the perpendicular magnetic field perturbation and the parallel current on the boundary is defined as

$$[\Delta \delta \mathbf{B}_{\perp}]_a = \delta \mathbf{B}_{\perp_{in}}(a) - \delta \mathbf{B}_{\perp_{out}}(a), \qquad [\Delta j_{\|}]_a = j_{\|_{in}}(a) - j_{\|_{out}}(a),$$

and its value are calculated from Eqs.(29)-(30), (39), (42) and (85)-(86) as

$$[\Delta \delta \mathbf{B}_{\perp}]_a = \frac{c\alpha}{u} \{(\alpha_1 - \alpha_2) \nabla_{\perp}\phi \mid_a \times \hat{\mathbf{z}} + \alpha_3 \frac{B_o u}{c} \hat{\mathbf{y}}\} \neq 0 \tag{87}$$

and

$$[\Delta j_{\|}]_a = \frac{c\alpha q B_o a k^2}{4\pi(p^2 + k^2)} \{\alpha_2 - \alpha_1\} \cos\theta \neq 0. \tag{88}$$

(3). Different allowed regions of the vortex propagating speed

Unlike other Rossby-drift vortices, the propagating speed of our first type solution given in subsection **III.A** can be complementary to the phase velocity of the corresponding linear modes, or it can also ovelap with the later. As for the intrinsic electromagnetic vortex, the calculation shows that it has much narrower allowed region of propagating speed.

The nonlinear dispersion relation for the Rossby-drift vortex is Eq. (44), which give the allowed region of propagating speeds of the vortices. Remembering that one positive root of equation (38) is sufficient for existence of the Rossby type vortex, so from (44) we can immediatly find that the allowed regions for vortex propagating speeds are determined by following conditions:

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 > 0; \qquad \beta_1\beta_3 \geq 0 \tag{89a}$$

or

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 < 0; \qquad \beta_1\beta_3 > 0 \tag{89b}$$

or

$$(\beta_2 + \beta_3 + \beta_4 - \beta_1) > 0; \qquad (\beta_2 + \beta_3 + \beta_4 - \beta_1)^2 > -4\beta_1\beta_3 > 0 \tag{89c}$$

The dispersion relation of the corresponding linear mode obtained by substituting $\tilde{n}, \phi, \psi$ with the form of propagating plane waves as $f(x, y, z, t) \sim e^{i(k_x x + k_y y + k_z z - \omega t)}$ into the linearized forms of Eqs.(13)-(15) and neglecting the electron mass effect is

$k_\perp^4 + k_\perp^2\{\rho_s^{-2}(1 - v_{de}/c_p) + [2\Omega\kappa_n + \frac{\omega_{ci}}{v_s^2}(\Omega^2 r_o + g(r_o))]/c_p - \rho_s^{-2}c_A^{-2}(\frac{k_y}{k_z})^2 c_p(c_p - v_{de})\}$

$$-\rho_s^{-2}c_A^{-2}(\frac{k_y}{k_z})^2(c_p - v_{de})[2\Omega\kappa_n + \frac{\omega_{ci}}{v_s^2}v_{de}(\Omega^2 r_o + g(r_o)] = 0 \qquad (90)$$

where

$$k_\perp^2 = k_x^2 + k_y^2, \qquad c_p = \frac{\omega}{k_y}.$$

If we take the correspondences between votex quantanties α, u, and linear mode quantanties $k_z/k_y, c_p$ then solve Eq.(89), we have

$$k_\perp^2 = \frac{1}{2}\{-(\beta_2 + \beta_3 + \beta_4 - \beta_1) \pm \sqrt{(\beta_2 + \beta_3 + \beta_4 - \beta_1)^2 + 4\beta_1\beta_3}\} \qquad (91)$$

where we substitute k_z/k_y for α and c_p for u in all β's. Due to the fact that

$$k_\perp^2 = k_x^2 + k_y^2 > 0,$$

the allowed regions for the linear mode phase velocity are determined by

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 > 0; \qquad \beta_1\beta_3 > 0 \qquad (92a)$$

or

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 < 0; \qquad \beta_1\beta_3 \geq 0 \qquad (92b)$$

or

$$\beta_2 + \beta_3 + \beta_4 - \beta_4 < 0; \qquad (\beta_2 + \beta_3 + \beta_4 - \beta_4)^2 > -4\beta_1\beta_3 > 0 \qquad (92c)$$

The allowed regions of vortex propagating speeds determined by Eqs. (89a)-(89c) and the regions of phase velocity for linear mode determined by (92a)-(92c) could be complemetary (*the regions determined by (89c)and (92c); the regions determined by (89a) and (92b) when* $\beta_1\beta_3 = 0$), as a common feature for different vortex of this type. But unlike other Rossby type vortex, they also can overlap (*the regions determined by (89a),(89b) and (92a),(92b) when* $\beta_1\beta_3 \neq 0$).

For the intrinsic electromagnetic vortex, the corresponding nonlinear dispersion relation (58)-(59) limits the propagating speeds of the vortices to a much narrower region, explicitly the region is determined by both conditions

$$\beta_2 + \beta_3 - \beta_1 > 0; \qquad (\beta_2 + \beta_3 - \beta_1)^2 > -4\beta_1\beta_3 > 0 \tag{93}$$

and

$$\beta_1(\beta_3 + C_2) > 0 \tag{94}$$

Remembering that the intrinsic magnetic vortex solution is obtained under a slow rotation assumption, we see that condition (93) is equivalent to condition (89c), but the further constraint given by condition (94) forces the allowed region for the propagating speed of the intrinsic electromagnetic vortex to be narrower, accordingly this region is not complementary to the allowed region of phase velocity.

In conclusion our study shows the existence of two types electromagnetic vortex in a rotating plasma immersed in a homogeneous magnetic field. These two types of vortices have localized coherent dipolar structure but the allowed region of the vortex propagation speeds for the Rossby-drift vortex is wider. We show that two spatial scales $k_1^2 \simeq \beta_2 + \beta_3 - \beta_1$ and $k_2^2 \simeq -\beta_1\beta_3/(\beta_2 + \beta_3 - \beta_1)$ exist for the new type of the vortex which is intrinsic to the electromagnetic perturbation in a magnetized plasma. As special cases we also show that the electron drift vortex and the ballooning vortex found in a nonrotating plasma [3,9] have their analogs in a rotating plasma. Inclusion of finite ion temperture and the ion parallel motion which are neglected in this work will bring more effects such as ion-acoustic wave coupling and finite Larmor radius effect into consideration.

Finally, we remark that several authors[8,11] have used the solitary vortex solutions to explain the formation of coherent vortex structure observed in space plasma. Since the localized, coherent vortices solutions obtained in this work can only exist in the edge of rotating plasma, we may, heuristically, expect to use them as candidate entity of the edge turbulence observed in some confinement experiments. However, to support this expectation much work remain to be done, for instance, for understanding the stability properties of these vortices under interaction both numerical and analytical studies of vortex-vortex collisions and vortex-wave interactions are required.

