Reply to Referee's Comments on

Semicollisional Drift-Tearing Modes in Toroidal Plasmas

(PF-15181)

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We appreciate the referee's valuable comments and have accordingly revised our manuscript, and thereby improved its clarity. The following are specific replies to the comments:

(1) As pointed out by the referee, there have been previous studies on semicollisional drift-tearing modes in cylindrical geometry and the basic flux-surface averaged equations are mathematically identical to the cylindrical case except for modified coefficients (when $H \to 0$). We have cited and discussed both papers (*Finn, et al.*, PF **26**, 962 (1983), *Hahm* and *Chen*, PF **28**, 2432 (1985)) in the appropriate places.

The following is the relationship between the present manuscript and the previous publications:

(a) *Finn et al.*, (PF **26**, 962 (1983));

The interchange term (the last term in Eq. (16) of Finn et al.) has been ignored in Eq. (A4). Therefore, the *instability threshold* Δ_c' (predicted by Eq. (A13)) which is determined entirely from 'drift wave type mode' contribution as $Q \to -iQ_*$ *does not contain average curvature effects.* Of course, the interchange term has been included in the 'localized mode' contribution, but the effect of the interchange term on the 'localized mode solution' (Eq. (A10)) is insignificant, and does not affect the instability threshold. Written in our notation, Eq. (A13) gives $r_0\Delta_c' \sim \beta\left(\frac{L_s}{L_n}\right)^{1/2}\frac{r_0}{\rho_s}$ which is the same as Bussac et al.'s (PRL **40**, 1500 (1978)) result. *In this sense, the physical content of Finn et al.'s Eq. (A13) is similar to that of our limiting case (Eq. (64)).*

(b) *Hahm* and *Chen* (PF **28**, 2432 (1985));

In the derivation, ion sound effects have been ignored since that paper's main issue is the semicollisional drift-interchange mode. However, for drift-tearing mode, ion sound effects are important as shown by Bussac et al., and later by Finn et al. The answer (Eq. (24) of PF **28**, 2432 (1985)) would predict $\Delta_c \to 0$ as $\omega \to \omega_{*e}$ even for finite $D < 0$ (i.e., underestimation of the average curvature effects by ignoring the ion sound term).

(2) We have made a number of additional comments about the assumptions and limitations of our model to clarify this point (in particular, descriptions of Eq. (1) and (5)).

(3) The terms 'Gp' and '$-G\phi$' are the ones the referee has mentioned. Please recall our notation, $\phi = \hat{\phi}$, but $p = \frac{i\omega\chi'}{P'}\hat{p} \propto \frac{\omega}{\omega_{*e}}\hat{p}$, where $\hat{p}$ is the physical pressure perturbation. That is, $G(p - \phi) \propto \omega\hat{p} - \omega_{*e}\hat{\phi}$.

Semicollisional Drift-Tearing Modes in Toroidal Plasmas

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Abstract

Semicollisional drift-tearing modes are studied analytically in toroidal plasmas. Corresponding differential equations for the eigenmodes are derived via the ballooning mode representation and flux-surface averaging. A dispersion relation is then obtained using asymptotic matching. It is found that the stabilizing effects of the good average curvature and finite plasma compression lead to a tearing instability threshold, Δ_c, which is independent of the resistivity.

I. Introduction

Tearing modes are important to tokamak plasma confinement since these instabilities may lead to disruptions and enhanced plasma transport. In the pioneering work of Furth, Killeen and Rosenbluth (FKR),[1] it has been shown that tearing instabilities occur when the parameter Δ', which characterizes the available magnetic free energy in the ideal magnetohydrodynamic (MHD) region, is positive. Noting that the work of FKR is based on resistive MHD description, there have been various follow-up studies of this mode including the additional effects which become important in the present-day and future high temperature tokamaks.

One such extension is the semicollisional drift-tearing mode.[2] When the tearing mode growth rate calculated from resistive MHD is smaller than the electron diamagnetic frequency, the mode becomes a drift tearing mode[3] with real frequency near $\omega_{*e} = T_e k_y / eBL_n$ where $L_n = -\left[\frac{d}{dr}\ell n\, n_0\right]^{-1}$ is the density scale length and k_y is the poloidal wavenumber. Further, when the resistivity becomes so small that the effective ion Larmor radius at the electron temperature ($\rho_s = \sqrt{T_e/M_i}M_i/eB$) is greater than the resistive layer width, the mode becomes the so-called semicollisional tearing mode.[2] Previous studies on semicollisional drift-tearing modes, however, have been mostly limited to the sheared slab geometry.[2,4]

On the other hand, it has been shown, using resistive MHD equations, that the good average curvature in a toroidal plasma can convert the purely growing tearing instability into an overstable mode and often stabilize it.[5] In this case, the usual tearing instability criterion, $\Delta' > 0$, is modified to $\Delta' > \Delta_c > 0$ where $\Delta_c \propto \eta_{\|}^{-1/3}|D_R|^{5/6}$, $\eta_{\|}$ is the parallel resistivity, and D_R, which is defined in the main text, characterizes the average curvature. For the typical tokamak parameters,[6] we have $r_0\Delta_c \sim 10$ for m=2, n=1 tearing mode and this is quite a large value. Thus, it indicates that in the resistive MHD limit, the average good curvature has a significant stabilizing influence on the stability of the tearing mode.[5]

Since, as noted earlier, the semicollisional rather than the usual resistive MHD description is more relevant for present-day tokamaks, it is, therefore, interesting to examine the average curvature effects on the semicollisional drift-tearing modes. Although semicollisional drift-tearing modes have been considered in cylindrical geometry in a number of papers,[7,8] their results do not give conclusive insight into the issue of average curvature effects. In Ref. 7, stabilizing effects due to the ion sound wave, which have been found earlier in sheared slab calculation,[4] have been noted. However, the stabilizing role of good average curvature has not been considered. On the other hand, in Ref. 8, a unified dispersion relation for semicollisional drift-tearing and drift-interchange modes has been derived in the absence of ion sound effects. As pointed out in that paper, neglect of the ion sound term has led to an underestimation of average curvature effects. This constitutes the main motivation of the present work.

The principal objectives of this paper are, thus, to derive the basic eigenmode equations which describe the semicollisional drift-tearing and drift-interchange modes in an axisymmetric toroidal geometry and, subsequently, the dispersion relation for the semicollisional drift-tearing modes including both the average curvature and plasma compressional effects.

