The Spectrum of Resistivity Gradient Driven Turbulence

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Abstract

The resistivity fluctuation correlation function and electrostatic potential spectrum of resistivity gradient driven turbulence are calculated analytically and compared to the results of three dimensional numerical calculations. Resistivity gradient driven turbulence is characterized by effective Reynolds' numbers of order unity. Steady-state solution of the renormalized spectrum equation yields an electrostatic potential spectrum $\left\langle \hat{\phi}^2 \right\rangle_{k_\theta} \sim k_\theta^{-3.25}$. Agreement of the analytically calculated potential spectrum and mean-square radial velocity with the results of multiple helicity numerical calculations is good. This comparison constitutes a quantitative test of the analytical turbulence theory used. The spectrum of magnetic fluctuations is also calculated, and agrees well with that obtained from the numerical computations.

I. Introduction

It is well known that large density and electrostatic potential fluctuations,[1-3] and significant particle transport[3,4] occur at the edge of a tokamak plasma. A detailed understanding of edge turbulence is important for exploiting improved confinement regimes, and for developing methods for plasma control and refueling.[5] However, the extensive experimental effort directed at analysis and characterization of tokamak edge plasmas also establishes such systems as ideal examples and test cases for basic turbulence studies and models.

Since edge plasma temperature and density parameter regimes make the incidence of dissipative or resistive phenomena likely, several such models have been proposed.[4,6-9] A model based on resistivity gradient driven (rippling mode) turbulence was found to have many attractive qualitative features.[6] However, the strong, linearly stabilizing effect of parallel thermal conduction cast doubt on the robustness and relevance of this model. Recently, fundamental work[9] on resistivity gradient driven turbulence has shown the theoretically predicted fluctuation levels to be comparatively insensitive to the value of the thermal conductivity. This result, which is contrary to intuition based on the linear theory, is derived from a model of fully developed resistivity gradient driven turbulence which is similar to that used to describe hydrodynamic shear flows, i.e., saturation by a dynamically regulated balance of energy input by gradient relaxation with dissipative thermal conduction. The dynamical regulation occurs by nonlinear modification of the mode structure, through which the resistivity $\hat{\eta}$ and potential $\hat{\phi}$ perturbations decouple from the current perturbation $\hat{J}_{||}$, thus effectively eliminating the vorticity equation from the nonlinear dynamics ($\hat{J}_{||} \sim 0$ in the regions of interest). The potential and resistivity fluctuations adjust so as to balance energy input from the average resistivity gradient with nonlinearly enhanced thermal dissipation. For this reason, saturated resistivity gradient driven turbulence has an effective Reynolds number of order unity. The predicted dependence of the saturated fluctuation levels on thermal conduction is weak, and the calculated fluctuation levels are comparable to those observed in experiments.

In this paper, the spectrum of resistivity gradient driven turbulence is analytically calculated. The theory presented in Ref. 9 was concerned with identification of the basic physical processes, and spatial and temporal scales, and with the calculation of the mean-square (i.e., spectrum integrated) fluctuation levels. Thus, a renormalized resistivity response calculation was utilized. However, to determine the fluctuation spectrum additional, more detailed information about the nonlinear dynamics, source, and dissipation is required. Therefore, a calculation of the fluctuation spectrum is of interest in two contexts: first, as a further step in the development of the resistivity gradient turbulence model; second, as a more stringent test of the analytical renormalized turbulence theory. The testing process is accomplished by comparison of the analytic theory with the results of nonlinear, multiple helicity numerical calculations.

In order to calculate the fluctuation spectrum, it is necessary to construct and solve the resistivity fluctuation correlation function ($\langle \hat{\eta}(1)\hat{\eta}(2)\rangle$) evolution equation. The renormalized two-point equation is derived by iterative closure of the triplet term $\left\langle \nabla\hat{\phi}(1) \times \hat{z} \cdot \nabla\hat{\eta}(1)\hat{\eta}(2)\right\rangle$. In the renormalized two-point equation, the effects of (nonlinear) mode coupling are represented by relative diffusion of resistivity $-D_- \partial^2 \langle \hat{\eta}(1)\hat{\eta}(2)\rangle / \partial x_-^2$, where $D_- \to 0$ as $1 \to 2$. The small scale inhomogeneity in turbulent diffusion is a consequence of the combined effects of incoherent and coherent mode coupling, and reflects the tendency of the electrostatic fluctuations to destroy resistivity correlation by relative convection, thus generating smaller scale turbulence. The two-point correlation function is thus determined by an equation of the form

$$\left\{ \frac{\partial}{\partial t} - \chi_{\|}(\nabla_{\|1}^2 + \nabla_{\|2}^2) - D_- \frac{\partial^2}{\partial x_-^2} \right\} \langle \hat{\eta}^2 \rangle = S_{1,2},$$

the solution of which may be expressed as $\langle \hat{\eta}^2 \rangle = \tau_{\mathrm{cl}} S_{1,2}$. Thus, here, fluctuation correlation is driven by the relaxation of the average resistivity gradient $\left(S_{1,2} = -\left\langle \nabla_y \hat{\phi}\hat{\eta}\right\rangle \partial \langle \eta \rangle / \partial r\right)$, and destroyed by relative diffusion (which represents turbulent shear stress) and thermal conduction, both of which combine to determine the correlation time $\tau_{\rm cl}$. Solution of the integral equation $\langle \hat{\eta}^2 \rangle = \tau_{\rm cl} S_{1,2}$ determines the fluctuation spectrum.

The processes discussed above are analogous to stirring, inertial range mode coupling, and viscous dissipation, respectively, in the more familiar case of Navier-Stokes turbulence. However, it is important to realize that, in contrast to the classic problem of homogeneous Navier-Stokes turbulence, the physical processes which determine the two-point correlation function and spectrum do not act in disjoint regions of wavenumber space. In particular, relaxation of the average resistivity gradient can drive fluctuations at *all* wavenumbers. In this vein, $\tau_{\rm cl}(x_-)$ may be viewed as a two-point response time (or lifetime), for scales of $0(|x_-|)$. Thus, the spectrum $\langle \hat{\eta}^2 \rangle$ is determined by the input rate $S_{1,2}$ multiplied by the lifetime $\tau_{\rm cl}$. Furthermore, since the (nonlinear) resistivity scale length is determined by the asymptotic balance of thermal conduction with turbulent diffusion, a Reynolds number Re of order unity is intrinsic to the case considered here. This is in clear contrast to the more familiar instance of large Reynolds number fluid turbulence. Finally, once the resistivity fluctuation correlation function is calculated, the fluctuation spectra can be straightforwardly obtained using Ohm's law and the decoupling ($\hat{J}_\parallel \sim 0$) approximation.

In this paper, the theory of two-point correlation for resistivity gradient driven turbulence is presented. The principal results are:

(i.) The resistivity fluctuation correlation function and potential and resistivity fluctuation wavenumber spectra are calculated.

(ii.) The two point theory corroborates the estimates of nonlinear space and time scales, and parameter scalings of fluctuation levels and diffusion obtained from the previous one-point theory.[9]

(iii.) the potential wavenumber spectrum fits a power law of the form $\left\langle \hat{\phi}^2 \right\rangle_{k_\theta} \sim k_\theta^{-3.25}$. The mean-square fluctuating radial velocity is $\langle \hat{v}_r^2 \rangle = 369a^2/\tau_R^2 (\tau_R = \mu_0 a^2/\eta_0$, the resistive decay time). These results are in good agreement with the results of nonlinear, multiple helicity calculations.

(iv.) The wavenumber spectrum of magnetic fluctuations has been calculated and agrees well with the results of the numerical calculations.

The remainder of this paper is organized as follows. In Sec. II, the basic model and physics of resistivity gradient driven turbulence are reviewed. In Sec. III, the renormalized two-point equation for the resistivity fluctuation correlation function is derived. The two-point correlation relaxation time and correlation function are calculated. The potential fluctuation wavenumber spectrum is calculated in Sec. IV. The comparison of the analytic theory with the results of the numerical calculations is presented in Sec. V. Section VI contains the conclusions.

II. Physics of Resistivity Gradient Driven Turbulence

The basic model of resistivity gradient driven turbulence is described in Ref. 9. Derived from reduced resistive magnetohydrodynamic equations in the electrostatic approximation, the model consists of the Ohm's law

$$-B_z \nabla_{\|}^{(0)} \hat{\phi} = \hat{\eta} J_{\|0} + \eta_0 \hat{J}_{\|} \tag{1}$$

the vorticity equation

$$\rho_m \frac{d}{dt} \nabla_{\perp}^2 \hat{\phi} = B_z \nabla_{\|}^{(0)} \hat{J}_{\|} \tag{2}$$

and an equation for the evolution of the resistivity $\hat{\eta}$

$$\frac{d\hat{\eta}}{dt} - \chi_{\|} \nabla_{\|}^{(0)2} \hat{\eta} = -\frac{\partial \hat{\phi}}{\partial y} \frac{d\eta_0}{dr}. \tag{3}$$

Here and throughout the remainder of this paper, the notation is the same as that used in Ref. 9. The linear stability theory of the rippling mode[10] is discussed in detail in Ref. 11. A crucial point to take note of is the strong stabilizing effect of electron thermal conduction ($\chi_{\|}$). In particular, for large $\chi_{\|}$ (the regime of relevance) the growth rate γ scales as $\gamma \sim \chi_{\|}^{-4/3}$. Thus, large $\chi_{\|}$ drastically reduces rippling mode growth rates, but does not result in complete stabilization since a small region where the effects of $\chi_{\|}$ are negligible always exists near the $\mathbf{k} \cdot \mathbf{B}_0 = 0$ resonant surface. Furthermore, $D = \gamma / k_{\perp}^2$-type estimates of rippling mode-induced transport scale as $D \sim \chi_{\|}^{-3}$. For this reason, rippling modes have received comparatively little attention.

