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ENHANCEMENT OF TOKAMAK ION TRANSPORT DUE TO ELECTRON COLLISIONS

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Abstract

In tokamaks where auxiliary ion heating leads to $T_i > T_e$, the neglect of electron collisions is no longer a good approximation in determining ion transport coefficients. The enhancement of the ion heat conduction due to electron collisions is determined for (a) the Pfirsch–Schluter regime and (b) the banana regime for the case where Z^i_{eff} is large. The enhancement of ion viscosity is particularly important; the contribution due to ie–collisions is approximately equal to the ii–collision term even for $T_i = T_e$.

I. Introduction

When making calculations to determine ion transport for tokamaks, such as heat conduction or viscosity, it has been the general practice to neglect the effect of collisions with electrons. This is a good approximation when $T_e > T_i$ since the electron collision frequency invariably appears combined with the coefficient (m_e/m_i); and $m_e\nu_e/m_i$ is smaller than ν_{ii} by the factor $\sqrt{2}(m_e/m_i)^{1/2}(T_i/T_e)^{3/2}$. This approximation will cease to be a good one for sufficiently large values of T_i/T_e and, in fact, it has long been recognized that in the slowing down of energetic ions injected into a plasma that if their energy exceeds $(m_i/m_e)^{1/3}T_e$, the dynamic friction with the electrons will exceed that with the plasma ions.[1] For electron collisions to become dominant for radial ion heat conduction in a tokamak the ratio T_i/T_e need not be as large as $(m_i/m_e)^{1/3}$. This is because ions in the ion distribution tail with velocity v experience a dynamic friction proportional to v^{-2} for ii-collisions but proportional to v for i-e collisions. As a result for transport processes which are strongly weighted by the ion energy the electron collisions will be relatively more important. Depending on the particular integral involved the electron term will have an extra numerical factor greater than unity.

In Section II the Pfirsch-Schluter ion heat conduction is determined by utilizing the published result of Herdan and Liley[2] for the ion heat conduction parallel to a magnetic field with electron collisions included. In this collisional regime the ion heat conduction is enhanced by the factor

$$[1+ \frac{15}{4}(\frac{m_e}{m_i})\frac{\nu_e}{\nu_{ii}}] = [1+ \frac{15\sqrt{2}}{4}(\frac{m_e}{m_i})^{1/2}(\frac{T_i}{T_e})^{3/2}]$$

and the electron contribution becomes the larger one for $T_i/T_i > 4$.

For the plateau regime, where the ion heat conduction is independent of the collision frequency, electron collisions will not change its magnitude; but since both the Pfirsch-Schluter and banana regime values are enhanced, the range of collision frequency for which the plateau formula applies will be reduced at both upper and lower limits. In the banana regime, to lowest order in $(r/R)^{1/2}$, the electron collisions do not contribute to the ion heat conduction because the important electron collisions involve energy scattering. Such scattering produces only a weak friction force parallel to the magnetic field for the trapped or nearly trapped particles, neoclassical transport being driven by this friction force. Energy scattering collisions give a contribution of order r/R and the enhancement of ion heat conduction due to ii-energy-scattering collisions was treated in reference 3. In Section III the method is given for obtaining the electron contribution to the ion heat conduction in the r/R order. The solution of a Spitzer-type problem is involved but here only a simplified version is solved which applies to the case where $Z^i_{eff} \gg 1$, where $Z^i_{eff} \equiv (n_i+\sqrt{2}\sum_z n_z Z^2)/n_i$. Two solutions are obtained. Case (a) involves the simple addition of the extra collision operator C_{ie} to the normal steady state ion drift-kinetic equation. Since this equation is not strictly valid - in the lowest order the term $C_{ie}(f_{io},f_{eo})$, which involves electron cooling of the ions, is unbalanced by any corresponding heating term or time derivative term involving $\partial T/\partial t$. A second example, case (b), is therefore solved in which

$C_{ie}(f_{io},f_{eo})$ is assumed balanced by the collisions with slowing down beam ions. For case (a) the electron collision contribution to the banana regime ion heat conduction (q_{ir}) is given by

$$(q_{ir})_{ie} = -24\, n_i(\frac{r}{R})(\nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 T_i')$$

and the enhancement over the simple banana regime formula for an impure hydrogen plasma is by the factor

$$[1+ \frac{46}{Z_{eff}^i} (\frac{r}{R})^{1/2} (\frac{m_e}{m_i})^{1/2} (\frac{T_i}{T_e})^{3/2}] \ .$$

For case (b), the numerical factors 24 and 46 are reduced to 12.2 and 24, respectively.

Section IV deals with the electron contribution to the ion viscosity for low collision frequency regimes (plateau or banana). It is first shown that the neoclassical contribution to $P_{\parallel r}$ is closely related to the $\sin\Theta$ component of the total ion heat conduction parallel to $\underset{\sim}{B}$, to be denoted by $\hat{q}_{i\parallel s}$ (Θ being the poloidal angle). One finds

$$P_{\parallel r} \underset{\sim}{>} - m\hat{q}_{i\parallel s}/2eBR \ .$$

The electron collisions drive $\hat{q}_{i\parallel s}$ because the collisional energy transfer from ions to electrons has $\cos\Theta$ components proportional to both $V_\parallel'$ and $V_\parallel T_i'/T_i$, where the prime denotes the radial gradient. For the particular balanced condition of case (b), the electron collisional contribution to $P_{\parallel r}$ is given by

$$(P_{\parallel r})_{ie} = -n_i\left(\frac{r}{R}\right)^2 \nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 m_i\left(\frac{6}{5}V_{\parallel}' + \frac{3}{2}\frac{T_i'}{T_i} V_{\parallel}\right) .$$

Assuming $T_i'/T_i \approx V_{\parallel}'/V_{\parallel}$, the ratio of this expression to the neoclassical formula allowing for only ion self collisions[4] is

$$\frac{(P_{\parallel r})_{ie}}{(P_{\parallel r})_{ii}} = 38\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}$$

which is close to unity for hydrogen even for T_i=T_e.