Analysis of the dispersion relation shows that semicollisional drift-tearing modes can be stabilized by plasma compression and good average curvature. Specifically, it is found that when $L_n/L_s > \beta/|D_R|$ ($\beta \equiv$ ratio of plasma pressure to magnetic pressure, L_s is shear scale length), the good average curvature effects are important and the stabilizing mechanism is similar to the resistive MHD case,[5] with the instability threshold,

$$r_0 \Delta_c \sim (-D_R \beta)^{1/2} \frac{r_0}{\rho_s},$$

where r_0 is the radial location of rational surface. Meanwhile, in the opposite limit, i.e., $L_n/L_s < \beta/|D_R|$, the plasma compression (including ion sound waves) plays the dominant stabilizing role as noted in the previous sheared slab calculation.[4] In this case, $r_0 \Delta_c \sim \beta \left(\frac{L_s}{L_n}\right)^{1/2} (1+2q^2)^{1/4} \frac{r_0}{\rho_s}$, where q is the safety factor. It is interesting to note that in both cases, Δ_c is independent of the resistivity; contrary to the resistive MHD prediction where $\Delta_c \propto \eta_{\|}^{-1/3}$. Semicollisional theory thus predicts values of Δ_c typically smaller than those predicted by the previous resistive MHD analysis.[5] Meanwhile, the perpendicular resistivity, which has been shown to give a substantial stabilizing effect on the pressure-driven drift-interchange mode,[8] is found to have an insignificant effect on the stability of the Δ'-driven semicollisional drift-tearing modes. We also note that, for the sake of simplicity, other potentially important effects; such as temperature gradients[9] and trapped particles[10] have been ignored in this work. In the presence of electron temperature gradients, finite thermal conductivity[11,12] as well as ion polarization drift[12] have been found to give stabilizing effects on semicollisional drift-tearing modes.

In Sec. II, we describe our theoretical model and present the reduced semicollisional equations. In Sec. III, the surface averaged eigenmode equations are derived from the reduced semicollisional equations in the ballooning representation.[13,14,15] In Sec. IV, the dispersion relation for the semicollisional drift-tearing mode is then derived via asymptotic analysis. In Sec. V, the corresponding stability properties are then examined in various limiting cases. Conclusions and discussions are given in Sec. VI.

II. Reduced Semi-Collisional Equations

In deriving the reduced equations, we assume cold ions ($T_i \ll T_e$) and hence, only T_e appears in the equations. However, $k_\perp \rho_s$ is kept finite so that we can consider the semi-collisional regime. As long as $\nu_{ei} \gg k_\| v_{\|e}, \omega$, fluid descriptions (for example, those of Braginskii)[16] are valid. We will present our equations in notations such that comparison with the conventional reduced resistive MHD equations[17] is straightforward.

The pressure evolution (or, equivalently, the continuity equation multiplied by T_e, because we are neglecting temperature fluctuations and equilibrium gradient) is given by

$$\frac{\partial}{\partial t} p + \vec{v}_E \cdot \nabla p + p \left(\vec{B} \cdot \nabla \frac{v_\|}{B} + \nabla \cdot \vec{v}_\perp \right) = 0, \tag{1}$$

where $\vec{v}$ is the bulk fluid velocity carried mainly by ions and $\vec{v}_E = \dfrac{\vec{E} \times \vec{B}}{B^2}$. The first and second terms in Eq. (1) represent, respectively, fluid convection due to explicit time dependence and $\vec{E} \times \vec{B}$ motion. The third term gives the coupling to ion sound waves. The last term represents perpendicular compression which includes ion polarization drift (i.e., semicollisional effects), perpendicular plasma transport as well as contributions due to the inhomogeneous magnetic field, i.e., ∇B and curvature drifts. We note that some authors[17,18] ignore perpendicular compression entirely and this simplification can not be justified even in the resistive MHD case, since perpendicular compression is larger than the parallel compression by a factor, $2q^2$. The vorticity equation is given by

$$\frac{\rho}{B^2} \frac{d}{dt} \nabla_\perp^2 \phi = \vec{B} \cdot \nabla \frac{j_\|}{B} + 2 \frac{\hat{b} \times \vec{K}}{B} \cdot \nabla p, \tag{2}$$

where ρ and ϕ are, respectively, mass density and electrostatic potential, $\hat{b} = \dfrac{\vec{B}}{B}$, $\vec{K} = (\hat{b} \cdot \nabla) \hat{b}$. Note that Eq. (2) can be derived from the equation of motion

$$\rho \frac{d}{dt} \vec{v} = -\nabla p + \vec{j} \times \vec{B}, \tag{3}$$

which gives

$$\vec{j}_\perp = \frac{\vec{B} \times \nabla p}{B^2} + \frac{\hat{b}}{B} \times \rho \frac{d}{dt} \vec{v} \approx \frac{\vec{B} \times \nabla p}{B^2} - \frac{\rho}{B^2} \frac{d}{dt} \nabla_\perp \phi. \tag{4}$$

In Eq. (4), the first term in the right hand side (RHS) is the diamagnetic drift and the second term is the polarization current. Therefore, the quasi-neutrality condition

$$\nabla \cdot \vec{j} = 0$$

becomes

$$\vec{B} \cdot \nabla \frac{j_{\parallel}}{B} + \nabla_{\perp} \cdot \left(\frac{\vec{B} \times \nabla p}{B^2} - \frac{\rho}{B^2} \frac{d}{dt} \nabla \phi \right) \approx \vec{B} \cdot \nabla \frac{j_{\parallel}}{B} - \frac{2\hat{b} \times \nabla p}{B^2} \cdot \nabla B - \frac{\rho}{B^2} \frac{d}{dt} \nabla_{\perp}^2 \phi = 0,$$

which can be further reduced to Eq. (2) using the force balance in the perpendicular direction; i.e.,

$$\nabla_{\perp} \left(p + \frac{B^2}{2} \right) \approx B^2 \vec{K},$$

where compressional Alfvén wave is assumed to be ignorable (i.e., $\omega \ll k_{\perp} v_A$).

The left hand side (LHS) of Eq. (2) is the inertial term. The first term in RHS represents the stabilizing influence of field line bending, while the second term in RHS is the pressure gradient multiplied by curvature (i.e., the ballooning driving term).

Neglecting electron inertia in the electron equation of motion, we have the generalized Ohm's law for an isothermal plasma given by

$$\eta \vec{j} = \frac{\partial}{\partial t} \vec{A} - \nabla \phi + \frac{1}{n_0 e} \nabla p + \vec{v}_e \times \vec{B}, \tag{5}$$

or, in terms of the fluid variables, using Eq. (3),

$$\eta \vec{j} = \frac{\partial}{\partial t} \vec{A} - \nabla \phi + \vec{v} \times \vec{B} - \frac{M_i}{e} \frac{d}{dt} \vec{v}. \tag{6}$$

Here, $\vec{A}$ is the vector potential. The parallel component of Eq. (5) yields

$$\eta_{\parallel} j_{\parallel} = \frac{\partial}{\partial t} A_{\parallel} - \hat{b} \cdot \nabla \phi + \frac{1}{n_0 e} \hat{b} \cdot \nabla p. \tag{7}$$

The last term on the RHS of Eq. (7) (which is neglected in the usual resistive MHD analysis) gives diamagnetic drift effects, polarization drift as well as the perpendicular plasma transport via Eq. (1). From Eq. (6), the perpendicular velocity is given as

$$\vec{v}_{\perp} = \frac{\hat{b} \times \nabla \phi}{B} - \frac{1}{B \Omega_i} \frac{d}{dt} \nabla_{\perp} \phi - \frac{\eta_{\perp}}{B^2} \nabla_{\perp} p. \tag{8}$$

Here, because $\beta \ll 1$, we have neglected $A_\perp \propto \delta B_\parallel$ and, hence, compressional Alfvén waves. The RHS of Eq. (8) contains $\vec{E} \times \vec{B}$ drift, polarization drift and the perpendicular plasma transport. The parallel velocity, meanwhile, can be obtained from Eq. (3); i.e.,

$$\rho \frac{d}{dt} v_\parallel = -\hat{b} \cdot \nabla p \tag{9}$$

Equation (9) contains coupling to the ion sound wave via the third term in Eq. (1). Finally, we need parallel Ampere's law

$$-\nabla_\perp^2 A_\parallel = j_\parallel \tag{10}$$

to close the system of equations.

