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HEALING OF MAGNETIC STOCHASTICITY AND ISLANDS BY SELF-CONSISTENT PLASMA CURRENTS

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Abstract

When resonant magnetic perturbations are imposed on a toroidally confined plasma, magnetic stochasticity can arise. It is shown that the transport of electrons in the stochasticity can lead to self-consistent magnetic fields which greatly reduce, or heal, the stochasticity in steady state. These currents occur if there is another plasma transport process operating in which the radial fluxes of electrons and ions are not automatically equal. The healing effect can be quite strong in stellarator plasma confinement devices for fusion applications.

Stochasticity and islands are important in many physical systems. In plasma physics, magnetic islands and stochastic behavior of the magnetic field can greatly increase the plasma transport.[1] To diffuse radially, electrons travel rapidly along the stochastic field lines. Parallel currents which can result from this parallel motion have been neglected in previous treatments.[2] Here we show that the resonant magnetic fields causing the stochasticity can be substantially modified by these currents. The stochasticity can be greatly reduced, or "healed", for a particularly simple case of interest: when the magnetic fields are time independent. This situation occurs in stellarators, where the stochasticity arises from external field coils[3] and equilibrium Pfirsch-Schlüter currents.[4,5]

A radial electron particle flux occurs in steady state only if there is another plasma transport process operating such that the electron and ion fluxes are not automatically equal (e.g., neoclassical transport in non-axisymmetric configurations[6]). Thus, the steady state has balancing charge losses from e.g., neoclassical processes, and parallel electron motion. We will show that healing of resonant magnetic perturbations occurs for neoclassical transport rates of interest for fusion applications in stellarator confinement devices. Also, it occurs for neoclassical transport rates much smaller than would arise from electron motion in the unhealed magnetic fields.

This healing is akin to dielectric shielding. When an external field is applied to a dielectric medium with a large susceptibility, the induced charges in the medium shield out the external field. The particle transport of electrons by parallel motion in a stochastic field gives it a large magnetic susceptibility to resonant magnetic perturbations.

The stability of a healed stochastic steady state is not considered here. We expect that magnetic perturbations away from the steady state will decay if the tearing mode stability parameter,[7] Δ', is sufficiently negative. This is indicated by previous analysis of tearing mode stability in a stochastic field,[8,9,10] although these are not strictly applicable since they neglect the self-consistent effects like those described here. The possibility that modes with $\Delta' > 0$ may be nonlinearly stabilized in some circumstances is under investigation.

The origin of the self-consistent parallel currents is most easily visualized in the case of partially overlapping islands (although our quantitative results are for the strongly overlapping limit). First consider Fig. 1(a), which shows a surface of section of an island, including a contour of constant radius. Note that the magnetic field points downward though the contour on the right side of the island, and upward on the left. Now consider an island surrounded by a stochastic region due to nearby islands of different helicity, Fig. 1(b). The magnetic field direction through the plane now fluctuates, but on average it is still downward on the right and upward on the left. Consider electrons moving parallel to the field. If there is a net radial flux of electrons downward through the contour, there must be a higher electron density on the right than on the left. However, for electrons traveling antiparallel to the field, a downward flux implies a higher density on the left. Thus equal radial fluxes of electrons with both directions implies a net current on the right and opposite current on the left. These currents have the same helicity as the island and so produce a similar magnetic perturbation.

Such a radial electron flux arises in steady state if there is another charge loss process which operates perpendicular to field lines, rather than parallel to field lines. For example, particles which are mirror trapped by variations of the magnetic field strength can transport across field lines by magnetic drifts in a non-axisymmetric geometry.[6] Note that the radial transport of such trapped particles does not itself produce large parallel currents along field lines, unlike the case of electron transport in stochastic fields, where parallel motion is intrinsic to the transport.

