DIFFUSION INDUCED BY CYCLOTRON RESONANCE HEATING

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ABSTRACT: The wave induced particle transport during the ion cyclotron resonance heating is studied in collisionless toroidal plasmas. It is shown that the previously neglected non-conservation of the toroidal angular momentum $I\!P_\phi$ caused by the toroidal wave component E_ϕ is necessary to allow particle diffusion and yields the leading diffusive contribution. While the induced ion transport for the rf power in contemporary experiments is of the order of the neoclassical value, that of fast alpha particles is quite large if resonance is present.

1. INTRODUCTION

The ion cyclotron–wave resonance heating (ICRH) of a plasma is one of the most important methods of plasma heating to high temperatures[1, 2, 3]. One of the main reasons is that this scheme can directly increase the proportion of reactive ions[4] for fusion, in a sense an alternative to the beam–sustained two–component plasma idea. Another reason is that high power radio frequency (rf) wave generators at the ion cyclotron waves are technically readily available. For these reasons and an experimental success[5] large tokamak fusion plasmas are planned to be heated by ICRH, including JET (the Joint

European Tokamak) and TFTR (the Toroidal Fusion Test Reactor). One of the potential problems associated with this method is the possibility of reduction of plasma confinement due to the cyclotron interaction with particles, i.e., the ion cyclotron–wave induced diffusion. It is thus crucial to examine the qualitative nature and the quantitative magnitude of this transport associated with ICRH. Our investigation is focused on ICRH, but also applicable to similar problems arising from the electron cyclotron–wave resonance heating (ECRH) for electron transport. The results can be applicable to controlling impurity ions via Alfvén wave induced diffusion. Such problems may take place in heating stellarators as well. Through our investigation we find that although the ion transport due to ICRH is of the same order of the neoclassical transport in the present large tokamak experiment parameters, the fusion α-particle transport induced by ICRH is far greater than the neoclassical value if there exists α-particle resonance.

High levels of rf power deposited in a plasma layer affect confinement by (i) driving the velocity distribution into a non-Maxwellian state and thus changing the distribution associated collisional diffusive flux and by (ii) inducing collisionless loss of particles through the stochastization of their trajectories caused by the wave. Case (i) has been investigated by a number of authors[6]; we instead focus our attention on case (ii), the directly induced collisionless diffusion.

The net diffusive flux of particles is intimately related to symmetry of the system. For collisionless confinement in three dimensions an equal number of exact or adiabatic invariants of motion must exist, regardless whether the confining field geometry has two degrees of symmetry (straight cylinder), one (axisymmetric torus, helically symmetric stellarator) or none (toroidal stellarator). However, the rate of diffusion caused by collisions or interaction with additional fields increases with decreasing degrees of symmetry as the particle orbits bifurcate into new classes[7, 8] characterized by higher radial excursions and thus higher diffusion levels. The transition from the classical diffusion in a straight cylinder to the neoclassical in a torus due to the increase of the mean step from the Larmor radius to the width of a banana orbit for the trapped particles is the basic example[9]. A relationship between degrees of symmetry and diffusion similar to but distinctively different from the collisional case holds for the case of the wave–induced collisionless diffusion under consideration as the average wave induced radial displacement for a trapped particle exceeds the displacement for a passing particle by a factor $1/\epsilon$, where ϵ is the inverse aspect ratio. Furthermore, destruction of axisymmetry by the wave enhances particle transport as compared to the case when only the magnetic moment and the total energy of the particle are allowed to change. We measure the diffusion rate for trapped particles by the radial shift in the guiding center turning point rather than the change in the width of the banana orbit used in previous work[10]. Thus in case that conservation of the canonical angular momentum $I\!P_\phi$ is assumed and finite Larmor radius effects are ignored in defining the turning point, no diffusion results, contrary to the conclusions of Ref. [10]. However, in devices with helical magnetic lines a finite toroidal component of the wave field E_ϕ exists, even with the parallel to the magnetic field component $E_{||}$ shorted out. This destroys the axisymmetry and its associated remaining invariant and thus introduces a neoclassical type of collisionless diffusion. The change in the magnetic moment yields a smaller contribution to diffusion entering through finite Larmor radius effects in the position of the turning point.