In summary, Eqs. (1),(2),(7),(8),(9), and (10) constitute the nonlinear reduced semicollisional equations for drift-tearing and resistive-ballooning modes in terms of the six variables $A_\parallel, \phi, p, v_\parallel, \vec{v}_\perp$ and $j_\parallel$.

In toroidal geometry, the poloidal direction is no longer a symmetry direction and coupling among different poloidal harmonics enters via the curvature terms. To facilitate the calculation, we adopt the modified Hamada coordinate system.[19] In this coordinate system, the equilibrium magnetic field is given by

$$\vec{B} = \nabla V \times (\psi' \nabla \theta - \chi' \nabla \varsigma) = \chi' \nabla (\varsigma - q\theta) \times \nabla V = \chi' \nabla \beta \times \nabla V, \tag{11}$$

where V, the volume contained within a toroidal magnetic surface, labels the surface; θ and ς are angle-like coordinates which increase by unity after one turn around the torus the short way and the long way, respectively. $\psi(V)$ and $\chi(V)$ are, respectively, the toroidal and poloidal magnetic fluxes, and primes denote differentiation with respect to V. $\beta \equiv \varsigma - q\theta$ is the angle-like variable which should not be confused with plasma β and labels the field lines on a particular flux surface. β will be used in place of the toroidal angle ς. This system V, θ, β has a unit Jacobian, $\nabla V \cdot \nabla \theta \times \nabla \beta = 1$, so that $\vec{B} \cdot \nabla \theta = \chi'(V), \vec{B} \cdot \nabla V = 0$ and $\vec{B} \cdot \nabla \beta = 0$. In these modified Hamada coordinates, the derivative along a field line which appears frequently in resistive MHD stability analysis, takes the following particularly simple form

$$\vec{B} \cdot \nabla f = \chi' \partial_\theta f|_{V,\beta}. \tag{12}$$

In addition, axisymmetry implies that $\left.\frac{\partial}{\partial\beta}\right|_{V,\theta} = \left.\frac{\partial}{\partial\varsigma}\right|_{V,\theta} = 0$, so β is an ignorable coordinate like ς.

Having introduced the coordinate system, we proceed to linearize the system of equations. The pressure evolution equation, Eq. (1), becomes

$$\frac{\partial}{\partial t}\tilde{p} + \vec{v}_E \cdot \nabla P + P\left(\vec{B}\cdot\nabla\frac{v_{\|}}{B} + \nabla\cdot\vec{v}_{\perp}\right) = 0, \tag{13}$$

where P and $\tilde{p}$ are, respectively, equilibrium and perturbed pressure. $\vec{B}$ denotes equilibrium magnetic field and all velocities are perturbed quantities. The vorticity equation, Eq. (2), becomes

$$\frac{\rho}{B^2}\frac{\partial}{\partial t}\nabla_{\perp}^2\phi = \vec{B}\cdot\nabla\frac{j_{\|}}{B} + 2\frac{\hat{b}\times\vec{K}}{B}\cdot\nabla\tilde{p}, \tag{14}$$

where ϕ and $j_{\|}$ are perturbed quantities. The parallel component of Ohm's law, Eq. (7), becomes

$$\eta_{\|}j_{\|} = \frac{\partial}{\partial t}A_{\|} - \hat{b}\cdot\nabla\phi + \frac{1}{n_0 e}(\hat{b}\cdot\nabla\tilde{p} + \hat{b}_1\cdot\nabla P), \tag{15}$$

where $\hat{b}_1$ denotes the direction of perturbed magnetic field and $\hat{b}$, along the equilibrium magnetic field. Finally, from Eqs. (8) and (9), the perpendicular and parallel velocities are given by

$$\vec{v}_{\perp} = \frac{\hat{b}\times\nabla\phi}{B} - \frac{1}{B\Omega_i}\frac{\partial}{\partial t}\nabla_{\perp}\phi - \frac{\eta_{\perp}}{B^2}\nabla_{\perp}\tilde{p}, \tag{16}$$

and

$$\rho\frac{\partial}{\partial t}v_{\|} = -\hat{b}\cdot\nabla\tilde{p} - \hat{b}_1\cdot\nabla P. \tag{17}$$

We now eliminate $v_{\|}, \vec{v}_{\perp}$ and $j_{\|}$ to obtain equations in terms of $\phi, \tilde{p}$ and $A_{\|}$ only. The pressure evolution equation, Eq. (13), then becomes

$$\begin{aligned}
&\vec{B}\cdot\nabla\frac{i}{\rho\omega B^2}\left(\vec{B}\cdot\nabla\tilde{p} - \frac{P'}{\chi'}\frac{\partial}{\partial\beta}A_{\|}B\right) + \frac{\eta_{\perp}}{B^2}\nabla_{\perp}^2\tilde{p} \\
&\quad + 2\frac{\vec{K}\times\vec{B}}{B^2}\cdot\nabla_{\perp}\phi - \frac{i\omega}{\Omega_i B}\nabla_{\perp}^2\phi - \left(\frac{P+B^2}{PB^2}\right)\left(-i\omega\tilde{p} + \frac{P'}{\chi'}\frac{\partial}{\partial\beta}\phi\right) = 0
\end{aligned} \tag{18}$$

where $\frac{\partial}{\partial t} = -i\omega$. The first term comes from the parallel compression and describes the coupling to ion sound wave. The next three terms originate from the perpendicular compression and correspond to the perpendicular plasma transport, curvature and ∇B drifts and the polarization drift, respectively. The last term is the convection term. Meanwhile, the vorticity equation, Eq. (14), becomes

$$-\frac{\rho i\omega}{B^2}\nabla_\perp^2\phi + \vec{B}\cdot\nabla\frac{\nabla_\perp^2 A_\| B}{B^2} + \frac{2\vec{K}\times\vec{B}}{B^2}\cdot\nabla\tilde{p} = 0, \tag{19}$$

and the parallel Ohm's law, Eq. (15), becomes

$$\eta_\| \nabla_\perp^2 A_\| B = \vec{B}\cdot\nabla\left(\phi - \frac{\tilde{p}}{n_0 e}\right) - i(\omega-\omega_*)A_\| B, \tag{20}$$

where $\omega_* \equiv \omega_{*e}$ and $\omega_* A_\| B$ originates from the $\hat{b}_1\cdot\nabla P$ term in Eq. (15). Equations (18), (19), and (20) are the final three linearized equations in terms of the three variables, $\phi, \hat{p}$ and $A_\| B$ only.