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The Electromagnetic Solitary Vortices In Rotating Plasma

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Abstract

The nonlinear equations describing drift-Alfven solitary vortices in a low β, rotating plasma are derived. Two types of solitary vortex solutions along with their corresponding nonlinear dispersion relations are obtained. Both solutions have the localized coherent dilopar structure. The first type of solution belongs to the family of the usual Rossby or drift wave vortex, while the second type of solution is intrinsic to the electromagnetic perturbation in a magnetized plasma and is a complicated structure. While the first type of vortex is a solution to a second order differential equation the second one is the solution of a fourth order differential equation intrinsic to the electromagnetic problem. The fourth order vortex solution has two intrinsic space scales in contrast to the single space scale of the previous drift vortex solution. With the second short scale length the parallel current density

at the vortex interface becomes continuous. As special cases the rotational electron drift vortex and the rotational ballooning vortex also are given.

I. Introduction

In a recent work by Horton et al [1] the elecrostatic flute solitary vortex in a low β rotating plasma is analyzed. Their dipolar vortex solution belongs to the usual Rossby or drift wave solitary vortex family[2,3]. In another recent work [4] analyzing the electromagnetic drift-Alfven vortex problem we show that generally the solution involves solving fourth order partial differential equations. These new solutions are quite different from the Rossby or drift wave vortices. In this work we generalize these two earlier results [1,4] in the following manner: by including the parallel electron dynamics and the perturbation of magnetic fields we generalize Ref.1 to describe the electromagnetic perturbations and by including plasma rotation in the equations studied in Ref.4 we look for the new type of vortex from a more complicated equation.

When the nonlinear parallel electron dynamics and the perpendicular magnetic field perturbation are included, a set of nonlinear equations describing the drift-Alfven vortices in a rotating plasma is derived. The main difference of this set of equations with the corresponding equations derived in the rest plasma [5,6] is contained in the new terms arising from plasma rotation which adds complexity to solving the equations.

Our analysis shows that, like in Refs.5-7, these new equations also allow the existence of the usual Rossby or drift wave type vortex. The corresponding nonlinear dispersion relations for this type of vortex also is obtained. Close examination of the Rossby-drift type vortex solution reveals that for the electromagnetic problem the drift vortex solutions in Refs.5-7 have a discontinuity of some physical quantities such as parallel current density $j_{\|}$ and the magnetic field perturbarion $\delta\mathbf{B}_{\perp}$ across the border of the interior and external regions.

As a remedy to this flaw we obtain a new type of vortex solution which envolves solving a fourth order linear partial differential equation for perturbation functions. While still being dipole-like in structure, this new type of vortex solution has a more complicated radial structure in both the inner and outer regions. The new solution, however, overcomes the discontinuity difficulty of $j_{\|}$ and $\delta\mathbf{B}_{\perp}$ across the boundary surface. The nonlinear dispersion relation for this type of vortex solution also is derived here for the first time. Unlike the first type of vortex, the second type of vortex appears to have no hydrodynamic analogy, whereupon we call it the intrinsic electromagnetic vortex to distiguish it from the previous one.

The arrangement of this work is as follows. In section **II** we derive the nonlinear equations. In section **III** we solve the derived equations and give the two types of vortex solutions explicitly. Finally, in section **IV** we discuss some properties of these solutions and give the conclusion.

II. Electromagnetic Drift-Alfven vortex equation

In this section we derive the nonlinear equations describing the coupling of the drift wave to the Alfven wave in a rotating plasma. The coupling of the compressional Alfven mode is neglected by using only the parallel component of the vector potential $A_\| = A_z(r,\theta,z,t)$ and the electrostatic potential $\phi(r,\theta,z,t)$.

The electromagnetic fields in the plasma are

$$\mathbf{E} = E_r(r)\hat{\mathbf{r}} - \nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}A_\|\hat{\mathbf{z}}, \qquad (II.1)$$

$$\mathbf{B} = B_o\hat{\mathbf{z}} + \nabla\times(A_\|\hat{\mathbf{z}}). \qquad (II.2)$$

where $E_r(r)$ is the equilibrium electric field which drives the plasma rotating with angular velocity

$$\mathbf{\Omega} = -\frac{cE_r}{rB_o}\hat{\theta},$$

and $\phi, A_\|$ are perturbation functions.

For low frequency $\omega \ll \omega_{ci} = eB_o/m_ic$ and large space scale $\lambda_\perp \gg \rho_i, \lambda_{De}$ motions the Maxwell field equations reduce to the quasineutrality condition

$$\nabla\cdot\mathbf{J} = \nabla_\perp\cdot\mathbf{J}_\perp + \mathbf{b}\cdot\nabla j_\| = 0, \qquad (III.3)$$

and to the Ampere's law

$$\nabla_\perp^2 A_\| = -\frac{4\pi}{c}j_\|. \qquad (II.4)$$

Due to the bending of the magnetic field the parallel direction is given by

$$\hat{\mathbf{b}} = \hat{\mathbf{z}} + \frac{1}{B_o}\nabla_\perp A_\| \times \hat{\mathbf{z}}$$

and the parallel gradient by

$$\nabla_\| = \hat{\mathbf{b}} \cdot \nabla = \partial_z - \frac{1}{B_o}[A_\|, \ ], \tag{II.5}$$

where $[f,g] = \hat{\mathbf{z}} \cdot \nabla f \times \nabla g$ is the Poisson bracket in the plane perpendicular to the equilibrium magnetic field.

The plasma dynamics is described by the two component fluid equations

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{V}_j) = o \tag{II.6}$$

$$m_j n_j \frac{d\mathbf{V}_j}{dt} = e_j n_j (\mathbf{E} + \frac{\mathbf{V}_j \times \mathbf{B}}{c}) - \nabla p_j + m_j n_j \mathbf{g} \tag{II.7}$$

$$(j = i, e)$$

with $T_j = const$ and $g = g(r)$ is gravity expressing the curvature of the equilibrium magnetic field line. In a previous study[1] the vortex solutions for the finite Larmor radius (FLR) electrostatic flute modes in the rotating plasma are analyzed. The FLR terms may be included here by using the results of that work. In this work we simplify the analysis by taking $T_i = 0$ which eliminates the FLR effects. We also neglect the coupling to the ion-acoustic mode which is given in Ref.3.

Solving Eq.(II.7) for the low frequency perturbed ($\omega \ll \omega_{ci}$) perpendicular plasma current gives

$$\mathbf{J}_\perp = \frac{m_i n_i c^2}{B_o^2}\frac{d\mathbf{E}_\perp}{dt} + \frac{c\hat{\mathbf{z}} \times \nabla p_e}{B_o} + \frac{m_i n_i c\mathbf{g} \times \hat{\mathbf{z}}}{B_o} \qquad (II.8)$$

where the first term is the ion polarization current and the third term is the ion gravity drift current. The currents due to $\mathbf{E} \times \mathbf{B}$ drift motion of ions and electrons canceal each other and only electron diamagnetic drift contributes to the perpendicular current by the second term.