A recent investigation[9] of resistivity gradient driven turbulence identified a crucial feature of the nonlinear dynamics and evolution. Specifically, the intrinsically radially asymmetric resistivity and potential fluctuations were observed to decouple from the current perturbation by moving off the rational surface. Hence $\hat{J}_{\|} \approx 0$ in the region of interest, and thus the stabilizing effect of field line bending is eliminated. Furthermore, the vorticity equation is therefore irrelevant, and the nonlinear mode structure departs from its linear analogue. Hence, it was useful to discuss the nonlinear physics in terms of the two energy-like integral quantities:

$$E_k = 1/2 \int d^3x |\nabla_\perp \hat{\phi}|^2 \tag{4}$$

$$E_T = 1/2 \int d^3x |\hat{\eta}|^2 \tag{5}$$

which satisfy, respectively, the relations:

$$\frac{\partial E_k}{\partial t} = -\frac{B_z}{\rho_m} \int d^3x \left[\hat{\phi}^* \nabla_\parallel^{(0)} \hat{J}_\parallel\right] \tag{6}$$

$$\frac{\partial E_T}{\partial t} = \int d^3x \left[-\hat{\eta}^* \nabla_y \hat{\phi} \frac{d\eta_0}{dr} - \chi_\parallel \left|\nabla_\parallel^{(0)} \hat{\eta}\right|^2\right]. \tag{7}$$

A stationary, saturated state is characterized by $\partial E_k/\partial t = \partial E_T/\partial t = 0$.

The decoupling of $\hat{J}_\parallel$ from $\hat{\phi}$ and $\hat{\eta}$ leaves $\hat{J}_\parallel = 0$ in the region of interest, and automatically results in $\partial E_k/\partial t = 0$. Furthermore, since Eq. (1) now reduces to:

$$-B_z \nabla_\parallel^{(0)} \hat{\phi} = \hat{\eta} J_0, \tag{8}$$

it follows that the condition $\partial E_T/\partial t = 0$ is satisfied if $\hat{\eta}$ is characterized by a radial scale length $\Delta_{\mathbf{k}}$ given by:

$$\Delta_{\mathbf{k}}^c \approx \left(\frac{L_s E_0}{L_\eta B_z}\right)^{1/3} (\chi_\parallel k_y^2 / L_s^2)^{-1/3}, \tag{9}$$

where $E_0 = \eta_0 J_0$. The scaling with m of the numerically calculated radial correlation length agrees well with that given in Eq. (9). Note that similar tests may be applied to laboratory fluctuation data. The fluctuation level required so that Eq. (9) is satisfied can be estimated using the renormalized resistivity evolution equation:

$$\frac{\partial \hat{\eta}_{\mathbf{k}}}{\partial t} + \chi_\parallel k_\parallel^2 \hat{\eta}_{\mathbf{k}} - D_{\mathbf{k}} \frac{\partial^2 \hat{\eta}_{\mathbf{k}}}{\partial x^2} = -i k_y \hat{\phi}_{\mathbf{k}} \frac{d\eta_0}{dr} \tag{10a}$$

where the diffusion coefficient $D_{\mathbf{k}}$ is given by

$$D_{\mathbf{k}} = \sum_{\mathbf{k}'} k_y'^2 \left\langle \hat{\phi}^2 \right\rangle_{\mathbf{k}'} \left[\gamma_{\mathbf{k}''} + \chi_\parallel k_\parallel''^2\right]^{-1} \tag{10b}$$

where $\mathbf{k}'' = \mathbf{k} + \mathbf{k}'$. Thus, the natural radial scale for 'decoupled' resistivity gradient driven turbulence is

$$\Delta_{\mathbf{k}} = \left[\frac{D_k}{\chi_{\|} k_y^2 / L_s^2} \right]^{1/4} . \tag{11}$$

Therefore, equating $\Delta_{\mathbf{k}} = \Delta_{\mathbf{k}}^c$ determines the level of diffusion so that $\partial E_T / \partial t = 0$ and saturation occurs. The necessary level of diffusion is given by:

$$D_{\mathbf{k}} \approx (L_s E_0 / L_\eta B_z)^{4/3} (\chi_{\|} k_y^2 / L_s^2)^{-1/3}. \tag{12}$$

Note that $D_{\mathbf{k}} \sim \chi_{\|}^{-1/3}$, an extremely weak scaling dependence. Finally, note that saturation occurs by the balance of nonlinearly regulated thermal dissipation with drive due to resistivity gradient relaxation; i.e., $D_{\mathbf{k}} / \Delta_{\mathbf{k}}^2 \sim \chi_{\|} (k_y^2 / L_s^2) \Delta_{\mathbf{k}}^2 \sim L_s E_0 / L_\eta B_z \Delta_{\mathbf{k}}$. Hence a Reynolds number of order unity is intrinsic to resistivity gradient driven turbulence.

III. Evolution of the Resistivity Correlation Function

The heuristic picture of saturation is represented rigorously by the steady state solution of the two-point resistivity correlation function equation, utilizing the decoupling condition. The two-point equation is

$$\frac{\partial}{\partial t}\langle\hat{\eta}(1)\hat{\eta}(2)\rangle - \chi_{\|}\left(\nabla^2_{\|1}+\nabla^2_{\|2}\right)\langle\hat{\eta}(1)\hat{\eta}(2)\rangle + T_{12} = S \tag{13}$$

where

$$T_{12} = \left\langle \nabla_1\hat{\phi}(1)\times\hat{n}\cdot\nabla_1\hat{\eta}(1)\hat{\eta}(2)\right\rangle + \left\langle \nabla_2\hat{\phi}(2)\times\hat{n}\cdot\nabla_2\hat{\eta}(2)\hat{\eta}(1)\right\rangle \tag{14}$$

is the triplet nonlinearity and

$$S = -\left\{\left\langle \nabla y_1\hat{\phi}(1)\hat{\eta}(2)\right\rangle + \left\langle \nabla y_2\hat{\phi}(2)\hat{\eta}(1)\right\rangle\right\}\frac{d\eta_0}{dr} \tag{15}$$

is the source, proportional to the resistivity gradient. The nonlinearity vanishes as the relative separation goes to zero, while the source does not. This leads to the production of short range correlation. The theory of two-point correlation provides the formalism for inverting the evolution operator of the renormalized two-point equation.[8,12] This inversion accounts for mode coupling (represented as an inhomogeneous relative diffusion process) as well as the coupling of dissipation (thermal conduction) and turbulent diffusion which determines the scales of correlation. Furthermore, it determines the stationary spectrum and thus accounts for the balance of free energy input with dissipation, consistent with the dynamics of two-point correlation and the effects of collective resonances.

Starting with the equation for two-point resistivity correlation

$$\left(\frac{\partial}{\partial t}\langle\hat{\eta}(1)\hat{\eta}(2)\rangle - \chi_{\|}\left(\nabla^2_{\|1}+\nabla^2_{\|2}\right)\langle\hat{\eta}(1)\hat{\eta}(2)\rangle + T_{12} = S\right),$$

we transform to a relative coordinate system $(\mathbf{x}_+, \mathbf{x}_-)$

$$\mathbf{x}_1 = \frac{1}{2}(\mathbf{x}_+ + \mathbf{x}_-)$$

$$\mathbf{x}_2 = \frac{1}{2}(\mathbf{x}_+ - \mathbf{x}_-)$$

where $\mathbf{x}_+$ describes the average position of the points 1 and 2, and $\mathbf{x}_-$ describes their relative position. To express the parallel heat conduction term as a function of the average and relative positions, we first write it as a Fourier expansion in y and z, for each radial position,

$$\left[\nabla_{\parallel 1}^2 + \nabla_{\parallel 2}^2\right] \langle \hat{\eta}(1)\hat{\eta}(2) \rangle = \left\langle \sum_{k_{1y},k_{1z}} \sum_{k_{2y},k_{2z}} \exp\left[ik_{1y}y_1 + ik_{2y}y_2 \right.\right.$$
$$\left.\left. + ik_{1z}z_1 + ik_{2z}z_2\right] \left\{ \frac{[x_1 - x_s(\mathbf{k}_1)]^2}{L_s^2} k_{1y}^2 + \frac{[x_2 - x_s(\mathbf{k}_2)]^2}{L_s^2} k_{2y}^2 \right\} (\hat{\eta}(\mathbf{k}_1)\hat{\eta}(\mathbf{k}_2)) \right\rangle.$$

To obtain the above expression, the wavenumbers corresponding to each position, $k_{\alpha z}$ and $k_{\alpha y} (\alpha = 1, 2)$, are chosen so that $\hat{b} \cdot \nabla_\alpha$ vanishes at the rational surface associated with that wavevector, i.e.,

$$\frac{B_z}{B} k_{\alpha z} + \frac{B_y(x_s^{(\alpha)})}{B} k_{\alpha y} = 0. \tag{16}$$