III. Flux-Surface Averaged Eigenmode Equations

Equations (18), (19) and (20) can be further reduced to ordinary differential equations by adopting the ballooning mode representation. That is, we let

$$\phi = \sum_{\ell=-\infty}^{\infty} \hat{\varphi}(\theta-\ell)e^{iS} \quad ,$$

$$\tilde{p} = \frac{\partial}{\partial\beta}\sum_{\ell=-\infty}^{\infty} \hat{p}(\theta-\ell)e^{iS} \quad ,$$

and

$$A_\| B = \sum_{\ell=-\infty}^{\infty} \hat{\psi}(\theta-\ell)e^{iS} \quad ;$$

where $S = -2\pi n(\beta + \ell q + \int kq' dv)$, and k is to be determined by a higher order radial non-local analysis which we are not considering here. In the ballooning-mode representation, the curvature terms vary along the magnetic field line (θ coordinate) with scaling length ~ 1 while the eigenmodes tend to extend along the field line to minimize the stabilizing influence of field line bending. Thus, there exist two distinct scale lengths in this problem and we can perform flux-surface averaging over the short connection length scale (~ 1) to derive the eigenmode equations in the long resistive scaling length ($\sim$ mode width$\sim \eta_{\|}^{-1/3}$, typically). Since the procedures are straightforward, we shall omit the detailed algebra[20] and simply present the resultant flux-surface averaged eigenmode equations. We adopt, wherever possible, notations which are identical to those used by Glasser, Greene and Johnson[5] (hereafter referred to as GGJ). Flux-surface averaged quantities are, thus, defined as

$$E = \frac{\langle B^2/|\nabla V|^2\rangle}{q'^2\chi'^2}\frac{P'}{\chi'}\left(\frac{\chi''}{\chi'^2}+\frac{2\pi RB_T q'}{\langle B^2\rangle}\right),$$

$$F = \frac{\langle B^2/|\nabla V|^2\rangle}{q'^2\chi'^2}\left(\frac{P'}{\chi'}\right)^2\left\{\left\langle\frac{1}{B^2}\right\rangle+\left(\frac{2\pi RB_T}{\chi'}\right)^2\left(\left\langle\frac{1}{B^2|\nabla V|^2}\right\rangle-\frac{\langle 1/|\nabla V|^2\rangle^2}{\langle B^2/|\nabla V|^2\rangle}\right)\right\},$$

$$H = \frac{\langle B^2/|\nabla V|^2\rangle}{q'^2\chi'^2}\frac{P'}{\chi'}\left\{2\pi RB_T q'\left(\frac{\langle 1/|\nabla V|^2\rangle}{\langle B^2/|\nabla V|^2\rangle}-\frac{1}{\langle B^2\rangle}\right)\right\},$$

$$M = \langle B^2/|\nabla V|^2\rangle\left\{\left\langle\frac{|\nabla V|^2}{B^2}\right\rangle+\left(\frac{2\pi RB_T}{\chi'}\right)^2\left(\left\langle\frac{1}{B^2}\right\rangle-\frac{1}{\langle B^2\rangle}\right)\right\},$$

$$M_\perp = \frac{\eta_\perp}{\eta_\|}+\left\langle\frac{B^2}{|\nabla V|^2}\right\rangle\left(\frac{2\pi RB_T}{\chi'}\right)^2\left(\left\langle\frac{1}{B^2}\right\rangle-\frac{1}{\langle B^2\rangle}\right),$$

$$K = \frac{q'^2\chi'^4}{MP'^2}\frac{\langle B^2\rangle}{\langle B^2/|\nabla V|^2\rangle},$$

$$G = \langle B^2\rangle/MP;$$

where E comes entirely from normal curvature, F contains both normal and geodesic curvature contributions, $M \approx 1+2q^2$ is the Pfirsch-Schlüter factor, and $\langle\ldots\rangle \equiv \oint d\theta(\ldots)$ denotes flux-surface averaging. Variables, meanwhile, are nondimensionalized as

$$x = \frac{\theta}{\theta_R},\qquad Q = \frac{-i\omega}{\gamma_R},\qquad Q_* = \frac{\omega_*}{\gamma_R},$$

$$\phi = \hat{\phi},\qquad ,\psi = \frac{-i\omega\theta_R}{\chi'}\left\langle\hat{\psi}\right\rangle,\qquad p = \frac{i\omega\chi'}{P'}\hat{p};$$

where

$$\theta_R = \frac{1}{2\pi}\left(\frac{n^2\tau_A}{\tau_R}\right)^{-1/3},\qquad \gamma_R = (2\pi\theta_R\tau_A)^{-1}$$

$$\tau_A = \left(\frac{\rho M}{4\pi^2\chi'^2}\right)^{1/2},\qquad \text{and}\qquad \tau_R = \frac{\langle B^2/|\nabla V|^2\rangle}{\eta_\| q'^2\langle B^2\rangle}.$$

The pressure evolution equation is then given by

$$(M_\perp/M)Qx^2p - \frac{\partial}{\partial x}(\psi + \frac{\partial}{\partial x}p) + Q^2\left(Hx\psi + (G+KF)p - (G-KE)\phi + iKQQ_*x^2\phi\right) = 0. \tag{21}$$

In Eq. (21), the first and second terms represent, respectively, the perpendicular transport and parallel compression, while Gp-$G\phi$ represents the convection of pressure. $Hx\psi$ originates from toroidal coupling of the Pfirsch-Schlüter current and the last term comes from the polarization current.

The parallel Ohm's law is given by

$$\frac{\partial}{\partial x}\left(\phi - \frac{\omega_*}{\omega}p\right) + \left(1 - \frac{\omega_*}{\omega}\right)\psi + \frac{1}{Q}(x^2\psi - Hxp) = 0. \tag{22}$$

In Eq. (22), the first two terms are electrostatic and electromagnetic parts of the parallel electric field. The terms with ω_* originate from the pressure gradient. The last term is the parallel current. In particular, HxP is the averaged Pfirsch-Schlüter current.

Finally, the vorticity equation (or the quasi-neutrality condition) becomes

$$Q^2x^2\phi - Q\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\phi - \frac{\omega_*}{\omega}p\right) + \left(1 - \frac{\omega_*}{\omega}\right)\psi\right) - (E+F)p - Hx\psi = 0. \tag{23}$$

In Eq. (23), the first term is the inertia term, the second term describes the field line bending and the third term is the interchange driving term, while the last term comes from toroidal coupling. We note that Eqs. (21) to (23) recover GGJ's results in the limit $Q_* \to 0$ (the polarization drift term also vanishes in this limit for fixed K). We also note that our equations do not recover those of Pogutse and Yurchenko[21] or Carreras and Diamond et al.[22] because we have assumed $\omega < C_s/qR$ in the flux-surface averaging; that is, equations in Refs. 21 and 22 correspond to the $\omega > C_s/qR$ limit.

Let us sidetrack for a while and discuss the effects of the purely toroidal term, H. In GGJ, it is found that H modifies the usual tearingmode growth rate scaling with respect to resistivity from $\eta^{3/5}$ to $\eta^{3-2H/5+2H}$; meanwhile, the interchange mode growth rate remains proportional to $\eta^{1/3}$. On the other hand, the ideal interchange driving term is given by $D_1 \equiv E+F+H-1/4$, while the resistive interchange driving term is given by $D_R \equiv E+F+H^2$. Since H is usually very small (order of E,F) for tokamaks with large aspect ratio $(1/\epsilon)$ and $\beta_p = 2P/B_p^2 \sim 0(1)$ so that ideal ballooning modes are stable, the resistive mode driving energy and the growth rate scaling is hardly affected by H. As we shall show later, H is also insignificant in the semicollisional regime.