2. COMPUTATION OF THE DIFFUSION COEFFICIENTS

Consider a wave propagating nearly perpendicularly to the magnetic field at the ion cyclotron resonance in a hot axisymmetric toroidal plasma. The position and velocity of a particle are given by $\vec{X} + \vec{\rho}$ and $\vec{V} + \vec{v}_\perp$ respectively with $\vec{X}$ and $\vec{V}$ the guiding center position and velocity $\vec{\rho}$ the gyroposition from the guiding center $\vec{v}_\perp = d/dt\vec{\rho}$. The trajectory of a trapped particle (banana orbit) is shown in Fig. 1(a) with the toroidal coordinate system $(\hat{e}_r, \hat{e}_\theta, \hat{e}_\phi)$. In a field aligned coordinate system with $(\hat{e}_\perp, \hat{e}_{||})$ making an angle α with $(\hat{e}_\theta, \hat{e}_\phi)$ respectively, we have $\vec{\rho} = \rho(\hat{e}_\perp \sin\chi + \hat{e}_r \cos\chi)$, $\chi = \int \Omega dt$, $\Omega = eB/mc$, $B = B_0(1 - \epsilon\cos\theta)$, $\rho = |\vec{v}_\perp|/\Omega$ and $\vec{V}$ is given to order ρ/R by $\vec{V} = V_{||}\hat{e}_{||} + \vec{V}_D$ with $V_{||} = v_{||} + (v_\perp^2/2\Omega)(1/B)(\partial B_\theta/\partial r)$, $\vec{V}_D = \Omega^{-1}\left[\left(\frac{1}{2}v_{||}^2 + v_\perp^2\right)/R\right](\hat{e}_{||} \times \hat{e}_R)$. R is the distance from the axis of the torus and r the minor radius. Without the wave the total energy $\mathcal{E} = 1/2mv_{||}^2 + \mu B$ and the canonical toroidal angular momentum

$$I\!P_\phi = \left\{m(\vec{X} + \vec{\rho}) \times (\vec{V} + \vec{v}_\perp) + (e/c)\Psi(\vec{X} + \vec{\rho})\right\} \cdot (\hat{e}_R \times \hat{e}_\phi), \tag{1}$$

with $\hat{e}_R$ along the major radius are exact invariants of the motion, while the magnetic moment $\mu = mv_\perp^2/2B$ is an adiabatic invariant. We consider trapped particles in the collisionless regime where the bounce frequency $\nu_b \simeq (2\pi)^{-1}(v_\perp \epsilon^{1/2}/qR)$ is much larger than the effective collision rate of $\nu_{ef} = \nu_{ei}/\epsilon$.

The diffusion of a trapped particle is characterized by the radial shift of the turning point. A relation between the turning point $\vec{X}^*$ and the invariants of the motion is obtained from the exact equation for $I\!P_\phi$, by expanding Eq. (1) to second order in Larmor radius and averaging over the gyroangle χ,

$$\langle I\!P_\phi \rangle = I\!P_\phi = -3(mc/e)\cos\theta^* \sin\alpha^* \mu + (e/c)\Psi(\vec{X}^*) + (1/2)(mc/e)\mu(\cos^2\alpha^*/B^*)\partial_r^2\Psi(\vec{X}^*) \tag{2}$$