The average $\langle\ \rangle$ may be completed by integrating over y_+ and z_+. Performing these integrations yields factors $\delta_{k_{1y},-k_{2y}}$ and $\delta_{k_{1z},-k_{2z}}$ which relate the two wavevectors ($\mathbf{k}_1 = -\mathbf{k}_2$). As a result, the double Fourier expansion is converted to a single one in the relative variables z_- and y_-. Furthermore, since Eq. (16) is invariant under $\mathbf{k} \to -\mathbf{k}$, both radial positions are related to a single rational surface defined by $(B_z/B)k_z + (B_y(x)/B)k_y = 0$. Writing x_1 and x_2 in terms of relative and average positions then yields

$$\left(\nabla_{\parallel 1}^2 + \nabla_{\parallel 2}^2\right) \langle \hat{\eta}(1)\hat{\eta}(2) \rangle = \sum_{k_y,k_z} \exp(ik_y y_- + ik_z z_-) \frac{1}{2} \left[\frac{(x_+ - 2x_s)^2 + x_-^2}{L_s^2} \right] k_y^2 \langle \hat{\eta}(1)\hat{\eta}(2) \rangle_{\mathbf{k}}$$
$$= \frac{1}{2} \left[\frac{(x_+ - 2x_s)^2 + x_-^2}{L_s^2} \right] \frac{\partial^2}{\partial y_-^2} \langle \hat{\eta}(1)\hat{\eta}(2) \rangle . \tag{17}$$

We further assume a radial structure which can be modeled by a Gaussian centered about a radial location x_0 with width $\Delta_{\mathbf{k}}$. Therefore,

$$\langle \hat{\eta}(1)\hat{\eta}(2) \rangle_{\mathbf{k}} \approx \frac{1}{4\pi \Delta_{\mathbf{k}}^2} \exp\left[-\frac{(x_1 - x_0)^2}{4\Delta_{\mathbf{k}}^2} - \frac{(x_2 - x_0)^2}{4\Delta_{\mathbf{k}}^2} \right] \langle \eta\eta \rangle_{\mathbf{k}}$$
$$= \frac{1}{4\pi \Delta_{\mathbf{k}}^2} \exp\left[-\frac{(x_+ - 2x_0)^2}{8\Delta_{\mathbf{k}}^2} - \frac{x_-^2}{8\Delta_{\mathbf{k}}^2} \right] \langle \eta\eta \rangle_{\mathbf{k}} .$$

The radial mode structure implies that $x_+ \approx 2x_0$, and effectively limits the quantity $(x_+ - 2x_s)$ to $2(x_0 - x_s)$. The quantity $(x_+ - 2x_s)$ is thus a measure of the shift of the mode relative to the rational surface and is of the order of the mode width $\Delta_{\mathbf{k}}$. The quantity x_-^2 represents correlation scales on the order of, or smaller than, the mode width $\Delta_{\mathbf{k}}$.

The triplet nonlinearity, Eq. (14) describes the spatially inhomogeneous turbulent scattering of resistivity by potential fluctuations. When a renormalized two-point equation is constructed from renormalized one-point equations, the inhomogeneities in the turbulent scattering are lost. However, direct renormalization of the triplet nonlinearity of the two-point equation, using standard procedures, is sufficient to retain the essential features of the triplet, including the inhomogeneities. Introducing Fourier expansions in the y and z directions, the triplet can be written as

$$T_{12} = \sum_{\mathbf{k}} \sum_{\mathbf{k}'} \sum_{\mathbf{k}''} \Bigg\langle \exp(ik_y y_1 + ik_z z_1) \exp(ik'_y y_1 + ik'_z z_1) \exp(ik''_y y_2 + ik''_z z_2) \left[ik_y \hat{\phi}_{\mathbf{k}}(1) \frac{\partial \hat{\eta}_{\mathbf{k}'}(1)}{\partial x_1} \hat{\eta}_{\mathbf{k}''}(2) - \frac{\partial \hat{\phi}_{\mathbf{k}}(1)}{\partial x_1} (ik'_y) \hat{\eta}_{\mathbf{k}'}(1) \hat{\eta}_{\mathbf{k}''}(2) \right] \Bigg\rangle + (1 \to 2).$$

Averaging over y_+ and z_+ yields factors of $\delta_{k_y + k'_y + k''_y, 0}$ and $\delta_{k_z + k'_z + k''_z, 0}$. As before one sum may thus be eliminated, with the result that the triplet is expressed in terms of test $(\mathbf{k})$, background $(\mathbf{k}')$ and 'beat' $(\mathbf{k} + \mathbf{k}')$ fluctuations. Iterative substitutions for the 'beat' fluctuations are then performed in order to close the two-point equation. The driven 'beat' resistivity, obtained from the basic resistivity equation, is

$$\hat{\eta}^{(2)}_{\mathbf{k}+\mathbf{k}'} = -\frac{1}{[\gamma_{k+k'} + (k_\parallel + k'_\parallel)^2 \chi_\parallel]} \left[ik'_y \hat{\phi}_{k'} \frac{\partial \hat{\eta}_k}{\partial x} + ik_y \hat{\phi}_k \frac{\partial \hat{\eta}_{k'}}{\partial x} \right]. \tag{18}$$

As explained in Ref. 7, the driven potential $\hat{\phi}^{(2)}_{k+k'}$ is neglected. Introducing a Markov approximation on the driven propagator and retaining radial diffusion only, we obtain the renormalized nonlinearity:

$$T_{12} = -\frac{\partial}{\partial x_-} D_- \frac{\partial}{\partial x_-} \langle \hat{\eta}(1) \hat{\eta}(2) \rangle . \tag{19}$$

where

$$D_{-} = 2D - D^{(1,2)} - D^{(2,1)}, \tag{20}$$

$$D = \sum_{k'} \left(\gamma_{k'} + k'^2_{\|}\chi_{\|}\right)^{-1} k'^2_y \left\langle \hat{\phi}_{-k'}(1)\hat{\phi}_{k'}(2) \right\rangle , \tag{21}$$

$$D^{(1,2)} = D^{(2,1)} = \sum_{k'} e^{ik'_y y_- + ik'_z z_-} \left(\gamma_{k'} + k'^2_{\|}\chi_{\|}\right)^{-1} k'^2_y \left\langle \hat{\phi}_{-k'}(1)\hat{\phi}_{k'}(2) \right\rangle , \tag{22}$$

The turbulent diffusion D_- is not homogeneous. The terms $D^{(1,2)}$ and $D^{(2,1)}$ depend strongly on the relative separation. This accounts for correlation at short separation, since fluid elements in close proximity experience nearly the same scattering force. At large relative separation $D^{(1,2)}$ and $D^{(2,1)}$ vanish and two fluid elements diffuse independently at a rate governed by D. The correlation peaks when the relative separation is less than the scales on which $D^{(1,2)}$ and $D^{(2,1)}$ differ from zero. These scales are referred to as the correlation scales. From Eq. (22) these scales are found to be the typical mode width scales in the three coordinates x, y and z. For relative separations (measured by $(x_-^2 + y_-^2 + z_-^2)^{1/2}$) less than the corresponding measure of the correlation scales $(k_{0x}^{-2} + k_{0y}^{-2} + k_{0z}^{-2})^{1/2}$, the relative diffusion coefficient D_- is quadratic in (x_-, y_-, z_-), i.e.,

$$D_- = 2D\left(k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2\right) \tag{23}$$

To complete the derivation of the renormalized two-point equation, it is useful to express the source S (Eq. (15)) in terms of the potential fluctuation spectrum $\left\langle \hat{\phi}_k(1)\hat{\phi}_{-k}(2) \right\rangle$, by using the decoupling condition $B_z \nabla_{\|}\hat{\phi} = -J_0\hat{\eta}$. This yields

$$S = \sum_k \frac{d\eta_0/dr}{J_0} B_z k_y^2 \frac{(x_+ - 2x_s)}{L_s} \left\langle \hat{\phi}_k^{(1)}\hat{\phi}_{-k}^{(2)} \right\rangle e^{ik_y y_-} e^{ik_z z_-} . \tag{24}$$

We note that the x_- contributions to $k_{\|}$ from the $\left\langle \dfrac{\partial\hat{\phi}_1}{\partial y_1}\hat{\eta}(2)\right\rangle$ and $\left\langle \dfrac{\partial\hat{\phi}_2}{\partial y_2}\hat{\eta}(1)\right\rangle$ parts of the source cancel. We also observe that since $\partial\eta_0/\partial r > 0$, the mode shifts outward relative to the rational surface x_s, so that $x_+ - 2x_s > 0$. Thus, in terms of the relative diffusion D_- and Eqs. (17) and (24) the renormalized two-point equation is

$$\left\{\frac{\partial}{\partial t}-\frac{\chi_{\|}}{2}\left[\frac{(x_+-2x_s)^2}{L_s^2}+\frac{x_-^2}{L_s^2}\right]\frac{\partial^2}{\partial y_-^2}-D_-\frac{\partial^2}{\partial x_-^2}\right\}\langle\hat{\eta}(1)\hat{\eta}(2)\rangle=\sum_k\frac{d\eta_0/dr}{J_0}B_zk_y^2\frac{(x_+-2x_s)}{L_s}$$
$$\left\langle\hat{\phi}(1)\hat{\phi}(2)\right\rangle_{\mathbf{k}}\exp(ik_yy_-+ik_zz_-). \tag{25}$$