Eliminating ψ from Eqs. (21), (22) and (23), and after some arrangements, we obtain

$$\frac{\partial}{\partial x}\frac{1}{1-\omega_*/\omega+x^2/Q}\left(\frac{\partial}{\partial x}\phi-\left(1+\frac{x^2}{Q}\right)\frac{\partial}{\partial x}p\right)+\frac{M_\perp}{M}Qx^2p+GQ^2(p-\phi)$$
$$+KQ^2(E\phi+Fp)+iKQ_*Q^3x^2\phi-\frac{H}{Q}\frac{\partial}{\partial x}\frac{xp}{1-\omega_*/\omega+x^2/Q}$$
$$+KH^2Q\frac{x^2}{1-\omega_*/\omega+x^2/Q}p-\frac{KHQ^2x}{1-\omega_*/\omega+x^2/Q}\frac{\partial}{\partial x}\left(\phi-(\omega_*/\omega)p\right)=0, \quad (24)$$

and

$$\frac{\partial}{\partial x}\frac{x^2}{1-\omega_*/\omega+x^2/Q}\frac{\partial}{\partial x}\left(\phi-\frac{\omega_*}{\omega}p\right)+\frac{Hx}{1-\omega_*/\omega+x^2/Q}\frac{\partial}{\partial x}(p-\phi)-Q^2x^2\phi$$
$$+\left\{D_R-\left(1-\frac{\omega_*}{\omega}\right)\frac{H(H-1)}{1-\omega_*/\omega+x^2/Q}+\left(1-\frac{\omega_*}{\omega}\right)Hx\frac{\partial}{\partial x}\left(\frac{1}{1-\omega_*/\omega+x^2/Q}\right)\right\}p=0.$$
$$(25)$$

For the sake of convenience, we shall investigate the effects of H in three separate regions. Note that since we are interested in low-β plasmas such that ideal ballooning or interchange modes are stable, we shall further order $\beta \sim \epsilon^2$ such that $E \sim F \sim H \sim \epsilon^2$.[6] We also note that the growth rate of the semicollisional tearing mode given by Drake and Lee[2] scales as $b_\theta^{1/3}\Delta'^{2/3}\left(\frac{n^2\tau_A}{\tau_R}\right)^{1/3}$, where $b_\theta = k_\theta^2\rho_s^2$, which shows that $Q \ll 1$ due to the smallness of b_θ unless Δ' is extremely large. The ideal MHD region corresponds to $x^2 \ll Q$, and in this limit, Eqs. (24) and (25) reduce to

$$\frac{\partial^2}{\partial x^2}(\phi-p)=0, \quad (26)$$

and

$$\frac{\partial}{\partial x}x^2\frac{\partial}{\partial x}\phi+(E+F+H)p=0. \quad (27)$$

Strictly speaking, Eqs. (26) and (27) do not describe the ideal region properly (flux-surface averaging has smeared out the possible ballooning effect in the ideal MHD region). However, in this paper, since we are mainly dealing with flute-like resistive modes in the ideal MHD stable low-β plasmas, Eqs. (26) and (27) are sufficient for our objectives. In Eq. (27), since $E+F+H \ll 1$, we can see that the first term dominates the ideal interchange driving term; which will, hence, be ignored in the present analyses ($E+F+H$ enters into the exponents of the Mercier solutions, not into Δ'). In the intermediate region, $x^2 \sim Q$, the resistive effects appear and we have

$$\frac{\partial}{\partial x}\frac{1}{1-\omega_*/\omega+x^2/Q}\left(\frac{\partial}{\partial x}\phi-\left(1+\frac{x^2}{Q}\right)\frac{\partial}{\partial x}p\right)=0, \tag{28}$$

and

$$\frac{\partial}{\partial x}\frac{x^2}{1-\omega_*/\omega+x^2/Q}\frac{\partial}{\partial x}\left(\phi-\frac{\omega_*}{\omega}p\right)=0. \tag{29}$$

In Eqs. (28) and (29), H does not appear in the equations due to its smallness with respect to unity. In the strongly resistive region, i.e., $x^2 \gg Q$, the inertial and resistive driving terms as well as the perpendicular transport term become important and the equations become

$$Q\frac{\partial^2}{\partial x^2}\left(\phi-\frac{\omega_*}{\omega}p\right)+D_R p-Q^2x^2\phi=0, \tag{30}$$

and

$$-\frac{\partial^2}{\partial x^2}p+\frac{M_\perp}{M}Qx^2p+GQ^2(p-\phi)+iKQ_*Q^2x^2\phi+KQ^2(E\phi+Fp)=0. \tag{31}$$

Again, terms containing H disappear because $1 \ll |1-\omega_*/\omega+x^2/Q|$. The above discussion thus indicates that H is generally negligible as far as low-$\beta(\sim \epsilon^2)$, near circular plasmas are concerned. For highly noncircular and high-$\beta(\sim \epsilon)$ plasmas, however, H may be large and introduce a major difference. Neglecting H, Eqs. (24) and (25) then reduce to

$$\frac{\partial}{\partial x}\frac{1}{1-\omega_*/\omega+x^2/Q}\left(\frac{\partial}{\partial x}\phi-\left(1+\frac{x^2}{Q}\right)\frac{\partial}{\partial x}p\right)+Qx^2p+GQ^2(p-\phi)$$
$$+KQ^2(E\phi+Fp)+iKQ_*Q^3x^2\phi=0, \tag{32}$$

and

$$\frac{f\partial}{\partial x}\frac{x^2}{1-\omega_*/\omega+x^2/Q}\frac{\partial}{\partial x}\left(\phi-\frac{\omega_*}{\omega}p\right)-Q^2x^2\phi+D_Rp=0. \tag{33}$$

Equations (32) and (33) are the basic eigenmode equations that we shall analyze in the next section.

IV. Derivation of the Dispersion Relation

In this section, we shall use asymptotic analysis to derive the dispersion relation for the semicollisional drift-tearing mode.

As stated in Sec. III, it is convenient to consider three different asymptotic regions. In the ideal MHD region where $x^2 \ll Q$, we have, from Eqs. (26) and (27), the large-x behavior given by

$$\phi = p \sim 1 + \frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R}\frac{1}{x}, \tag{34}$$

where Δ', in general, must be obtained numerically from the ideal MHD equations and $\hat{s} = r_0\frac{d}{dr}\ell nq$ is the shear. Note that the trivial solution of Eqs. (26) and (27) which does not contribute to the dispersion relation has been neglected.

The intermediate region where $x^2 \sim Q$, is described by Eqs. (28) and (29). By integrating Eqs. (28) and (29) directly, we obtain

$$\phi = -\frac{B_0}{x} + B_1 + \frac{\omega_*}{\omega}B_4 + \left(\frac{B_0}{Q} - \frac{\omega_*}{\omega}B_2\right)x, \tag{35}$$

and

$$p = \frac{B_0}{x} + B_4 - B_2x, \tag{36}$$

where the B's are constants of integration to be determined by the asymptotic matching procedure.