2. Pfirsch–Schluter Ion Heat Conduction

In the general transport equation obtained by Herdan and Liley for a collisional plasma,[2] they used Grad's thirteen-moment approximation and included both ie-and ii-collisions for ion transport. From their Eq. (54), allowing for the definitions of their non-standard symbols, the ion heat conduction parallel to a magnetic field is given by

$$q_{i\parallel} = -\frac{3.1 p_i \nabla_{\parallel} T_i}{m_i \nu_{ii} + \frac{15}{4} m_e \nu_e} \qquad (1)$$

where ν_e, ν_{ii} have the usual definitions

$$\nu_e = \frac{4(2\pi)^{1/2} n e^4 \ell n\Lambda}{3m_e^{1/2} T_e^{3/2}} , \quad \nu_{ii} = \frac{4\pi^{1/2} n e^4 \ell n\Lambda}{3m_i^{1/2} T_i^{3/2}} . \tag{2}$$

Introducing the energy-weighted friction force $\underset{\sim}{G}$ defined by

$$\underset{\sim}{G} \equiv \int d^3v \; m_i \underset{\sim}{v} \left(\frac{m_i v^2}{2} - \frac{5T_i}{2}\right) C(f_i) \tag{3}$$

where C is the collision operator, Eq. (1) can be rewritten as the balancing of two energy weighted forces

$$-\frac{5}{2} p_i \nabla_{\parallel} T_i = \left(\frac{4}{5} m_i \nu_i + 3m_e \nu_e\right) q_{i\parallel} = -G_{\parallel} \tag{4}$$

where the equality to $-G_{\parallel}$ follows because $-(5/2)p_i \nabla_{\parallel} T_i$ is the energy weighted force due to $\nabla_{\parallel} T_i$ which corresponds to the definition of $\underset{\sim}{G}$ in Eq. (3).

The surface-averaged Pfirsch-Schluter radial ion heat conduction is given by[3].

$$(q_{ir})_{ps} = -\frac{1}{eB_{\Theta}} \overline{\left(2\frac{r}{R}\cos\Theta G_{\parallel}\right)} \tag{5}$$

where the overbar indicates the Θ-average. Since the $\cos\Theta$ component of $q_{i\parallel}$, the Pfirsch-Schluter heat flow, is given by

$$q_{i\parallel} = -5\,\frac{p_i T_i'}{eB_\Theta}\,\frac{r}{R}\cos\Theta\ , \tag{6}$$

substituting from Eqs. (4) and (6) in Eq. (5) yields

$$(q_{ir})_{ps} = -2\left(\frac{r}{R}\right)^2 n_i\left(\nu_{ii} + \frac{15}{4}\frac{m_e}{m_i}\nu_e\right)\rho_{i\Theta}^2 T_i' \tag{7}$$

where $\rho_{i\Theta}^2 \equiv 2m_i T_i/e^2B_\Theta^2$. This formula is for a pure hydrogen plasma.

Using Eqs. (2), the ratio of the two collision terms satisfies

$$\frac{15}{4}\,\frac{m_e\nu_e}{m_i\nu_{ii}}\left[= \frac{15\sqrt{2}}{4}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}\right] \geqslant 1 \quad \text{for} \quad \frac{T_i}{T_e} \geqslant 4$$

in the case of hydrogen. The coefficient 2 in Eq. (7) will not be completely accurate because of the limitation of the 13-moment approximation. A more accurate calculation by Hazeltine and Hinton[5] with only ii-collisions gave the value 1.6.

3. Banana Regime Heat Conduction

A. Method for Solving the Drift-Kinetic Equation

To obtain the electron contribution in the banana regime it is necessary to solve the steady-state ion drift kinetic equation, namely,

$$v_\parallel\,\frac{B_\Theta}{B}\,\frac{\partial f_{i1}}{r\partial\Theta} + v_{dr}\,\frac{\partial f_{io}}{\partial r} = C(f_{i1}) \tag{8}$$

with i-e collisions included in the linearized collision operator C. Here f_{io} is the Maxwellian velocity distribution, f_{i1} is the perturbation to f_i which is first order in $\rho_{i\Theta}/L$, $v_{dr} = (m_i v_\parallel/e)\partial(v_\parallel/B)/r\partial\Theta$ and the other symbols have their usual meaning. Concentric magnetic surfaces with circular cross section are assumed and the velocity space coordinates to be used are $\varepsilon \equiv (v^2/2)+(e\phi/m)$ and $\lambda \equiv hv_\perp^2/v^2=2\mu B_o/v^2$, where $h = B_o/B = 1+(r/R)\cos\Theta$.

Since the collision term is small in the banana regime, to zeroth order in the collision frequency, Eq. (8) can be integrated to give the standard form[6]

$$f_{i1} = -\frac{m_i h v_\parallel}{eB_\Theta}\frac{\partial f_{io}}{\partial r} + g \tag{9}$$

where g is independent of Θ and is to be found by solving the constraint equation

$$\oint \frac{d\Theta}{v_\parallel} C(f_{i1}) = 0, \tag{10}$$

which is obtained by integration from Eq. (8). As is well known,[6] it follows from Eq. (10) that g is zero in the trapped region.

The procedure to be adopted to solve Eq. (10) is an adaptation of the method used by Hazeltine et al. to obtain the higher order correction to the neoclassical electrical conductivity.[7] In Eq. (9) g is expanded in powers of $\delta^{1/2} \equiv (r/R)^{1/2}$ in the form

$$f_{i1} = \left(-\frac{m_i v_\parallel h}{eB_\Theta}\frac{\partial f_{io}}{\partial r} + g^o\right) + g^1 + \ldots \quad (11)$$

The two terms in the brackets are of zero order in $\delta^{1/2}$, g^1 is first order, etc. In Eq. (11) the terms in each order are now rewritten in the form

$$f_1 = \left(\frac{mv_\parallel hV_\parallel}{T_i} f_o + h^{(o)}\right) + (f_* + h^{(1)}) + \ldots \quad (12)$$

where $hV_\parallel$ is the mean ion velocity parallel to $\underset{\sim}{B}$ in lowest order and $h^{(0)}$ is "localized". This means that although the magnitude of $h^{(0)}$ is $O(\delta^{1/2})$, the derivative $v_\parallel \partial h^{(0)}/\partial\lambda$ is localized being $O(1)$ in the trapped region and $O(\delta)$ in the untrapped region. Similarly f_* is chosen such that $h^{(1)}$ is "localized", having magnitude $O(\delta)$ with $v_\parallel \partial h^{(1)}/\partial\lambda$ being $O(\delta^{1/2})$ in the trapped region.

