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Self-Consistency Constraints on Turbulent Magnetic Transport and Relaxation in Collisionless Plasma

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Abstract

Novel constraints on collisionless relaxation and transport in drift-Alfvén turbulence are reported. These constraints arise due to the consideration of mode coupling and incoherent fluctuations and the proper application of self-consistency conditions. The result that electrostatic fluctuations *alone* regulate transport in drift-Alfvén turbulence follows directly. Quasilinear transport predictions are discussed in light of these constraints.

Transport caused by turbulent magnetic fluctuations is considered to be an important agent for relaxation and confinement degradation in magnetically confined plasmas. Previous investigations of turbulent magnetic transport typically have utilized quasilinear models of fluctuation dynamics, and have neglected self-consistent field effects.[1–3] In this letter, novel constraints on magnetic transport in fully developed collisionless plasma turbulence are described. These constraints arise from the role of self-consistency conditions (i.e., quasi-neutrality and Ampere's law) in models of the dynamics of drift-Alfvén microturbulence which are more complete than quasilinear theory. In particular, it is argued that the self-consistency constraint imposed by Ampere's law, along with proper consideration of the role of mode coupling and incoherent fluctuations in the dynamics of relaxation, together lead to the conclusion that transport and relaxation in drift-Alfvén turbulence are regulated by the *electrostatic* fluctuations. Previous transport models, such as that recently advanced by Kadomtsev and Pogutse,[1] are then reconsidered in light of these constraints. Throughout this letter, it is assumed that the drift kinetic equation (DKE) governs electron dynamics, and that ion dynamics, are described by a warm, low frequency response.

The DKE relates the dynamics of phase space density fluctuations (δf) to the relaxation of the average distribution function ($\langle f \rangle$) through the expression $\int dv_{\|} d\mathbf{x} \partial/\partial t(\delta f^2) = -\int dv_{\|} d\mathbf{x} \partial/\partial t(\langle f \rangle^2)$. Predictions of plasma transport and relaxation are thus direct consequences of the nature of the fluctuation dynamics. In particular, it is noteworthy that in fully developed Vlasov turbulence, shear stresses generate granular, incoherent fluctuations which are macroparticle-like, localized phase space 'blobs', analogous to fluid eddys rather than to waves.[4–6] Under certain rather general conditions, such blobs can even support localized self-trapping potentials (i.e., positive for electrons) and hence have lifetimes which exceed the average correlation time.[7] In general, the incidence of incoherent fluctuations is indicative of the dynamical significance of mode coupling processes. Thus, magnetic transport driven by fully developed turbulence *cannot* be described by quasilinear theory, which intrinsically neglects the effects of mode coupling and localized fluctuations.

Here, two related models of incoherent drift-Alfvén fluctuation dynamics and induced transport are described. The first is concerned with the evolution of an isolated phase space blob $\tilde{f}$ in a drift-Alfvén system. In the second model, statistical averaging is used to construct a Lenard-Balescu turbulent collision integral for the relaxation of $\langle f \rangle$ due to 'fully developed' (i.e., many blobs and collective resonances) drift-Alfvén microturbulence. While the statistical model is more representative of fully developed turbulence, the isolated blob model helps develop physical intuition. Both yield qualitatively similar insights into self-consistency constraints on relaxation and transport. In both cases, the results are independent of detailed approximations made in treating the turbulence dynamics.

In the first model, an isolated, localized electron phase space density blob $\tilde{f}$ with velocity $u_{\|}$ at position $\mathbf{x}_0$ is considered. The blob has correlation length $\Delta v_{\|}$ in velocity (of order the trapping width) and Δx, Δy, Δz in position space, where $\Delta z \sim L_{\|}$ is the parallel length scale for the system. For a background distribution $\langle f \rangle = \big\langle f(v_{\|}, x) \big\rangle$, the

DKE states that $\tilde{f}$ evolves according to

$$\int dv_{\|} \int dx \frac{\partial}{\partial t}(\tilde{f}^2) = -2 \int dv_{\|} dx \frac{d}{dt}(\tilde{f} \langle f \rangle). \tag{1}$$

