Tearing Mode Driven by Equilibrium Parallel Electric Field

Franco Cozzani and *Swadesh Mahajan*
Institute for Fusion Studies
The University of Texas at Austin
Austin, Texas 78712

I. Introduction

In a recent paper,[1] it has been shown that the presence of the parallel equilibrium electric field $E_{\parallel}^{(0)}$ (i.e., parallel equilibrium current $j_{\parallel}^{(0)}$) in the inner resistive layer of the tearing mode[2,3] enters significantly in the description of the electron dynamics in the hydrodynamic regime,[4] a result of considerable relevance, since it is customary[4–10] to ignore the equilibrium field $E_{\parallel}^{(0)}$ in the relevant eigenmode equations. In Ref. 1, it has been shown that the leading effect of the equilibrium current comes from perturbations of the Spitzer-Braginskii resistivity in the generalized Ohm's law, which couples to the perturbed electrostatic potential through the presence of electron temperature gradients.

Although the presence of such a new term alters the structure of the eigenmode equations,[5] it has been nevertheless possible to analytically derive a mode dispersion relation, which reduces exactly to the well known[5,8] result for the collisional drift tearing mode when the equilibrium current term is neglected, describing a new instability, driven by the new $j_{\parallel}^{(0)}$ term rather than the usual Δ' term.[2,3] The new growth rate γ has been obtained under the additional assumption $\gamma < \omega_0$, ω_0 being the real mode frequency in the absence of equilibrium current,[8] in order to avoid a numerical evaluation of the dispersion relation. However, it is noteworthy that, whenever the new growth rate is evaluated for typical parameters of present day tokamak experiments, the condition $\gamma < \omega_0$ is usually not satisfied.

In Sec. III of the present paper, the mode dispersion relation obtained in Ref. 1 is therefore studied numerically and it is found that the growth rate is indeed of the order of ω_0, a value significantly larger than the typical tearing mode growth rates found in previous studies. Also, the fact that the equilibrium current yields such a large growth rate extends the validity of the hydrodynamic (i.e., strongly collisional) description well beyond the previously classical limit,[8,10] this important point is discussed at the end of Sec. III. The extension of the analysis to the semi-hydrodynamic regime[4–10] of the tearing mode is discussed in Sec. IV, and in particular the effect of thermal conduction on the new instability is assessed.

II. Eigenmode Equations

The electron dynamics inside the inner layer of the tearing mode in the collisional and semicollisional regime is correctly represented by the following moment equations,[7,8] expressing particle conservation, momentum balance along the total magnetic field and energy balance, respectively.

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{u}) = 0, \tag{1}$$

$$\alpha'' \frac{\partial}{\partial t}(nm_e\hat{b}\cdot\mathbf{u}) = -ne\hat{b}\cdot\mathbf{E} - \hat{b}\cdot\nabla_{pe} - (ne)^2\eta\hat{b}\cdot\mathbf{u}\left[1 + \frac{3}{2}\alpha''\left(\frac{m_e}{\eta ne^2}\right)\frac{\partial}{\partial t}\ell nT_e\right]$$
$$- \alpha n\hat{b}\cdot\nabla T_e\left[1 - \frac{3\alpha'}{\nu}\frac{\partial}{\partial t}\ell nT_e - \frac{\alpha'}{\nu}\frac{\partial}{\partial t}\ell n(\hat{b}\cdot\nabla T_e)\right], \tag{2}$$

$$\frac{3}{2}n\left(\frac{\partial T_e}{\partial t} + \mathbf{u}\cdot\nabla T_e\right) + nT_e\nabla\cdot\mathbf{u} + \nabla\cdot\left[\alpha nT_e\hat{b}\hat{b}\cdot\mathbf{u} - \chi\hat{b}\hat{b}\cdot\nabla T_e\right] - \nabla\cdot\left[\frac{5}{2}\frac{nT_e}{eB}\hat{b}\times\nabla T_e\right]$$
$$- \hat{b}\cdot\mathbf{u}\left[\alpha n\hat{b}\cdot\nabla T_e + (en)^2\eta\hat{b}\cdot\mathbf{u}\right] = 0; \tag{3}$$

where

$$\mathbf{u} = \hat{b}\hat{b}\cdot\mathbf{u} + \frac{\mathbf{E}\times\hat{b}}{B} - \frac{\hat{b}\times\nabla p_e}{enB}. \tag{4}$$