Because of the "localization" of $h^{(0)}$, in the zeroth order approximation to the constraint equation, C can be replaced by the approximate operator

$$C^o \equiv 2\nu_{PA} h\left(1-\frac{\lambda}{h}\right)^{1/2}\frac{\partial}{\partial\lambda}\lambda\left(1-\frac{\lambda}{h}\right)^{1/2}\frac{\partial}{\partial\lambda} \quad (13)$$

giving

$$\oint \frac{d\Theta}{v_\parallel} C^o(h^{(0)}) = 0 \quad (14)$$

with

$$h^{(0)} = -\frac{m_i v_\parallel h}{T_i}\left(V_\parallel f_{io} + \frac{T_i}{eB_\Theta}\frac{\partial f_{io}}{\partial r}\right) + g_o$$

from Eqs. (11) and (12). Also using Eq. (8), the mean radial diffusion for the ions in lowest order is

$$\Gamma_i^{(0)} = \int \frac{d\Theta}{2\pi} \int v_{dr}\ f_{i1}^{(0)} d^3v = -\frac{m_i}{eB_\Theta} \int \frac{d\Theta}{2\pi} \int v_\parallel\ C(f_{i1}^{(0)}) d^3v$$

$$\simeq -\frac{m_i}{eB\Theta} \int \frac{d\Theta}{2\pi} \int v_\parallel\ C^o(h^{(0)}) d^3v \qquad (15)$$

where the approximation is possible because $C(f_{i1}^{(0)}) = C(h^{(0)})$ and because the dominant part of $C(h^{(0)})$ is $C^o(h^{(0)})$. Because of detailed ambipolar balancing,[8] Γ_i must equal the electron neoclassical diffusion (Γ_{eNC}). Here Γ_{eNC} will be assumed small compared with the separate ion terms in Eq. (15); this is equivalent to assuming $(m_e/m_i)^{1/2}(T_i/T_e)^{3/2} \ll 1$. Thus the approximate ambipolarity condition is

$$\int \frac{d\Theta}{2\pi} \int m_i v_\parallel\ C^o(h^{(0)}) d^3v = 0\ . \qquad (16)$$

This equation determines the radial electric field which is rapidly established to produce the neoclassical ambipolarity.[8] Since the assumption $T_e/T_i \ll 1$ will also be made, the magnitude of this ratio is limited by the double inequality

$$1 \gg T_e/T_i \gg (m_e/m_i)^{1/3} \; . \tag{17}$$

(If T_e/T_i is too small to satisfy Eq. (17), the zero in Eq. (16) must be replaced by $\Gamma_{eNC}eB_\Theta$).

Following Hazeltine et al.,[7] in first order, f_* is chosen to be the solution of the equation

$$C(f_*) = -(C-C^o)(h^{(0)}) \tag{18}$$

which is a Spitzer type problem. The first order constraint equation then reduces to

$$\oint \frac{d\Theta}{v_\parallel} C(h^{(1)}) \approx \oint \frac{d\Theta}{v_\parallel} C^o(h^{(1)}) = 0 \tag{19}$$

and only $h^{(1)}$ in this order contributes to the neoclassical transport. Substituting from Eq. (13) into Eq. (19), since $h^{(1)} = g^1 - f_*$, one obtains

$$\frac{\partial g^1}{\partial \lambda} = \left(\frac{\langle \sqrt{1-\lambda/h} \, \frac{\partial f_*}{\partial \lambda} \rangle}{\langle \sqrt{1-\lambda/h} \rangle} \right)_u$$

or

$$\frac{\partial h^1}{\partial \lambda} = - \frac{\partial f_*}{\partial \lambda} + \left(\frac{\langle \sqrt{1-\lambda/h} \, \frac{\partial f_*}{\partial \lambda} \rangle}{\langle \sqrt{1-\lambda/h} \rangle} \right)_u \tag{20}$$

where the angular brackets denote the usual magnetic surface average

$$\langle A\rangle \equiv \int \frac{d\Theta}{2\pi} hA$$

and the subscript u denotes that this term applies only for the untrapped region and is to be taken as zero for the trapped region. Since away from the trapped or near trapped region

$$\frac{\langle\sqrt{1-\lambda/h}\,\frac{\partial f_*}{\partial\lambda}\rangle}{\langle\sqrt{1-\lambda/h}\rangle} \simeq \frac{\partial f_*}{\partial\lambda} ,$$

making $\partial h^{(1)}/\partial\lambda \simeq 0$, it follows that $\partial f_*/\partial\lambda$ is needed only for the trapped or near trapped region as the transport integrals depend on $\partial h^{(1)}/\partial\lambda$.

For the case of a pure hydrogen plasma, with electron collisions neglected, the well-known solutions to the zeroth order Eqs. (14) and (16) are[6]

$$E_r^{(0)} = V_{\parallel}B_{\Theta} + \frac{T_i}{e}\left[\frac{n_i'}{n_i} - \frac{T_i'}{T_i}\left(\frac{3}{2} - \frac{\bar{\nu}_i}{\nu_i^*}\right)\right] \tag{21}$$

and

$$h^{(0)} = -\frac{m_i v f_{io}}{eB_{\Theta}}\frac{T_i'}{T_i}\left(\frac{v^2}{v_{T_i}^2} - \frac{\bar{\nu}_i}{\nu_i^*}\right)\left(h\sqrt{1-\lambda/h} - \int_{\lambda}^{\frac{1}{1+\delta}} \frac{d\lambda}{2\langle\sqrt{1-\lambda/h}\rangle}\right) \tag{22}$$

where

$$\frac{\bar{\nu}_i}{\nu_i^*} \equiv \frac{\int d^3v \nu_{iPA} v_\parallel^2 (v^2/v_{T_i}^2) f_{io}}{\int d^3v \nu_{iPA} v_\parallel^2 f_{io}} = 1.33 \ , \tag{23}$$

ν_{iPA} being the pitch-angle collision frequency for C_{ii}^o given by

$$\nu_{iPA} = \frac{3}{2}\left(\frac{\pi}{2}\right)^{1/2} \nu_{ii} \left(\frac{v_{T_i}}{v}\right)^3 \left[\left(1 - \frac{v_{T_i}^2}{2v^2}\right)\Phi\left(\frac{v}{v_{T_i}}\right) + \frac{v_{T_i}}{2v}\Phi'\right] \ . \tag{24}$$

Here Φ is the error function and $v_{T_i} = (2T_i/m_i)^{1/2}$. Also, the mean radial ion heat conduction in this order is given by

$$q_{ir}^{(o)} = -0.68\delta^{1/2} n_i \rho_{i\Theta}^2 T_i' \ . \tag{25}$$

In the next order the ambipolarity condition is

$$\int \frac{d\Theta}{2\pi} \int v_\parallel \, C_{ii}^o(h^{(1)}) d^3v = 0 \tag{26}$$

and Hazeltine et al. found that the solution to Eqs. (18), (19) and (26) gave $V_\parallel^{(1)} = 0$, a more correctly $E_r^{(1)} = 0$, since it is E_r which changes rapidly to satisfy Eq. (26).