In order to determine the stationary fluctuation spectrum, it is necessary to find the steady-state solution of Eq. (25). This requires inversion of the evolution operator

$$\mathcal{L}=\frac{\partial}{\partial t}-\frac{\chi_{\|}}{2}\left[\frac{(x_+-2x_s)^2}{L_s^2}+\frac{x_-^2}{L_s^2}\right]\frac{\partial^2}{\partial y_-^2}-D_-\frac{\partial^2}{\partial x_-^2},$$

which contains turbulent relative diffusion and the parallel thermal conductivity terms. To approximate the solution, it is sufficient to determine the correlation decay time τ_{cl} associated with this operator. This decay time, the average lifetime of the two-point correlation $\langle\hat{\eta}(1)\hat{\eta}(2)\rangle$, is given by the average time for two points originally separated by a distance $(x_-^2+y_-^2+z_-^2)^{1/2}\ll(k_{0x}^{-2}+k_{0y}^{-2}+k_{0z}^{-2})^{1/2}$ to diffuse to a separation equivalent to a correlation scale. The time evolution of the relative position of the two points can be determined from the moments of the Green's function, g, which satisfies the equation

$$\left(\frac{\partial}{\partial t}-\frac{\chi_{\|}}{2}\left[\frac{(x_+-2x_s)^2}{L_s^2}+\frac{x_-^2}{L_s^2}\right]\frac{\partial^2}{\partial y_-^2}-D_-\frac{\partial^2}{\partial x_-^2}\right)g=0. \tag{26}$$

Taking the moments of Eq. (26), we obtain

$$\frac{\partial}{\partial t}\left\langle x_-^2\right\rangle=4D\left(k_{0y}^2\left\langle y_-^2\right\rangle+k_{0z}^2\left\langle z_-^2\right\rangle+k_{0x}^2\left\langle x_-^2\right\rangle\right), \tag{27a}$$

$$\frac{\partial}{\partial t}\left\langle y_-^2\right\rangle=\chi_{\|}\left[\frac{(x_+-2x_s)^2}{L_s^2}+\frac{\left\langle x_-^2\right\rangle}{L_s^2}\right] \tag{27b}$$

$$\frac{\partial}{\partial t}\left\langle z_-^2\right\rangle=0, \tag{27c}$$

where $\langle A(t)\rangle=\int dx_-\int dy_-\int dz_-A(t)g$. From Eqs. (27) we note that $\left\langle x_-^2\right\rangle$ grows exponentially. Therefore we eliminate $\left\langle y_-^2\right\rangle$ and $\left\langle z_-^2\right\rangle$ in Eq. (27a) using (27b) and (27c), and obtain the evolution equation for $\left\langle x_-^2\right\rangle$:

$$\frac{\partial^2}{\partial t^2}\left\langle x_-^2\right\rangle-4k_{0x}^2D\frac{\partial}{\partial t}\left\langle x_-^2\right\rangle-4k_{0y}^2D\chi_{\|}\frac{\left\langle x_-^2\right\rangle}{L_s^2}=4Dk_{0y}^2\chi_{\|}\left[\frac{(x_+-2x_s)^2}{L_s^2}\right]. \tag{28}$$

Equation (28) is inhomogeneous. The inhomogeneous term corresponds to decay of resistivity correlation by parallel thermal diffusion. The particular solution of Eq. (28) is

$$\left\langle x_-^2 \right\rangle_p = (x_+ - 2x_s)^2. \tag{29}$$

It is the homogeneous solution which describes the exponential separation of neighboring fluid elements by turbulent scattering. The homogeneous part of $\left\langle x_-^2 \right\rangle$ is solved as an initial value problem in which the initial values of $\left\langle x_-^2 \right\rangle$, $\left\langle y_-^2 \right\rangle$ and $\left\langle z_-^2 \right\rangle$ $\left(\left\{ \left\langle x_-^2 \right\rangle, \left\langle y_-^2 \right\rangle, \left\langle z_-^2 \right\rangle \right\} \Big|_{t=0} = \left\{ x_-^2, y_-^2, z_-^2 \right\} \right)$ determine the first and second derivatives of $\left\langle x_-^2 \right\rangle$ at the initial time; i.e.,

$$\frac{\partial}{\partial t} \left\langle x_-^2 \right\rangle \Big|_{t=0} = 4D(k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2) \tag{30a}$$

$$\begin{aligned} \frac{\partial^2}{\partial t^2} \left\langle x_-^2 \right\rangle \Big|_{t=0} &= 16 k_{0x}^2 D^2 (k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2) \\ &\quad + 4Dk_{0y}^2 \chi_\parallel \frac{\chi_-^2}{L_s^2} + 4Dk_{0y}^2 \chi_\parallel \frac{(x_+ - 2x_s)^2}{L_s^2} \end{aligned} \tag{30b}$$

The homogeneous solution of Eq. (28) is thus given by

$$\left\langle x_-^2 \right\rangle_h = \tilde{A} e^{u_+ t} + \tilde{B} e^{u_- t} \tag{31}$$

where $u_\pm$ are the two roots of the quadratic characteristic equation generated by the trial solution e^{ut}, i.e., $u^2 - 4k_{0x}^2 Du - 4\frac{k_{0y}^2 D\chi_\parallel}{L_s^2} = 0$. The two roots are given by

$$u_\pm = 2Dk_{0x}^2 \pm 2 \left[(k_{0x}^2 D)^2 + \frac{Dk_{0y}^2 \chi_\parallel}{L_s^2} \right]^{1/2} . \tag{32}$$

$\tilde{A}$ and $\tilde{B}$ are determined by requiring that Eq. (31) satisfy the initial conditions given in Eqs. (30). Thus,

$$\tilde{A} = \left(\frac{1}{u_+} \frac{\partial^2}{\partial t^2} \left\langle x_-^2 \right\rangle \Big|_{t=0} - \frac{u_-}{u_+} \frac{\partial}{\partial t} \left\langle x_-^2 \right\rangle \Big|_{t=0} \right) (u_+ - u_-)^{-1} \tag{33a}$$

$$\tilde{B} = \left(\frac{u_+}{u_-} \frac{\partial}{\partial t} \left\langle x_-^2 \right\rangle \bigg|_{t=0} - \frac{1}{u_-} \frac{\partial^2}{\partial t^2} \left\langle x_-^2 \right\rangle \bigg|_{t=0} \right) (u_+ - u_-)^{-1}. \tag{33b}$$

We note from Eq. (32) that $u_- < 0$, so that at large $t(t > u_-^{-1})$ $\left\langle x_-^2 \right\rangle$ is determined primarily by the exponentially growing term. For initial separations which are much smaller than the correlation scales, t becomes large before $\left\langle x_-^2 \right\rangle$ reaches the correlation scale. We may thus write

$$\left\langle x_-^2 \right\rangle \cong \tilde{A} e^{u_+ t} - (x_+ - 2x_s)^2, \tag{34}$$

an approximation permissible when t is of the order of the correlation lifetime, τ_{cl}, the time required for $\left\langle x_-^2 \right\rangle$ to reach the scale k_{0x}^{-2}. We obtain τ_{cl} from Eq. (33) by solving for t such that $\left\langle x_-^2 \right\rangle = k_{0x}^{-2}$. Thus

$$\tau_{\mathrm{cl}} = \frac{1}{u_+} \ln \left\{ \frac{k_{0x}^{-2} + (x_+ - 2x_s)^2}{\tilde{A}} \right\}, \tag{35}$$

where $\tilde{A}$ is given by Eqs. (33a), (32) and (30).

Following Ref. 8, it is useful and instructive to define a Reynolds number as the ratio of the nonlinear term to the linear dissipative term in the resistivity equation. Thus

$$\mathrm{Re} = \frac{D k_{0x}^4 L_s^2}{k_{0y}^2 \chi_\|} = k_{0x}^2 D \left(k_{0y}^2 \chi_\| / k_{0x}^2 L_s^2 \right)^{-1} \approx \tau_\| / \tau_c. \tag{36}$$

The Reynolds number parametrizes the inertial and dissipation ranges. In the inertial range corresponding to $\mathrm{Re} \gg 1$, the nonlinearity dominates dissipation and the dynamics are conservative. In the dissipation range, for which $\mathrm{Re} < 1$, the dissipation is larger than the nonlinearity. For the temperature gradient driven turbulence model considered here, the nonlinearity effectively regulates the balance of gradient relaxation with dissipation by determining the radial scale of the fluctuations; i.e., $\chi_\| (k_{0y}^2 / L_s^2) \Delta_{\mathbf{k}}^2 \sim D_{\mathbf{k}} / \Delta_{\mathbf{k}}^2 \sim \dfrac{L_s}{L_\eta} \dfrac{E_0}{B_z \Delta_{\mathbf{k}}}$ which effectively sets $\Delta_{\mathbf{k}}(D_{\mathbf{k}})$ and $D_{\mathbf{k}}$. This requires that $\tau_\| = [(\chi_\| (k_{0y}^2 / L_s^2) \Delta_{\mathbf{k}}^2]^{-1} \approx \tau_c = (D/\Delta_{\mathbf{k}}^2)^{-1}$, so that stationary resistivity gradient driven turbulence intrinsically has Reynolds number of order unity ($\mathrm{Re} \sim 1$) and lies between the purely inertial and dissipation ranges. Though it has been touched upon already, it is worthwhile observing that, unlike homogeneous Navier-Stoke turbulence, Reynolds' numbers of order unity are not inconsistent with fully developed resistivity-gradient-driven turbulence. We recall that relaxation of the average resistivity gradient directly drives fluctuations throughout the spectrum. In addition, the modes are closely spaced and strongly overlapping, so that nonlinear interaction is significant. This, and the fact that dissipation of energy is achieved principally through nonlinear broadening of turbulent fluctuations rather than through a cascade process imply that the Reynolds' number, as defined above, is necessarily of order unity for fully developed turbulence of this type.