The speed of light c is set equal to unity for convenience; α, α', α'' are numerical transport coefficients defined in Ref. 7 and 8; η is the Spitzer-Braginskii resistivity; χ is the thermal conductivity along the ambient magnetic field; ν is the electron collision frequency; $\hat{b}$ a unit vector in the direction of the total magnetic field $\mathbf{B}$. The physical origin and relevance of the time-dependent thermal force terms in Eq. (2) have been discussed in detail in Ref. 7; here, we shall only remark that they are responsible for the known ∇T_e-driven tearing mode growth rate[9] in the final dispersion relation, and they are not necessary to obtain the new instability discussed in the present paper. Upon linearization of Eqs. (1)-(3), we obtain a first equation for the perturbed electrostatic potential $\tilde{\phi}$ and the parallel component of the perturbed vector potential $\tilde{A}_{||}$, following standard approximations[5,6] of the drift-tearing eigenmode theory:

$$\psi'' = \sigma_*(x^2)\left[\psi - \left(x - 2\Lambda(x^2)\right)\phi\right]. \tag{5}$$

The generalized conductivity $\sigma_*(x^2)$ is given by[8,10]

$$\sigma_*(x^2) = \frac{-i\tau_s}{a^2}\left\{\frac{\Omega_2 + (\omega - \omega_n^*)i\alpha''' s\frac{k_{||}^2 D}{\omega}}{(1 + i\alpha''' s\frac{k_{||}^2 D}{\omega})(1 + is\frac{k_{||}^2 D}{\omega}) + \frac{2}{3}\hat{\alpha}(1+\alpha)is\frac{k_{||}^2 D}{\omega}}\right\}. \tag{6}$$

The equilibrium current appears in the factor $\Lambda(x^2)$ defined by

$$\Lambda(x^2) = \frac{1}{2}\left\{\frac{R}{-i\Omega_2 + \alpha''' s(\omega - \omega_n^*)\frac{K_{||}^2 D}{\omega}}\right\}. \tag{7}$$

In Eq. (6) and (7) we defined:

$$\begin{aligned}
\psi &\equiv \frac{\omega \tilde{A}_{||}}{\kappa_{||}'} \\
\omega_n^* &\equiv -\frac{\kappa_\perp}{eB}\frac{n'}{n}T_e \\
\omega_T^* &\equiv -\frac{\kappa_\perp}{eB}T_e' \\
\Omega_2 &\equiv \omega - \omega_n^* - \hat{\alpha}\omega_T^* \\
\hat{\alpha} &\equiv 1 + \alpha + i\alpha\alpha'\frac{\omega}{\nu} \\
\tau_s &\equiv \frac{\omega_p^2 a^2 s}{\nu} \\
D &\equiv \frac{T_e}{m_e \nu} \\
s &= \eta(Z=1)/0.51\eta(Z_{eff}) \\
R &\equiv \frac{3}{s}\frac{u_e}{v_e}\frac{\nu}{\kappa_{||}' v_e}\omega_T^* \\
\frac{\chi}{n} &\equiv \frac{3}{2}\alpha''' sD,
\end{aligned} \tag{8}$$

v_e is the electron thermal velocity; ω_p is the plasma frequency, a is the plasma radius. The effect of the equilibrium current appears in R through the electron drifting velocity u_e defined by