B. Solution for Impure Plasma

In order to simplify the problem of obtaining the electron contribution to q_{ir}, the assumption will be made that Z_{eff} is large. For such an impure plasma the lowest order solutions are somewhat modified. Thus since ν_{izPA} has the simpler form [c.f. Eq. (24)]

$$\nu_{izPA} = (3\sqrt{\pi}/4)(v_{T_i}/v)^3 \nu_{iz} \ , \tag{27}$$

where

$$\nu_{iz} = \sqrt{2}(n_z Z^2/n_i)\nu_{ii} \ ,$$

one finds

$$\overline{\nu}_{iz}/\nu^*_{iz} = 1 \ , \tag{28}$$

where $\overline{\nu}_{iz}$, ν^*_{iz} have similar definitions to those of $\overline{\nu}_i$, ν^*_i implied in Eq. (23). This value for $\overline{\nu}_{iz}/\nu^*_{iz}$ must be used in the expressions for $E_r^{(0)}$ and $h^{(0)}$ in Eqs. (21), (22) and the expression for $q_{ir}^{(o)}$ is changed to

$$q_{ir}^{(0)} = -0.73\ \delta^{1/2} n_i \nu_{iz} \rho_{i\Theta}^2 T_i' \ . \tag{29}$$

Considering now the effect of electron collisions, in the zeroth order their effect is small. Thus the pitch angle scattering collision frequency for ie-collisions is $\nu_{iePA} = \nu_e(m_e/m_i)(2T_e/m_i v^2)$ so that

$$\frac{\nu_{iePA}}{\nu_{izPA}} \simeq \frac{n_e}{n_z Z^2} \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{v}{v_{T_i}} \frac{T_e}{T_i}\right) . \tag{30}$$

The assumption $(m_e/m_i)^{1/2}(T_i/T_e)^{3/2} << 1$ has already been made in Eq. (17) and it will be assumed that T_i/T_e is sufficiently large such that values of v/v_{T_i} greater than T_i/T_e will be unimportant because of the Maxwellian exponential. Thus the ratio in Eq. (30) will be small compared with unity even without the factor $n_e/n_z Z^2$.

In the $\delta^{1/2}$ order, energy scattering is involved and the ratio of the ie-collision frequency for dynamic friction (ν_{ieES}) to the ii-collision frequency for energy scattering is

$$\frac{\nu_{ieES}}{\nu_{iiES}} \simeq \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{T_i}{T_e}\right)^{3/2} \left(\frac{v}{v_{T_i}}\right)^3 . \tag{31}$$

This is substantially larger than the ratio in Eq. (30) and can be of order unity for the ions of the distribution tail which are important for heat conduction. Of interest therefore is the part of the first order perturbation f_* which is driven by electron collisions and which is the solution of (c.f. Eq. (18))

$$\begin{aligned} C(f_*) &= -C_{ie}(h^{(0)}, f_{eo}) - C_{ie}(f_{io}, f_{e1}) \\ &\simeq -C_{ie}(h^{(0)}, f_{eo}) \simeq -\nu_e\left(\frac{m_e}{m_i}\right)\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^3 h^{(0)}\right) . \end{aligned} \tag{32}$$

In Eq. (32) f_{e1} is the perturbation of f_e caused by $h^{(0)}$, i.e. it is the solution of

$$C(f_{e1}) = -C_{ei}(f_{eo}, h^{(0)}) \quad . \tag{33}$$

Since the assumption has been made that $Z_{eff} >> 1$, it follows from Eq. (33) that f_{e1} contains the small factor $(Z_{eff})^{-1}$ and that $C_{ie}(f_{io}, f_{e1})$ is smaller than $C_{ie}(h^{(0)}, f_{eo})$ by this factor. Hence only the larger contribution is retained in Eq. (32). Also the last approximation in Eq. (32) involves the omission of an extra term smaller by the factor T_e/T_i. The most important simplification which arises from the assumption of large Z_{eff} is that $C(f_*)$ can be replaced by $C^o(f_*)$ in Eq. (32) to a good approximation. This simplifies greatly this Spitzer-type problem to be solved for f_*.

An important point which must be made is that since $T_e < T_i$ and since we are retaining terms of order $\nu_e(m_e/m_i)$ the constraint equation which is zero order in $\rho_{i\Theta}/L$ will contain the non-zero term $\oint (d\Theta/v_\parallel)C_{ie}(f_{io})$; this term will be unbalanced unless either an ion heating term is included or $\partial f_{io}/\partial t$ is non-zero. In general, whatever balancing process is operating on f_{io}, it will also operate on f_{i1} and affect the ion transport. It will balance, at least to some extent, the effect of $C_{ie}(f_{i1})$. Here two values of q_{ir} will be obtained; (a) the linear contribution due to C_{ie} with other processes omitted; this will come from the solution of Eq. (32) and (b) the net value of q_{ir} for the case where $C_{ie}(f_{io})$ is assumed balanced by the (heating) collisions with injected beam ions. For case (b) an extra collision term is needed on the right hand side of Eq. (32) to allow for beam ion collisions.

C. Case (a)

Returning to the solution of Eq. (32), after substituting for $h^{(0)}$ from Eqs. (22), (28) and replacing $C(f_*)$ by $C^o(f_*)$ the equation to be solved is

$$2\nu_{izPA}\, h\sqrt{1-\lambda/h}\, \frac{\partial}{\partial\lambda}\, \lambda\sqrt{1-\lambda/h}\, \frac{\partial f_*}{\partial\lambda} = A(v)\big(h\sqrt{1-\lambda/h} - \int_\lambda^{\frac{1}{1+\delta}} \frac{d\lambda}{2<\sqrt{1-\lambda/\Theta}>}\big) \quad (34)$$

where

$$A(v) = \frac{\nu_e m_e}{eB_\Theta}\, \frac{T_i'}{T_i}\, \frac{1}{v^2}\, \frac{\partial}{\partial v}\, v^4\big(\frac{v^2}{v_{T_i}^2} - 1\big) f_{io}\ . \quad (35)$$

Multiplying Eq. (34) by $(1-\frac{\lambda}{h})^{-1/2}$ and integrating with respect to λ from 0 to λ yields

$$2\nu_{izPA}\lambda h\sqrt{1-\lambda/h}\, \frac{\partial f_*}{\partial\lambda} = A(v)\big(h\lambda - \int_0^\lambda \frac{d\lambda}{\sqrt{1-\lambda/h}} \int_\lambda^{\frac{1}{1+\delta}} \frac{d\lambda}{2<\sqrt{1-\lambda/h}>}\big)$$

$$= A(v)\big[h\lambda + \Big|2h\, \sqrt{1-\lambda/h} \int_\lambda^{\frac{1}{1+\delta}} \frac{d\lambda}{2<\sqrt{1-\lambda/h}>}\Big|_0^\lambda + \int_0^\lambda \frac{h\sqrt{1-\lambda/h}\ d\lambda}{<\sqrt{1-\lambda/h}>}\big]. \quad (36)$$