In terms of the Reynolds number Re, we can write $\tau_{\rm cl}$ as

$$\tau_{\rm cl} = \frac{1}{2k_{0x}^2 D[1+(1+{\rm Re}^{-1})^{1/2}]} \ell n \left(\frac{1}{a_x k_{0x}^2 x_-^2 + a_y k_{0y}^2 y_-^2 + a_z k_{0z}^2 z_-^2 + a} \right), \tag{37}$$

where

$$\left.\begin{aligned} a_x &= \frac{1+\frac{1}{2}{\rm Re}^{-1} + (1+{\rm Re}^{-1})^{1/2}}{1+{\rm Re}^{-1}+(1+{\rm Re}^{-1})^{1/2}} \left[1+k_{0x}^2(x_+ - 2x_s)^2\right]^{-1} \\ a_y = a_z &= \frac{1}{(1+{\rm Re}^{-1})^{1/2}} \left[1+k_{0x}^2(x_+ - 2x_s)^2\right]^{-1}, \\ a &= \frac{1}{2} \frac{{\rm Re}^{-1}}{(1+{\rm Re}^{-1}+(1+{\rm Re}^{-1})^{1/2})} \left[\frac{k_{0x}^2(x_+ - 2x_s)^2}{1+k_{0x}^2(x_+ - 2x_s)^2}\right]. \end{aligned}\right\} \tag{38}$$

The factor multiplying the logarithmic function in $\tau_{\rm cl}$ represents the one-point or asymptotic (large separation) response. We note from the definition of Re that the results of the one-point theory[9] are recovered for Re ~ 1. Since the equations for relative diffusion, Eqs. (19)-(21), imply that the radial correlation scale k_{0x}^{-1} is the mode width in x while the correlation scale in the y direction k_{0y}^{-1} corresponds to a typical wavelength in the spectrum, we observe that Re $= 1$ implies that $k_{0x}^{-4} = \Delta_{\mathbf{k}_0}^4 = DL_s^2/(k_{0y}^2 \chi_\parallel)$, consistent with Eq. (11). The one-point response correctly asymptotes to the familiar turbulent response time $(k_{0x}^2 D)^{-1}$ in the limit of large Reynolds number where relative $\mathbf{E} \times \mathbf{B}_0$ convection alone determines decay of correlation. In the opposite limit, $\tau_{\rm cl} \sim (\tau_\parallel \tau_c)^{1/2}$ (i.e., the geometric mean of the dissipative and turbulent relaxation times), as previously determined for the case of dissipative drift waves.[8] The logarithmic factor yields a peaked correlation function for small separation, with the peaking becoming increasingly sharp as $\mathrm{Re} \to \infty$, in which case $a_x, a_y, a_z \to 1$, $a \to 0$, and the correlation function is singular as $\mathbf{x}_- \to 0$. The correlation lifetime function for several Reynolds numbers is plotted in Fig. 1.

IV. Calculation of Fluctuation Spectra

Using $\tau_{\rm cl}$ given in Eq. (37), we obtain the stationary spectrum equation from the steady-state solution of the two-point resistivity equation, Eq. (25),

$$\langle \hat{\eta}(1)\hat{\eta}(2)\rangle = \tau_{\rm cl} S = \tau_{\rm cl} \sum_k \frac{d\eta_0/dr}{J_0} B_z k_y^2 \frac{(x_+ - 2x_s)}{L_s} \left\langle \hat{\phi}_k(1)\hat{\phi}_{-k}(2)\right\rangle . \tag{39}$$

We note that since two-points are correlated only if $(k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2)^{1/2} < 1$, the approximation $\exp(ik_y y_- + ik_z z_-) \simeq 1$ has been used in the source term S. Similarly, the x-dependence in $\left\langle \hat{\phi}_k(1)\hat{\phi}_{-k}(2)\right\rangle$ is also neglected. Using the decoupling condition, we can write Eq. (39) as

$$B_z^2 \frac{1}{J_0^2} \left\langle \nabla_{\parallel 1}\hat{\phi}(1)\nabla_{\parallel 2}\hat{\phi}(2)\right\rangle = \tau_{\rm cl} S. \tag{40}$$

The stationary spectrum condition determines the fluctuation level at which the decay of correlation by thermal conduction and turbulent diffusion matches drive by the relaxation of the temperature gradient. To obtain the wavenumber spectrum we use the Fourier representation of the left-hand side of Eq. (40),

$$B_z^2 \frac{1}{J_0^2} \frac{(x_1 - x_s)(x_2 - x_s)}{L_s^2} \sum_k k_y^2 \left\langle \hat{\phi}_k(1)\hat{\phi}_{-k}(2)\right\rangle \exp(ik_y y_- + ik_z z_-) = \tau_{\rm cl} S. \tag{41}$$

Fourier transforming both sides of Eq. (40) and writing x_1 and x_2 in terms of x_+ and x_- we obtain

$$\frac{B_z^2}{4J_0^2} \frac{[(x_+ - 2x_s)^2 - x_-^2]}{L_s^2} k_y^2 \left\langle \hat{\phi}(1)\hat{\phi}(2)\right\rangle_k = \frac{1}{(2\pi)^2} \int dy_- \int dz_- \tau_{\rm cl} \exp(-ik_y y_- - ik_z z_-) S. \tag{42}$$

To evaluate summations over k, we take the continuum limit so that summations become integrals, and also assume that the radial structure of $\left\langle \hat{\phi}(1)\hat{\phi}(2)\right\rangle_k$ is Gaussian, i.e.,

$$\left\langle \hat{\phi}(1)\hat{\phi}(2)\right\rangle_{\mathbf{k}} = \left\langle \hat{\phi}\hat{\phi}\right\rangle_k \frac{1}{4\pi\Delta_k^2} \exp\left\{ -\frac{[x_1 - x_0]^2 + [x_2 - x_0]^2}{4\Delta_{\mathbf{k}}^2} \right\}, \tag{43}$$

which can be rewritten as

$$\left\langle \hat{\phi}(1)\hat{\phi}(2)\right\rangle_{\mathbf{k}} = \left\langle \hat{\phi}\hat{\phi}\right\rangle_k \frac{1}{4\pi\Delta_{\mathbf{k}}^2} \exp\left[-\frac{(x_+ - 2x_0)^2}{8\Delta_{\mathbf{k}}^2} - \frac{x_-^2}{8\Delta_{\mathbf{k}}^2} \right].$$

Note that the radial asymmetry of the fluctuations is accounted for in Eq. (43). Integrating the left-hand side of Eq. (43) over x_+ and x_- yields

$$\frac{B_z^2}{J_0^2}\frac{2(x_0-x_s)^2}{L_s^2}k_y^2\left\langle\hat{\phi}_k\hat{\phi}_{-k}\right\rangle = \int_{-\infty}^{\infty}dx_+\int_{-\infty}^{\infty}dx_-\int dy_-\int dz_- e^{-ik_y y_- - ik_z z_-}$$

$$\tau_{\rm cl}(x_+,x_-,y_-,z_-)\frac{B_z}{J_0}\frac{d\eta_0}{dr}\frac{1}{(2\pi)^2}\frac{(x_+-2x_s)}{L_s}\int d\mathbf{k}' k_y'^2\frac{\left\langle\hat{\phi}_{k'}\hat{\phi}_{-k'}\right\rangle}{4\pi\Delta_{\mathbf{k}}^2}\exp\left[-\frac{(x_+-2x_0)^2}{8\Delta_{\mathbf{k}}^2}\right],$$

where the x_-, y_- and z_- integrations involve only $\tau_{\rm cl}$. Details of these integrations are given in the appendix. The result is

$$\int dx_-\int dy_-\int dz_-\tau_{\rm cl}\exp(-ik_y y_- - ik_z z_-)$$
$$= \frac{1}{2k_{0x}^2 D}G(a)F(k_\perp)\frac{8\pi\Delta_{\mathbf{k}}}{\sqrt{a_x a_y a_z}k_{0y}k_{0z}}\frac{1}{[1+(1+{\rm Re}^{-1})^{1/2}]} \tag{44}$$

where

$$G(a) = \frac{\sqrt{1-a}}{9} - \frac{4a\sqrt{1-a}}{9} + \frac{1}{3}a^{3/2}\cos^{-1}(\sqrt{a}) \tag{45}$$

$$F(k_\perp) = \frac{I(k_\perp)}{I(k_\perp=0)} \tag{46}$$

$$I(k_\perp) = \int_0^{\sqrt{1-a}}dk k(1-a-k^2)^{1/2}J_0(kk_\perp)$$
$$-\int_0^{\sqrt{1-a}}dk k\sqrt{k^2+a}\left(\cos^{-1}\sqrt{k^2+a}\right)J_0(kk_\perp) \tag{47}$$

and

$$k_\perp = \left[k_y^2/(a_y k_{0y}^2) + k_z^2/(a_z k_{0z}^2)\right]^{1/2}.$$

The quantities a, a_x, a_y, a_z all depend on x_+ (Eqs. (38)). The function $F(k_\perp)$ has been numerically calculated and is plotted in Fig. 2. To very good approximation, $F(k_\perp)$ decays as a Gaussian over the first two decades of the range of decay, and can be fit well by the functional form $F(k_\perp) = \exp[-k_\perp^2/16.6]$, for ${\rm Re}=1$. Writing the x_+ dependence explicitly yields,

$$F(k_\perp) = \exp\left\{-\left[\frac{k_y^2}{k_{0y}^2}+\frac{k_z^2}{k_{0z}^2}\right]\frac{\sqrt{2}}{16}[1+k_{0x}^2(x_+-2x_s)^2]\right\}.$$