We now estimate the size of the self-consistent currents for the case of significantly overlapping islands. Consider for simplicity a sheared slab geometry periodic in y and z, and where x plays the role of the radial direction. Initially there is a magnetic field $\mathbf{B}_0 = B_0 \left[\hat{z} + \left(x/L_s\right) \hat{y}\right]$. Externally produced, time independent resonant magnetic perturbations $\delta\mathbf{B}_{\text{ext}}$ are then imposed, which, along with the plasma response $\delta\mathbf{B}_{\text{ind}}$, result in stochasticity. Label their Fourier harmonics in the y and z direction by the wavenumber $k = k_y\hat{y} + k_z\hat{z}$. The total magnetic field $\mathbf{B}_0 + \delta\mathbf{B}$ has a slight tilt in the radial direction due to $\hat{x} \cdot \delta\mathbf{B} = \hat{x} \cdot \delta\mathbf{B}_{\text{ind}} + \hat{x} \cdot \delta\mathbf{B}_{\text{ext}}$. An individual particle with parallel velocity $v_{||}$ has a radial velocity $v_{||}\hat{x} \cdot \delta\mathbf{B}/B$, where $B = |B_0|$. The average radial electron current from stochastic motion is therefore

$$\langle j_x \rangle = \left\langle \delta j_{||} \hat{x} \cdot \delta\mathbf{B}/B \right\rangle \tag{1}$$

where $\delta j_{||}$ is the parallel electron current, and the average is over a surface of constant radius, and over the statistics of the fluctuations.

Breaking Eq. (1) into the various helicities, we have

$$\langle j_x \rangle = \sum_k \left\langle \delta j_{\|}^{-k} \hat{x} \cdot \delta \mathbf{B}^k / B \right\rangle . \tag{2}$$

We now heuristically estimate the importance of self-consistency effects. The motion of a single particle is only correlated with a given harmonic near its respective rational surface; call the correlation distance Δx_c. Thus at a given radius, only terms from nearby rational surfaces contribute. Denote the typical spacing between rational surfaces as λ; then Eq. (2) is roughly

$$\langle j_x \rangle B \sim \frac{\Delta x_c}{\lambda} \hat{x} \cdot \delta B^k \delta j_{\|}^{-k}. \tag{3}$$

Thus the magnitude of $\delta j_{\|}^k$ is

$$\delta j_{\|}^k \sim j_x \frac{\lambda}{\Delta x_c} \frac{B}{\hat{x} \cdot \delta \mathbf{B}^k}. \tag{4}$$

The induced radial magnetic perturbation $\hat{x} \cdot \delta B_{\text{ind}}^k$ can be simply computed using the "constant ψ"[7] approximation of tearing mode theory. If $\Delta x_c k_\perp \ll 1$, where $k_\perp$ is the wavenumber perpendicular to $\vec{B}_0$, $|(k_y \hat{y} + k_z \hat{z}) \times \mathbf{B}_0| / B_0$, then

$$\frac{\Delta'}{k_\perp} \hat{x} \cdot \delta \mathbf{B}_{\text{ind}}^k = \frac{4\pi}{c} \int dx j_{\|}^k \sim \frac{4\pi}{c} \delta j_{\|}^k \Delta x_c \tag{5}$$

where Δ' is the tearing mode stability parameter. Typically $-\Delta'/k_\perp$ is somewhat greater than one.

Self-consistency is important when $\delta B_{\text{ind}} \sim \delta B_{\text{ext}}$. This gives the criterion

$$\left(\frac{\hat{x} \cdot \delta \mathbf{B}_{\text{ext}}^k}{B} \right) \left(\frac{\hat{x} \cdot \delta \mathbf{B}^k}{B} \right) \lesssim \frac{4\pi}{c} \frac{\langle j_x \rangle \lambda}{B} \frac{k_\perp}{\Delta'}. \tag{6}$$

Note that Δx_c canceled out of this result. We will see that healing necessarily occurs for

$$\left(\frac{\hat{x} \cdot \delta B_{\text{ext}}^k}{B} \right)^2 \lesssim \frac{4\pi}{c} \frac{\langle j_x \rangle \lambda}{B} \frac{k_\perp}{\Delta'}, \tag{7}$$

but is also possible for larger $\delta \mathbf{B}_{\text{ext}}$, since $\hat{x} \cdot \delta \mathbf{B}$ can be smaller than $\hat{x} \cdot \delta \mathbf{B}_{\text{ext}}$ in Eq. (6) due to the healing effect.