Noting from lowest order neoclassical theory[6] that

$$< \int_0^{1+\delta} d\lambda\big[\frac{1}{\sqrt{1-\lambda/h}} - \big(\frac{1}{<\sqrt{1-\lambda/h}>}\big)_u\big] > = 2\sqrt{2}(0.69)\delta^{1/2}, \quad (37)$$

so that

$$\int_0^{\frac{1}{1+\delta}} \left(\frac{d\lambda}{<\sqrt{1-\lambda/h}>}\right)_u \left[= \int_0^{1+\delta} \left(\frac{d\lambda}{<\sqrt{1-\lambda/h}>}\right)_u\right] \simeq 2-2\sqrt{2}(0,69)\delta^{1/2} \ , \tag{38}$$

Eq. (36) reduces to

$$2\nu_{izPA}\lambda h \sqrt{1-\lambda/h}\, \frac{\partial f_*}{\partial \lambda} = A(v)\Big[\lambda - 2(1-0.69\sqrt{2}\delta^{1/2})$$

$$+ h\sqrt{1-\lambda/h} \int_\lambda^{\frac{1}{1+\delta}} \frac{d\lambda}{<\sqrt{1-\lambda/h}>} + \int_0^\lambda \frac{h\sqrt{1-\lambda/h}\,d\lambda}{<\sqrt{1-\lambda/h}>}\Big] \ . \tag{39}$$

The value of the left-hand side is needed to order $\delta^{1/2}$ for the trapped region, where λ varies from $(1+\delta)$ to $(1+\delta)^{-1}$, and for the near trapped region. For this range of λ and to the required accuracy $\lambda=1$, the first integral in the square brackets is zero and the second integral is unity, so that

$$\left(\nu_{izPA}\lambda h \sqrt{1-\lambda/h}\, \frac{\partial f_*}{\partial \lambda}\right)_T = 0.69\sqrt{2}\delta^{1/2} A(v) \tag{40}$$

where the subscript T denotes the value for the trapped or near trapped region.

The contribution to ion diffusion caused by $h^{(1)}$ is given by

$$(\Gamma_{iNC})_{ie} = - <\frac{m_i}{eB_\Theta} \int h v_\parallel\, C^o(h^{(1)}) d^3v> \ .$$

After substituting from Eqs. (13) and (20) and integrating by parts with respect to λ, this becomes

$$(\Gamma_{iNC})_{ie} = - < \frac{m_i}{eB_\Theta} \iint v\nu_{izPA}\, \lambda\Big[- \frac{\partial f_*}{\partial\lambda} + \Big(\frac{<\sqrt{1-\lambda/h}\,\frac{\partial f_*}{\partial\lambda}>}{<\sqrt{1-\lambda/h}>}\Big)_u\Big] 2\pi v^2 dv d\lambda >$$

$$= - \frac{m_i}{eB_\Theta} \int 0.69\sqrt{2\delta}\; A(v) 2\pi v^3 dv < \int \Big[\frac{1}{\sqrt{1-\lambda/h}} - \Big(\frac{1}{<\sqrt{1-\lambda/h}>}\Big)_u\Big] d\lambda > \tag{41}$$

using Eq. (40). Finally from Eqs. (35) and (37), the integrals yield

$$(\Gamma_{iNC})_{ie} = -2.1\; n_i\; \Big(\frac{r}{R}\Big)\Big(\nu_e \frac{m_e}{m_i}\Big)\; \rho_{i\Theta}^2 \frac{T_i'}{T_i} \;. \tag{42}$$

Since for the impure plasma considered here most of the positive friction force exerted on the electrons due to $h^{(0)}$ will be balanced by a negative friction force from the impurity ions; there will not be an electron diffusion term with comparable magnitude to match $(\Gamma_{iNC})_{ie}$. Hence, the ambipolarity condition requires a first order correction to the radial electric field, namely $E_r^{(1)}$. The diffusion contribution due to this electric field in lowest order is found to be

$$\Gamma_i(E_r^{(1)}) = 0.73 n_i \Big(\frac{r}{R}\Big)^{1/2} \nu_{iz}\rho_{i\Theta}^2 \Big(\frac{eE_r^{(1)}}{T_i}\Big) \;. \tag{43}$$

Balancing the two contributions from Eqs. (42) and (43) requires

$$\frac{eE_r^{(1)}}{T_i} = 2.9\left(\frac{n_e}{n_z Z^2}\right)\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}\left(\frac{r}{R}\right)^{1/2}\left(\frac{T_i'}{T_i}\right) . \tag{44}$$

The contributions to the radial heat conduction due to $h^{(1)}$ and $E_r^{(1)}$ are

$$q_{ir}(h^{(1)}) = - \left\langle\frac{m_i}{eB_\Theta} \int hv_\parallel\left(\frac{mv^2}{2} - \frac{5}{2} T_i\right)C^o(h^{(1)})d^3v\right\rangle = -21\ n_i\left(\frac{r}{R}\right) \nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 T_i' . \tag{45a}$$

and

$$q_{ir}(E_r^{(1)}) = -0.73\left(\frac{3T_i}{2}\right)n_i\left(\frac{r}{R}\right)^{1/2}\nu_{iz}\rho_{i\Theta}^2\left(\frac{eE_r^{(1)}}{T_i}\right)$$

$$= -2.1\left(\frac{3T_i}{2}\right)n_i\left(\frac{r}{R}\right)\nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 \frac{T_i'}{T_i} . \tag{45b}$$

using Eq. (44). Note that $q_{ir}(E_r^{(1)})$ has the opposite sign to $\Gamma_i(E_r^{(1)})$ because the total heat flow is less than $5\Gamma_i T_i/2$. The resultant heat conduction caused by ie-collisions is the sum of Eqs. (45a) and (45b), namely

$$(q_{ir})_{ie} = -24n_i\left(\frac{r}{R}\right)\nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 T_i' \tag{45}$$

The large numerical coefficient arises because of the high power of v occurring in the v-integral in $q_{ir}(h^{(1)})$.