Expanding $\langle f \rangle$ around x_0 $\left(\langle f \rangle = \langle f(x_0, u_{\|}) \rangle + (x - x_0) \partial \langle f \rangle / \partial x|_{x_0, u_{\|}} \right)$ and noting that, for drift-Alfvén turbulence, $dx/dt = c\hat{E}_\theta / B_0 + v_{\|} \hat{B}_r / B_0$, it follows that:

$$\int dv_{\|} \int dx \frac{\partial}{\partial t}(\tilde{f}^2) = -2 \frac{\partial \langle f \rangle}{\partial x}|_{x_0, u_{\|}} \left\{ \frac{c}{B_0} \left\langle \hat{E}_\theta \hat{n}_e \right\rangle_b - \frac{\left\langle \hat{B}_r \hat{J}_{\| e} \right\rangle_b}{|e| B_0} \right\}. \tag{2}$$

Here, $\hat{E}_\theta = -\nabla_\theta \hat{\phi}$, $\hat{B}_r = \nabla_\theta \hat{A}_{\|}$, where $\hat{\phi}$ and $\hat{A}_{\|}$ are the electrostatic and the parallel component of the vector potential, respectively. Also, $\tilde{n}_e = \int dv_{\|} \tilde{f}$, $\tilde{J}_{\| e} = -|e| \int dv_{\|} v_{\|} \tilde{f}$, $\langle \ \rangle_b$ denotes an average over the blob volume, and energy scattering has been ignored for convenience. Equation (2) thus implies that:

$$\int dv_{\|} dx \frac{\partial \tilde{f}^2}{\partial t} = -2 \frac{\partial f_0}{\partial x}|_{x_0, u_{\|}} \left\{ \frac{c}{B_0} \left\langle \hat{E}_\theta \tilde{n}_e \right\rangle_b - \frac{\left\langle \hat{B}_r \tilde{J}_{\| e} \right\rangle_b}{|e| B_0} \right\} \tag{3a}$$

and, with quasi-neutrality ($\hat{n}_e = \hat{n}_i$) and Ampere's law ($\hat{J}_{\| e} = -\nabla_\perp^2 \hat{A}_{\|}$, for negligible ion current), that:

$$= -2 \frac{\partial f_0}{\partial x}|_{x_0, u_{\|}} \left\{ \frac{c}{B_0} \left\langle \hat{E}_\theta \hat{n}_i \right\rangle_b + \frac{\left\langle \hat{B}_r \nabla_\perp^2 \hat{A}_{\|} \right\rangle_b}{|e| B_0} \right\}. \tag{3b}$$

Equation (3a) states that $\tilde{f}$ evolves (and thus $\langle f \rangle$ relaxes) by cross-field convection and fluctuating currents flowing along magnetic perturbations, while Eq. (3b) indicates how self-consistency constraints regulate the relaxation mechanisms. In particular, since $\hat{\mathbf{B}} = \nabla \hat{A}_{\|} \times \mathbf{n}$ and $\nabla \cdot \hat{\mathbf{B}} = 0$, it follows that $\left\langle \hat{B}_r \nabla_\perp^2 \hat{A}_{\|} \right\rangle_b = -\left\langle \partial / \partial r (\hat{B}_r \hat{B}_\theta) \right\rangle_b$, which ultimately contributes only surface terms of $0(\Delta x / L_x) << 1$. Hence, in this simple drift-Alfvén system, magnetic fluctuations do not result in evolution of $\tilde{f}$ nor in the relaxation of $\langle f \rangle$. Physical insight into this rather counter-intuitive result may be gained by noting the similarity of the above argument to that used to establish the ambipolarity of magnetic transport.[8] However, here Ampere's law (with $\hat{J}_{\parallel i} = 0$) and the granularity (i.e., localization in phase space) of $\tilde{f}$ imply that the transport processes associated with the relaxation of *all moments* of $\langle f \rangle$ are similarly constrained over scales of Δx, the radial correlation length, $\left(\text{i.e., } \left\langle \hat{B}_r \hat{J}_{\parallel e} \right\rangle_b \approx - \left\langle \hat{B}_r \hat{B}_\theta \right\rangle \Big|_{x_0 - \Delta x}^{x_0 + \Delta x} \rightarrow 0(\Delta x / L_x) \right)$. This in turn severely restricts the role of magnetic drift-Alfvén fluctuations in the dynamics of transport and relaxation. It is also instructive to note that the familiar quasilinear result $\int dv_\parallel dx \; \partial / \partial t (\delta f^2) = \int dv_\parallel dx D (\partial f_0 / \partial x)^2$ (here D is the quasilinear diffusion coefficient for magnetic turbulence) can be recovered by discarding fluctuation granularity by replacing $\tilde{f}$ in the right-hand side of Eq. (1) with f^c, the linear coherent response. This observation is further evidence that the constraint on magnetic transport and relaxation discussed above arises as a consequence of self-consistency (Ampere's law) and the granularity of $\tilde{f}$. Finally, it should be noted that the new constraint arises solely through the presumption of granularity and Ampere's law, and is insensitive to detailed consideration of the structure and origin of $\tilde{f}$.