Assume that the plasma rotates uniformly, i.e, $\Omega = -\frac{cE_r}{rB_o} = const.$ which implies that $E_r(r) = \frac{B_o\Omega}{c}r$, and let $n = n_i = n_e = n_o(r) + \delta n(r, \theta, z, t)$. In the rotating frame comoving with the bulk plasma where the equilibrium electric field is transformed out and only the perturbation fields are left, we can write the quasineutrality condition (II.3), electron continuity equation and electron parallel momentum balance equation as

$$\frac{m_i c^2}{B_o^2}\nabla \cdot \{n\frac{\partial}{\partial t}\nabla_\perp \phi + \frac{c}{B}n[\phi, \nabla_\perp \phi]\} - \frac{2c^2 m_i \Omega}{B_o^2}[n, \phi] - \frac{m_i c}{B_o}[n,\ U - \frac{r^2\Omega^2}{2}] - \nabla_{||} j_{||} = 0 \qquad (II.9)$$

$$\frac{\partial \delta n}{\partial t} - \frac{c}{B_o}\frac{dn_o}{dr}\frac{\partial \phi}{r\partial \theta} + \frac{c}{B_o}[\phi, \delta n] - \frac{1}{e}\nabla_{||} j_{||} = 0, \qquad (II.10)$$

$$m_e n(\frac{\partial}{\partial t} + \frac{c}{B_o}[\phi,\ ])v_{||e} = -enE_{||} - T_e \nabla_{||} n. \qquad (II.11)$$

where we neglect the ion contribution to the parallel current, hence $j_{\|} = -env_{\|e}$. Suppose that the space scale of the pertubations $\phi, A_{\|}$ and δn is much smaller than the length scale of the equilibrium quantity such as

$$r_n = \left| \frac{1}{n_o}\frac{dn_o}{dr} \right|^{-1} \gg \left| \frac{\nabla \delta n}{\delta n} \right|^{-1}, \left| \frac{\nabla \phi}{\phi} \right|^{-1}, \left| \frac{\nabla A_{\|}}{A_{\|}} \right|^{-1},$$

then equation (II.9) can be written as

$$\frac{m_i n_o c^2}{B_o^2}\{\frac{\partial}{\partial t} + \frac{c}{B_o}[\phi,\ ]\}\nabla_{\perp}^2\phi - \frac{2m_i c^2\Omega}{B_o^2}\frac{dn_o}{dr}\frac{\partial\phi}{r\partial\theta} + \frac{m_i c}{B_o}(\Omega^2 r + g(r))\frac{\partial\delta n}{r\partial\theta} - \nabla_{\|}j_{\|} = 0. \tag{II.12}$$

Since we are interested in the study of localized perturbations in the region far away from the center of the rotating plasma column which is the usual case in some magnetized rotating plasma experiments, we can use local Cartesian coordinates in the region of vortex instead of the cylindrical coordinates. By using the coordinate transformation

$$r - r_o \to x, \quad r(\theta - \theta_o) \to y, \quad z \to z$$

where $(r_o, \theta_o, 0)$ are the cylinderical coordinates of a reference point inside the localized region under study, we can rewrite Eqs.(II.10)-(II.12) as

$$\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}\tilde{n} + \frac{c\kappa_n}{B_o}\partial_y\phi + \frac{c}{4\pi n_o e}\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\nabla_{\perp}^2 A_{\|} = 0, \tag{II.13}$$

$$m_e\{\partial_t + \frac{c}{B_o}[\phi,\ ]\}v_{\|e} = e\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\phi + \frac{e}{c}\partial_t A_{\|} - T_e\{\partial_z - \frac{1}{B_o}[A_{\|},\ ]\}\tilde{n} + \frac{T_e\kappa_n}{B_o}\partial_y A_{\|}, \tag{II.14}$$

$$\{\partial_t + \frac{c}{B_o}[\phi, \,]\}\nabla_\perp^2\phi + 2\Omega\kappa_n\partial_y\phi + \frac{B_o}{c}(\Omega^2 r_o + g(r_o))\partial_y\tilde{n} + \frac{c_A^2}{c}\{\partial_z - \frac{1}{B_o}[A_\|, \,]\}\nabla_\perp^2 A_\| = 0, \tag{II.15}$$

where

$$\tilde{n} \equiv \frac{\delta n}{n_o}, \qquad \kappa_n \equiv \frac{1}{r_n} = -\frac{1}{n_o}\frac{dn_0(r)}{dr},$$

and in third term of Eq.(II.15) we have used the localized value for the centrifugal force and gravity as an approximation which can be justified by the smallness of the scale length of the perturbations.

We look for the solutions in the form of a translating helical vortex with

$$\begin{aligned} \phi(x,y,z,t) &= \phi(x,\eta) \\ A_\|(x,y,z,t) &= A_\|(x,\eta) \\ \tilde{n}(x,y,z,t) &= \tilde{n}(x,\eta) \\ \eta &= y + \alpha z - ut, \end{aligned} \tag{II.16}$$

and for convenience we introduce a new potential function $\psi(x,y,z,t) = \psi(x,\eta)$ such that

$$\partial_z\psi = \partial_z\phi + \frac{1}{c}\partial_t A_\| \tag{II.17}$$

from which

$$E_z = -\partial_z\psi.$$

Substituting (II.16) into (II.17) we find that

$$A_\| = \frac{c\alpha}{u}(\phi - \psi). \tag{II.18}$$

Substituting (II.16),(II.18) in Eqs.(II.13)-(II.15) and neglecting the electron mass effect in Eq.(II.14) yields

$$\hat{L}_1\tilde{n} - \frac{ec^2v_s^2\alpha^2}{T_e\omega_{pi}^2u^2}\hat{L}_2\nabla_\perp^2(\phi-\psi) - \frac{e}{T_e}\frac{v_{de}}{u}\partial_\eta\phi = 0, \qquad (II.19)$$

$$\hat{L}_2(\psi - \frac{T_e}{e}\tilde{n}) + \frac{v_{de}}{u}\partial_\eta(\phi-\psi) = 0, \qquad (II.20)$$

$$\hat{L}_1\nabla_\perp^2\phi - \frac{2\Omega\kappa_n}{u}\partial_\eta\phi - \frac{B_o}{cu}(\Omega^2 r_o + g(r_o))\partial_\eta\tilde{n} - \frac{c_A^2\alpha^2}{u^2}\hat{L}_2\nabla_\perp^2(\phi-\psi) = 0, \quad (II.21)$$

where

$$\hat{L}_1 = \partial_\eta - \frac{c}{B_o u}[\phi, \ ]$$
$$\hat{L}_2 = \partial_\eta - \frac{c}{B_o u}[\phi-\psi, \ ] \qquad (II.22)$$

and

$$v_{de} = \frac{cT_e\kappa_n}{B_o e}, \quad v_s^2 = \frac{T_e}{m_i}, \quad \omega_{pi}^2 = \frac{4\pi n_o e^2}{m_i}$$

It is interesting to note that Eqs. (II.19)-(II.21) are a quite general set of equations. Special cases of these equations have been discussed by different authors. As examples we list the following: the $\Omega = 0, g = 0$ case was analyzed in Refs.4,5; the $\Omega = 0, g = 0, \kappa_n = 0$ case was treated in Refs.6,7; the $\Omega = 0, g \neq 0, \alpha = 0, \phi = \psi$ case was discussed in Ref.8; the $\Omega = 0, g \neq 0, \alpha \neq 0, \psi = 0,\ \rho_s^2 = v_s^2/\omega_{ci}^2 \to 0$ case was discussed in Ref.9; and finally, the problem solved in Ref.1 corresponds to the case where $\Omega \neq 0, g \neq o, \phi = \psi$ and $\alpha = 0$.

In this work we intend to analyze this more general set of equations. To further reduce the basic equations, we solve equation (II.20) first. From the property of Poisson bracket it is easy to see that the relation

$$\frac{T_e}{e}\tilde{n} = \frac{v_{de}}{u}\phi + (1 - \frac{v_{de}}{u})\psi \tag{II.23}$$

satisfies Eq.(2.20).