D. Case (b)

Here the simple example case is considered where the cooling effect of $C_{ie}(f_{io})$ is assumed balanced by the heating due to injected beam ions. The distribution function for the beam ions (f_b) is taken to be isotropic in pitch angle with all particle velocities being greater than those of interest for the ions in f_{io}. This means that the collision frequency for dynamic friction due to ib-collisions is zero. (There are no beam ions within the velocity space sphere of radius v.) Also if G is the Rosenbluth potential

$$G \equiv \int d^3v' f_b(v')|\underset{\sim}{v}'-\underset{\sim}{v}| \ ,$$

G reduces to

$$G = 4\pi \int_v^\infty f_b(v')v'^2 dv'\left(\frac{v^2}{3v'} + v'\right)$$

and the parallel velocity diffusion coefficient is

$$\frac{D_\parallel}{2} = \frac{1}{2}\frac{\partial^2 G}{\partial v^2} = \frac{4\pi}{3}\int_v^\infty f_b(v')v'dv' = \frac{n_b}{3}\left\langle\frac{1}{v_b}\right\rangle \tag{46}$$

where n_b is the density of beam particles and $\langle 1/v_b\rangle$ is the average value of v^{-1} for the beam particles. The required collision operator is then

$$C_{ib}(f_{io}) = \frac{\gamma n_b}{3}\left\langle\frac{1}{v_b}\right\rangle \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 \frac{\partial f_{10}}{\partial v}\right) \tag{47}$$

where $\gamma=4\pi e^4 \ell n\Lambda/m_i^2$.

To zeroth order in $\rho_{i\Theta}/L$ the drift-kinetic equation required

$$(C_{ii}+C_{iz})(f_{io}) = -\nu_e\left(\frac{m_e}{m_i}\right) \frac{1}{v^2} \frac{\partial v^3}{\partial v} \left(f_{io} + \frac{T_e}{m_i v} \frac{\partial f_{io}}{\partial v}\right)$$

$$- \frac{\gamma n_b}{3} \left\langle\frac{1}{V_b}\right\rangle \frac{1}{v^2} \frac{\partial}{\partial v}\left(v^2 \frac{\partial f_{1o}}{\partial v}\right). \tag{48}$$

If

$$T_z = T_i \ ,$$

and

$$\frac{\gamma n_b}{3} \left\langle\frac{1}{V_b}\right\rangle \frac{m_i}{T_i} = \nu_e\left(\frac{m_e}{m_i}\right)\left(1 - \frac{T_e}{T_i}\right) \ , \tag{49}$$

then the Maxwellian with temperature T_i is a solution for f_{io} in Eq. (48), since the two sides of the equation vanish separately. This simple condition is unrealistic for the case considered here where f_{io} is assumed to be Maxwellian for all values of v; it implies that all the heat received from the beam particles is passed to the electrons. However, for the two component (non Maxwellian) ion distributions observed in PDX,[9] the experimental results suggest that the two terms on the right-hand side of Eq. (48) do balance for the higher energy part of f_{io}. Here, the equality of Eq. (49) is assumed to hold and the balancing of the two collision terms is taken merely as an example to illustrate that the physical process balancing the electron cooling can modify the ion transport.

Turning to the operation of C_{ie} and C_{ib} on $h^{(0)}$, which is written in the form $h^{(0)} = \hat{h} f_{io}$, since the pitch angle scattering part of C_{ib} can also be shown to be small,

$$(C_{ie}+C_{ib})(h^{(0)}) = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\nu_e\left(\frac{m_e}{m_i}\right)\frac{T_e}{m_i} + \frac{\gamma n_b}{3}\left\langle\frac{1}{V_b}\right\rangle\right) v^2 f_{io}\frac{\partial\hat{h}}{\partial v}]$$

$$= \frac{1}{v^2}\frac{\partial}{\partial v}\left[\nu_e\left(\frac{m_e}{m_i}\right)\frac{T_i}{m_i} v^2 f_{io}\frac{\partial\hat{h}}{\partial v}\right]$$

$$= \left(h\sqrt{1-\lambda/h} - \int_{\lambda}^{\frac{1}{1+\delta}} \frac{d\lambda}{2\langle\sqrt{1-\lambda/h}\rangle}\right)\left(-\frac{m_i T_i'}{eB_\Theta T_i}\right)\frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{\nu_e m_e}{2m_i}\left(3v^4 - v_{T_i}^2 v^2\right) f_{io}\right] , \quad (50)$$

using Eqs. (22), (28) and (49).

Replacing the right-hand side of Eq. (34) by the expression in Eq. (50) and following through the analysis of subsection C, one obtains

$$(q_{ir})_{ie+ib} = -12.2\, n_i\left(\frac{r}{R}\right)\nu_e\frac{m_e}{m_i}\rho_{i\Theta}^2 T_i' . \quad (51)$$

Comparing this value with that of Eq. (45), it is seen that the effect of the beam ion collisions is to reduce q_{ir} by approximately 50%.

4. Ion Viscosity

To illustrate qualitatively how the electron collisions contribute to ion viscosity, consider the neoclassical formula for the ion pressure tensor component $P_{\parallel r}$

$$P_{\parallel r} = \int \frac{d\Theta}{2\pi} \int m_i v_\parallel v_{dr} f_i \; d^3v$$

$$= -\frac{m_i}{2eBR} \int \sin\Theta \, \frac{d\Theta}{\pi} \int v_\parallel \left(\frac{1}{2} mv^2\right)(1+\xi^2) f_1 d^3v \tag{52}$$

where $\xi \equiv v_\parallel/v$. If one omits the ξ^2 term, the d^3v integral is simply the total ion heat flow parallel to $\underset{\sim}{B}$, namely $\hat{q}_{i\parallel} = q_{i\parallel} + (5/2)\, T_i V_{i\parallel}$, and the Θ-integral picks the $\sin\Theta$ component of $\hat{q}_{i\parallel}$ to be denoted by $\hat{q}_{i\parallel s}$. Hence

$$P_{\parallel r} \gtrsim -\frac{m_i \hat{q}_{i\parallel s}}{2eBR} \,. \tag{53}$$

From the $\cos\Theta$ component of the ion heat balance equation, to first order in $\rho_{i\Theta}/L$,

$$\frac{B_\Theta}{B} \frac{\partial}{r\partial\Theta} (\hat{q}_{i\parallel s}\sin\Theta) - \frac{3}{2}\frac{E_r}{B} \frac{\partial}{r\partial\Theta}(\tilde{p}_{is}\sin\Theta) = -\tilde{Q}_{iec}\cos\Theta \tag{54}$$

where $\tilde{Q}_{iec}$ is the $\cos\Theta$ component of Q_{ie}, the collisional energy transfer from ions to electrons, and $(\tilde{p}_{is}\sin\Theta)$ is the part of the ion pressure which varies as $\sin\Theta$.