$$j_{\parallel}^{(0)} = -neu_e. \tag{9}$$

We recall[1] that the factor R comes from resistivity perturbations, i.e., the term $\tilde{\eta} j_{\parallel}^{(0)} = -\frac{3}{2}\frac{\tilde{T}_e}{T_e}\eta j_{\parallel}^{(0)}$ in Eq. (2), which couple to $\tilde{\phi}$ through Eq. (3). This is the leading effect of the equilibrium current in Eq. (5); small corrections to the mode growth rate of the form $i\eta j_{\parallel}^{(0)^2}/nT_e$ have been neglected, representing a weak damping of temperature fluctuations due to Ohmic heating, a branch of the final dispersion relation which is not connected with the tearing mode. Also, terms arising from parallel equilibrium current which give merely Doppler shifts to the real frequency of the mode of the form $\kappa_{\parallel} j_{\parallel}^{(0)}/en$ have been systematically neglected in the generalized conductivity given by Eq. (6). A second coupled equation in ϕ, ψ is obtained by the standard[5,6] result expressing the quasineutrality condition:

$$x_A^2 \phi'' = x\psi'', \tag{10}$$

where

$$x_A^2 \equiv \frac{\omega(\omega + \omega_i^*)}{(\kappa_{\parallel}' v_A)^2}, \qquad \omega_i^* \equiv -\frac{\kappa_{\perp}}{eB}\frac{p_i'}{n_i}; \tag{11}$$

v_A is the Alfvén velocity. In deriving Eq. (10), radial gradients of the equilibrium current have been neglected. (See also, Appendix I.) It is standard[8] to refer to the scaling $\kappa_{\parallel}^2 D/\omega < 1$ as collisional regime and to the scaling $\kappa_{\parallel}^2 D/\omega \sim 1$ as semicollisional regime, respectively; let us note that when the equilibrium current is neglected, one finds from the conventional analysis for the collisional case that the frequency Ω_2 is quite smaller than $\omega - \omega_n^*$. The form of the conductivity given by Eq. (6) suggests therefore the existence of a third scaling, intermediate between the collisional and the semicollisional, in which, due to the smallness of Ω_2, simultaneously $\kappa_{\parallel}^2 D/\omega < 1$ and $\Omega_2 \sim (\omega - \omega_n^*)\kappa_{\parallel}^2 D/\omega$ hold. The analysis of these particular two terms conductivity has been carried on, for example, in Ref. 10. (See also, Appendix II.) Anticipating the results of Sec. III, we note however that when the equilibrium current is retained, the frequency Ω_2 turns out to be precisely of order of the drift frequency ω^*, making the intermediate scaling of Ref. 10 no longer relevant. The inclusion of the equilibrium current, which yields such a large growth rate, a result of obvious relevance *per se*, has therefore the further effect of extending the validity of the collisional description of the tearing mode. We shall indeed find that, because of the presence of the equilibrium current, some present day tokamaks are quite well described by a collisional model, in which the conductivity given by Eq. (6) reduces to a constant.

III. Collisional Tearing Mode

Upon neglect of terms $\kappa_{\|}^2 D/\omega$ in Eqs. (6) and (7), the eigenmode equations given by Eqs. (5) and (10) reduce to Eqs. (21) and (22) of Ref. 1; let us just recall the dispersion relation obtained:

$$\overline{\sigma}_*^3 x_A^2 = -\left(\frac{3}{4}\right)^4 \left(\frac{1}{4\sqrt{2}-5}\right) \left[\frac{\Delta'}{\pi^{1/2}} - 4i\pi^{1/2}\frac{\hat{\lambda}}{x_A^2}\right]^4. \tag{12}$$

In Eq. (12) the current dependent term is given by

$$\begin{aligned} \hat{\lambda} &\equiv (\mathrm{sign} Im\lambda)\lambda, \\ \lambda &\equiv \frac{1}{2}\frac{R}{-i\Omega_2}; \end{aligned} \tag{13}$$