Substituting (II.23) into (II.19) yields

$$\hat{L}_1(1 - \frac{v_{de}}{u})\psi = \frac{c^2 v_s^2 \alpha^2}{\omega_{pi}^2 u^2}\hat{L}_2\nabla_\perp^2(\phi - \psi) \tag{II.24}$$

Due to the fact that $\hat{L}_1\psi = \hat{L}_2\psi$ Eq.(II.24) reduces to

$$\hat{L}_2\{\psi - \frac{c^2 v_s^2 \alpha^2}{\omega_{pi}^2 u(u - v_{de})}\nabla_\perp^2(\phi - \psi)\} = 0. \tag{II.25}$$

Substituting the (II.23),(II.24) into Eq.(II.21) gives

$$\hat{L}_1\{\nabla_\perp^2\phi - \frac{\omega_{pi}^2 c_A^2}{c^2 v_s^2}(1 - \frac{v_{de}}{u})\psi\} = [\frac{2\Omega\kappa_n}{u} + \frac{\omega_{ci}v_{de}}{v_s^2 u^2}(\Omega^2 r_o + g(r_o))]\partial_\eta\phi$$

$$+\frac{\omega_{ci}}{v_s^2 u}(1 - \frac{v_{de}}{u})(\Omega^2 r_o + g(r_o))\partial_\eta\psi. \tag{II.26}$$

Eqs. (II.25)-(II.26) compose the electromagnetic vortex equations of the rotating plasma, in the next section we will give the solutions of these two equations.

III. The Solitary Vortex Solutions

The basic equations (II.19)-(II.21) or their reduced form allow a wide group of solitary vortex solutions. Limiting consideration to dipole-like vortex solutions, we find two types of solutions. The first type of solution corresponds to the usual Rossby or drift wave vortex. The second type of solution is intrinsic to the electromagnetic perturbation and there is no analogy of it in hydrodynamics or the analysis of an electrostatic perturbation in a magnetized plasma. In this section we give these two types of the solutions.

III.1 Rossby or Drift Wave Vortex Solutions

For simplifying the notations we rewrite Eqs. (II.25)-(II.26) as

$$\hat{L}_2\{\nabla_\perp^2(\phi-\psi)-\beta_1\psi\}=0, \qquad (III.1)$$

$$\hat{L}_1\{\nabla_\perp^2\phi-\beta_2\psi\}-\beta_3\partial_\eta\phi-\beta_4\partial_\eta\psi=0, \qquad (III.2)$$

where

$$\beta_1=\frac{\omega_{pi}^2 u(u-v_{de})}{c^2 v_s^2\alpha^2}$$

$$\beta_2=\frac{\omega_{pi}^2 c_A^2}{c^2 v_s^2}(1-\frac{v_{de}}{u})$$

$$\beta_3=\frac{2\Omega\kappa_n}{u}+\frac{\omega_{ci}v_{de}}{v_s^2 u^2}[\Omega^2 r_o+g(r_o)]$$

$$\beta_4=\frac{\omega_{ci}}{v_s^2 u}(1-\frac{v_{de}}{u})[\Omega^2 r_o+g(r_o)].$$

Considering the nonlinear third order differential structures of Eqs. (III.1)-(III.2) and noticing that the essence of the technique for seeking a Rossby vortex type solution is to reduce the nonlinear equation to linear second order differential equations in two region separated by a circle with radius a as a characteric space scale of the vortex in the $x-\eta$ plane leads us to take the following steps.

We define the polar coordinates in $x-\eta$ plane as

$$r^2 = x^2 + \eta^2, \quad \theta = \tan^{-1}\frac{\eta}{x}$$

and suppose that the two perturbation potentials ϕ and ψ have a linear algebric relation in both $r > a$ and $r < a$ regions as

$$\psi(r,\theta) = \alpha_1\phi(r,\theta) \quad (r > a) \tag{III.3}$$

$$\psi(r,\theta) = \alpha_2\phi(r,\theta) + \alpha_3\frac{B_o u}{c} r\cos\theta \quad (r < a), \tag{III.4}$$

where α_1, α_2 and α_3 are some real constant parameters.

Since we are looking for regular localized solutions, the potential functions must meet following requirements:

$$\phi, \psi \to 0 \quad \text{as} \quad r \to \infty \tag{III.5}$$

and

$$\phi, \psi = finite \quad \text{when} \quad r = 0. \tag{III.6}$$

Substituting Eqs.(III.3)-(III.4) into (III.1)-(III.2) and considering the conditions (III.5)-(III.6), along with proper choice of integration constants yields the linear equations satisfied by ϕ in both regions:

$$\nabla_\perp^2\phi - k^2\phi = 0, \quad (r > a) \qquad (III.7)$$

$$\nabla_\perp^2\phi + p^2\phi + q\frac{B_o u}{c} r\cos\theta = 0 \quad (r < a) \qquad (III.8)$$

where

$$k^2 = \frac{\alpha_1\beta_1}{1-\alpha_1} > 0 \qquad (III.9)$$

$$p^2 = \frac{\alpha_2\beta_1}{1-\alpha_2} + \frac{(\alpha_2+\alpha_3)[(\beta_2+\beta_4)(\alpha_2-1)+\beta_1]+(\alpha_2-1)}{2-\alpha_2+\alpha_3} > 0 \qquad (III.10)$$

$$q = \frac{\alpha_3\beta_1}{\alpha_2-1} + \frac{(1+\alpha_3)(\alpha_2-\alpha_3)[(\beta_2+\beta_3)+\beta_1/(\alpha_2-1)]+\beta_2(1+\alpha_3)}{2-\alpha_2+\alpha_3} \qquad (III.11)$$

and the parameter α_1 now is determined as one of the roots of the quadratic equation

$$\alpha_1^2 - [1-(\beta_1+\beta_3)/(\beta_2+\beta_4)]\alpha_1 - \beta_3/(\beta_2+\beta_4) = 0 \qquad (III.12)$$

which satisfies the condition (III.9) while α_2, α_3 still are free.

Following the standard procedure to solve Eqs.(III.7)-(III.8) under conditions (III.5)-(III.6) and matching the inner and outer solutions on the border $r = a$ for continuity of the function ϕ up to second derivatives gives

$$\begin{aligned}\phi_{out}(r,\theta) &= -\frac{qa}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c}\cos\theta \quad (r>a)\\ \phi_{in}(r,\theta) &= \{\frac{k^2}{(p^2+k^2)}\frac{a}{r}\frac{J_1(pr)}{J_1(pa)}-1\}\frac{q}{p^2}\frac{B_o u}{c} r\cos\theta \quad (r<a)\end{aligned} \qquad (III.13)$$

The relation between k and p is given by

$$\frac{1}{pa}\frac{J_2(pa)}{J_1(pa)}+\frac{1}{ka}\frac{K_2(ka)}{K_1(ka)}=0, \qquad (III.14)$$

Among the seven parameters $u, a, \alpha, p, k, \alpha_2$ and α_3 introduced to obtain the solutions (III.13) only three of them, say u, a α, are free, the other four can be determined when the three parameters are given. For given u, a α, the parameter ka is determined by Eq.(III.9), and then we can obtain pa by solving (III.14). Once p, k were determined we can use Eqs. (III.10),(III.11) and the continuity condition of ψ on the border $r=a$

$$\alpha_1\phi_{out}\mid_{r=a}=\alpha_2\phi_{in}\mid_{r=a}+\alpha_3\frac{B_o u}{c}a\cos\theta \qquad (III.15)$$

to fix the parameters α_2 and α_3. After completing this procedure the dipole vortex solution $\phi(r,\theta)$ with three free parameters a, u, α is obtained.