The electron $\langle j_x \rangle$ is the same size as the ion $\langle j_x \rangle$ in steady state. For neoclassical transport in stellarators, the latter can be estimated from the global particle confinement time, τ_p, by $j_x \sim ern/\tau_p$, with r the plasma radius and n the particle density. For typical fusion reactor plasma parameters, $r \sim 2m$, $B \sim 5T$, and n can be derived from $\beta = 8\pi nT/B^2 = .1$, with temperature $T = 10keV$. We estimate τ_p from the Lawson criterion $n\tau_p = 2 \times 10^{20}\,\mathrm{sec}/m^3$, and take $\lambda \sim r/4$, which is crudely true for low mode number perturbations. From Eq. (7), $\hat{x} \cdot \delta\mathbf{B}_{\mathrm{ext}}/B \sim 10^{-2}$, which can give very large overlapping islands and stochastic transport.

We now give a more quantitative analysis of stochastic healing when the stochastic electron transport can be described using quasi-linear theory. Electrons can be described with the drift-kinetic equation. We neglect variations in the magnetic field strength for simplicity, so the velocity space can be parameterized by the parallel and perpendicular velocity $v_\parallel$ and $v_\perp$. The distribution function f obeys

$$\frac{\partial}{\partial t} f\left(\mathbf{x}, v_\parallel, v_\perp, t\right) + v_\parallel \hat{b} \cdot \nabla f + \frac{q}{m} \hat{b} \cdot \mathbf{E} \frac{\partial f}{\partial v_\parallel} = 0 \tag{8}$$

where $\hat{b}$ is a unit vector in the direction of the magnetic field, and $\mathbf{E}$ is the electric field; note that the $\mathbf{E} \times \mathbf{B}$ drift usually present in Eq. (8) is not important in what follows.

All quantities g are split into parts averaged over a flux surface $\langle g \rangle$ and fluctuating parts δg. Fluctuating quantities are Fourier analyzed in y and z

$$\frac{\partial}{\partial t} \langle f \rangle + \frac{\partial}{\partial x} \sum_k \langle v_\parallel \delta b_x^{-k} \delta f^k \rangle + \frac{q}{m} \sum_k \left\langle \delta E_\parallel^{-k} \frac{\partial \delta f^k}{\partial v_\parallel} \right\rangle = 0, \tag{9}$$

$$\frac{\partial \delta f}{\partial t} + v_\parallel \left(k_z - k_y \frac{x}{L_s} \right) \delta f^k + v_\parallel \delta b_x^k \frac{\partial}{\partial x} \langle f \rangle + \frac{q}{m} \delta E_\parallel^k \frac{\partial \langle f \rangle}{\partial v_\parallel} + \text{Higher Order Nonlinear Terms} = 0, \tag{10}$$

where we take $\delta E_\parallel = \delta b_x \langle E_x \rangle$. The second term in Eq. (9) is the divergence of the radial particle flux from stochasticity (the third term is usually very small).

For steady state stochastic fields in the quasilinear limit, the higher order nonlinear terms may be replaced by a diffusion operator,[2] $D\partial^2/\partial x^2$. We take Δx_c to be thin so that δb_x and average quantities can be treated as spatially constant. We then Fourier analyze Eq. (10) in x, resulting in a first order ordinary differential equation in the transform variable. Standard techniques show that

$$\int dx\, dv v_\parallel \delta f = -\pi \delta b_x \sqrt{\frac{2T}{\pi m_e}} \left| \frac{L_s}{k_y} \right| \left(\frac{dn}{dx} + \frac{n}{2T} \frac{dT}{dx} + \frac{ne \langle E_x \rangle}{T} \right) \tag{11}$$

and $\Delta x_c = \left(L_s D / k_y v_e\right)^{1/3}$. Note that Eq. (11) is independent of D, for roughly the same reason that $\int dx \delta j_{||}$ was independent of Δx_c in the heuristic argument.