The $\tilde{p}_{is}$ term leads to a contribution to $P_{\| r}$ given by $m_i \Gamma_i E_r / B_\Theta$ where Γ_i is the neoclassical ion diffusion; this small contribution has been known for many years.[10] The new contribution considered here comes from $\tilde{Q}_{iec}$; substituting from Eq. (54) into Eq. (53)

$$P_{\| r} \gtrsim \frac{r m_i \tilde{Q}_{iec}}{2ReB_\Theta} . \tag{55}$$

More accurate neoclassical theory leads to the formula[4]

$$P_{\| r} = \frac{m_i}{eB_\Theta r} \frac{\partial}{\partial r} r < \int \frac{1}{2} mh^2 v_\|^2 v_{dr} f_i d^3v> - <\frac{m_i^2}{eB_\Theta} \int \frac{h^2 v_\|^2}{2} C(f_i) d^3v> . \tag{56}$$

The first term has already been treated in reference [4] in obtaining the neoclassical formula for $P_{\| r}$ due to ii-collisions; this term is closely related to the divergence of the radial heat conduction and for sufficiently large T_i/T_e some modifications would be necessary to allow for ie-collision as described in Section 3. Here we are concerned with the extra contribution to the second term due to C_{ie} which has not been treated previously. Only the balanced case given by Eq. (49) will be considered; otherwise a fictitious contribution is obtained from $C_{ie}(f_o)$. Also neutral beam heating is involved in those tokamak experiments which exhibit large unexplained ion viscosity. The ions will be assumed to be in either the plateau or banana regimes and in the appendix it is shown that the perturbation to f_i which is even in $v_\|$ and varies as $\cos\Theta$ is g_c^+ given by

$$g_c^+ = f_o \frac{r}{R} \cos\Theta\{ \frac{m_i V_\parallel^2}{T_i} - \frac{(\mu B+\hat{v}_\parallel^2+V_\parallel^2)}{\Omega_\Theta} \frac{m_i V_\parallel'}{T_i}$$

$$- \frac{2V_\parallel}{\Omega_\Theta}[\frac{n_i'}{n_i} + \frac{T_i'}{T_i}(\frac{m_i \hat{v}^2}{2T_i} - \frac{3}{2})]\} - \frac{e\tilde{\Phi}}{T_i}(1 - \frac{m_i V_\parallel'}{eB_\Theta}) f_0 \ . \tag{57}$$

Here $V_\parallel$ is the mean ion velocity parallel to $\underset{\sim}{B}$ at radius r, being equal to the mean toroidal velocity to good approximation; $\tilde{\Phi}$ is the part of the electrostatic potential which varies with Θ, $\hat{v}_\parallel = v_\parallel - V_\parallel$ and $\hat{v}^2 = 2\mu B+\hat{v}_\parallel^2$. For simplicity the assumption has been made that $E_r/B_\Theta \simeq V_\parallel$, which corresponds to the case observed in the tokamak PDX where the impurity poloidal velocity $[\simeq (B_\Theta V_\parallel/B)-(E_r/B)]$ was found to be very small.[11]

Omitting the parts of f_e, f_b which vary as $\cos\Theta$, the contribution due to $C(g_c^+,f_{eo}) + C(g_c^+,f_{bo})$ in Eq. (56) reduce to

$$P_{\parallel r} = - \frac{m^2}{eB_\Theta} <\int \frac{h^2 v_\parallel^2}{2} \frac{r}{R} \cos\Theta[\nu_e \frac{m_e}{m_i} \frac{1}{v^2} \frac{\partial}{\partial v} (\frac{T_i}{m_i} v^2 f_{io} \frac{\partial}{\partial v} (\frac{g_c^+}{f_{io}})] d^3v> \ . \tag{58}$$

After substituting from Eq. (57) one obtains

$$P_{\parallel r} = -n_i(\frac{r}{R})^2 \nu_e \frac{m_e}{m_i} \rho_{i\Theta}^2 m_i (\frac{6}{5} V_\parallel' + \frac{3T_i'}{2T_i} V_\parallel) \ . \tag{59}$$

Allowing for the poloidal nonuniformity of n_e introduces an extra collision term in Eq. (57) given by $(\tilde{n}_{ec}\cos\Theta/\overline{n}_e)C_{ie}(\overline{f}_{io},\overline{f}_{eo})$, where $\tilde{n}_{ec}\cos\Theta$ is the part of of n_e which varies as $\cos\Theta$ and the overbars have been added to emphasize the Θ-average is involved. This term will not depend on the sign of $V_\parallel'$ or $V_\parallel$ but on the sign of $\tilde{n}_{ec}$ which will generally be positive irrespective of the sign of $V_\parallel$, giving a positive contribution to $P_{\parallel r}$. The corresponding term due to $\tilde{n}_b(\Theta)$ will be given only approximately by $(\tilde{n}_{bc}\cos\Theta/\overline{n}_b)C(f_{io},f_{bo})$, the approximation being associated with the fact that $\langle V_b^{-1}\rangle$ will in general be somewhat smaller for the ions in $\tilde{f}_{bc}$. This will generate a negative contribution to $P_{\parallel r}$ (assuming $\tilde{n}_{bc}$ is positive). However, the beam ions associated with $\tilde{f}_{bc}$ will contribute directly to $P_{\parallel r}$ in the same way as g_c^+ in Eq. (58). In fact a more accurate value of $P_{\parallel r}$ would be obtained by combining $\tilde{f}_{bc}$ and g_c^+ and taking the term $C(\tilde{f}_{bc}+g_c^+)$. The positive contribution coming from $\tilde{f}_{bc}$ in general will more than compensate for the negative contribution due to $C(f_{io},\tilde{f}_{bc})$ and hence Eq. (59) is an underestimate.

5. Summary and Conclusions

The results of Sections 2–4 can be summarized as follows:

1. From Eq. (7) the ion heat conduction in the Pfirsch–Schluter regime is given by

$$(q_{ir})_{PS} = -2n_i(r/R)^2\rho_{i\Theta}^2\nu_{ii}T_i'[1+5.3(\frac{m_e}{m_i})^{1/2}(\frac{T_i}{T_e})^{3/2}] \ . \tag{60}$$

This formula is for a pure hydrogen plasma. For an impure plasma one would expect the expression in the square brackets to be replaced by $[Z^i_{eff} + 5.3(m_e/m_i)^{1/2}(T_i/T_e)^{3/2}]$ with $Z^i_{eff} \equiv (n_i+\sqrt{2}\sum_z n_z Z^2)/n_i$.

2. For the banana regime, assuming the cooling effect of the electron collisions is balanced by neutral beam heating, the ion heat conduction is

$$(q_{ir})_{ban} = -0.73 n_i \left(\frac{r}{R}\right)^{1/2} \rho^2_{i\Theta}\nu_{ii}\left[Z^i_{eff} + 24\left(\frac{r}{R}\right)^{1/2}\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}\right] . \quad (61)$$

This formula is only accurate for $Z^i_{eff} >> 1$; it will overestimate the electron contribution as $Z^i_{eff} \to 1$. (When the neutral beam particle collisions are omitted the numerical coefficient 24 becomes 46).