Δ' is the usual stability parameter of the tearing mode theory.[2,3] The collisional "conductivity" $\overline{\sigma}^*$ is given by Eq. (6) upon neglect of $\frac{\kappa_{\|}^2 D}{\omega}$ terms, i.e.,

$$\overline{\sigma}^* \equiv \frac{-i\Omega_2 \tau_s}{a^2}. \tag{14}$$

The dispersion relation has been obtained variationally,[1,5] the variational parameter α, where $\alpha^{-1/2}$ measures the mode width, being given by

$$\alpha^{1/2} = \frac{4/3\,\overline{\sigma}_*}{\frac{\Delta'}{\pi^{1/2}} - 4i\pi^{1/2}\frac{\hat{\lambda}}{x_A^2}}. \tag{15}$$

The roots in Eq. (12) must consistently satisfy $Re\alpha^{1/2} > 0$; $Re\alpha > 0$, as given by Eq. (15). Also, in the derivation of Eq. (12) we assumed

$$\left|\frac{2\lambda}{x_A}\right| < 1, \tag{16}$$

$$|\alpha^{1/2} x_A| < 1, \tag{17}$$

which is, the shift due to current is smaller than the Alfvén layer whereas the radial width of the mode is larger than the Alfvén layer. Furthermore, the validity of the collisional scaling $\kappa_{||}^2 D/\omega < 1$ requires

$$|\alpha^{1/2} x_r| > 1, \tag{18}$$

where $x_r = \left(\omega/\kappa_{||}'^2 D\right)^{1/2}$ measures the scale over which the semihydrodynamic conductivity varies.[4,5] Let us now proceed to discuss the dispersion relation Eq. (12).

Contact with the classical case is established by setting $\lambda = 0$ in Eq. (12); we therefore obtain the collisional drift tearing mode dispersion relation[5,6]

$$\overline{\sigma}_*^3 x_A^2 = -\left(\frac{3}{4}\right)^4 \left(\frac{1}{4\sqrt{2}-5}\right)\left(\frac{\Delta'}{\pi^{1/2}}\right)^4. \tag{19}$$

We find, in complete agreement with Ref. 8, that in this hydrodynamic regime, the real frequency and the previously known ∇T_e driven part of the tearing mode rate are given by

$$\omega_o = \omega_n^* + (1+\alpha)\omega_T^* \tag{20}$$

$$\gamma \nabla T_e \equiv \alpha\alpha' \frac{\omega_o}{\nu} \omega_T^*. \tag{21}$$

In Eq. (20) and (21), α' are the numerical transport coefficients of Ref. 7. The right-hand side of Eq. (19) containing Δ' provides, in this drift ordering $\gamma/\omega_o < 1$, a small damping to the growth rate Eq. (21); we recall from Eq. (15) that the mode is consistent when $\Delta' < 0$. Continuous departure from the usual result given by Eq. (19) follows by assuming $\lambda \neq 0$ but

$$|\Delta' x_A| > 4\pi |\lambda/x_A|, \tag{22}$$

in Eq. (12). We can then treat the λ-term as a perturbation of the classical result; in particular the $(-i\Omega_2)$-factor in the definition of λ, is given by the usual magnetic part of γ. Expanding to the lowest significant order we find

$$\overline{\sigma}_*^3 x_A^2 = -\left(\frac{3}{4}\right)^4 \left(\frac{1}{4\sqrt{2}-5}\right)\left(\frac{\Delta'}{\pi^{1/2}}\right)^4 \left[1 \pm \frac{i16\pi\lambda}{\Delta' x_A^2} - \frac{96\pi^2\lambda^2}{(\Delta' x_A^2)^2}\right]. \tag{23}$$

From the definition of $\overline{\sigma}_*$, given by Eq. (14), we see that the first order change in the eigenvalue affects the real frequency of the mode (the $\pm$ sign depending on the sign of

$Im\lambda$, which in the scaling Eq. (22) depends in turn whether the classical mode is stable or unstable), whereas the growth rate is affected only to second order in the small quantity $\lambda/(\Delta' x_A^2)$. It appears that the effect of the equilibrium current is to (slightly) destabilize the mode.