Substituting (III.13) into (III.3)-(III.4) gives the potential function ψ in both regions as

$$\begin{aligned}\psi_{out}(r,\theta) &= -\alpha_1\frac{qa}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c}\cos\theta \quad (r>a)\\ \psi_{in}(r,\theta) &= \{\alpha_2[\frac{k^2}{p^2+k^2}\frac{a}{r}\frac{J_1(pr)}{J_1(pa)}-1]\frac{q}{p^2}-\alpha_3\}\frac{B_o u}{c} r\cos\theta \quad (r<0)\end{aligned} \qquad (III.16)$$

Substituting (III.13) and (III.16) into (II.24) the corresponding of density perturbation is obtained as

$$\tilde{n}_{out}(r,\theta) = -\frac{T}{e}[\frac{v_{de}}{u}(1-\alpha_1)]\frac{q}{p^2+k^2}\frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c} a\cos\theta \quad (r > a)$$

$$\tilde{n}_{in}(r,\theta) = \frac{T_e}{e}\{[\frac{v_{de}}{u}(1-\alpha_2)+\alpha_2][\frac{k^2}{p^2+k^2}\frac{a}{r}\frac{J_(pr)}{J_(pa)} - 1]\frac{q}{p^2} \qquad (III.17)$$

$$+\alpha_3(1-\frac{v_{de}}{u})\}\frac{B_o u}{c} r\cos\theta \quad (r < 0)$$

Eqs. (III.13),(III.16) and (III.17) represent three perturbation functions with the same dipole vortex structure as the Rossby vortex in a rotating fluid[2]. This vortex moves with the constant speed u in the $x-z$ plane without changing its shape. The velocity vector forms an angle $\gamma = \tan^{-1}(1/\alpha)$ with the direction of the equilibrium magnetic field direction. Henceforth we call this solution the Rossby type vortex solution.

The allowed region of propagating speeds u for these vortices is determined by condition (III.9) hence we call it the nonlinear dispersion relation of the vortices. After solving equation (III.12) this nonlinear dispersion relation can be written explicitly in terms of the coefficients of Eqs. (III.1)-(III.2) as

$$k^2 = \frac{1}{2}[\beta_2+\beta_3+\beta_4-\beta_1 \pm \sqrt{(\beta_2+\beta_3+\beta_4-\beta_1)^2+4\beta_1\beta_3}] > 0. \qquad (III.18)$$

When there is no rotation and gravity, i.e. $\beta_3 = \beta_4 = 0$, (III.18) degenerates to

$$-k^2 = (1-\frac{v_{de}}{u})(1-\frac{\alpha^2 c_A^2}{u^2}) < 0 \qquad (III.19)$$

which is the nonlinear dispersion relation of usual drift-Alfven vortex [4,5]. The full relation (III.18) is considerable more complicated than (III.19) and we leave further discussion of this nolinear dispersion relation to Section **IV**.

III.2 The Intrinsic Electromagnetic Vortex Solution

In subsection **III.1** we obtained the Rossby type vortex solutions of Eqs. (III.1)-(III.2) and the corresponding nonlinear dispersion relation. Although this type of solution shares many nice properties with the Rossby-drift wave vortex, it has the unsatisfactory property shared with the other electromagnetic vortex solutions obtained in previous studies[5,6,7] that some physical quantities related with higher derivatives of the perturbation potential such as the perturbed parallel current $j_{||}$ and the perpendicular magnetic perturbation $\delta\mathbf{B}_{\perp}$ are discontinuous across the boundary between the interior and external regions. The reason for this discontinuity is related with the fact that the method for seeking the Rossby-drift vortex solution lowers the order of the basic differetial equations and which, in turn, limits the freedom for choosing enough parameters to meet the requirement of continuity for higher derivatives of the perturbation functions on the boundary.

As an effort to eliminate the discontinuity, in this subsection we try to construct another type of vortex solution for Eqs. (III.1)-(III.2) which envolves solving the full fourth order linear differential equation.

In first looking for the new solutions we suppose that the rotation of plasma is sufficiently slow and the gravity is sufficiently weak that we can neglect the terms connected with centrifugal force and gravity. Under this condition we drop the last tem of Eq.(III.1) and only keep the Coriolis effect in coefficient β_3, hence Eq.(III.2) becomes

$$\hat{L}_1\{\nabla_\perp^2\phi - \beta_2\psi - \beta_3\phi\} = 0, \qquad (III.20)$$

where

$$\beta_3 = \frac{2\Omega\kappa_n}{u}.$$

Integrating Eqs. (III.1) and (III.20) yields

$$\nabla_\perp^2(\phi - \psi) - \beta_1\psi = C_1(\phi - \frac{B_o u}{c}x), \qquad (III.21)$$

$$\nabla_\perp^2\phi - \beta_2\psi - \beta_3\phi = C_2(\phi - \psi - \frac{B_o u}{c}x), \qquad (III.22)$$

where C_1, C_2 are integration constants.

Similar to the procedure of seeking the Rossby type vortex, we solve Eqs. (III.21)-(III.22) in two regions under constraints (III.5)-(III.6). Imposing the condition (III.5) on Eqs. (III.21)-(III.22) on the external region ($r > a$) gives

$$C_1 = C_2 = 0. \qquad (III.23)$$

Substituting (III.23) into (III.21)-(III.22) and eliminating ψ from them yeilds

$$\nabla_\perp^4\phi + \gamma_1\nabla_\perp^2\phi + \gamma_2\phi = 0 \quad (r > a), \qquad (III.24)$$

where

$$\gamma_1 = \beta_1 - \beta_2 - \beta_3$$
$$\gamma_2 = -\beta_1\beta_3 \qquad (III.25)$$

In the inner region where $r < a$, for simplicity we choose

$$C_1 = 0,\ C_2 \neq 0. \qquad (III.26)$$

Then (III.21)-(III.22) can be reduced to

$$\nabla_\perp^4 \phi + \gamma_3 \nabla_\perp^2 \phi + \gamma_4 \phi + \gamma_5 \frac{B_o u}{c} r \cos\theta = 0 \quad (r < a), \qquad (III.26)$$

where

$$\gamma_3 = \beta_1 - \beta_2 - (\beta_3 + C_2)$$

$$\gamma_4 = -\beta_1(\beta_3 + C_2) \qquad (III.27)$$

$$\gamma_5 = C_2 \beta_1$$

Solving Eqs.(III.24) and (III.26) in both regions with constraints (III.5)-(III.6) and assuming the solution with form of $\phi(r,\theta) = \Phi(r)\cos\theta$ gives

$$\begin{aligned} \phi_{out}(r,\theta) &= \{A_1 \frac{K_1(\lambda_1 r)}{K_1(\lambda_1 a)} + A_2 \frac{K_1(\lambda_2 r)}{K_1(\lambda_2 a)}\} \cos\theta \quad (r > a) \\ \phi_{in}(r,\theta) &= \{A_3 \frac{J_1(\lambda_3 r)}{J_1(\lambda_3 a)} + A_4 \frac{I_1(\lambda_4 r)}{I_1(\lambda_4 a)} - \frac{\gamma_5}{\gamma_4}\frac{B_o u}{c} r\} \cos\theta \quad (r < a) \end{aligned} \qquad (III.28)$$

where I_1 is modified Bessel function, A_1, A_2, A_3, A_4 are intgration constants, and

$$\lambda_{1,2}^2 = \frac{1}{2}[-\gamma_1 \pm \sqrt{\gamma_1^2 - 4\gamma_2}] \qquad (III.29)$$

$$\lambda_{3,4}^2 = \frac{1}{2}[\sqrt{\gamma_3^2 - 4\gamma_4} \mp \gamma_3] \qquad (III.30)$$