To perform the integration over x_+, we note that $F(k_\perp)$ varies slowly and is therefore of order unity for the region of variation of $\exp[-(x_+ - 2x_0)^2/8\Delta_{\mathrm{k}}^2]$. Therefore, the radial mode structure fixes x_+ at the position of the mode. The integration replaces x_+ with $2x_0$ (noting $dx \to \sqrt{8\pi}\Delta_{\mathrm{k}}$). Thus, the spectrum is given by:

$$k_y^2 \left\langle \hat{\phi}_k \hat{\phi}_{-k} \right\rangle = \frac{L_s}{L_\eta} \left(\frac{E_0}{B_z}\right) \frac{1}{(x_0 - x_s)} \frac{1}{\sqrt{2\pi^3}} \frac{1}{\sqrt{a_x a_y a_z}} \frac{1}{k_{0z} k_{0y}}$$
$$\frac{1}{k_{0x}^2 D} \int d\mathrm{k}' k_y'^2 \left\langle \hat{\phi}_{k'} \hat{\phi}_{-k'} \right\rangle \frac{G(a) F(k_\perp)}{[1 + (1 + \mathrm{Re}^{-1})^{1/2}]} \Bigg|_{x_+ = 2x_0} \tag{48}$$

where x_+ is evaluated at $x_+ = 2x_0$ throughout. Thus,

$$\left.\begin{aligned} a &= \frac{\frac{1}{2}\mathrm{Re}^{-1}}{[1 + \mathrm{Re}^{-1} + (1 + \mathrm{Re}^{-1})^{1/2}]} \frac{k_{0x}^2 (2x_0 - 2x_s)^2}{[1 + k_{0x}^2 (2x_0 - 2x_s)^2]} \\ a_x &= \frac{[1 + \frac{1}{2}\mathrm{Re}^{-1} + \sqrt{1 + \mathrm{Re}^{-1}}]}{[1 + \mathrm{Re}^{-1} + \sqrt{1 + \mathrm{Re}^{-1}}]} \frac{1}{[1 + k_{0x}^2 (2x_0 - 2x_s)^2]} \\ a_z = a_y &= \frac{1}{\sqrt{1 + \mathrm{Re}^{-1}}} \frac{1}{[1 + k_{0x}^2 (2x_0 - 2x_s)^2]}. \end{aligned}\right\} \tag{49}$$

The spectrum as a function of k_y for fixed k_z is plotted in Fig. 3. The wavenumber dependence of the spectrum $k_y^2 \left\langle |\hat{\phi}|^2 \right\rangle_k$ is contained entirely in the function $F(k_\perp)$. $F(k_\perp)$ is the Fourier transform in k_y and k_z of the two-point decay time τ_{cl}, which is a logarithmic function of $a_x k_{0x}^2 x_-^2 + a_y k_{0y}^2 y_-^2 + a_z k_{0z}^2 z_-^2$. Several features of the spectrum may be directly attributed to the functional form of τ_{cl}. As mentioned previously, for $k_\perp < 7 \left(k_\perp = \sqrt{k_y^2/k_{0y}^2 a_y + k_z^2/k_{0z}^2 a_z}\right)$, the spectrum has a Gaussian shape. The width of the Gaussian depends on the Reynolds number. For large Reynolds number, an inertial range exists and the $k_\perp$ spectrum width is independent of Reynolds number, to lowest order in Re^{-1}. For small Reynolds number, the width decreases with the one fourth root of decreasing Reynolds number. The Gaussian shape characterizes the spectrum only for values of $k_y/k_{0y} \lesssim 1.5 (k_\perp \lesssim 7)$, over which the spectrum decays by nearly two orders of magnitude. For $k_y/k_{0y} > 1.5$, asymptotic evaluation of the Fourier transform of τ_{cl} indicates that the spectrum decays as $k_\perp^{-2}$. In this region $F(k_\perp)$ oscillates about zero.

This behavior is attributable to the fact that the Fourier transform of $\tau_{\rm cl}$ is cut off beyond $k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2 = 1$ because of the expansion of D_- made in Eq. (23). As these oscillations occur in a region where the energy content of the spectrum is negligible, they are of no consequence.

Although the spectrum shape is best approximated by a Gaussian, it is useful to compare it to a power law (i.e., $k_y^{-\alpha}$) fit for purposes of comparison with the spectrum obtained from the numerical calculations. A least-squares fit of the spectrum over approximately one decade in k_y/k_{0y}, which corresponds to two decades of decay from the peak, yields a power law dependence of $k_y^{-1.25}$. The power law fit with $\alpha \cong 1.25$ is in excellent agreement with the spectrum obtained from the numerical calculations. Detailed comparison of the numerically calculated spectrum with this power law is presented in the section describing the numerical results. The theoretical spectrum, showing the least-squares fit is given in Fig. 4.

Another check of the accuracy of the theoretically predicted spectrum is the comparison of the predicted mean-square radial velocity with that obtained from the numerics. The mean-square radial velocity is obtained by integrating the $\langle \hat{v}_r \rangle_{\mathbf{k}}$ spectrum over k_y and k_z. Thus integrating both sides of Eq. (47) over k_y and k_z, the mean-square radial velocity $\bar{E} = \langle v_r^2 \rangle = \int dk_y \int dk_z k_y^2 \left\langle \hat{\phi}^2 \right\rangle_k$ cancels out. Solving for D and noting that $x_0 - x_s \sim 2\Delta_{\mathbf{k}} \sim 2k_{0x}^{-1}$ and $\mathrm{Re} = D\Delta_{\mathbf{k}}^{-4} L_s^2/(k_{0y}^2 \chi_\parallel) \approx 1$ imply that $\Delta_{\mathbf{k}} = D^{1/4} L_s^{1/2}/(\chi_\parallel^{1/4} k_{0y}^{1/2})$ yields:

$$D = \left(\frac{E_0 L_s}{B_z L_\eta}\right)^{4/3} \frac{1}{(\chi_\parallel k_{0y}^2/L_s^2)^{1/3}} \left\{ \frac{\frac{1}{2} G(a) \int \frac{dk_y}{k_{0y}} \frac{dk_z}{k_{0z}} F(k_\perp)}{\sqrt{2\pi^3 a_x a_y a_z}[1 + (1 + \mathrm{Re}^{-1})^{1/2}]} \right\}^{4/3} . \quad (50)$$

This scaling is identical to that of the result of the one-point theory, with the additional numerical factor given in the brackets. We evaluate this factor and compare with the simulation results by computing the mean-square radial flux $\bar{E}$. Integrating both sides of

Eq. (48) to form $\bar{E}$ on the left-hand side, and substituting for D using the relation

$$D \approx \int dk_z \int dk_y \left\{ \frac{k_y^2 \left\langle \hat{\phi}^2 \right\rangle_k}{4\Delta_{\mathbf{k}}^2 \chi_\| k_y^2 / L_s^2} \right\},$$

obtained from the definition of D, Eq. (21), yields

$$\bar{E} = 2\left(\frac{E_0 L_s}{B_z L_\eta}\right) \Delta_{\mathbf{k}}^3 \frac{k_y^2}{L_s^2} \chi_\| \frac{\int \frac{dk_z}{k_{0z}} \int \frac{dk_y}{k_{0y}} F(k_\perp) G(a)}{\sqrt{2\pi^3 a_x a_y a_z}[1 + (1 + \mathrm{Re}^{-1})^{1/2}]}.$$

For $\Delta_{\mathbf{k}}$ we use the Reynolds number definition evaluated at Re $= 1$, i.e., $\Delta_{\mathbf{k}} = D^{1/4} L_s^{1/2} / (\chi_\|^{1/4} k_{0y}^{1/2})$, and substitute for D from Eq. (50). The result is

$$\bar{E} = 2\left(\frac{E_0 L_s}{B_z L_\eta}\right)^2 \left\{ \frac{G(a) \int \frac{dk_z}{k_{0z}} \int \frac{dk_y}{k_{0y}} F(k_\perp)}{\sqrt{2\pi^3 a_x a_y a_z}(1 + \sqrt{1 + \mathrm{Re}^{-1}})} \right\}^2 \tag{51}$$

With the Gaussian approximation $(F(k_\perp) = \exp[-k_\perp^2/16])$ the integral $(a_y a_z)^{-1/2} \int dk_z / k_{0z} \int dk_y / k_{0y} F(k_\perp)$ equals 16π. Evaluating the remaining factors yields

$$\bar{E} = \int dk_y \int dk_z \left\langle \hat{\phi}_k \hat{\phi}_{-k} \right\rangle = 1.89 \left(\frac{L_s E_0}{L_\eta B_z}\right)^2. \tag{52}$$

In terms of the resistive decay time $\tau_R = \mu_0 a^2 / \eta_0$, the mean-square radial flux is

$$\bar{E} = 369\ a^2 / \tau_R^2 \tag{53}$$

which in excellent agreement with the numerical prediction of $\bar{E} = 375 \pm 25\ a^2/\tau_R^2$. It should be cautioned, however, that approximations and uncertainties in the magnitude of modewidths and mean wavenumbers produce considerble uncertainty in the mean-square radial flux. It is worthwhile to notice that the two-point theory prediction $\bar{E}_{\mathrm{2pt.}} = 1.89(L_s E_0 / L_\eta B_z)^2$ is in good agreement with the one-point theory prediction[9] $\bar{E}_{\mathrm{1pt.}} = 1.55(L_s E_0 / L_\eta B_z)^2$. This follows from the fact that since Re ≈ 1, $\tau_{\mathrm{cl}}(\mathbf{x}_-)$ is not sharply peaked as $|\mathbf{x}_-| \to 0$. Hence $\bar{E}_{\mathrm{2pt.}}$, which is effectively a measure of the relative coordinate-integrated correlation time $\int d\mathbf{x}_- \tau_{\mathrm{cl}}(\mathbf{x}_-)$, does not differ significantly from

$\bar{E}_{1\mathrm{pt.}}$. By way of contrast, a more distinct disparity would be expected in the case where Re $\gg 1$.