Finally, in the strongly resistive region, where $x^2 \gg Q$, we have Eqs. (30) and (31). Further nondimensionalizing variables via

$$y^4 = Qx^4, \quad \hat{D} = D_RQ^{-3/2}, \quad \hat{G} = GQ^{3/2},$$

$$\overline{K} = KQ^3, \quad \hat{K} = KQ_*Q^2, \quad \text{and} \quad \tilde{K} = KQ_*^2Q,$$

Equations (30) and (31) become

$$\frac{\partial^2}{\partial y^2}\left(\phi - \frac{\omega_*}{\omega}p\right) + \hat{D}p - y^2\phi = 0, \tag{37}$$

and

$$\left(\frac{\partial^2}{\partial y^2} - \frac{M_\perp}{M}y^2\right)p + \hat{G}(\phi - p) - i\hat{K}y^2\phi - \overline{K}(\hat{E}\phi + \hat{F}p) = 0. \tag{38}$$

Noting that $\tilde{K} \gg 1$ for semicollisional drift-tearing modes with $\omega \simeq \omega_*$, we, therefore, adopt the subsidiary orderings $\tilde{K}^{1/2} \sim \hat{G} \sim \hat{K}\hat{D} \gg 1$, and proceed to solve Eqs. (37) and (38) using the method of dominant balance.[23] Note also that, within these subsidiary orderings, effects due to ion sound wave and average curvature can be included perturbatively. Equations (37) and (38) are then found to possess two types of solutions.

A) Nearly electromagnetic solution:

Here, $\phi \sim \overline{K}^{-1}p \ll p$ and $y^2 \sim \tilde{K}^{1/2} \gg 1$, Eqs. (37) and (38) can then be approximated as

$$-\frac{\omega_*}{\omega}\frac{\partial^2}{\partial y^2}p - y^2\phi + \hat{D}p = 0, \tag{39}$$

and

$$-\frac{M_\perp}{M}y^2 p - \hat{G}p - i\hat{K}y^2\phi - \overline{K}\hat{F}p = 0. \tag{40}$$

Note that the ion sound term is absent in Eq. (40). Combining Eqs. (39) and (40) we have

$$\tilde{K}\frac{\partial^2}{\partial y^2}p_A - \frac{M_\perp}{M}y^2 p_A - (\hat{G} + \overline{K}\hat{F} + i\hat{K}\hat{D})p_A = 0. \tag{41}$$

The solution of Eq. (41) which decays as $x \to \infty$ is the Parabolic cylinder function,[24] i.e.,

$$p_A = U\left(b\left(\frac{M_\perp}{M}\right)^{-1/2}, \left(4\frac{M_\perp/M}{\tilde{K}}\right)^{1/4} y\right),$$

where

$$b \equiv \frac{\hat{G} + \overline{K}\hat{F} + i\hat{K}\hat{D}}{2\tilde{K}^{1/2}}.$$

Hence, the small-x behavior is given by

$$p_A \approx p_A(0)\left(1 - \frac{2Q^{1/4}(M_\perp/M)^{1/4}}{\tilde{K}^{1/4}}\frac{\Gamma\left(\frac{3}{4} + \frac{b}{2}(M_\perp/M)^{-1/2}\right)}{\Gamma\left(\frac{1}{4} + \frac{b}{2}(M_\perp/M)^{-1/2}\right)}x\right), \tag{42}$$

and

$$\phi_A \approx 0. \tag{43}$$

B) Nearly adiabatic solution:

For this solution, we have $\phi \approx (\omega_*/\omega)p$, $y^2 \sim \tilde{K}^{-1/2} \ll 1$. (Note that the non-adiabatic density response $\phi - \frac{\omega_*}{\omega}p \approx 0$.) However, it turns out that the exactly adiabatic solution does not contribute to the matching procedure so that we have to derive the non-adiabatic corrections perturbatively. In the zeroeth order, we have, from Eqs. (37) and (38),

$$\phi_B^0 = \frac{\omega_*}{\omega} p_B^0, \tag{44}$$

and

$$\frac{\partial^2}{\partial y^2} p_B^0 + \hat{G}(\phi_B^0 - p_B^0) - \hat{K} y^2 \phi_B^0 - \overline{K}(\hat{E}\phi_B^0 + \hat{F} p_B^0) = 0. \tag{45}$$

Combining Eqs. (44) and (45) yields

$$\frac{\partial^2}{\partial y^2} p_B^0 - \tilde{K} y^2 p_B^0 - \left((\hat{G} + \overline{K}\hat{F}) \left(1 - \frac{\omega_*}{\omega}\right) - i\hat{K}\hat{D} \right) p_B^0 = 0. \tag{46}$$

The solution is given by the Parabolic cylinder function,[24]

$$p_B^0 = U\left(a, (4\tilde{K})^{1/4} y\right),$$

where

$$a \equiv \frac{(\hat{G} + \overline{K}\hat{F}) \left(1 - \frac{\omega_*}{\omega}\right) - i\hat{K}\hat{D}}{2\tilde{K}^{1/2}}.$$

In the first order, Eq. (37) gives

$$\frac{\partial^2}{\partial x^2} \left(\phi_B^1 - \frac{\omega_*}{\omega} p_B^1\right) - y^2 \phi_B^0 + \hat{D} p_B^0 = 0 \tag{47}$$

Here, we note that the last term (interchange term) of Eq. (47) represents the crucial effects of average curvature, and has not been included in the previous semicollisional calculations.[4,7] Defining $z = \tilde{K}^{1/4} y$ and using Eq. (44), Eq. (47) can be written as

$$\frac{\partial^2}{\partial z^2} \left(\phi_B^1 - \frac{\omega_*}{\omega} p_B^1\right) = \left(\frac{\omega_*}{\omega} \tilde{K}^{-1} z^2 - \hat{D} \tilde{K}^{-1/2}\right) p_B^0. \tag{48}$$

Integrating Eq. (48) and imposing the proper boundary condition, we have

$$\phi_B^1 - \frac{\omega_*}{\omega} p_B^1 = \frac{\omega_*}{\omega} \tilde{K}^{-1} \int_z^\infty dt (t-z) t^2 p_B^0(t)$$
$$- \hat{D}\hat{K}^{-1/2} \int_z^\infty dt (t-z) p_B^0(t).$$

Hence, the small-y limit is given by

$$\phi_B^1 - \frac{\omega_*}{\omega} p_B^1 \approx p_B^0(0) \left\{ \frac{\omega_*}{\omega} \tilde{K}^{-1} I_3 - \hat{D}\tilde{K}^{-1/2} I_1 + \left(\hat{D}\tilde{K}^{1/2} I_0 - \frac{\omega_*}{\omega} \tilde{K}^{-1} I_2 \right) \tilde{K}^{1/4} y \right\}. \quad (49)$$

In Eq. (49), definite integrals I_n's are defined by

$$I_n \equiv \frac{1}{p^0(0)} \int_0^\infty dt p^0(t) t^n. \quad (50)$$

By using Eq. (46) and integrating by parts, the I_n's can be shown to satisfy the following recursion relations

$$I_2 = \Gamma\left(\frac{3}{2} + \frac{a}{2}\right) \Big/ \Gamma\left(\frac{1}{4} + \frac{a}{2}\right) - 2aI_0,$$

and

$$I_3 = 1 - 2aI_1.$$

From Eq. (46), it is also easy to derive the limiting values of I_0 i.e.,

$$\begin{cases} I_0 \sim \frac{1}{\sqrt{2}} \Gamma(3/4), & a \to 0 \\ I_0 \sim \frac{1}{\sqrt{2a}}, & a \to \infty. \end{cases}$$

The general solutions for p and ϕ in the strongly resistive region can, thus, be written as linear combinations of type A and type B solutions; i.e.,

$$p = a_A p_A + a_B p_B \approx a_A p_A + a_B \left(p_B^0 + p_B^1 \right), \quad (51)$$

$$\phi = a_A \phi_A + a_B \phi_B \approx a_B \left(\frac{\omega_*}{\omega} p_B^0 + \phi_B^1 \right), \quad (52)$$

where the a's are the constants to be determined and we have used Eq. (44). Here, we remark again that while we keep only the zeroth order type A solutions, we need to keep the type B solutions of the first order since the zeroth order solution does not contribute to the dispersion relation.