To keep $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ real, we need the following conditions:

$$\gamma_1 < 0; \qquad \gamma_1^2 > 4\gamma_2 > 0 \qquad (III.31)$$

and

$$\gamma_4 < 0. \qquad (III.32)$$

Substituting (III.28) into (III.22), considering the choice of C_1, C_2 in both regions gives

$$\psi(r,\theta)_{out} = \frac{1}{\beta_2}\{(\lambda_1^2 - \beta_3)A_1\frac{K_1(\lambda_1 r)}{K_1(\lambda_1 a)} + (\lambda_2^2 - \beta_3)A_2\frac{K_1(\lambda_2 r)}{K_1(\lambda_2 a)}\}\cos\theta, \quad (r > a)$$
$$\psi(r,\theta)_{in} = \beta_2\{-(\lambda_3^2 + C_2 + \beta_3)A_3\frac{J_1(\lambda_3 r)}{J_1(\lambda_3 a)} + (\lambda_4 - C_2 - \beta_3)A_4\frac{I_1(\lambda_4 r)}{I_1(\lambda_4 a)}$$
$$+ \frac{B_o u}{c}r\}\cos\theta. \ (r < a)$$
$$(III.33))$$

For determining the integration constants $C_2,\ A_1,\ A_2,\ A_3,\ A_4$ we impose the following matching conditions on the boundary for the solutions (III.28) and (III.33):

$$\phi_{in}\mid_a = \phi_{out}\mid_a \qquad (III.34a)$$

$$\frac{\partial\phi_{in}}{\partial r}\mid_a = \frac{\partial\phi_{out}}{\partial r}\mid_a \qquad (III.34b)$$

$$\nabla_\perp^2\phi_{in}\mid_a = \nabla_\perp^2\phi_{out}\mid_a \qquad (III.34c)$$

$$\psi_{in}\mid_a = \psi_{out}\mid_a \qquad (III.35a)$$

$$\frac{\partial\psi_{in}}{\partial r}\mid_a = \frac{\partial\psi_{out}}{\partial r}\mid_a \qquad (III.35b)$$

$$\nabla_\perp^2\psi_{in}\mid_a = \nabla_\perp^2\psi_{out}\mid_a . \qquad (III.35c)$$

Substituting Eqs. (III.28),(III.33) into (III.34a)-(III.35a), after some algebra, we have determined A_1, A_2, A_3, A_4 as

$$\begin{aligned}
A_1 &= \frac{a_{11}a_1 + a_{12}a_2}{\Gamma}\frac{B_o ua}{c}K_1(\lambda_1 a) \\
A_2 &= \frac{a_{21}a_1 + a_{22}a_2}{\Gamma}\frac{B_o ua}{c}K_1(\lambda_2 a) \\
A_3 &= \frac{a_{31}a_1 + a_{32}a_2}{\Gamma}\frac{B_o ua}{c}J_1(\lambda_3 a) \\
A_4 &= \frac{a_{41}a_1 + a_{42}a_2}{\Gamma}\frac{B_o ua}{c}I_1(\lambda_4 a)
\end{aligned} \qquad (III.36)$$

where

$$\Gamma = (\lambda_1^2 - \lambda_2^2)[(\lambda_3 J_2(\lambda_3 a)I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_1 a)K_1(\lambda_2 a)$$

$$+(\lambda_3^2 + \lambda_4^2)[(\lambda_1 K_2(\lambda_1 a)K_1(\lambda_2 a) - \lambda_2 K_2(\lambda_2 a)K_1(\lambda_1 a))]J_1(\lambda_3 a)I_1(\lambda_4 a), \quad (III.37)$$

$$a_1 = 1 - \frac{\gamma_5}{\gamma_4}(1 + \beta_3/C_2) \qquad (III.38)$$

$$a_2 = 1 - \frac{\gamma_5}{\gamma_4}\beta_3/C_2 \qquad (III.39)$$

$$a_{11} = -\{(\lambda_3^2 + \lambda_4^2)\lambda_2 K_2(\lambda_2 a)J_1(\lambda_3 a)I_1(\lambda_4 a)$$

$$+\lambda_2^2[\lambda_3 J_2(\lambda_3 a)I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_2 a)\} \qquad (III.40)$$

$$a_{12} = \lambda_3\lambda_4[\lambda_4 J_2(\lambda_3 a)I_1(\lambda_4 a) - \lambda_3 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_2 a), \qquad (III.41)$$

$$a_{21} = \lambda_1^2[\lambda_3 J_2(\lambda_3 a)I_1(\lambda_4 a) + \lambda_4 I_2(\lambda_4 a)J_1(\lambda_3 a)]K_1(\lambda_1 a)$$

$$+(\lambda_3^2+\lambda_4^2)\lambda_1 K_2(\lambda_1 a)J_1(\lambda_3 a)I_1(\lambda_4 a), \tag{III.42}$$

$$a_{22}=\lambda_3\lambda_4[\lambda_3 I_2(\lambda_4 a)J_1(\lambda_3 a)-\lambda_4 J_2(\lambda_3 a)I_1(\lambda_4 a)K_1(\lambda_1 a), \tag{III.43}$$

$$a_{31}=\lambda_1\lambda_2[\lambda_1 K_2(\lambda_2 a)K_1(\lambda_1 a)-\lambda_2 K_2(\lambda_1 a)K_1(\lambda_2 a)]I_1(\lambda_4 a), \tag{III.44}$$

$$a_{32}=(\lambda_1^2-\lambda_2^2)\lambda_4 I_2(\lambda_4 a)K_1(\lambda_1 a)K_1(\lambda_2 a)$$

$$+\lambda_4^2[\lambda_1 K_2(\lambda_1 a)K_1(\lambda_2 a)-\lambda_2 K_2(\lambda_2 a)K_1(\lambda_1 a)], \tag{III.45}$$

$$a_{41}=\lambda_1\lambda_2[\lambda_2 K_2(\lambda_1 a)K_1(\lambda_2 a)-\lambda_1 K_2(\lambda_2 a)K_1(\lambda_1 a)]J_1(\lambda_4 a), \tag{III.46}$$

$$a_{42}=(\lambda_1^2-\lambda_2^2)\lambda_3 J_2(\lambda_3 a)K_1(\lambda_1 a)K_1(\lambda_2 a)$$

$$+\lambda_3^2[\lambda_1 K_2(\lambda_1 a)K_1(\lambda_2 a)-\lambda_2 K_2(\lambda_2 a)K_1(\lambda_1 a)]J_1(\lambda_3 a). \tag{III.47}$$

Notice that with the choice of constants $C_1=0$ Eq. (III.21) for both inner and external regions becomes the same equation as

$$\nabla_\perp^2\psi=\nabla_\perp^2\phi-\beta_1\psi, \tag{III.48}$$

so condition (III.35c) is automatically satisfied.