However, the most interesting physics appears to be described by the scaling opposite to Eq. (22), namely

$$|\Delta' x_A| < 4\pi|\lambda/x_A|. \tag{20}$$

Indeed, even though all this section is based on the assumption $|\lambda/x_A| < 1$, the $m \geq 2$ tearing mode is characterized by $|\Delta' x_A| < 1$ and therefore the approximate scaling to use whenever current is included is likely to be Eq. (24) rather than Eq. (22).

Let us then neglect Δ' in Eq. (12) and (15); now, of course, the $(-i\Omega_2)$ factor appearing in λ has to be computed self-consistently. The dispersion relation reduces to

$$\overline{\sigma}_*^7 x_A^{10} = -\pi^2\left(\frac{3}{2}\right)^4\left(\frac{1}{4\sqrt{2}-5}\right)\frac{R^4\tau_s^4}{a^8}. \tag{25}$$

which we can write in the convenient adimensional form:

$$\mu^7(\mu - i)^5\left(\mu - i - i\frac{\omega_i^*}{\omega_o}\right)^5 = A, \tag{26}$$

where

$$\mu = \frac{-i\Omega_2}{\omega_0}, \tag{27}$$

$$\omega_0 \equiv \omega_n^* + \hat{\alpha}\omega_T^*, \tag{28}$$

$$A = \frac{\pi^2}{s^7}\left(\frac{3}{2}\right)^6\frac{9}{4\sqrt{2}-5}\left(\frac{u_e}{v_e}\right)^4\left(\frac{m_e}{m_i}\right)^5\left(\frac{L_n}{L_s}\right)^6\frac{\omega_n^{*6}\omega_T^{*4}\nu^7}{\beta^8\omega_0^{17}}. \tag{29}$$

In Eq. (29), L_n, L_s are the density and shear scale length, respectively; β has been defined as $4\pi p_e/B^2$, instead that with the usual 8π factor. Let us note that, as imposed by the validity of fluid equations, the ratio $\frac{\omega}{\nu}$ always satisfies $\frac{\omega}{\nu} < 1$ (i.e., in the present contest we have referred to a "large" growth rate due to equilibrium current in the sense $\gamma \sim \omega^*$, but nevertheless always assumed $\gamma < \nu$); we can therefore ignore the factor $i\alpha\alpha'\frac{\omega}{\nu}\omega_T^*$ in the definition of ω_0, as given by Eq. (28), and treat $\omega_0 \approx \omega_n^* + (1+\alpha)\omega_T^*$ as a real positive quantity.

In order to satisfy the consistency conditions $Re\alpha^{1/2} > 0$; $Re\alpha > 0$, as given by Eq. (15), the roots of Eq. (26) must satisfy

$$Re\{i\mu^2(\mu - i)(\mu - i - i\frac{\omega_i^*}{\omega_0})\} sign Im(\mu) > 0, \tag{30}$$

$$Re\{-\mu^4(\mu - i)^2(\mu - i - i\frac{\omega_i^*}{\omega_0})^2\} > 0. \tag{31}$$

To establish contact with the result of Ref. 1, let us neglect μ with respect to i in Eqs. (26), (30), and (31), i.e., let us assume that the equilibrium current term gives rise to a growth rate (and a real frequency shift) much smaller than the real frequency of the classical drift tearing mode.

Also, let us neglect the ion temperature by setting $\omega_i^* = 0$ in the above equations. Equation (26) reduces to

$$\mu = (-1)^{1/7} A^{1/7}, \tag{32}$$

whereas Eq. (30) and (31) reduce to

$$Re\{-i\mu^2\} sign Im(\mu) > 0, \tag{33}$$

$$Re\{-\mu^4\} > 0. \tag{34}$$

Of the seven roots of $(-1)^{1/7}$, one finds from Eqs. (33) and (34) that $e^{\pm i\pi/7}$ (growing roots) are consistent and acceptable solutions of Eq. (32) are therefore

$$\mu = e^{\pm i\pi/7} \frac{\pi^{2/7}}{s}(2.06)\left(\frac{u_e}{v_e}\right)^{4/7}\left(\frac{m_e}{m_i}\right)^{5/7}\left(\frac{L_n}{L_s}\right)^{6/7} \frac{\omega_n^{*6/7}\omega_T^{*4/7}}{\omega_0^{17/7}} \frac{\nu}{\beta^{8/7}}. \tag{35}$$