Let us assume that all the resonant δB^k have nearly the same amplitude and k_y. To find the particle flux from the spatially overlapping resonance regions, we space average over a distance $L > \Delta x_c$, and use Eq. (11),

$$\Gamma_e = \frac{1}{2L}\int_{-L}^{+L} dx \sum_k \int \left\langle v_{||}\delta b_x^{-k}\delta f^k\right\rangle dv = Dn\left(\frac{1}{n}\frac{dn}{dx} + \frac{1}{2T}\frac{dT}{dx} + \frac{e\left\langle E_x\right\rangle}{T}\right) \tag{12}$$

with

$$D = \left(\frac{2T}{\pi m_e}\right)^{1/2} \pi L_s \left|\frac{\delta B^k}{B}\right|^2 \frac{1}{k_\perp \lambda}. \tag{13}$$

Equations (11)–(12) agree with previous results;[8,11] note that the electron flux vanishes for a particular radial electric field, corresponding to a positive plasma potential. Thus

$$\Gamma_e \approx D\frac{n}{L_\perp}\left(1 - E_r/E_0\right)$$

where $L_\perp$ is a perpendicular gradient scale length, and E_0 is the value of the radial electric field for which $\Gamma_e = 0$.

The induced magnetic perturbation δB_{ind} caused by this transport can be computed from Eqs. (5) and (10)

$$\hat{x}\cdot\delta\mathbf{B}_{\text{ind}}^k = i\kappa_0\left(1 - E_r/E_0\right)\hat{x}\cdot\delta\mathbf{B}^k \tag{14}$$

where

$$\kappa_0 = 4\pi n\sqrt{\frac{2T\pi}{me}}eL_s/cBL_\perp\Delta'. \tag{15}$$

Thus, δB_{ind} is related to the stochastic electron particle flux. For parameters of interest in fusion research $\kappa_0 \sim 10^2$. Since $\delta\mathbf{B} = \delta\mathbf{B}_{\text{ext}} + \delta\mathbf{B}_{\text{ind}}$, we have

$$\hat{x}\cdot\delta B^k = \hat{x}\cdot\delta B_{\text{ext}}^k \Big/ \left[1 + i\kappa_0\left(1 - \frac{E_r}{E_0}\right)\right]. \tag{16}$$

Thus as the particle flux increases, δB decreases due to the induced parallel currents. Since D depends on δB, Eqs. (11)–(12) become

$$\Gamma_e \approx \sqrt{\frac{2T}{m_e}}L_x\left(\frac{\hat{x}\cdot\delta B_{\text{ext}}^k}{B}\right)^2 \frac{1 - E_r/E_0}{1 + \kappa_0^2\left(1 - E_r/E_0\right)^2}. \tag{17}$$

A graph of this appears in Fig. 2. Near $E_r = E_0$, Γ_e is an increasing function of E_r, until $1 - E_r/E \sim 1/\kappa_0$; then Γ_e decreases due to the healing.

In steady state, $\Gamma_e = J_n/e \equiv \Gamma_n$, where J_n is the charge flux from all other non-intrinsically ambipolar processes (e.g., non-axisymmetric neoclassical transport). Without such a process, $\Gamma_e = 0$, which implies $E_r = E_0$, and thus no healing. Consider the case when various amounts of Γ_n are present, as shown in Fig. 2. We suppose that $\Gamma_n = 0$ when $E_r = E_n$, and generally $E_n \neq E_0$; in fact they usually have different signs. For small Γ_n, labeled Γ_n^1 in Fig. 2 ,there are three roots of $\Gamma_e = \Gamma_n$; only the two near E_n and E_0 are stable (stable here means that if E_r is perturbed away from the root, the resulting charge flux causes E_r to relax back). The root near E_n has substantial healing, and the one near E_0 has negligible healing. For sufficiently large values of Γ_n, such as Γ_n^2 in Fig. 2, only the root with healing is present. The criterion for this situation is in fact equivalent to Eq. (7). Note that since κ_0 is large, a non-ambipolar transport mechanism much weaker than that which would arise from the unhealed magnetic stochasticity can cause the latter to heal. Also, note that even for less transport than that given by Eq. (7) , one root (the one near E_n) has healing; this corresponds with Eq. (6) when $\delta B < \delta B_{\text{ext}}$.