Now we perform asymptotic matching. First, by comparing the ideal solution, Eq. (34) and intermediate solutions, Eqs. (35) and (36), we obtain

$$-B_0 = \frac{r_0 \Delta'}{2\pi^2 m \hat{s} \theta_R}, \tag{53}$$

and

$$\frac{\omega_*}{\omega} B_4 + B_1 = B_4 = 1. \tag{54}$$

The large-x behavior of Eqs. (35) and (36) can be written as

$$\phi = 1 + \left(\frac{-r_0 \Delta'}{2\pi^2 m \hat{s} \theta_R} - \frac{\omega_*}{\omega} B_2 \right) x, \tag{55}$$

and

$$p = 1 - B_2 x. \tag{56}$$

Second, we match the small-x behavior of Eqs. (51) and (52) with Eqs. (55) and (56). By equating the constant terms, we get

$$1 = a_A p_A(0) + a_B p_B(0) \approx a_A p_A(0) + a_B p_B^0(0), \tag{57}$$

and

$$1 = \phi(0) \approx \frac{\omega_*}{\omega} a_B p_B^0(0). \tag{58}$$

We can eliminate the undetermined coefficient B_2 by matching $\phi - \frac{\omega_*}{\omega} p$. From Eqs. (55) and (56), we have

$$\phi - \frac{\omega_*}{\omega} p = 1 - \frac{\omega_*}{\omega} - \frac{r_0 \Delta'}{2\pi^2 m \hat{s} \theta_R} x. \tag{59}$$

On the other hand, from Eqs. (51) and (52), we have, in the small-x limit,

$$\begin{aligned}
\phi - \frac{\omega_*}{\omega} p &\approx -a_A \frac{\omega_*}{\omega} p_A + a_B \left(\phi_B^1 - \frac{\omega_*}{\omega} p_B^1 \right) \\
&\approx -a_A \frac{\omega_*}{\omega} p_A(0) \left\{ 1 - \frac{2Q^{1/4} (M_\perp / M)^{1/4}}{\tilde{K}^{1/4}} \frac{\Gamma\left(\frac{3}{4} + \frac{b}{2} (M_\perp / M)^{-1/2} \right)}{\Gamma\left(\frac{1}{4} + \frac{b}{2} (M_\perp / M)^{-1/2} \right)} x \right\} \\
&+ a_B \left\{ \left(\phi_B^1(0) - \frac{\omega_*}{\omega} p_B^1(0) \right) + p_B^0(0) \left(\hat{D} \tilde{K}^{-1/2} I_0 - \frac{\omega_*}{\omega} \tilde{K}^{-1} I_2 \right) \tilde{K}^{1/4} Q^{1/4} x \right\}.
\end{aligned} \tag{60}$$

Matching Eqs. (59) and (60) we have

$$\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R} Q\left(1-\frac{\omega_*}{\omega}\right)^{-1}\frac{\omega_*}{\omega} \approx \frac{\omega_*}{\omega}\frac{2Q^{1/4}(M_\perp/M)^{1/4}}{\tilde{K}^{1/4}}\frac{\Gamma\left(\frac{3}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)}{\Gamma\left(\frac{1}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)}$$

$$+\frac{a_B p_B^0(0)}{a_A p_A(0)}\left(\hat{D}\tilde{K}^{-1/2}I_0 - \frac{\omega_*}{\omega}\tilde{K}^{-1}I_2\right)\tilde{K}^{1/4}Q^{1/4}, \tag{61}$$

where $\phi_B^1(0) - \frac{\omega_*}{\omega}P_B^1(0)$ has been neglected compared to $P_A(0)$. Using Eqs. (57) and (58) to eliminate $a_B p_B^0(0)/a_A p_A(0)$ in Eq. (61), we finally have the dispersion relation,

$$\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R} Q^{-5/4} = 2\left(1-\frac{\omega_*}{\omega}\right)\frac{(M_\perp/M)^{1/4}}{\tilde{K}^{1/4}}\frac{\Gamma\left(\frac{3}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)}{\Gamma\left(\frac{1}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)}$$

$$+\left\{\tilde{K}^{-3/4}\left(\frac{\Gamma\left(\frac{3}{4}+\frac{a}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{a}{2}\right)} - 2aI_0\right) - \frac{\omega}{\omega_*}\hat{D}\tilde{K}^{-1/4}I_0\right\}, \tag{62}$$

where I_0 is defined by Eqs. (46) and (50). Equation (62) is thus the desired dispersion relation for drift-tearing modes including the effects of ion sound wave, perpendicular compression, average curvature and perpendicular resistivity. Since Eq. (62), in general, is still too complicated to be illuminating, we shall consider various limiting cases in the next section.

V. Limiting Cases of the Dispersion Relation

We note that the crucial effects of ion sound wave and average curvature are contained mainly in the second term of RHS of Eq. (62), except for the contribution of curvature via b in the first term of RHS. Therefore, we first consider the case with $\tilde{K}^{-1/2}\hat{D} \ll 1$ such that the effects of average curvature are negligible. Equation (62) as well as a and b then reduce to

$$\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R}Q^{-5/4} = 2\left(1-\frac{\omega_*}{\omega}\right)\frac{(M_\perp/M)^{1/4}}{\tilde{K}^{1/4}}\frac{\Gamma\left(\frac{3}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)}{\Gamma\left(\frac{1}{4}+\frac{b}{2}(M_\perp/M)^{-1/2}\right)} + \tilde{K}^{-3/4}\left(\frac{\Gamma\left(\frac{3}{4}+\frac{a}{2}\right)}{\Gamma\left(\frac{1}{4}+\frac{a}{2}\right)} - 2aI_0\right), \tag{63}$$

$b \cong \hat{G}/2\tilde{K}^{1/2}$, and $a \cong b\left(1-\frac{\omega_*}{\omega}\right)$. Noting that the second term in RHS is formally smaller than the first term by $O(\tilde{K}^{-1/2})$, we expect that the second term, i.e., the ion sound effects, will be appreciable only when $\left|1-\frac{\omega_*}{\omega}\right| \ll 1$.

As to the magnitude of b, we know from Eq. (41) that $|b| \gg (M_\perp/M)^{1/2}$ corresponds to small perpendicular resistivity limit, and $|b| \ll (M_\perp/M)^{1/2}$ corresponds to an anomalously large perpendicular resistivity limit (i.e., $(\eta_\perp/\eta_\parallel + 2q^2)/(1+2q^2) \gg L_s/L_n$). We then have the following simplified dispersion relations corresponding to the various limits.