In the second model, relaxation and transport due to fully developed collisionless drift-Alfvén turbulence is examined using statistical turbulence theory. The relaxation of $\langle f \rangle$ is described by a Lenard-Balescu turbulent collision integral (LBTCI), which contains a drag operator as well as the usual quasilinear diffusion term.[4–6] The drag operator represents the role of incoherent fluctuations in $\langle f \rangle$ relaxation. The relationship between coherent and incoherent fluctuations imposed by the self-consistency constraints profoundly affects the predicted transport and relaxation rates.

For drift-Alfvén turbulence, $\langle f \rangle$ evolves according to

$$\frac{\partial \langle f \rangle}{\partial t} = \sum_{\mathbf{k},\omega} \mathrm{Re}\left\{ \frac{i|e|}{T_e} \left(\omega - \omega_{*e}^{T}\right) \left\langle \left(\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel\right) \hat{h} \right\rangle_{\mathbf{k},\omega} \right\}, \tag{4}$$

where $\hat{h}$ is the nonadiabatic piece of the distribution function and ω_{*e}^{T} is the (thermal) diamagnetic frequency. Note that the correlation $\left\langle \left(\hat{\phi} - \frac{v_\parallel}{c} \hat{A}_\parallel\right) \hat{h} \right\rangle$, which determines $\langle f \rangle$ relaxation, is essentially that studied previously in Eqs. (2), (3) for the case of an isolated drift-blob. However, for fully-developed turbulence $\hat{h} = \hat{h}^c + \tilde{h}$, where $\hat{h}^c$ is the familiar coherent response component. By definition, $\hat{h}^c$ can be written as $\hat{h}^c = \hat{h}^c_{\mathbf{k},\omega} = R^{\phi}_{\mathbf{k},\omega} \hat{\phi}_{\mathbf{k},\omega} + R^{A}_{\mathbf{k},\omega} \hat{A}_{\parallel \mathbf{k},\omega}$, where R^{ϕ} and R^{A} refer to the generalized nonlinear electron coherent response functions. Substituting $\hat{h} = \hat{h}^c_e + \tilde{h}_e$ into Eq. (4) yields:

$$\begin{aligned} \frac{\partial \langle f \rangle}{\partial t} = \sum_{\mathbf{k},\omega} \mathrm{Re} \Bigg\{ \frac{i|e|}{T_e} (\omega - \omega_{*e}^{T}) \Bigg[\Bigg(& R^{\phi}_{\mathbf{k},\omega} \left\langle \hat{\phi}^2 \right\rangle_{\mathbf{k},\omega} \\ & - \frac{v_\parallel}{c} R^{A}_{\mathbf{k},\omega} \left\langle \hat{A}_\parallel^2 \right\rangle_{\mathbf{k},\omega} + R^{A}_{\mathbf{k},\omega} \left\langle \hat{\phi} \hat{A}_\parallel \right\rangle_{\mathbf{k},\omega} - \frac{v_\parallel}{c} R^{\phi}_{\mathbf{k},\omega} \left\langle \hat{A}_\parallel \hat{\phi} \right\rangle_{\mathbf{k},\omega} \Bigg) \\ & + \left(\left\langle \hat{\phi} \tilde{h} \right\rangle_{\mathbf{k},\omega} - \frac{v_\parallel}{c} \left\langle \hat{A}_\parallel \tilde{h} \right\rangle_{\mathbf{k},\omega} \right) \Bigg] \Bigg\}, \end{aligned} \tag{5}$$