where the asterisks denote quantities at the turning point defined by $V_\parallel^* = 0$ and the relations $\mu = (1/2)\rho^2\Omega e/c$, and $\left|1/r^2\partial^2\Psi/\partial\theta^2\right| \ll \left|\partial^2\Psi/\partial r^2\right|$ have been used. If finite Larmor radius effects are omitted from (2) and if it is assumed that only μ changes due to heating with $I\!P_\phi$ fixed as constant, as in previous work[10], then the turning point remains always on the same flux surface $\Psi(\vec{X}^*) \equiv \Psi^* = (c/e)I\!P_\phi$ and *no* diffusion of the average position occurs although the banana width changes between successive passages through resonance. The more accurate description, Eq. (2), shows, however, that a shift of the turning point is produced by both $\delta\mu$ and $\delta I\!P_\phi$ resulting in a neoclassical type of diffusion. We shall show that the change in $I\!P_\phi$ by the toroidal component of the wave $E_\phi = E_\parallel \cos\alpha - E_\perp \sin\alpha$, $\alpha \cong B_\theta/B_t \cong \epsilon/q$ yields the leading diffusive contribution even when $E_\parallel$ is shorted out by the large parallel conductivity and that a small residual diffusion due to $\delta\mu$ persists in the limit $E_\phi \to 0$. The wave is $\vec{E} = \vec{E}_0 \sin(\int \vec{k}\cdot d\vec{x} - \omega t)$ with $\vec{k} = (k_r, k_\perp, 0)$, $k_\perp = k_\theta \cos\alpha - k_\phi \sin\alpha$, $k_\theta = m/r$, $k_\phi = n/R$, $\int \vec{k}\cdot d\vec{x} = k\rho\sin(\chi+\varsigma) + (n+m/q)\phi$, $k = (k_r^2 + k_\perp^2)^{1/2}$, $\varsigma = \tan^{-1}(k_r/k_\perp)$ and $\vec{E}_0$ is split into longitudinal and transverse part $\vec{E}_0 = E_L(\vec{k}/k) + E_T(\vec{k}\times\hat{e}_\parallel/k)$ with $E_L = -\nabla U$, $E_T = -1/c\partial\vec{A}/\partial t$ according to the Coulomb gauge. The jumps $\delta\mu$ and $\delta I\!P_\phi$ received during the passage through the resonant region are calculated using

$$\frac{d}{dt}\mu B = e\vec{v}_\perp \cdot \vec{E} = ev_\perp\left[E_L\cos(\chi+\varsigma) - E_T\sin(\chi+\varsigma)\right] \times \cos\left(k\rho\sin(\chi+\varsigma) + (n+m/q)\phi - \omega t\right) \tag{3}$$

$$\frac{d}{dt}I\!P_\phi = -e\left(\frac{\partial U}{\partial \phi}\right) - \frac{e}{c}\vec{v}\cdot\left(\frac{\partial \vec{A}}{\partial \phi}\right) = eR\left[E_L - \frac{v_\perp k}{\omega}\sin(\chi+\varsigma)E_T\right]$$
$$\times\cos\left(k\rho\sin(\chi+\varsigma) + (n+m/q)\phi - \omega t\right)\cos\varsigma\sin\alpha, \qquad (4)$$

where the term $\partial/\partial\phi(\vec{k}\times\hat{e}_{||}/k)$ is neglected in the last part of Eq. (4) for $n \gg 1$, and $R = (\vec{X}+\vec{\rho})\cdot\hat{e}_R = R_G + \rho\cos\theta\sin\psi$. Only the resonant terms of the general form $J_N(k\rho)\sin[N\chi + (n+m/q)\phi - \omega t + c]$ are retained with N such that at resonance

$$d/dt[N\chi + (n+m/q)\phi - \omega t + c] = N\Omega_0 - k_{||}v_0 - \omega = 0. \qquad (5)$$

The $k_{||}v_{||0}$ term can be dropped for nearly perpendicular propagation as $(k_{||}v_{||}/N\Omega) \ll k\rho\sqrt{2\epsilon} < 1$, where the subscript refers to the resonance point.