Several approximations are made in order to invert the evolution operator of the two-point equation. These include the expansion of D_- for small relative separation, the solution of the moment equations for the regime of exponential separation, and the cut-off imposed on τ_{cl} for $k_{0x}^2 x_-^2 + k_{0y}^2 y_-^2 + k_{0z}^2 z_-^2 > 1$. These approximations certainly affect the shape of the spectrum. It is worth noting, however, that despite these approximations the least-squares fit of a power law to the spectrum and the integrated spectrum agree well with the simulation results. This means that approximations notwithstanding, the physics retained in the approximate expressions for the relative diffusion and correlation lifetime is sufficient to accurately describe the distribution of energy in the steady-state spectrum. This is not fortuitous. Turbulence driven by gradients, as pointed out, is driven at all scales present in the spectrum. The correlation scales are fixed at scales comparable to the driving scales by typical wavenumbers in the spectrum.

Although resistivity gradient driven turbulence is predominantly electrostatic, there are also current and magnetic field fluctuations localized near the mode rational surface. The spectrum of magnetic fluctuations is of special interest, in view of experimental studies of magnetic turbulence currently in progress.[13] Near the resonant surface, the current fluctuation is given by

$$\hat{J}_{\|} \cong -J_0(r)\hat{\eta}/\eta_0. \tag{54}$$

Integrating over x, recalling that $x_{0_{\mathbf{k}}} \sim \Delta_{\mathbf{k}}$ at saturation and that $\Delta' \sim -2k_y$, yields

$$\delta B_{r_{\mathbf{k}}} \approx J_0(r)\Delta_{\mathbf{k}}\frac{\hat{\eta}_{\mathbf{k}}}{\eta_0}. \tag{55}$$

Thus, the magnetic fluctuation spectrum is approximately given by $\langle \delta B_r^2 \rangle_{\mathbf{k}} \approx J_0^2 \Delta_{\mathbf{k}}^2 \langle (\hat{\eta}/\eta_0)^2 \rangle_{\mathbf{k}}$. Hence, since $\Delta_{\mathbf{k}} \sim k^{-2/3}$ and $J_0\hat{\eta}_{\mathbf{k}} \sim \Delta_{\mathbf{k}}\hat{v}_{r_{\mathbf{k}}}/L_s$, it follows that $\langle \delta B_r^2 \rangle_{\mathbf{k}} \sim \Delta_{\mathbf{k}}^4 \langle \hat{v}_r^2 \rangle_{\mathbf{k}} / L_s^2 \sim k^{-3.9}$. This power law fit is in reasonable agreement with the results of the numerical calculations.

V. Results of Numerical Calculations

Numerical calculations of resistivity driven turbulence[8] have been done using the reduced resistive magnetohydrodynamic equations and an evolution for the resistivity, i.e.,

$$\frac{\partial \psi}{\partial t} - B_z \nabla_{\|} \phi = \eta J_z \tag{56}$$

$$\rho_m \frac{d}{dt} \nabla_{\perp}^2 \phi = B_z \nabla_{\|} J_z \tag{57}$$

$$\frac{d\eta}{dt} = \nabla_{\|}(\chi_{\|} \nabla_{\|} \eta). \tag{58}$$

Although the electrostatic approximation is used in the analytical theory, no further approximations are made in the numerical calculations.

Equations (56)-(58) are solved by a nonlinear, three-dimensional (3-D) initial value code KITE.[14] The numerical method used in this code is a finite difference representation for the radial variable r, and spectral representations in the poloidal angle θ and toroidal angle ς. The equilibrium and physical parameters considered[9] correspond to those of the Macrotor tokamak. The electron temperature profile is given by:

$$T_{e0}(r) = T_{e0}(0)[1 + (r/r_0)^2]^{-4/3}. \tag{59}$$

The resistivity profile is then obtained using the Spitzer relation. The equilibrium current profile, J_{0z}, is chosen to be consistent with a resistive equilibrium. The safety factor q is determined by taking $q(0) = 1$, while $q(a) = 3.4$ for $r_0 = 0.632a$. The resistivity driven turbulence is localized in an annular region in minor radius between $r = .65a$ and $r = .9a$. The dimensionless parameters $S = 10^5$ (evaluated at the magnetic axis) and $\bar{\chi}_{\|} = 2.5 \times 10^5$ are chosen to correspond to a peak electron density of $n = 10^{13}\text{cm}^{-3}$, a toroidal field $B_z = 2kG$, and an electron temperature $T_e = 15eV$ at $r = .775a$, the center of the (annular) region where the resistivity gradient-driven turbulence is localized.

As discussed in Sec. II, a crucial element in the theory of resistivity gradient-driven turbulence is the nonlinear radial scale length $\Delta_{\mathbf{k}} = (D_{\mathbf{k}} L_s^2/\chi_{\|} k_y^2)^{1/4} =$

$(L_s E_0/L_\eta B_z)^{1/3}(\chi_\| k_y^2/L_s^2)^{-1/3}$. Hence, it is worthwhile to determine whether the numerically calculated radial correlation length is consistent with the theoretically predicted $\Delta_{\mathbf{k}}$. A particularly convenient but detailed test is to examine the scaling of $\Delta_{\mathbf{k}}$ with poloidal mode number m. Assuming that the potential fluctuation spectrum has a Gaussian radial structure (i.e., as in Eq. (42)), the $\mathbf{k} = (m, n)$ harmonic component can be written as $\left\langle\hat{\phi}\right\rangle_{\mathbf{k}} = \left\langle\hat{\Phi}^2\right\rangle_{\mathbf{k}} (4\pi\Delta_{\mathbf{k}})^{-1}\exp[-x^2/2\Delta_{\mathbf{k}}^2]$, where $\hat{\phi}_{\mathbf{k}}$ is the (m,n)-harmonic component obtained from the numerical simulation and $\Delta_{\mathbf{k}}$ is the spectral width. $\Delta_{\mathbf{k}}$ is easily determined by $\Delta_{\mathbf{k}} = (2\pi)^{-1/2}\int_{r_1}^{r_2} dx \left\langle\hat{\phi}^2\right\rangle_{\mathbf{k}} / \max[\left\langle\hat{\phi}^2\right\rangle_{\mathbf{k}}]$, where the (numerical) integration is performed over the annular region $r_1 < r < r_2$ where the turbulence is localized. Figure 5 shows the numerically calculated $\Delta_{\mathbf{k}}$ (determined after $\hat{\phi}_k$ reaches the saturated state) plotted versus poloidal mode number m. $\Delta_{\mathbf{k}}$ is calculated at different times, for a 177 mode calculation, and the results are then time averaged. The solid line depicts the theoretically predicted $\Delta_{\mathbf{k}} \sim m^{-2/3}$ relation. The numerically calculated and theoretically predicted m dependency of $\Delta_{\mathbf{k}}$ agree well. Finally, it is useful to note that $\Delta_{\mathbf{k}}$ for the large m modes ($m \gtrsim 60$) is limited by the grid spacing size δ_g, where $\delta_g = 2 \times 10^{-3}a$ in this calculation.

In order to test the results of the theoretical analysis presented in this paper, the numerically calculated fluctuation spectrum must be determined. In Fig. 6a, the results at the conclusion of the 177 mode calculation are presented. The spectrum plots may be interpreted in the following way. All modes with the same n harmonic number are represented by a common symbol (triangle, circle, etc.) and a solid line is drawn connecting them. From each set of common-n number modes, the m harmonic number corresponding to the largest $\left\langle\hat{\phi}^2\right\rangle_{m,n}$ amplitude is selected, and the corresponding $\left\langle\hat{\phi}^2\right\rangle_{m,n}$ value is plotted. Note that since each group of common-n modes has a clearly identifiable maximum element (i.e., corresponding to a certain m value in the common-n group), this procedure is effectively equivalent to plotting the n-number integrated spectrum versus m. Assuming that the spectrum can be written in the form $\left\langle\hat{v}_r^2\right\rangle_m \sim m^{-\alpha}$, a least-squares fit is used to determine α from the numerical calculations. The result obtained is $\alpha = 1.2$, in good agreement with the analytic theory. Figure 6b shows plots of the common-n group maxima and corresponding fit (broken line) versus m, while in Fig. 6a, the amplitudes of all modes are plotted on a semilogarithmic scale with the corresponding fit also depicted by a broken line. The (dominant) m=5, n=2 mode is included in the fit but is not shown on the spectrum plots because of its large value. Finally, it should also be mentioned that the numerically calculated mean-square radial velocity is in good agreement with the analytical prediction of the two-point theory.