where the first four terms constitute the usual quasilinear diffusion operator, containing magnetic, electrostatic and off-diagonal, respectively. The last two terms constitute the drag operator, and are induced by incoherent fluctuations. Note that the first quasilinear term and first drag term govern all transport moments arising from $\mathbf{E} \times \mathbf{B}$ motion ($\hat{E}_\theta$ perturbations in correlation with moments of the distribution $\hat{f}$), whereas the second quasilinear term and second drag term govern all transport moments arising from magnetic flutter ($\hat{B}_r$ in correlation with moments of $v_\parallel \hat{f}$).

The fluctuations $\hat{h}^c_{\mathbf{k},\omega}$ and $\tilde{h}_{\mathbf{k},\omega}$ are related by Ampere's law and quasineutrality, which respectively imply that $d^{A,A}_{\mathbf{k},\omega}\hat{A}_{\|\mathbf{k},\omega}+d^{A,\phi}_{\mathbf{k},\omega}\hat{\phi}_{\mathbf{k},\omega} = -\frac{4\pi|e|}{c}\int dv_\| v_\| \tilde{h}_{\mathbf{k},\omega} = -\frac{4\pi}{c}\tilde{J}_{\|\mathbf{k},\omega}$ and that $d^{\phi,\phi}_{\mathbf{k},\omega}\hat{\phi}_{\mathbf{k},\omega} + d^{\phi,A}_{\mathbf{k},\omega}\hat{A}_{\|\mathbf{k},\omega} = -\int dv_\| \tilde{h}_{\mathbf{k},\omega} = -\tilde{n}_{\mathbf{k},\omega}$. Here $d^{A,A}$, $d^{A,\phi}$, $d^{\phi,A}$, and $d^{\phi,\phi}$ are dielectric tensor elements obtained from velocity moments of the electron and ion coherent phase space density responses. It is thus possible to express $\hat{A}_\|$ and $\hat{\phi}$ in terms of $\tilde{J}_\|$ and $\tilde{n}$, as:

$$\hat{A}_{\|\mathbf{k},\omega} = \mathcal{L}^{-1}_{\mathbf{k},\omega}\left[d^{\phi,\phi}_{\mathbf{k},\omega}\frac{4\pi}{c}\tilde{J}_{\|\mathbf{k},\omega} - d^{A,\phi}_{\mathbf{k},\omega}\tilde{n}_{\mathbf{k},\omega}\right] \tag{6a}$$

$$\hat{\phi}_{\mathbf{k},\omega} = \mathcal{L}^{-1}_{\mathbf{k},\omega}\left[d^{A,A}_{\mathbf{k},\omega}\tilde{n}_{\mathbf{k},\omega} - d^{\phi,A}_{\mathbf{k},\omega}\frac{4\pi}{c}\tilde{J}_{\|\mathbf{k},\omega}\right]. \tag{6b}$$

Setting $\mathcal{L} = d^{\phi,A}d^{A,\phi} - d^{A,A}d^{\phi,\phi} = 0$ determines the eigenfrequencies of the system. Equations (6a,b) indicate that the collective plasma response shields incoherent density and current fluctuations. This shielding mechanism underlies the relationship between $\hat{h}^c_{\mathbf{k}}$ and $\tilde{h}_{\mathbf{k}}$, which follows from Eqs. (6a,b) and the definition of $\hat{h}^c_{\mathbf{k}}$. Note that Eqs. (6a,b) assume that the collective resonances (where $\mathrm{Re}\mathcal{L}_{\mathbf{k},\omega} = 0$) are nonlinearly over-saturated or stable.