(i) $|a| \ll 1$ and $|b| \gg (M_\perp/M)^{1/2}$. The effects of perpendicular resistivity are negligible and we have

$$\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R}Q^{-5/4} = \left(1-\frac{\omega_*}{\omega}\right)\left(\frac{G}{K}\right)^{1/2}\frac{Q^{1/4}}{Q_*} + \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}\frac{Q^{-3/4}}{K^{3/4}Q_*^{3/2}}. \tag{64}$$

The first term in the RHS is Drake and Lee's result[2] and the second term represents the ion-sound stabilizing effects previously obtained by Bussac et al.[4] We also note that Eq. (64) is similar to Finn et al.'s[7] result (Eq. (A13)). For this case, the tearing mode instability condition is then given by

$$\Delta' > \Delta_c > 0,$$

where, assuming $\omega \approx \omega_*$,

$$r_0\Delta_c = r_0\Delta_{cs} \equiv \frac{\sqrt{2}\pi^2\Gamma(3/4)m\hat{s}\theta_R}{\Gamma(1/4)}K^{-3/4}Q_*^{-1},$$

or in terms of physical parameters,

$$r_0\Delta_{cs} \cong 0.37\beta\frac{r_0}{\rho_s}\left(\frac{L_s}{L_n}\right)^{1/2}(1+2q^2)^{1/4}, \tag{65}$$

where the last factor represents the enhancement of effective inertia in a toroidal geometry, and modifies the sheared slab[4] or cylinder[7] results. It is interesting to note that Δ_{cs} is independent of the resistivity. Therefore, as $\eta_\parallel \to 0$, the stability criterion becomes independent of $\eta_\parallel$, while the growth rates for unstable modes are reduced.

(ii) $|a| \ll 1, |b| \ll (M_\perp/M)^{1/2}$. The effects of perpendicular resistivity are large and we have

$$\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R}Q^{-5/4} = 2\left(1-\frac{\omega_*}{\omega}\right)\frac{(M_\perp/M)^{1/4}}{\tilde{K}^{1/4}}\frac{\Gamma(3/4)}{\Gamma(1/4)} + \frac{\Gamma(3/4)}{\Gamma(1/4)}\frac{Q^{-3/4}}{K^{3/4}Q_*^{3/2}}. \tag{66}$$

In Eq. (66), although the first term in RHS is now somewhat different from Drake and Lee's result,[2] which has been derived neglecting the perpendicular resistivity, ω remains close to ω_*. The second term is identical to that of case (i), Eq. (64). Therefore the growth rate is roughly a factor of $b^{1/2}(M_\perp/M)^{-1/4} \sim (L_s/L_n)^{1/2}\left(\frac{1+2q^2}{\eta_\perp/\eta_\parallel + 2q^2}\right)^{1/4}$ smaller than that of case (i) when $\Delta' > \Delta_{cs}$ meanwhile, Δ_c is the same as case (i) as long as $\omega \approx \omega_*$. In summary, we have found that the stabilizing influences of ion sound wave on drift-tearing modes can be characterized by the quantity Δ_{cs} given in Eq. (65), which is independent of the parallel and perpendicular resistivities.

Now we consider the effects of average curvature. As stated before, the curvature effects become important when $\hat{K}^{1/2}\hat{D} \gg 1$. Hence, we consider the case with $|a| \gg 1$ and $|b| \gg (M_\perp/M)^{1/2}$. In this limit, after some algebra, Eq. (62) reduces to the following expression

$$\begin{aligned}\frac{r_0\Delta'}{2\pi^2 m\hat{s}\theta_R} &= \frac{(G+KF)^{1/2}}{K^{1/2}Q_*}\left(Q+\frac{iKD}{G+KF}Q_*\right)^{1/2}(Q+iQ_*)\\ &-\frac{i^{1/2}Q^{1/2}}{2KQ_*^{3/2}}\frac{(G+KF)\left(1-\frac{\omega_*}{\omega}\right)+KD_R}{\left((G+KF)\left(1-\frac{\omega_*}{\omega}\right)-KD_R\right)^{1/2}}.\end{aligned} \tag{67}$$

In Eq. (67), the first term in the RHS represents slight modifications from Drake and Lee's results[2] due to D_R and F. The tearing mode instability criterion is given by $\Delta' > \Delta_c > 0$, where Δ_c in this case is given by

$$r_0\Delta_c = r_0\Delta_{cD} \equiv \frac{\pi(-D_R)^{1/2}m\hat{s}\theta_R}{K^{1/2}Q_*},$$

here, again, $\omega \approx \omega_*$ is again assumed; i.e., $(G+KF)\left(1-\frac{\omega_*}{\omega}\right) \ll KD_R$. In terms of physical parameters, Δ_{cD} can be expressed as

$$r_0\Delta_{cD} \cong 0.35(-D_R\beta)^{1/2}\frac{r_0}{\rho_s}. \tag{68}$$

Note also that Δ_{cD}, which characterizes the stabilizing influence of good average curvature, is also independent of resistivity. It is interesting to note that this result differs from the resistive MHD case[5] where the corresponding Δ_c due to the good average curvature is proportional to $\eta_\parallel^{-1/3}$ and, hence, can be very large for small resistivity. It is instructive to compare the magnitudes of Δ_{cs} and Δ_{cD}. From Eqs. (65) and (68), we have

$$\frac{\Delta_{cD}}{\Delta_{cs}} \cong \left(\frac{L_n}{L_s}\right)^{1/2}\left(\frac{-D_R}{\beta}\right)^{1/2}(1+2q^2)^{-1/4}$$

which, for typical tokamak parameters, tends to be slightly smaller than unity i.e., the plasma compression stabilizing effects tend to be slightly more important.

VI. Conclusions and Discussions

In this paper, we have analyzed the semicollisional drift-tearing modes in a toroidal geometry. By employing the ballooning mode representation and performing flux-surface averaging over the short connection length scale on which the curvature varies, we have derived the flux-surface averaged eigenmode equations, Eqs. (21)-(23), which cover both the semicollisional and collisional regimes. This is one of the contributions of this paper. Our equations also recover those of GGJ's in the usual resistive MHD limit.[5] From these eigenmode equations, the stability of semicollisional drift-tearing modes has been studied analytically via asymptotic matching. For typical tokamak parameters, the perpendicular resistivity gives a negligible contribution and the mode could be completely stabilized if either $r_0 \Delta' \leq 0.37\beta \frac{r_0}{\rho_s} \left(\frac{L_s}{L_n}\right)^{1/2} (1+2q^2)^{1/4}$ due to the plasma compressional effects or $r_0 \Delta' \leq 0.35(-\beta D_R)^{1/2} \frac{r_0}{\rho_s}$ due to the good average curvature and plasma compression. We again emphasize that Δ_c for the semicollisional drift-tearing mode is independent of resistivity, while the conventional resistive MHD theory (GGJ) predicts the $\eta_{\parallel}^{-1/3}$ dependence of Δ_c. It, therefore, may be concluded that the resistive MHD results[5] tend to be over-optimistic with regard to the stability of linear tearing modes in the present and future high-temperature tokamaks where the resistivity is extremely small.

Finally, let us reiterate that in the present work, we have neglected various other potentially important effects; such as temperature gradients, trapped particles, and thermal conductivities, in order to clarify the stability roles of ion-sound waves and average curvature on semicollisional drift-tearing modes in a tokamak plasma. Since there already have been various studies on those effects in the highly collisional regime[9,10] one would expect it to be relatively straightforward to extend the present semicollisional theory including those additional effects.

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