Note that for extremely strong healing, $\kappa_0 \left(1 - E_0/E_r\right) \gg 1/\Delta' \Delta x_c$, the constant ψ approximation may break down. Then the final amount of healing may be less than predicted by Eq. (15), though it is still substantial.

The main assumption in the above is that the islands overlap. However, the resonant perturbations may begin with non-overlapping islands, or Eq. (16) may imply this (and thus exceed its range of validity). Thus, we now examine the non-overlapping, single helicity limit.

Self-consistent currents arise in a manner similar to the partially overlapping case. Electrons flow across flux surfaces by some non-intrinsically ambipolar process until they reach the region near the island separatrix. If the transport along field lines is very rapid, they cross the island by parallel motion along lines near the separatrix, producing parallel currents. We again assume slab geometry, and here consider perturbations independent of z. The total current passing through the radial contour in Fig. 1 by flow along the separatrix is $I_{||} B_\perp / B$, where $I_{||}$ is the total current along the separatrix, and $B_\perp$ is the magnetic field perpendicular to z, $B_\perp = \frac{x}{L_s} B_0 \hat{y} + \delta B$. If the current entering from the top of Fig. 1 is $I_\perp$, we have

$$I_\| \frac{B_\perp}{B} = I_\perp. \tag{18}$$

This allows us to estimate the size of the induced magnetic field using the "constant ψ" approximation. There is one difficulty with this; $B_\perp$ varies along the separatrix, and vanishes near the x-point. Thus there are current spikes there. If the island is left-right symmetric, however, the current spike to the left of the x-point has the opposite sign to that from the nearby spike just to the right. The magnetic field from this current dipole can be shown to be of the same size as that caused by currents from the region away from the x-point, where $B_\perp \sim B_0 \Delta x_I / L_x$, with Δx_I the island width, $\Delta x_I^2 \sim L_s \hat{x} \cdot \delta B / B k_y$. Parity considerations show that the induced field is 90° out of phase with the total field causing the island, so we have

$$\frac{\hat{x} \cdot \delta B_{\text{ind}}}{B} = Ci \frac{\hat{x} \cdot \delta B}{|\hat{x} \cdot \delta B|} \frac{L_s}{\Delta x_I} \frac{4\pi I_\perp k_\perp}{cBr\Delta'} \tag{19}$$

where C is a numerical factor of order unity, which could be obtained by exactly solving the equations for the parallel current.

The equation $\hat{x} \cdot (\delta\mathbf{B} - \delta\mathbf{B}_{\text{ind}} = \delta\mathbf{B}_{\text{ext}})$ can now be manipulated into the form

$$\left(\frac{\hat{x} \cdot \delta B}{B}\right)^2 + \frac{L_s^2}{\Delta x_i^2} \left(\frac{4\pi C I_\perp}{crB}\right)^2 \left(\frac{k_y}{\Delta'}\right)^2 = \left(\frac{\hat{x} \cdot \delta B_{\text{ext}}}{B}\right)^2. \tag{20}$$

Using the expression for island width in terms of $\hat{x} \cdot \delta B$, one can show that this equation has solution only for $\left(I_\| k_\perp / cBr\Delta'\right) \sqrt{L_s k_y} \lesssim 1$; this corresponds to $\delta\mathbf{B}_{\text{ind}} \lesssim \delta\mathbf{B}_{\text{ext}}$. When $I_\perp$ exceeds this, there is no steady state solution with an island width large enough for Eq. (18) to be valid. Then equilibrium is lost and the resonant magnetic field evolves in time, so that additional parallel currents are produced by an inductive magnetic field. Numerical calculations are in progress with a model incorporating the above effects as well as an inductive field. Preliminary results indicate that in the single helicity case the island simultaneously rotates and decays, eventually reaching a small amplitude. Results for more extensive parameter scans will be reported in detail in the future, both for the above single helicity case, and for the multiple helicity case with marginally overlapping islands.