Condition (III.35b) gives a relation

$$\lambda_1^2\frac{\lambda_1 K_2(\lambda_1 a)}{K_1(\lambda_1 a)}A_1+\lambda_2^2\frac{\lambda_2 K_2(\lambda_2 a)}{K_1(\lambda_2 a)}A_2+(\lambda_3^2+C_2)\frac{\lambda_3 J_2(\lambda_3 a)}{J_1(\lambda_3 a)}A_3$$

$$+(\lambda_4^2 - C_2)\frac{\lambda_4 I_2(\lambda_4 a)}{I_1(\lambda_4 a)} A_4 = 0 \qquad (III.49)$$

through which we can determine constant C_2 for given free parameters u, a, α.

Obviously, substituting Eqs. (III.28) and (III.33) into (II.24) we can give the explicit expression for corresponding density pertubation function $\tilde{n}(r, \theta)$.

Eqs. (III.28) and (III.33) give the intrinsic electromagnetic solitary vortex solutions in the rotating plasma under slow rotation assumption we made in the begining of this subsection. After all integration constants were determined by the coefficients of Eqs. (III.1) and (III.20), the constraint conditions (III.31)-(III.32) play the role of nonlinear dispersion relation for this type of vortex, which will determine the allowed region of the vortices. We leave the discussion of this relation in the next section.

III.3. Special Cases

As we mentioned in section **II**, the Eqs. (II.19)-(II.21) are a quite general set of nonlinear equations describing the dynamics of a magnetized plasma. Beside the special cases analyzed previously which we listed there, here we present two interesting special cases which have not been studied yet. These two special cases correspond to the electron drift vortex and the ballooning vortex in a rotating plasma.

III.3a. The Rotational Electron Drift Vortex

When $A_{\|} = 0$, but $\Omega \neq 0$, $g \neq 0$ and $\alpha \neq 0$, we are treating the nonflute case of electrostatic perturbation in a rotating plasma. For this case it is convenient to start from Eqs.(II.10)-(II.12). Neglecting electron mass, from Eq.(II.11) we get the adiabatic electron density

$$\tilde{n} = \frac{e\phi}{T_e}. \qquad (III.50)$$

Substututing (III.50) into (II.10) and (II.12), eliminating $\nabla_{\|} j_{\|}$ yields the single equation for ϕ

$$\partial_t\phi - \rho_s^2\{\partial_t + \frac{c}{B_o}[\phi, \;]\}\nabla_\perp^2\phi + \{v_{de} - \rho_s^2[2\Omega\kappa_n + \frac{eB_o}{c}(\Omega^2 r_o + g(r_o))]\}\partial_y\phi = 0. \qquad (III.51)$$

Comparing Eq.(III.51) with the Eq.(10) of Ref. 3, we see that (III.51) is the nonlinear electron drift wave equation modified by the plasma rotation.

Following the procedure of Ref.3 the localized solution of the perturbation potential ϕ in form of (II.16) is given as

$$\phi_{out}(r,\theta) = \frac{K_1(kr)}{K_1(ka)}\frac{B_o u}{c} a\cos\theta, \quad (r > a)$$
$$\phi_{in}(r,\theta) = \{\frac{k^2+p^2}{p^2}\frac{r}{a} - \frac{k^2}{p^2}\frac{J_1(pr)}{J_1(pa)}\}\frac{B_o u}{c} a\cos\theta, \quad (r < a) \qquad (III.52)$$

where p, k are related by the relation

$$\frac{1}{ka}\frac{K_2(ka)}{K_1(ka)} + \frac{1}{pa}\frac{J_2(pa)}{J_1(pa)} = 0.$$

The nonlinear dispersion relation is given by

$$k^2 = \frac{1}{\rho_s^2}\{1 - \frac{v_{de}}{u} + \frac{2\kappa_n\rho_s^2\Omega + (\Omega^2 r_o + g(r_o))/\omega_{ci}}{u}\} > 0 \qquad (III.53)$$

From Eq.(III.53) we see that the allowed region of the usual electron drift vortex propagating speeds is restrained by the plasma rotation through the last term. So we call this solution a rotational electron drift vortex.

III.3b. The Rotational Ballooning Vortex

If we follow Ref. 10, take the limit

$$\psi = 0, \quad \rho_s^2 \to 0,$$

then from Eq. (II.20) we have

$$\tilde{n} = \frac{v_{de}}{u}\frac{e\phi}{T_e}. \qquad (III.54)$$

Substituting (III.54) into Eq.(II.21) gives

$$(u^2 - \alpha^2 c_A^2)\hat{L}_1\nabla_\perp^2\phi = \Gamma^2\partial_\eta\phi \qquad (III.55)$$

where

$$\Gamma^2 = \kappa_n[g(r_o) + (\Omega^2 r_o + 2\Omega u)] \qquad (III.56)$$

Comparing Eqs. (III.55)-(III.56) with Eq. (9) in Ref.9, we see that the Eq.(III.55) describes the so-called ballooning vortex in rotating plasma with the same potential function as (III.52). And the nonlinear dispersion is given by

$$\begin{aligned} k^2 &= \frac{\Gamma^2}{u^2 - \alpha^2 c_A^2} \\ &= \kappa_n[g(r_o) + \Omega^2 r_o + 2\Omega u]/(u^2 - \alpha^2 c_A^2) \end{aligned} \qquad (III.57)$$

Since the analogy of this vortex with the one analyzed in Ref.9 for the rest plasma, we call it the rotational balooning vortex.

IV. Discussion and Conclusion

In previous section we give two types of solitary vortex solutions in a rotating plasma. In this section we discuss some properties of these solutions.

(1). Common features of two types of vortices

From direct observation of expressions (III.13),(III.16) for the Rossby type vortex and expressions (III.28),(III.33) for the intrinsic electromagnetic vortex we can see that both solutions have the common features of a localized dipole structure; moving in the $y-z$ plane with constant speed u in the direction at an angle $\gamma = \tan^{-1}\frac{1}{\alpha}$ respect to the equilibrium magnetic field direction without changing their shape. The localized structures decay to zero as $r \to \infty$ with the asymptotic form $e^{-\lambda r}/\sqrt{r}$, where for Rossby-drift vortex $\lambda = k$ while for the intrinsic electromagnetic one $\lambda = min\{\lambda_1, \lambda_2\}$, which can be seen from Eqs.(III.13) and (III.33).