The LBTCI can be simplified by substituting Eq. (6a,b) into Eq. (5) and assuming moderate or weak spectral broadening ($\Delta\omega \leq \omega$). This allows the electron response functions $R^{\phi}_{\mathbf{k},\omega}$ and $R^{A}_{\mathbf{k},\omega}$ to be written in terms of the ballistic propagator $2\pi\delta(\omega - k_\| v_\|)$, and the correlation function to be written as $\left\langle \tilde{h}^2 \right\rangle_{\mathbf{k},\omega} = 2\pi\delta(\omega - k_\| v_\|)\left\langle \tilde{h}^2 \right\rangle_{\mathbf{k}}$. Using these properties and multiplying the drag terms by the unit operator $\mathcal{L}^{-1}\mathcal{L} = \mathcal{L}^{-1}(d^{A,\phi}d^{\phi,A} - d^{\phi,\phi}d^{A,A})$, it follows that the magnetic 'flutter' diffusion term $v_\|^2\left\langle (\hat{B}_r/B_0)^2 \right\rangle \delta(\omega - k_\| v_\|)$ of the LBTCI is exactly cancelled by the electron-electron piece of the $\left\langle \hat{A}_\| \tilde{h} \right\rangle$ drag term (i.e., that part proportional to the electron susceptibility contribution to $\mathrm{Im}d^{A,A}$ and $\mathrm{Im}d^{\phi,A}$). The electrostatic diffusion term is similarly cancelled by the electron-electron piece of $\left\langle\hat{\phi}\tilde{h}\right\rangle$. The surviving terms in the LBTCI include 'cross' diffusion and 'nonresonant' drag operators, both proportional to Re $d_{\mathbf{k},\omega}$, as well as electron-ion drag terms, proportional to $\mathrm{Im}d^{\phi,\phi}_{(\mathrm{ion})}$ and Im $d^{A,\phi}_{(\mathrm{ion})}$. Consistent with the assumption of moderate spectral broadening, the collective resonance $\mathbf{k} = \mathbf{k}(k_{\|}u_{\|})$ dominates the integration over $\mathbf{k}$. Thus, the susceptibilities $d_{\mathbf{k},\omega}$ are evaluated on resonance, where their real parts are negligible. Therefore only the electron-ion drag terms associated with $\mathbf{E}\times\mathbf{B}$ motion survive, and Eq. (5) reduces straight-forwardly to

$$\frac{\partial\langle f\rangle}{\partial t} = \sum_{\mathbf{k},\omega} -\frac{|e|}{T_e}\frac{(\omega-\omega^T_{*e})}{|k_{\|}|}2\pi\delta(\omega-k_{\|}u_{\|})[\mathrm{Re}\ S_{\mathbf{k},\omega}] \tag{7a}$$

where

$$S_{\mathbf{k},\omega} = \mathcal{L}^{-1}_{\mathbf{k},\omega}\left\langle\hat{\phi}\tilde{h}\right\rangle_{\mathbf{k}} \times \left[d^{A,A}_{\mathbf{k},\omega}\mathrm{Im}d^{\phi,\phi}_{\mathbf{k},\omega\,(\mathrm{ion})}\right] \tag{7b}$$

in the usual case of negligible ion current ($\hat{J}^i_{\|} \rightarrow 0$, $\mathrm{Im}d^{A,\phi}_{\mathbf{k},\omega\,(\mathrm{ion})} \approx 0$). Finally, it should be noted that the LBTCI reduces to previously derived results[6] in the electrostatic limit.