One distinguishes between two cases for the calculation of $\delta I\!P_\phi$ and $\delta\mu$. Case (a). The change in ρ during one passage is small, $k\delta\rho \ll \pi$. Then the argument $k\rho$ of the Bessel function can be assumed constant over one passage, thus Eqs. (3)-(4) are integrated applying the stationary phase approximation. This yields $k\delta\rho = 2(eE_0k/m\Omega_0)(\pi/\gamma_N)^{1/2}$ with the detuning rate, $\gamma_N = d/dt[\ell n\Omega(t)] = \kappa_B v_{||}$, $\kappa_B = (\epsilon/qR_0)\sin\theta_0$ the scale length of the magnetic field. Defining the resonant width $L_R = (1/N)(B_0/E_0)^2(v_{ph}/c)^2\rho$ as the length over which $k\rho$ changes by π, the applicability criterion for case (a) is written as $R_0q/\epsilon \ll L_R$. Thus the magnetic field must change considerably within a length shorter than the distance L_R required for a large change in $k\rho$; L_R is small for high k and small v_{ph}/c (i.e., near a wave resonance). Case (b) with $R_0q/\epsilon \geq L_R$ meaning that $k\delta\rho \gtrsim \pi$. The change in the argument of the Bessel function leads to modulation much faster than the detuning due to the change in Ω and the particle executes many gyrations in the resonant regime, diffusing stochastically over one passage. For lower hybrid heating with typical TFTR parameters (given in the next section) we obtain a resonant length of a few centimeters much less than the scale length of the magnetic field and case (b) applies. For ion cyclotron heating one finds under the same plasma parameters that L_R is a few meters thus stationary phase approximation can be applied.

The phase in the resonant terms of the rhs in Eq. (5) is expanded as

$$\Phi(t) = \Phi_0 + \kappa_B\Omega_0\left[\frac{1}{2}v_{||}t^2 + \frac{1}{6}\dot{v}_{||}t^3\right], \qquad (6)$$

with $\Phi_0 = N\chi_0 + N\varsigma - \omega t_0$, where the second term inside the bracket comes from the free streaming and the third term, coming from the parallel acceleration $\mu\nabla B$, need to be considered only when the turning point $v_\parallel \cong 0$ falls very close to the resonant layer. Keeping R_0 and B constant during the integration we find

$$\delta I\!P_\phi = 2e\left(\pi/\gamma_N \Omega_0\right)^{1/2} R_0 \left\{E_L J_N(k\rho)\cos\Phi_0' + (v_\perp k/\omega) E_T \left(N J_N(k\rho)/k\rho\right)\cos\Phi_0''\right\} \times \cos\varsigma \sin\alpha, \quad (7)$$

$$\delta\mu = 2\left(ev_\perp/B_0(1-\epsilon\cos\theta_0)\right)\left(\pi/\gamma_N\Omega_0\right)^{1/2}\left\{E_L\cos\Phi_0' + E_T\cos\Phi_0''\right\}\left(N J_N(k\rho)/k\rho\right), \quad (8)$$

where Φ_0' and Φ_0'' are related to the phase difference Φ_0 between the gyroangle and the wave phase at the resonance by $\Phi_0' = \Phi_0 + \pi/4$ and $\Phi_0'' = \Phi_0 + 3\pi/4$.

We now obtain the displacement of the guiding center turning point perpendicular to the flux surface, $\delta r^* = \delta\Psi^*/|\nabla\Psi^*|$ as a function of the induced changes $\delta I\!P_\phi, \delta\mu$. The quantities $\nabla^2\Psi \cong R\nabla(1/R\nabla\Psi) = RI_\phi$ and $\alpha = \epsilon/q(\Psi)$ are both flux functions in the low β axisymmetric Grad–Shafranov equilibrium, thus Eq. (2) is of the form $F(I\!P_\phi, \mu, \theta^*, \Psi^*) = 0$. Differentiating Eq. (2) and using the energy conservation

$$\frac{1}{2}mv_\parallel^2 + \mu B(1-\epsilon\cos\theta_0) = \mu B(1-\epsilon\cos\theta^*), \quad (9)$$

to express $\delta\theta^*$ in terms of $\delta I\!P_\phi, \delta\mu$ we find the turning point displacement due to the wave resonance as

$$\delta r^* = (R^*B^*)^{-1}\left\{\delta I\!P_\phi + \left[(2mc^2/e^2)\cos\theta_0\sin\alpha^* - \frac{1}{2}(R^* I_\phi^*/\Omega^*)\right]\delta\mu\right\} \quad . \quad (10)$$