(2). Differences between the two types of vortices

Comparing (III.28), (III.33) with (III.13) and (III.16) we find that the intrinsic electromagnetic vortex is a more complicated structure than the Rossby-drift vortex in both its inner and outer regions. The amplitude of the intrinsic electromagnetic vortex is a linear combination of both regular and modified Bessel functions J_1 and I_1 in the interior region and the combination of two first order Mcdonald functions with different decay lengths in external region. In contrast, the amplitude of the

Rossby-drift vortex is single $J_1(pr)$ in the interior region and $K_1(kr)$ in the external region. Related with this difference, most significantly, these two types of vortex are different physically. From Eqs.(II.2), (II.4) and (II.18) we have

$$\delta \mathbf{B}_\perp = \frac{c\alpha}{u} \nabla(\phi - \psi) \times \hat{\mathbf{z}} \tag{IV.1}$$

and

$$j_\| = -\frac{c^2\alpha}{4\pi u} \nabla_\perp^2 (\phi - \psi). \tag{IV.2}$$

Due to the fact that for the intrinsic electromagnetic vortex $\nabla_\perp \psi$ and $\nabla_\perp^2 \psi$ are continuous on the border $r = a$, hence from Eqs. (IV.1)-(IV.2) we see that this type of vortex keeps $j_\|$ and $\delta \mathbf{B}_\perp$ continuous across the boundary. In contrast, the Rossby type vortex both $\delta \mathbf{B}_\perp$ and $j_\|$ have jumps on the boundary because for it only ψ but not it's derivatives is continuous across the border. The jump of parallel current on the boundary is defined as $[\Delta j_\|]_a = j_{\|_{in}}(a) - j_{\|_{out}}(a)$ and its value is calculated as

$$[\Delta j_\|]_a = \frac{c\alpha q B_o a k^2}{4\pi(p^2 + k^2)} \{2 - \alpha_1 - \alpha_2\} \cos\theta. \tag{IV.3}$$

(3). Different allowed regions of the vortex propagating speed

Unlike other Rossby-drift vortices, the propagating speed of our first type solution given in subsection **III.1** can be complementary to the phase velocity of the corresponding linear modes, or it can also ovelap with the later. As for the intrinsic electromagnetic vortex, the calculation shows that it has much narrower allowed region of propagating speed.

The nonlinear dispersion relation for the Rossby-drift vortex is Eq. (III.18), which give the allowed region of propagating speeds of the vortices. Remembering that one positive root of equation (III.12) is sufficient for existence of the Rossby type vortex, so from (III.18) we can immediatly find that the allowed regions for vortex propagating speeds are determined by following conditions:

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 > 0; \qquad \beta_1\beta_3 \geq 0 \tag{IV.4a}$$

or

$$\beta_2 + \beta_3 + \beta_4 - \beta_1 < 0; \qquad \beta_1\beta_3 > 0 \tag{IV.4b}$$

or

$$(\beta_2 + \beta_3 + \beta_4 - \beta_1) > 0; \qquad (\beta_2 + \beta_3 + \beta_4 - \beta_1)^2 > -4\beta_1\beta_3 > 0 \tag{IV.4c}$$

The dispersion relation of the corresponding linear mode obtained by substituting $\tilde{n}, \phi, \psi$ in the form of propagating plane waves as $f(x,y,z,t) \sim e^{i(k_x x + k_y y + k_z z - \omega t)}$, into the linearized forms of Eqs.(II.13)-(II.15) and neglecting the electron mass effect is

$$k_\perp^4 + k_\perp^2 \{ \frac{\omega_{pi}^2 c_A^2}{c^2 v_s^2}(1 - v_{de}/c_p) + 2\Omega\kappa_n/c_p + \frac{\omega_{ci}}{v_s^2}(\Omega^2 r_o + g(r_o))/c_p - \frac{\omega_{pi}^2}{c^2 v_s^2}\frac{k_y}{k_z}^2 c_p(c_p - v_{de})\}$$
$$- \frac{\omega_{pi}^2}{c^2 v_s^2}\frac{k_y}{k_z}^2 (c_p - v_{de})[2\Omega\kappa_n + \frac{\omega_{ci}}{v_s^2} v_{de}(\Omega^2 r_o + g(r_o)] = 0 \tag{IV.5}$$

where

$$k_\perp^2 = k_x^2 + k_y^2, \qquad c_p = \frac{\omega}{k_y}.$$

If we take the correspondences between votex quantanties α, u, and linear mode quantanties $k_z/k_y, c_p$ then solve Eq.(IV.5), we have

$$k_\perp^2 = \frac{1}{2}\{-(\beta_2+\beta_3+\beta_4-\beta_1) \pm \sqrt{(\beta_2+\beta_3+\beta_4-\beta_1)^2+4\beta_1\beta_3}\} \qquad (IV.6)$$

where we substitute k_z/k_y for α and c_p for u in all β's. Due to the fact that

$$k_\perp^2 = k_x^2 + k_y^2 > 0,$$

the allowed regions for the linear mode phase velocity are determined by

$$\beta_2+\beta_3+\beta_4-\beta_1 > 0; \quad \beta_1\beta_3 > 0 \qquad (IV.7a)$$

or

$$\beta_2+\beta_3+\beta_4-\beta_1 < 0; \quad \beta_1\beta_3 \geq 0 \qquad (IV.7b)$$

or

$$\beta_2+\beta_3+\beta_4-\beta_4 > 0; \quad (\beta_2+\beta_3+\beta_4-\beta_4)^2 > -\beta_1\beta_3 > 0 \qquad (IV.7c)$$

The allowed regions of vortex propagating speeds determined by Eqs. (IV.4a)-(IV.4c) and the regions of phase velocity for linear mode determined by (IV.7a)-(IV.7c) could be complemetary (*the regions determined by (IV.4c)and (IV.7c); the regions determined by (IV.4a) and (IV.7b) when* $\beta_1\beta_3 = 0$), as a common feature for different vortex of this type. But unlike other Rossby type vortex, they also can overlap (*the regions determined by (IV.4a),(IV.4b) and (IV.7a),(IV.7b) when* $\beta_1\beta_3 \neq 0$).

For the intrinsic electromagnetic vortex, the corresponding nonlinear dispersion relation (III.31)-(III.32) limits the propagating speeds of the vortices to a much narrower region, explicitly the region is determined by both conditions

$$\beta_2 + \beta_3 - \beta_1 > 0; \qquad (\beta_2 + \beta_3 - \beta_1)^2 > -4\beta_1\beta_3 > 0 \qquad (IV.8)$$

and

$$\beta_1(\beta_3 + C_2) > 0 \qquad (IV.9)$$

Remembering that the intrinsic magnetic vortex solution is obtained under a slow rotation assumption, we see that condition (IV.8) is equivalent to condition (IV.4c), but the further constraint given by condition (IV.9) forces the allowed region for the propagating speed of the intrinsic electromagnetic vortex to be narrower, accordingly this region is not complementary to the allowed region of phase velocity.

In conclusion our study shows the existence of two types electromagnetic vortex in a rotating plasma immersed in a homogeneous magnetic field. These two types of vortices have localized coherent dipolar structure but the allowed region of the vortex propagation speeds for the Rossby-drift vortex is wider. We show that two spatial scales $k_1^2 \simeq \beta_2 + \beta_3 - \beta_1$ and $k_2^2 \simeq -\beta_1\beta_3/(\beta_2 + \beta_3 - \beta_1)$ exist for the new type of the vortex which is intrinsic to the electromagnetic perturbation in a magnetized plasma. As special cases we also show that the electron drift vortex and the ballooning vortex found in a rest plasma [3,9] have their analogs in a rotating plasma. For understanding the stability properties of these vortices under interaction numerical simulation studies of their collisions are necessary. Inclusion of finite ion temperture and the ion parallel motion which are neglected in this work will bring more effects such as ion-acoustic wave coupling and finite Larmor radius effect into consideration.

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