It is worthwhile to examine the sensitivity of the numerically calculated spectrum to the number of modes retained in the calculation. A similar calculation of α, performed using only 108 modes, yields the result α=1.6. This indicates that the spectrum decay rate decreases as the number of modes retained increases, until convergence is achieved. The larger value of α for the case of fewer modes is probably due to the concomittant reduction in the number of nonlinear couplings. Such a reduction makes energy transfer to small scales more difficult, thus resulting in an energy accumulation at large scales (small k_y) and an increase in the calculated value of α. The α-calculation for a 108 mode calculation demonstrates the convergence of the numerical calculations as the number of modes retained increases, since it also results in the prediction α=1.6. Figure 7 presents α as a function of time for various numbers of modes. Finally, the 108 mode calculation has been performed with finer grid spacing ($\Delta r = 10^{-3}a$ in the annular region where fluctuations occur) and yields approximately the same value of α, thus demonstrating convergence of the calculation with respect to the variation of grid spacing size.

In addition, by considering the flux function $\hat{\psi}$ instead of the stream function $\hat{\phi}$, a similar calculation can be performed in order to determine the spectrum decay rate of radial magnetic field fluctuations, i.e., to determine α_M for $\left\langle \hat{B}_r^2 \right\rangle \sim m^{-\alpha_M}$. The results are shown in Fig. 8. In this case $\alpha_M \approx 4.4$, which is in reasonable agreement with the theoretical prediction $\alpha_M \approx 3.9$. Such agreement supports the validity of the decoupling approximation $\hat{J}_{\|} \approx 0$, used in the development of the analytical theory.

VI. Conclusions

Resistivity gradient driven turbulence provides an interesting model in light of observed fluctuations in the tokamak edge. A previous study has shown that this model is indeed relevant because thermal conductivity only weakly enters the scalings of diffusivities and fluctuation levels in the steady state, in contrast to intuition based on linear theory.[9] Fluctuation levels predicted from that study are consistent with edge measurements. Here we have extended that study to include calculation of the fluctuation spectra of the potential and magnetic field. These spectra are calculated from the theory of two-point resistivity correlation. In the saturated state, the resistivity and potential fluctuations effectively decouple from the current perturbation. Hence, the system of equations describing the turbulence is particularly simple and conducive to a detailed check on the analytic theory. This check has been made by comparing the theory with nonlinear, multiple helicity numerical simulations.

The principal results of this paper have been the calculation of the resistivity correlation function and wavenumber spectrum of the potential, resistivity and magnetic field. The two-point theory has corroborated the estimates of the one-point theory.[9] The potential wavenumber spectrum fits a power law of the form $\left\langle \hat{\phi}^2 \right\rangle_{k_\theta} \sim k_\theta^{-3.25}$. The mean-square radial velocity is $\langle \hat{v}_r^2 \rangle = 369 \; a^2/\tau_R^2$. Both of these results are in good agreement with the results of the simulations. We conclude that for resistivity gradient driven turbulence, the physics retained in the theory of two-point resistivity correlation with the usual approximations is adequate to accurately account for the distribution of energy in the spectrum in the dynamically regulated steady state.

Appendix: The Fourier Transform of the Radially Averaged Correlation Lifetime

We outline the evaluation of

$$L = \int dx_- \int dy_- \int dz_- e^{-i(k_y y_- + ik_z z_-)} \tau_{\rm cl}$$
$$= \frac{1}{2k_{0x}^2 D} \frac{1}{[1+(1+\mathrm{Re}^{-1})^{1/2}]} \int dx_- \int dy_- \int dz_- e^{-i(k_y y_- + k_z z_-)}$$
$$\ln\left\{ \left[a_x x_-^2 k_{0x}^2 + a_y y_-^2 k_{0y}^2 + a_z z_-^2 k_{0z}^2 + a \right]^{-1} \right\} \qquad (A-1)$$

where a, a_x, a_y, a_z are given in Eqs. (38). Integrating over x_- (from 0 to $[1 - a_y k_{0y}^2 y_-^2 - a_z k_{0z}^2 z_-^2 - a]^{1/2} / a_x^{1/2} k_{0x}]$) we obtain[12]

$$L = \frac{1}{k_{0x}^2 D} \cdot \frac{1}{a_x^{1/2} k_{0x}} \int dy_- \int dz_- e^{-i(k_y y_- + k_z z_-)} \left[(1-\ell^2)^{1/2} - |\ell| \cos^{-1} |\ell| \right] \qquad (A-2)$$

where $\ell^2 = a_y k_{0y}^2 y_-^2 + a_z k_{0z}^2 z_-^2 + a$. The elliptical symmetry of the integrand suggests transforming to a polar system given by

$$\left.\begin{aligned} y_- &= \frac{\kappa}{a_y^{1/2} k_{0y}} \sin\theta \\ z_- &= \frac{\kappa}{a_z^{1/2} k_{0z}} \cos\theta \\ dy_- dz_- &= \frac{-\kappa d\kappa d\theta}{(a_y a_z)^{1/2} k_{0y} k_{0z}}. \end{aligned}\right\} \qquad (A-3)$$

Under this transformation the angle integration readily yields a Bessel function,

$$\int_0^{2\pi} d\theta \exp[i(k_y y_- + k_z z_-)] = \int_0^{2\pi} d\theta \exp[-i\kappa k_\perp \sin\theta] = 2\pi J_0(k_\perp \kappa),$$

giving

$$L = \frac{4\pi[1+(1+\mathrm{Re}^{-1})^{1/2}]^{-1}}{(k_{0x}^2 D) k_{0x} k_{0y} k_{0z} (a_x a_y a_z)^{1/2}} \int_0^{\sqrt{1-a}} d\kappa \kappa J_0(k_\perp \kappa)$$
$$\left[(1-\kappa^2-a)^{1/2} - \sqrt{\kappa^2+a} \cos^{-1} \sqrt{\kappa^2+a} \right] \qquad (A-4)$$

where $k_\perp^2 = k_y^2 / a_y k_{0y}^2 + k_z^2 / a_z k_{0z}^2$. Normalizing the integral to unity at $k_\perp = 0$, we obtain

$$L = \frac{4\pi[1+(1+\mathrm{Re}^{-1})^{1/2}]^{-1}}{(k_{0x}^2 D) k_{0x} k_{0y} k_{0z} (a_x a_y a_z)^{1/2}} G(a) F(k_\perp) \qquad (A-5)$$

where

$$F(k_\perp) = I(k_\perp)/I(k_\perp = 0),$$

$$I(k_\perp) = \int_0^{\sqrt{1-a}} d\kappa\kappa J_0(k_\perp \kappa) \left[(1-\kappa^2-a)^{1/2} - \sqrt{\kappa^2+a}\cos^{-1}\sqrt{\kappa^2+a}\right] \cdot$$

and

$$G(a) = I(k_\perp = 0) = \frac{\sqrt{1-a}}{9} - \frac{4a\sqrt{1-a}}{9} + \frac{1}{3}a^{3/2}\cos^{-1}\sqrt{a}.$$

Noting that $\Delta_k = k_{0x}^{-1}$ we obtain Eq. (44).

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Figure Captions

Fig. 1: The two-point correlation lifetime as a function of relative radial separation. In the steady state the lifetime is effectively proportional to the resistivity correlation $\langle \hat{\eta}(1)\hat{\eta}(2) \rangle$.

Fig. 2: $F(k_\perp)$, the normalized Fourier transform of the radially averaged correlation lifetime. This function gives the k dependence of the spectrum.

Fig. 3: The spectrum as a function of poloidal wavenumber.

Fig. 4: The spectrum showing the least squares fit of a power law $k^{-\alpha}$. The index is found to be $\alpha = 1.25$. The k^{-2} falloff represents the envelope of oscillations in the spectrum attributable to the cutoff of $\ln\left\{\left[k_{0x}^2 a_x x_-^2 + k_{0y}^2 a_y y_-^2 + k_{0z}^2 a_z z_-^2 + a\right]^{-1}\right\}$ for arguments less than 1.

Fig. 5: Nonlinear radial scale length $\Delta_{\mathbf{k}}$ as a function of poloidal mode number m determined numerically at saturation. The solid line is the theoretical prediction.

Fig. 6: Wavenumber spectrum $k_\theta^2 \left\langle \hat{\phi}_k^2 \right\rangle$ as a function of poloidal mode number m ($k_\theta = m/r$). In a), modes with a common-n number are represented by like symbols connected by solid lines. The broken line is the best power-law fit to the maximum amplitude mode in each set of modes with common n. In b), maximum amplitude modes in each common-n set are plotted as a function of m with best power-law fit, $m^{-1.2}$, indicated by the broken line.

Fig. 7: Spectral index α as a function of time for simulations with various numbers of modes.

Fig. 8: Magnetic fluctuation wavenumber spectrum, $k_\theta^2 \left\langle |\psi_k|^2 \right\rangle$, plotted as a function of $m \left(k_\theta = \frac{m}{r}\right)$. In a), modes with a common n number are represented by like symbols connected by solid lines. The broken line is the best power-law fit to the maximum amplitude mode in each set of modes with common n. In b), maximum amplitude modes in each common-n set are plotted as a function of m with best power-law fit, $m^{-4.4}$, indicated by the broken line.