It is reasonable to assume that the heating process is stochastic as the stochasticity threshold with the last invariant $I\!P_\phi$ destroyed is lower than the threshold with constant $I\!P_\phi$ evaluated in Ref. [10] and shown to be sufficiently small. The angle Φ_0 entering the calculation of $\delta I\!P_\phi$, $\delta\mu$ in Eqs. (7)-(8) becomes decorrelated within a banana time while θ_0, θ^*, R^* change much slower on a diffusion time scale and remain constant over a time span long enough to perform averaging $\langle\ \rangle$ over Φ_0. The diffusion coefficient $D_{\rm rf}$ is given by

$$D_{\rm rf} = 4\pi\int_{\sqrt{2\epsilon}\sin\theta_0/2}^{\sqrt{2\epsilon}} dv d\lambda v^2\cos\lambda\left\langle\delta r^2\right\rangle 2\nu_b f(v,\lambda),$$

assuming that on the average a particle receives two kicks per banana orbit; λ is the pitch angle $\lambda = \tan^{-1}(v_{\|}/v_{\perp})$ at the minimum B position. The lower limit of integration corresponds to banana orbits with their tips touching the resonant surface at θ_0 while the upper limit corresponds to marginally trapped (untrapped) particles. Particles with λ within the thin separatrix layer $\sqrt{2\epsilon} \pm \delta_1$, $\delta_1 \cong (E_{\perp}c/Bv)(\Omega/\kappa_B v_{\|})^{1/2} \ll 1$ can scatter from trapped to untrapped and vice versa within one bounce yielding a radial shift $\delta r \cong \Delta_b =$ banana width; however, their bouncing period tends to infinity and they need not be considered. Particles within the thin layer $\sqrt{2\epsilon}\sin(\theta_0/2) \pm \delta_2$, $\delta_2 \sim 1/4(\pi\kappa_B\rho)^{1/3} \ll 1$ have their turning point $v_{\|} \cong 0$ within the resonant regime thus the acceleration term $\dot{v}_{\|} = -\mu\kappa_B\Omega m^{-1}$ dominates in the phase expansion Eq. (6) and the resulting jumps $\delta I\!P_{\phi}$,$\delta\mu$ are greater from the results in Eqs. (7)-(8), (with $v_{\|} \sim \sqrt{\epsilon}v_{\perp}$) by a factor $\sqrt{\pi}Ai(0)4^{1/3}\epsilon^{1/4}(\kappa_B\rho_L)^{-1/6}$, $Ai(0) = 0.355$. Due to the smallness of δ_2, the above enhanced local contribution does not change overall diffusion significantly. The velocity distribution $f(v,\lambda)$ represents a steady state reached when the rate of wave induced velocity diffusion is balanced by the rate of pitch angle ion-ion scattering and ion-electron frictional slowing down. This balance can be achieved without violating the original assumption $\nu_b \gg \nu_{ii} \gg \nu_{ie}$, as from Eq. (7) the wave induced pitch angle scattering frequency scales as $\nu_{\rm rf} \sim (E_0c/Bv)^2\nu_b \ll \nu_b$. In excluding time dependence from $f(v,\lambda)$ it is assumed that velocity relaxation occurs on a much faster time scale than particle transport; this is generally true for the collisional processes and we will show it is valid for the wave induced diffusion as well. For $f(v,\lambda)$ far from Maxwellian such as the asymptotic forms obtained by Stix[1] (or by Bernstein and Baxter[11] if electron diffusion is considered), numerical integration is necessary; for near Maxwellian distribution we use Eqs. (7)-(8) in the rhs of Eq. (10), discard terms of order $\epsilon^2(\rho/r_0) \sim \epsilon(\rho/R_0)$ or higher, set $N = 1$ and use the small argument expansion for the Bessel function to obtain the diffusion coefficient