The growth rate given by Eq. (35), in agreement with Ref. (1) is

$$\gamma = \frac{2.57}{s}\left(\frac{u_e}{v_e}\right)^{4/7}\left(\frac{m_e}{m_i}\right)^{5/7}\left(\frac{L_n}{L_s}\right)^{6/7}\left(\frac{\omega_n^*}{\omega_0}\right)^{6/7}\left(\frac{\omega_T^*}{\omega_0}\right)^{4/7}\frac{\nu}{\beta^{8/7}}. \tag{36}$$

Let us now proceed to the study of the dispersion relation in its full form, given by Eq. (26), which must be carried out numerically; the simple form of Eq. (26) makes however such an evaluation quite straightforward.

In Figs. 1-4, we plot the roots of Eq. (26) which meet the consistency conditions Eq. (30) and (31), as a function of the parameter A. Recalling Eq. (27) and (28), the real part of μ gives the growth (damping rate) due to the equilibrium current, normalized to $\omega_0 = \omega_n^* + (1+\alpha)\omega_T^*$, the real mode frequency in absence of current. The imaginary part of μ represents the real frequency correction to ω_0, due to the equilibrium current. Figs. 1 and 2 refer to a cold ion model ($T_i = 0$), whereas Figs. 3 and 4 refer to the $T_i = T_e$ case. Values of the measured physical parameters for a typical discharge[11] in the TEXT tokamak have been used to determine the relevant plasma parameters at the $q = 2$ rational surface, q being the safety factor, which is found to be located at approximately two thirds of the minor radius. At $q = 2$ in the TEXT machine, the electron temperature has fallen to about one fifth the central value and the density to about one third the central value. The electron drift frequencies ω_n^*, ω_T^* are $1.2 \cdot 10^4 sec^{-1}$, $1.6 \cdot 10^4 sec^{-1}$, respectively; the frequency ω_0 to which the roots found in this section are normalized is therefore $4.4 \cdot 10^4 sec^{-1}$. Because of the fairly low temperature (and because the effective ion charge Z_{eff} is equal to 2.3 ± 0.7) the collision frequency ν_{ei} is quite high, of the order of $7 \times 10^5 sec^{-1}$, making meaningful a fluid description of the electron response.

The values obtained for the TEXT tokamak are then scaled for the Alcator, PLT, TFTR machines, respectively, from corresponding central values[12] of temperature and density. The estimated values of the parameter A, the frequency ω_0, and the ratio ω_i^*/ω_0 for the above mentioned machines are given in Table I.

	TEXT	ALCATOR	PLT	TFTR
A	3	4	1.9	1.2
ω_0 (sec^{-1})	$4.4 \cdot 10^4$	$3.5 \cdot 10^4$	$4 \cdot 10^4$	$1.5 \cdot 10^4$
ω_i^*/ω_0	0.40	0.56	1.60	0.95

Table I

In addition to satisfying the consistency conditions $Re\alpha^{1/2} > 0$, $Re\alpha > 0$, it is necessary to check the acceptable roots μ against the various approximations used in deriving the dispersion relation, Eq. (26); these conditions, given by Eqs. (16), (17), (18) and (24), characterize in effect the region of parameter space where the mode exists. Expressing the adimensional quantities appearing in the above mentioned equations in terms of the plasma parameters and of the particular root of interest, we obtain:

$$\left|\alpha^{1/2}x_r\right| = \frac{6\sqrt{2}}{\pi^{1/2}}s^2\left(\frac{u_e}{v_e}\right)^{-1}\left(\frac{m_i}{m_e}\right)\left(\frac{L_s}{L_n}\right)^2\left(\frac{\omega_0}{\nu}\right)^{3/2}\beta^2 \times \mu^2(\mu-1)^{3/2}\left(\mu - i - i\frac{\omega_i^*}{\omega_0}\right) \tag{37}$$