3. Combining the neoclassical formula for $P_{\| r}$ due to ii-collisions,[4] the classical contribution[2,12] and the contribution found here due to ie-collisions from Eq. (59)

$$P_{\| r} = -0.1 n_i \left(\frac{r}{R}\right)^2 \rho^2_{i\Theta}\nu_{ii} m_i \left\{\left[1+\frac{6}{q^2} + 17\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}\right]V_\|' + 21\left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_i}{T_e}\right)^{3/2}\frac{V_\| T_i'}{T_i}\right\} . \quad (62)$$

(A term found in reference 4 which is third order in $\rho_{i\Theta}/L$ and related to T_i' but independent of $V_\|'$ and $V_\|$ has been omitted for simplicity. Note that the classical term, the one containing q^{-2}, exceeds the neoclassical term due to ii-collisions unless $q^2 \geqslant 6$, where $q \equiv rB/RB_\Theta$. The ratio of the two terms was incorrectly stated by a factor of 4 in reference 4.

The above results all assume that the ion distribution in lowest order, f_{io}, is Maxwellian. However, there is experimental evidence that at least in hot ion plasmas where $\rho_{i\Theta}$ is a significant fraction ($\gtrsim 0.2$) of the minor radius, the distribution is non-Maxwellian and has an enhanced tail. (For a summary of the experimental evidence see reference 13.) Such distributions can be explained as a result of neoclassical transport[13] and even with only ii-collisions have increased transport. But for the enhanced tail particles electron collisions will be particularly important. The further enhancement of ion transport due to ie-collisions with such distributions will be the subject of a future paper. In addition, as discussed in Section 4, the slowing down beam ions, which constitute another form of non-Maxwellian tail, will also make important contributions to ion transport when the neutral beam injection powers are large.

Appendix

In this appendix the part of f_i which is even in $v_\parallel$ and varies as $\cos\Theta$ is derived to first order in r/R from the drift-kinetic equation. Using velocity space coordinates μ and $v_\parallel$ and assuming in lowest order that f_o is a moving Maxwellian with velocity $V_\parallel$ parallel to $\underset{\sim}{B}$, with $f_i=f_o+g$, the drift-kinetic equation to order (r/R) gives

$$\left(\frac{B_\Theta}{B} v_\parallel - \frac{E_r}{B}\right) \frac{\partial g}{r\partial\Theta} + C(g) = -\left(\frac{B_\Theta}{B} v_\parallel - \frac{E_r}{B}\right)\left(\frac{\partial f_o}{r\partial\Theta} - \frac{\mu}{v_\parallel} \frac{\partial B}{\partial\Theta} \frac{\partial f_o}{\partial v_\parallel}\right)$$

$$- v_{dr}\left(\frac{\partial f_o}{\partial r} + \frac{eE_r}{mv_\parallel} \frac{\partial f_o}{\partial v_\parallel}\right) + \frac{B_\Theta e}{Bm}\left(\frac{\partial\tilde{\Phi}}{r\partial\Theta}\right) \frac{\partial f_o}{\partial v_\parallel} . \qquad \text{(A1)}$$

with f_o given by

$$f_o = \frac{n}{(\pi^{1/2} v_T)^3} \exp\{-\frac{m}{T}[\mu B + (v_\parallel - V_\parallel)^2/2]\} .$$

In this appendix only the hydrogen ions are considered and the subscript "i" has been omitted.

The simple case is considered where $E_r/B_\Theta \simeq V_\parallel$. This was observed experimentally in PDX where the impurity poloidal velocity $[\simeq(B_\Theta V_\parallel/B) - (E_r/B)]$ was found to be very small.[11] Also, since the part of g which varies as $\cos\Theta$ will be independent of the collision frquency in lowest order, a simple Krook model is taken for the collision operator. Changing the velocity variable from $v_\parallel$ to $\hat{v}_\parallel$ where

$$\hat{v}_\parallel = v_\parallel - V_\parallel$$

and $\partial/\partial r$ transforms to $(\partial/\partial r - V_\parallel' \partial/\partial \hat{v}_\parallel)$, Eq. (A1) becomes

$$\frac{B_\Theta \hat{v}_\parallel}{Br}\frac{\partial g}{\partial \Theta} + \nu g = f_o\Big(\frac{B_\Theta \hat{v}_\parallel}{Br}\mu\frac{\partial B}{\partial \Theta}\frac{mV_\parallel}{T(\hat{v}_\parallel + V_\parallel)}$$

$$- \frac{[\mu B + (\hat{v}_\parallel + V_\parallel)^2]}{\Omega R}\sin\Theta\Big[\frac{n'}{n} + \frac{T'}{T}\Big(\frac{m\hat{v}^2}{2T} - \frac{3}{2}\Big) - \frac{\hat{v}_\parallel e E_r}{T(\hat{v}_\parallel + V_\parallel)} + \frac{m\hat{v}_\parallel V_\parallel'}{T}\Big]$$

$$- \frac{B_\Theta}{B}\frac{\hat{v}_\parallel e}{rT}\frac{\partial \tilde{\Phi}}{\partial \Theta} + \frac{1}{Br}\frac{\partial \tilde{\Phi}}{\partial \Theta}\Big[\frac{n'}{n} + \frac{T'}{T}\Big(\frac{m\hat{v}^2}{2T} - \frac{3}{2}\Big) + \frac{m\hat{v}_\parallel V_\parallel'}{T}\Big]\Big) \qquad (A2)$$

with $\hat{v}^2 = 2\mu B + \hat{v}_\parallel^2$. The part of g which is both even in $\hat{v}_\parallel$ and varies as $\cos\Theta$ is denoted by g_c^+. After integrating Eq. (A2) with respect to Θ and taking the $\cos\Theta$ component, the small collision term can be neglected giving

$$g_c^+ = f_o \frac{r}{R} \cos\Theta \{ \frac{mV_\parallel^2}{T} - \frac{(\mu B + \hat{v}_\parallel^2 + V_\parallel^2)}{\Omega_\Theta} \frac{mV_\parallel'}{T}$$

$$- \frac{2V_\parallel}{\Omega_\Theta} [\frac{n'}{n} + \frac{T'}{T} (\frac{m\hat{v}^2}{2T} - \frac{3}{2})]\} - \frac{e\tilde{\Phi}}{T} (1 - \frac{mV_\parallel'}{eB_\Theta}) f_o$$

where $\Omega_\Theta = eB_\Theta/m$ and $E_r/B_\Theta = V_\parallel$ has been used.

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