$$D_{\rm rf} = C(3\pi/4)\left(e^2/m^2\Omega^3\right)E_0^2\epsilon^{-1/2}\left[k_\perp^2\rho^2 - 2k_\perp\rho^2/r_0 + (\rho/r_0)^2\right], \qquad (11)$$

with C a numerical factor of $0(1)$

$$C = \int_{\sin\frac{\theta_0}{2}+\delta_2}^{1-\delta_1} du(1+2\epsilon u^2)^{-5/2}K(u^2)^{-1}\left(u^2 - \sin(\theta_0/2)\right)^{-1/2}$$

and $1/r_0 = \partial/\partial_r(\ell n B_\theta)$ (r_0 is the minor radius of the torus). The diffusion is proportional to the total rf-energy density $E_0^2 = E_L^2 + E_T^2$ and for given E_0^2 the expression is insensitive to the details of possible mode conversion.

The first term inside the bracket in Eq. (11) results from the change $\langle \delta I\!P_\phi \rangle$ in the angular momentum due to the toroidal field component $E_\phi \cong E_\perp \epsilon/q$ and in the regime of short wavelengths $kr_0 \ll 1$ it dominates the contribution from the change in the magnetic moment $\langle \delta\mu \rangle$ given by the third term, as well as the contribution from the cross-correlation $\langle \delta I\!P_\phi \delta\mu \rangle$ given by the second term. (During electron cyclotron heating $E_\parallel$ can be significant and with $E_\phi \sim E_\parallel$ the electron diffusion is enhanced by a factor $(q/\epsilon)^2$). In case of wave propagation along the minor radius $k_\perp = k\cos\varsigma = 0$ (Fig. 1b), $\delta I\!P_\phi$ in Eq. (4) vanishes leaving a residual neoclassical-type diffusion due to $\langle \delta\mu^2 \rangle$.

A heating time τ_H can be defined by $\tau_H \cong v^2/\tilde{D}_{\rm rf}$ and $\tilde{D}_{\rm rf}$ given by $\tilde{D}_{\rm rf} \cong \nu_b \langle \Delta v_\perp^2 \rangle$ and Eq. (8). Defining the diffusion time $\tau_D \cong r_0^2/D_{\rm rf}$ one obtains the ratio $\tau_H/\tau_D \cong (\rho/r_0)^2 k^2 \rho^2 \ll 1$, which justifies our assumption of separation between velocity and spatial diffusion time scales.

The diffusion coefficient for passing particles can be obtained using the same procedure. Computation of the radial shift for δr^{**} at the point of minimum $v_\parallel$ (i.e., $\theta = \pi$) leads to $\delta r^{**} \cong \epsilon \delta r^*$ thus $D_{\rm passing} \sim \epsilon^{3/2} D_{\rm trapped}$.

3. CONCLUSIONS

It is of importance to define the amplitude E_{nc} at which the induced diffusion becomes comparable to the neoclassical, $D_{\rm rf}/D_{nc} \sim 1$ where $D_{nc} = q^2\rho^2\nu_{ei}\epsilon^{-3/2}$. Using the leading term in Eq. (11) one finds approximately that for ions

$$E_{nc}^i = (v_{ph}/v_A) B_0 (\nu_{ei}/\Omega_i)^{1/2} \epsilon^{-1/2} q \quad . \tag{12}$$

For TFTR-type parameters $T_i = 10^4 eV, B = 5T, n_i = 3 \times 10^{13} cm^{-3}, k = \omega/v_{ph}, \epsilon = .15$, and $q = 1.5$ we have $E_{nc} \sim (v_{ph}/v_A) \times 60 V/cm$. Thus, given that the actual diffusion rate exceeds the neoclassical by an order of magnitude and if the electric field near resonance is not to exceed a few tens of V/cm during large scale ICRH the directly wave induced diffusion poses no real threat to confinement.