$$\left|\alpha^{1/2}x_A\right| = \frac{6\sqrt{2}}{\pi^{1/2}}s^2\left(\frac{u_e}{v_e}\right)^{-1}\left(\frac{m_i}{m_e}\right)^{3/2}\left(\frac{L_s}{L_n}\right)^2\left(\frac{\omega_0}{\nu}\right)^2\beta^{5/2} \times \mu^2(\mu-i)^3\left(\mu - i - i\frac{\omega_i^*}{\omega_0}\right)^{3/2}, \tag{38}$$

$$\left|\frac{2\lambda}{x_A}\right| = \frac{1}{\sqrt{2}s}\left(\frac{u_e}{v_e}\right)\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{\nu}{\omega_0}\right)\beta^{-1/2} \times \frac{1}{\mu(\mu-i)^{1/2}\left(\mu - i - i\frac{\omega_i^*}{\omega_0}\right)^{1/2}}, \tag{39}$$

$$\left|\frac{\Delta' x_A}{2\pi}\right| = \frac{|\Delta' a|\omega_0}{2\pi k_\|' a v_A}(\mu-i)^{1/2}\left(\mu - i - i\frac{\omega_i^*}{\omega_0}\right)^{1/2}. \tag{40}$$

In Eqs. (37)-(40) we made use of the fact that, typically, $\omega_n^* \simeq \omega_T^* \simeq \frac{1}{3}\omega_0$. Also, recall that the particular root μ is itself a function of the relevant plasma parameters, through the quantity A defined by Eq. (29). The validity of the approximations given by Eqs. (16), (17), (18), and (24) can therefore be assessed only after the dispersion relation has been solved, and its acceptable roots (in the sense of Eqs. (30) and (31)) determined.

In Table II, we give the values of the adimensional quantities appearing in Eqs. (37)-(40) for the fastest growing root of Eq. (26), consistent with Eqs. (30) and (31); the value of $\left|\frac{\Delta' x_A}{2\pi}\right|$ is normalized to $|\Delta a|$; the roots for the different machines have been calculated for values of $\frac{\omega_i^*}{\omega_0}$ corresponding to Table I.

	TEXT	ALCATOR	PLT	TFTR
$\left\|\frac{\Delta' x_A}{2\pi}\right\| / \|\Delta' a\|$	$6.0 \cdot 10^{-4}$	$4.3 \cdot 10^{-4}$	$4.6 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
$\left\|\frac{2\lambda}{x_A}\right\|$	$1.8 \cdot 10^{-2}$	$9.5 \cdot 10^{-3}$	$2.9 \cdot 10^{-2}$	$6.0 \cdot 10^{-2}$
$\|\alpha^{1/2} x_A\|$	0.43	0.40	0.40	0.36
$\|\alpha^{1/2} x_r\|$	2.3	5.2	0.78	0.64

Table II

It is evident from Table II that the approximations used in the previous analysis are well satisfied, with the exception of the condition $|\alpha^{1/2} x_e| > 1$, expressing the validity of the collisional scaling, for the PLT and TFTR tokamaks. The extension of the present analysis to the semicollisional regime is discussed in the next section.

References

1. F. Cozzani, S. Mahajan, IFS Report #163.
2. H.P. Furth, J. Killeen, M.N. Rosenbluth, Phys. Fluids **6**, 459 (1963).
3. R.B. White, Handbook of Plasma Physics, Chapter 3.5, eds. M. Rosenbluth and R. Sagdeev, North Holland (1983).
4. J.E. Drake and Y.C. Lee, Phys. Fluids **20**, 1341 (1977).
5. S.M. Mahajan et al., Phys. Fluids **22**, 2147 (1979).
6. R.D. Hazeltine and D.W. Ross, Phys. Fluids **21**, 1140 (1978).
7. A.B. Hassam, Phys. Fluids **23**, 38 (1980).
8. A.B. Hassam, Phys. Fluids **23**, 2493 (1980).
9. R.D. Hazeltine et al., Phys. Fluids **18**, 1778 (1976).
10. J.F. Drake et al., Phys. Fluids **26**, 2509 (1983).
11. W. Rowan, private communication.
12. R.D. Hazeltine, private communication.