Equation (7a,b) indicates that the quasilinear magnetic flutter transport ($\sim v^2_{\|}\left\langle(\hat{B}_r/B_0)^2\right\rangle_{\mathbf{k}}\delta(\omega-k_{\|}v_{\|})$) does not contribute to the relaxation of $\langle f\rangle$, and thus does not result in electron energy or momentum transport! Insight into this result can be gained by noting that $\partial\langle f_e\rangle/\partial t \sim$ Im $d^{\phi,\phi}_{\mathbf{k},\omega\,(\mathrm{ion})}$, which indicates that electron phase space relaxation is proportional to the dissipative ion response to the electrostatic potential. Indeed, if such dissipative ion coupling is absent, the LBTCI vanishes and $\langle f\rangle$ cannot relax. This result is analogous to those obtained using collisional and collisionless Lenard-Balescu equations for a one-dimensional electron-ion plasma.[9] In that system, constraints of momentum and energy conservation on the interaction (collisional or collisionless) of localized phase space fluctuations imply that like 'particle collisions' leave the final state identical to the initial state, thus precluding relaxation of $\langle f \rangle$. This results in a similar cancellation of electron-electron terms in the LBTCI, leaving $\partial \langle f \rangle / \partial t$ proportional to Im χ_i, where χ_i is the ion susceptibility. Here, since $\hat{J}_{\|}^{i} \to 0$ the only electron-ion coupling occurs through Im $d^{\phi,\phi}_{\mathbf{k},\omega(\text{ion})}$. Hence, it is not surprising that an analogous cancellation of the magnetic flutter diffusion term (a purely electron-electron term) occurs, leaving $\langle f \rangle$ relaxation and transport to be determined by electrostatic mechanisms.

The two models of collisionless drift-Alfvén dynamics, the isolated blob and fully-developed turbulence models, respectively, give consistent, complementary insights into the effects of the same self-consistently constraints on relaxation and transport. In the case of an isolated blob, Ampere's law and the granularity (i.e., localization in phase space) of $\tilde{f}$ lead to the result that $\partial/\partial t \left\langle \tilde{f}^2 \right\rangle \sim \left\langle \hat{B}_r \hat{J}_{\|e} \right\rangle \sim \left\langle \partial/\partial r (\hat{B}_\theta \hat{B}_r) \right\rangle \to 0$, to $0(\Delta x / L_x)$. In the case of fully developed turbulence, Ampere's law and the proper consideration of granular, incoherent fluctuations in the dynamics of $\langle f \rangle$ result in the cancellation of diffusive magnetic flutter terms (in the LBTCI) by electron-electron drag terms $\left(\sim \left\langle \hat{A}_{\|} \tilde{h} \right\rangle\right)$. Both results indicate that *transport and relaxation in drift-Alfvén turbulence are regulated by electrostatic fluctuations.*

It is interesting to reconsider theories of anomalous transport due to magnetic flutter in light of the discussion presented here. In particular, a recent paper[1] by Kadomtsev and Pogutse (K and P) treats transport caused by small scale, high frequency ($\Delta x \sim c/\omega_{pe}$,

$\omega \sim v_{Te}/qR$) electromagnetic turbulence. Tacitly assuming that the transport-causing small scales are nonlinearly driven by larger scales via cascading, K and P then use quasi-linear theory (with dissipation due to electron Landau resonance) and mixing length estimates to derive the thermal conductivity $\chi_e \sim \epsilon(c^2/\omega_{pe}^2)v_{Te}/qR$. However, dissipative ion coupling is ignored throughout their analysis. Thus, by way of contrast, a parallel calculation following the discussion presented here yields the result that χ_e vanishes! The discrepancy is due to the fact that K and P invoke mode coupling to drive the transport-causing scales, but compute χ_e using quasilinear theory. This procedure ignores incoherent fluctuations and thus clearly is not internally consistent. Thus, the discrepancy discussed here dramatically underscores the importance of phase space granulation and self-consistency constraints.

Finally, it is important to note that several restrictions apply to the discussion presented here. First, these considerations apply only to collisionless Alfvénic microturbulence, where the electron susceptibility is determined by collisionless, parallel dynamics. Hence, it is not surprising that dramatic differences between models incorporating parallel trapping and granulation, and those based on unperturbed orbits (i.e., quasi-linear theory) are uncovered. However, these considerations do *not* apply to magnetic transport resulting from collisional[10] or macroscopic (i.e., resistive MHD) turbulence. In those cases, the question of possible modification to the Ohm's law by magnetic turbulence (i.e., the nature of the electron viscosity) must be addressed.[3] Furthermore, stationary turbulence is assumed throughout. Nonstationary turbulence (such as in the case of growing waves) permits the exchange of energy and momentum between waves and incoherent fluctuations, thus allowing different relaxation mechanisms. Finally, these considerations do not straightforwardly lend themselves to the study of magnetic transport induced by external perturbations, such as an applied helical coil.

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