The 3.5 MeV α-particles produced by fusion experience the same cyclotron resonances as the deuterium. Therefore the calculational procedures we have taken for ions are applicable to α-particles as well except that, due to the large Larmor radius ρ_α, we should revive the original Bessel function, replacing the square brackets in Eq. (11) by $4\left\langle J_1^2(k_\perp\rho_\alpha)\right\rangle\left[1-(k_\perp r_0)^{-2}\right]$ and multiplying Eq. (12) by $k_\perp\rho_\alpha/2\left\langle J_1^2(k_\perp\rho_\alpha)\right\rangle^{1/2}$. We note here that a description with terms of order of $(\rho/R)^2$ more acurate than the guiding center description yields additional corrections no larger than $\epsilon(\rho/R)^2$ to the diffusion coefficient. This correction is much smaller than the terms kept in Eq. (11), thus our analysis remains valid for α-particles. From Eq. (11) we find that $D_{\rm rf}^\alpha/D_{\rm rf}^d \sim 4\left\langle J_1^2(k_\perp\rho_\alpha)\right\rangle/(k_\perp\rho_d)^2$ which can be much larger than unity. From Eq. (12) we can write down the equivalent critical rf-power density P_{nc} above which the wave induced diffusion dominates the neoclassical one for a given particle species and find that $P_{nc}^\alpha/P_{nc}^d \sim (1/4)k_\perp^2\rho_\alpha^2\left\langle J_1^2(k_\perp\rho_\alpha)\right\rangle^{-1}(T_\alpha/T_d)^{-3/2} \ll 1$. It is clear that α-particles suffer severe diffusion if resonance is present. This α pump-out during heating near the deuterium-tritium hybrid resonance may be avoided when tritium is a minority (then $\omega_{\rm hybrid} \sim \Omega_t$) by having the deuterium resonant surface outside the plasma. After ignition, on the other hand, with the α particles produced mainly in the plasma core, this wave induced transport may be utilized for rapid and preferential α heat distribution through the plasma volume to avoid heat accumulation and the heat runaway instability as well as for the ash transport. Applications of the present theory to trapped particles of astrophysical plasmas such as in the Van Allen belt might be also relevant although the toroidal direction plays a different role, as the basic physics is similar.

It is of interest to observe that the present wave-diffusion process preserves the Onsager symmetry[12], because it is microscopically reversible. In the presence of plasma turbulence, however, the Onsager symmetry may be broken in a way similar to the neoclassical transport case[13] studied by Molvig et al. It is to be noted that the present process causes a preferential ion diffusive flux $\Gamma_{\rm rf}^i \gg \Gamma_{\rm rf}^e$ leaving a possibility of an ambipolar potential build-up, while the neoclassical process has $\Gamma_{nc}^i = \Gamma_{nc}^e$.