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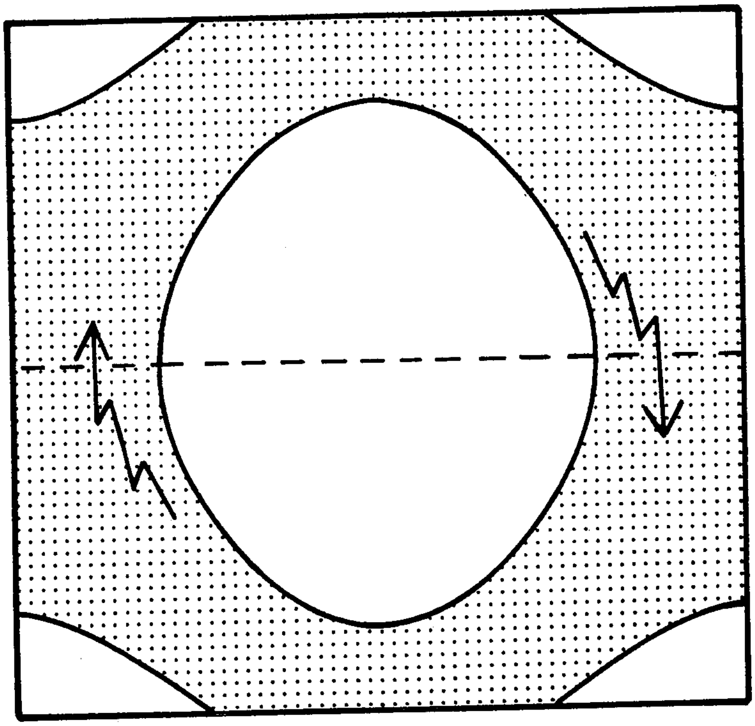
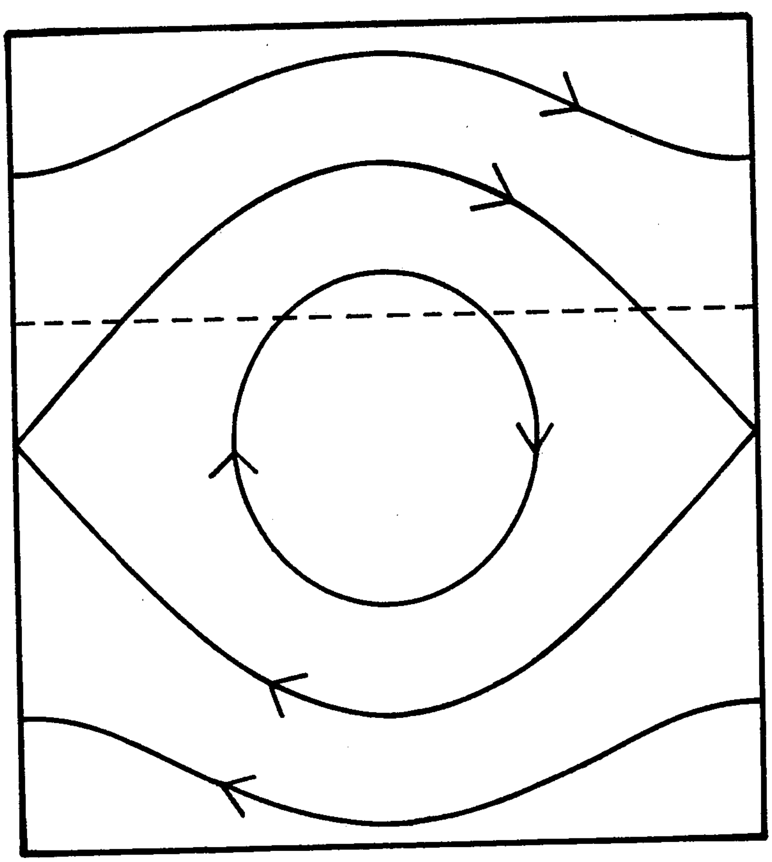
Figure Captions