Appendix I

In deriving Eq. (10), a term involving radial gradients of the equilibrium current has been neglected. If such a term is retained, Eq. (10) becomes

$$x\psi'' = x_A^2\phi'' + \frac{4\pi j_{\parallel}^{(0)'} L_s}{B}\psi. \qquad (AI1)$$

In Eq. (AI1), the shear length L_s is defined to be $L_s \equiv k_{\perp}/k_{\parallel}'$.

Unfortunately, the extra $j_{\parallel}^{(0)'}$ term in (AI1) destroys the symmetry of the eigenmode equations given by Eq. (AI1) and Eq. (5) to the extent that the problem is no longer soluble (or: is now ill-posed). It is however possible to argue that this current gradient term can be safely neglected in the interior analysis of the tearing layer. The following argument follows closely from Ref. 13.

Making use of the constant-ψ approximation, we can replace the $j_{\parallel}^{(0)'}$ term in (AI1) by $\frac{4\pi j_{\parallel}^{(0)'} L_s}{B}\psi_0$, where ψ_0 denotes the value of the exterior solution (i.e., the one obtained solving for ψ in the ideal MHD region, outside the tearing layer) evaluated at $x = 0$.

We obtain from (AI1)

$$\psi\Delta' = \int_{-\infty}^{\infty} \frac{x_A^2}{x}\left(-E' + \frac{4\pi j_{\parallel}^{(0)'} L_s}{B x_A^2}\psi_0\right) dx, \qquad (AI2)$$

where[2-3]

$$\Delta' \equiv \psi_0^{-1} \int_{-\infty}^{\infty} \psi'' dx \equiv \frac{\psi'(+\infty) - \psi'(-\infty)}{\psi_0}$$

and $E' = -\phi'$ is the radial electric field. Equation (AI2) shows that the effect of the $j_{\parallel}^{(0)'}$ term is merely to remove the singularity in the integrand of Eq. (AI2) due to the $j_{\parallel}^{(0)}$-driven odd part of E, $E_-(x)$, without affecting the value of Δ', i.e.

$$E_-'(0) = \frac{4\pi j_{\parallel}^{(0)'} L_s}{B x_A^2}\psi_0. \qquad (AI3)$$

Furthermore, it is possible to estimate the value of the $j_{\parallel}^{(0)'}$ term in Eq. (AI1) for the particular mode under present consideration, and convince one that the additional term is indeed numerically negligible.

Making use of the constant-ψ approximation in Eq. (AI1), the $j_{\parallel}^{(0)'}$ term is negligible with respect to the $x\psi''$ term provided that

$$\left|\frac{\alpha^{1/2}B}{4\pi j_{\parallel}^{(0)'}L_s}\right| >> 1, \tag{AI4}$$

where $\left(\alpha^{1/2}\right)^{-1}$ is the variational parameter appearing in the present analysis, measuring the radial mode width.

Proceeding as in Sec. III we find

$$\left|\frac{\alpha^{1/2}B}{4\pi j_{\parallel}^{(0)'}L_s}\right| = \frac{2s^2}{\pi^{1/2}}\left(\frac{m_i}{m_e}\right)\left(\frac{u_e}{v_e}\right)^{-2}\beta\left(\frac{L_0}{L_n}\right)\left(\frac{\omega_0}{\nu}\right)^2$$
$$\times\, \mu^2(\mu - i)\left(\mu - i - i\frac{\omega_1^*}{\omega_0}\right), \tag{AI5}$$

where $L_0 \equiv j_{\parallel}^{(0)}/j_{\parallel}^{(0)'} \simeq L_n$ and μ is the particular root of interest of Eq. (26). In Table (A1) we give the value of the adimensional quantity appearing in Eq. (AI5) for the fastest growing root of Eq. (26), analogously with Table II.