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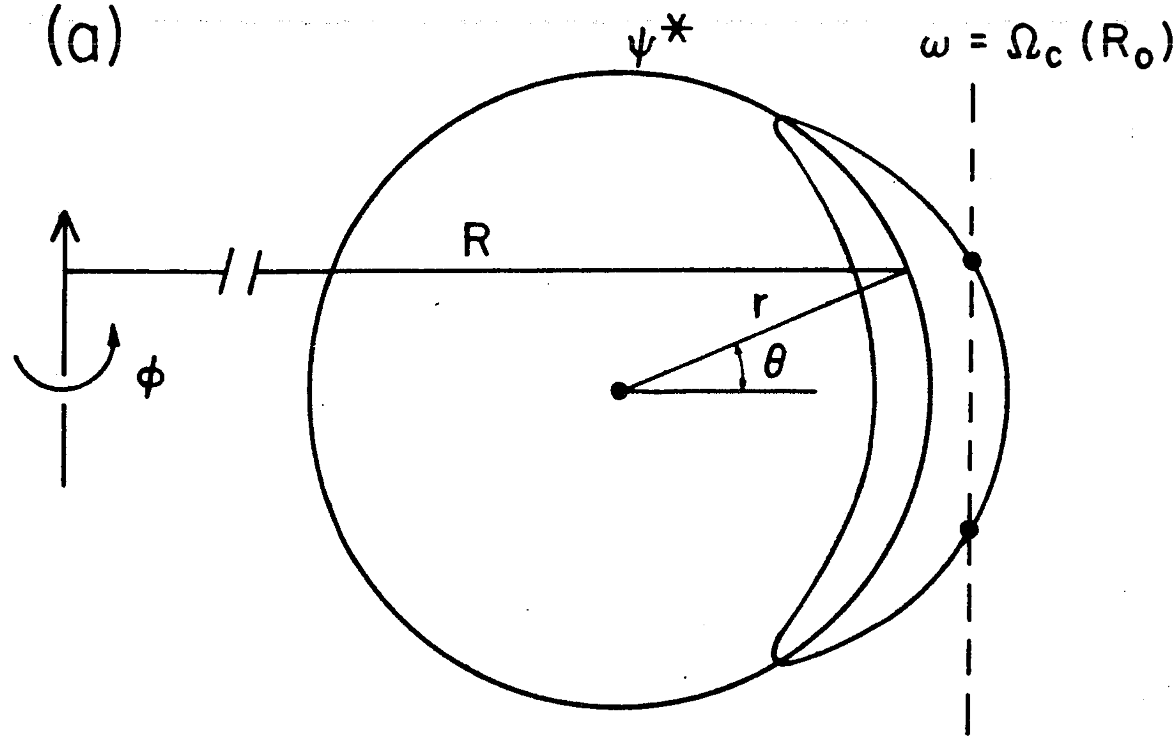
FIGURE CAPTION

1. (a) Guiding center orbit for a trapped particle. (b) Geometry for induced diffusion. The vectors $\hat{e}_\perp$, $\hat{e}_r$ the wave vector $\vec{k}$ and the gyroposition $\vec{\rho}$ lie on the same plane perpendicular to $\vec{B}(\hat{e}_{||})$.



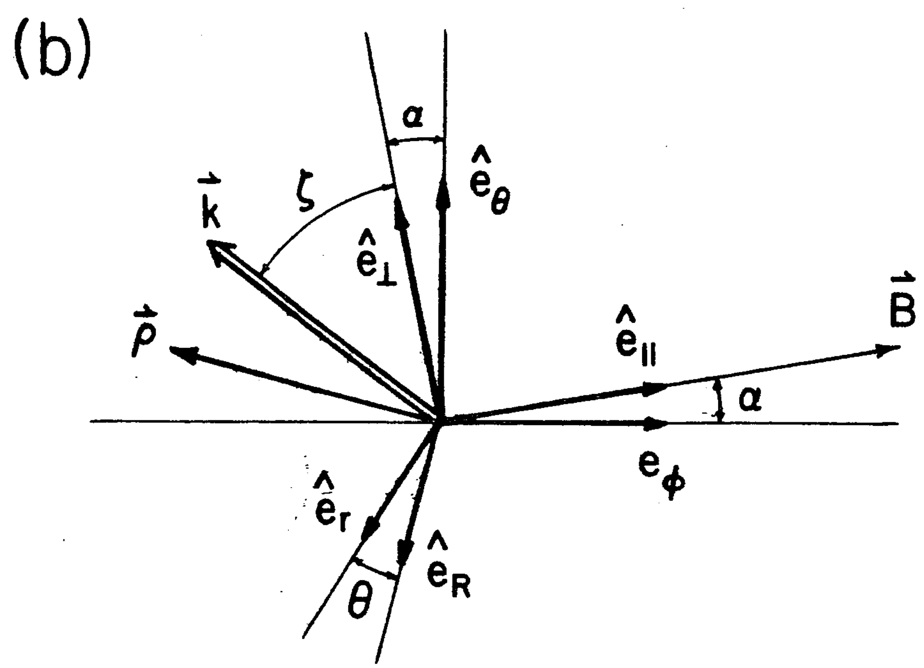


FIGURE 1