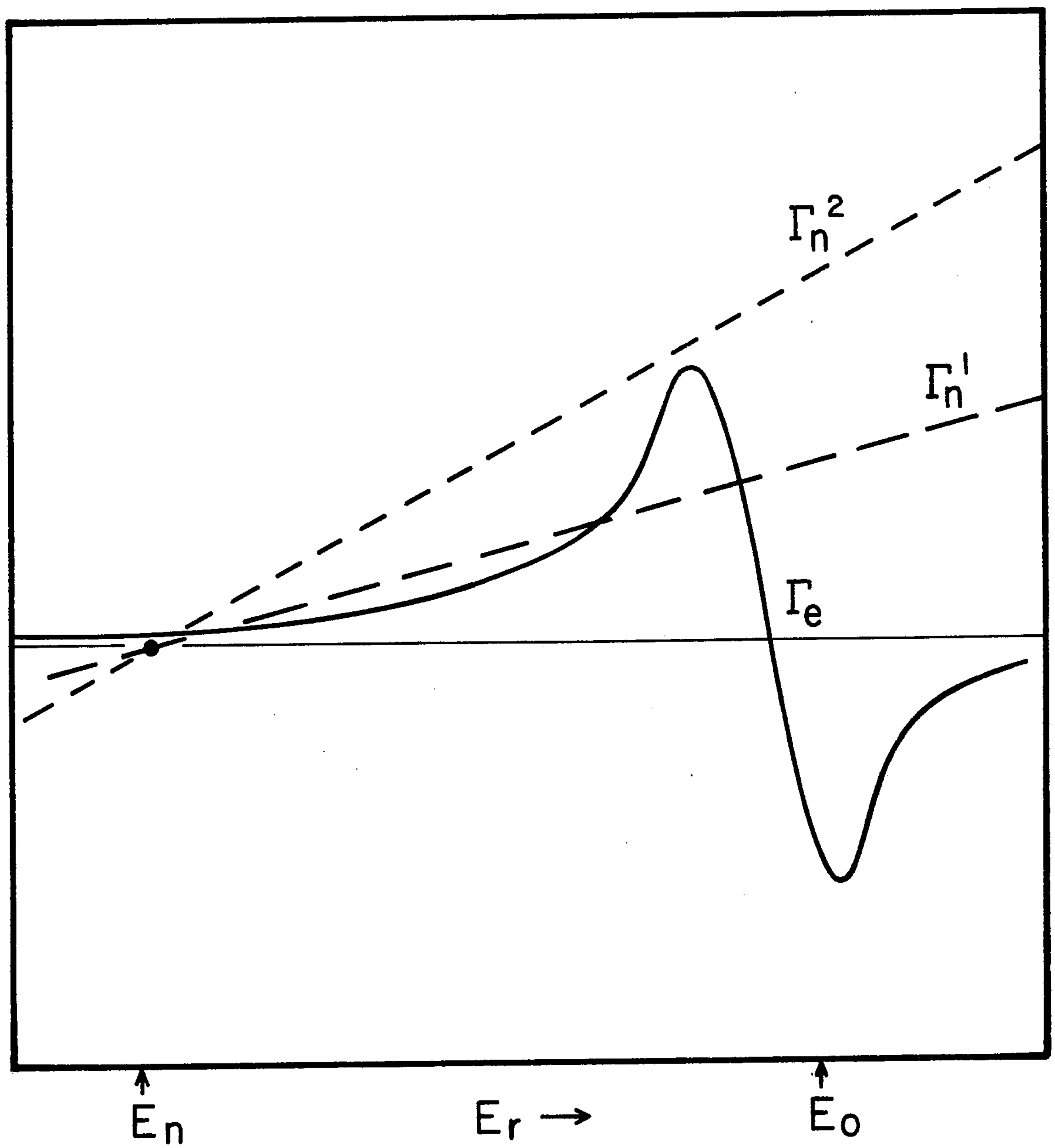
Fig. 1(a) Cross-section of magnetic island, the main component of $\mathbf{B}$ is out of the page.

Fig. 1(b) Magnetic island with some stochasticity.

Fig. 2 Radial particle fluxes from stochasticity Γ_e, and other processes Γ_n, versus radial electric field.







Γn²
Γn¹
Γe
En
Er →
Eo